CHAPTER 9

TRIGONOMETRIC RATIOS AND IDENTITIES

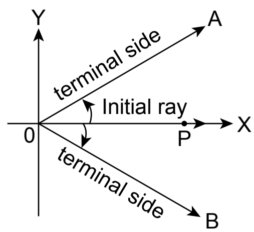
9.1 INTRODUCTION

The word 'trigonometry' is derived from two Greek words: (i) trigon (means a triangle) and (ii) metron (means a measure). Therefore, trigonometry means science of measuring the sides of angles and study of the relations between side and angles of triangle.

9.2 ANGLE

Angle is defined as the measure of rotation undergone by a given revolving ray OX in a plane, about its initial point O. The original ray OX is called the *initial side* and the final position (OP) of the ray after rotation is called the *terminal side* of the angle ∠XOP. The point of rotation (O) is called the *vertex*.

9.2.1 Rules for Signs of Angles



- If initial ray OA rotates to terminal ray OA, then angle = θ (rotation anti clockwise)
- If initial ray OA rotates to terminal ray OB, then angle = –θ (rotation clockwise), where θ is the measure of rotation.

9.2.2 Measurement of Angle

The measurement of angle is done under the following three systems of measurement of angles:

9.2.2.1 Sexagesimal or english system

1 right angle = 90^0 (degrees); 1^0 = 60' (minutes);
1' = 60'' (seconds)

9.2.2.2 Centesimal or french system (Grade)

1 right angle = 100^g (grades); 1^g = 100' (minutes)
1' = 100'' (seconds)

Remark:

The minutes and seconds in the sexagesimal system are different them the respective minutes and seconds in the centesimal system. Symbols in both there systems are also different.

9.2.2.3 Radian measure or circular measurement

One radian corresponds to the angle subtended by arc of length r (radius) at the centre of the circle with radius r. Since the ratio is independent of the size of a circle it follows that a radian is a dimensionless quantity.

Length of an arc of a circle: $\text{Angle } \theta \text{ (in radians)} = \frac{l}{r} = \frac{\text{arc length}}{\text{radius}}$

Relation between radian and degree: $\pi^c = 180°$

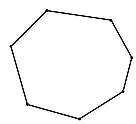
In hand working tips

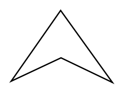
- The unit radian is denoted by c (circular measure) and it is customary to omit this symbol c. Thus, when an angle is denoted as $\frac{\pi}{2}$, it means that the angle is $\frac{\pi}{2}$ radians where p is the number with approximate value 3.14159.
- $\frac{D}{180°} = \frac{G}{200} = \frac{R}{\pi}$, where D, G and R denotes degree, grades and radians respectively
- The angle between two consecutive digits in a clock is 30° (π/6 rad). The hour hand rotates through an angle of 30° in one hour.
- The minute hand rotate through an angle of 6° in one minute.

9.3 POLYGON AND ITS PROPERTIES

A closed figure surrounded by n straight lines is called a polygon. It is classified in two ways.

- A closed figure surrounded by **n** straight lines.
- If all sides of a polygon are equal, then it is regular polygon.
- **Convex Polygon:** A polygon in which all the internal angles are smaller than 180°.
- **Concave Polygon:** A polygon in which at least one internal angles is larger than 180°.





Properties:

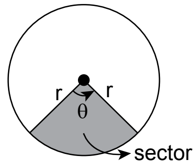
- An angle is called **reflexive angle** if it is greater than or equal to 180° or π radians.
- Sum of all internal angles of a convex polygon = $(n - 2)\ \pi^c = (n - 2)\ 180°$.
- Each internal angle of regular polygon of n sides = $\frac{(n-2)\pi}{n}$.

Nomenclature of Polygons

Name of Polygon	Number of Sides	Name of Polygon	Number of Sides	Name of Polygon	Number of Sides
1. Triangle	3	**7.** Nonagon	9	**13.** Penta-decagon	15
2. Quadrilateral	4	**8.** Decagon	10	**14.** Hexa-decagon	16
3. Pentagon	5	**9.** Hendecagon	11	**15.** Hepta-decagon	17

Name of Polygon	Number of Sides	Name of Polygon	Number of Sides	Name of Polygon	Number of Sides
4. Hexagon	6	**10.** Duodecagon	12	**16.** Octa-decagon	18
5. Heptagon	7	**11.** Tri-decagon	13	**17.** Nona-decagon	19
6. Octagon	8	**12.** Tetra-decagon	14	**18.** Ico-sagon	20

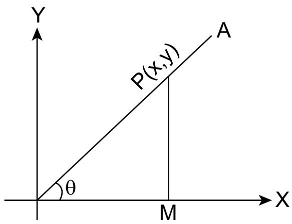
Circular Sector:

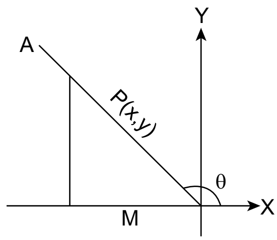


- Perimeter of a circular sector of sectoral angle $\theta^c = r(2 + \theta)$.
- Area of a circular sector of sectoral angle $q^c = \frac{1}{2} r^2 q$.

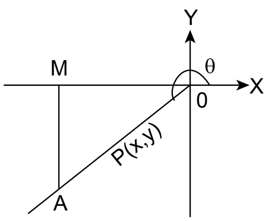
9.4 TRIGONOMETRIC RATIOS

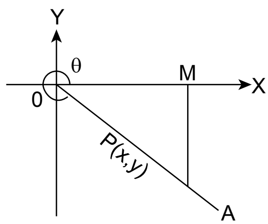
Consider an angle θ = ∠XOA as shown in figure. P be any point other than O on its terminal side OA and let PM be perpendicular from P on x-axis. Let length OP = r, OM = x and MP = y. We take the length OP = r always positive while x and y can be positive or negative depending upon the position of the terminal side OA of ∠XOA.





In the right-angled triangle OMP, we have Base = OM = x, perpendicular = PM = y and, Hypotenuse = OP = r.





We define the following trigonometric ratios, which are also known as trigonometric functions:

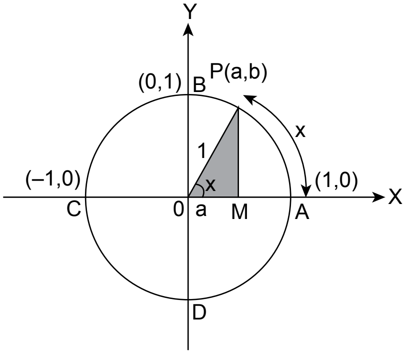
$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{y}{r}; \ \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{x}{r}$$

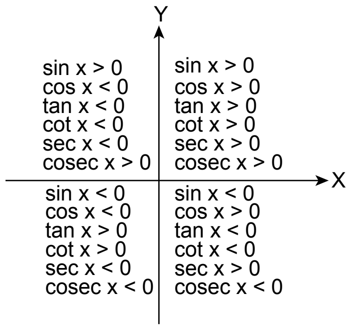
$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}; \ \text{cosec}\,\theta = \frac{\text{Hypotaneuse}}{\text{Perpendicular}} = \frac{r}{y}$$

$$\sec\theta = \frac{\text{Hypotaneuse}}{\text{Base}} = \frac{r}{x}; \ \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$$

9.4.1 Signs of Trigonometric Ratios

Consider a unit circle with centre at origin of the coordinate axes. Let P(a, b) be any point on the circle with angle AOP = x radian, i.e., length of arc AP = x as shown in the following figure:





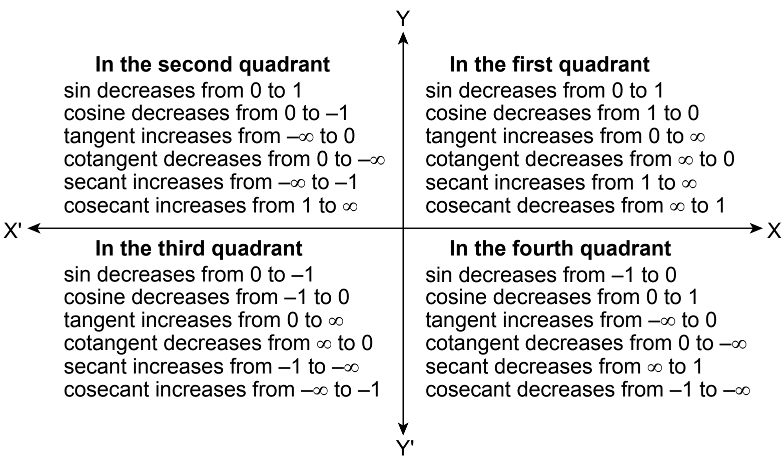
We defined cos x = a and sin x = b. Since ΔOMP is a right triangle, we have $OM^2 + MP^2 = OP^2$ or $a^2 + b^2 = 1$.

Thus, for every point on the unit circle, we have $a^2 + b^2 = 1$ or $\cos^2 x + \sin^2 x = 1$. Accordingly, we can judge the sign of a trigonometric function by comparing it with the sign of respective coordinates in that particular quadrant.

Remark:

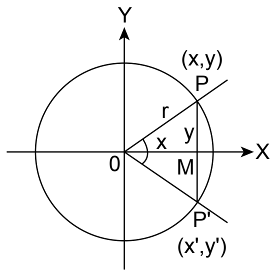
The sign conventions can be kept in mind by the sentence "After School To College"; where A stands for All, S stands for Sine, T stands for Tangent, C stands for Cosine.

9.4.2 Range of Trigonometric Ratios



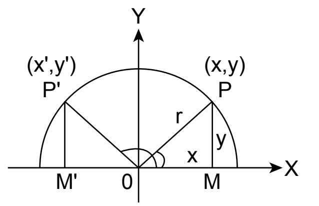
9.4.3 Trigonometric Ratios of Allied Angles

9.4.3.1 Trigonometric ratios of –θ



Sin(–θ) = –sinθ	cos(–θ) = cosθ	tan(–θ) = –tanθ	cot(–θ) = –cotθ	sec(–θ) = secθ	cosec(–θ) = –cosecθ

9.4.3.2 Trigonometric ratios of π – θ



Sin(π – θ) = sinθ	cos(π –θ) = –cosθ	tan(π – θ) = –tanθ	cot(π – θ) = –cotθ	sec(π– θ) = –secθ	cosec(π–θ) =cosecθ

Similarly

Sin(π + θ)= –sinθ	**cos(π + θ) = –cosθ**	**tan(π + θ) = tanθ**	**cot(π + θ) = cotθ**
$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$	$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$	$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$	$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$
$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$	$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$	$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$	$\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

sec(π + θ) = –secθ	**cosec(π + θ)= –cosec θ**
$\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$	$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$
..	$\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=\sec\theta$

Think yourself! Try to evaluate the conversions for f(270 – θ), f(270 + θ), f(360 – θ), f(360 + θ), where f is a trigonometric function.

Generalized Results: The values of trigonometric functions of any angle can be represented in terms of an angle in the first quadrant as follows. Let $A = n.\frac{\pi}{2} \pm \theta$; where n ∈ Z, $0 \le \theta < \frac{\pi}{2}$. Then

(i) Sin π = 0, cosn π = $(-1)^n$

(ii) $\sin\left(n.\frac{\pi}{2}+\theta\right) = \begin{cases} (-1)^{\frac{(n-1)}{2}}\cos\theta; & \text{if n is odd} \\ (-1)^{n/2}\sin\theta; & \text{if n is even} \end{cases}$

(iii) $\cos\left(n.\frac{\pi}{2}+\theta\right) = \begin{cases} (-1)^{\frac{(n+1)}{2}}\sin\theta; & \text{if n is odd} \\ (-1)^{n/2}\cos\theta; & \text{if n is even} \end{cases}$

(iv) $\tan\left(n.\frac{\pi}{2}\pm\theta\right) = \begin{cases} \pm\tan\theta; & \text{if n is even} \\ \pm\cot\theta; & \text{if n is odd} \end{cases}$

(v) $\cot\left(n.\frac{\pi}{2}\pm\theta\right) = \begin{cases} \pm\cot\theta; & \text{if n is even} \\ \pm\tan\theta; & \text{if n is odd} \end{cases}$

(vi) $\sec\left(n.\frac{\pi}{2}\pm\theta\right) = \begin{cases} \pm\sec\theta; & \text{if n is even} \\ \pm\text{cosec}\theta; & \text{if n is odd} \end{cases}$

Think, and fill up the blank blocks in the following table

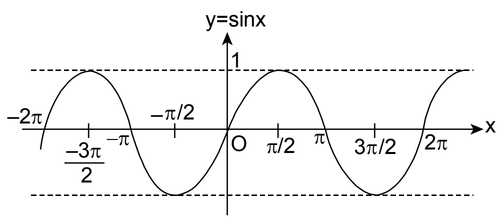
Angles Functions	0	π/6	π/4	π/3	π/2	2π/3	5π/6	π	7π/6	4π/3	..	5π/3	11π/6	2π
sinθ	0	1/2	1/√2	√3/2	1									
cosθ	1	√3/2	1/√2	1/2	0									
tanθ	0	1/√3	1	√3	N.D. (∞)									
cotθ	∞	√3	1	1/√3	0									
secθ	1	2/√3	√2	2	N.D. (∞)									
tanθ	∞	2	√2	2/√3	1									

9.5 GRAPHS OF DIFFERENT TRIGONOMETRIC RATIOS

9.5.1 y = sin x

x	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	p
sin x	0	1/2	1/√2	√3/2	1	√3/2	1/√2	1/2	0

Properties:



P.1 Domain of sinx is R and range is [–1, 1]

P.2 sinx is periodic function with period 2π.

P.3 Principle domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

P.4 It is an odd function.

P.5 It is a continuous function and increases in first and fourth quadrants while decreases in second and third quadrants.

9.5.2 y = cos x

X	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	p
cos x	1	√3/2	1/√2	1/2	0	–1/2	–1/√2	–√3/2	–1

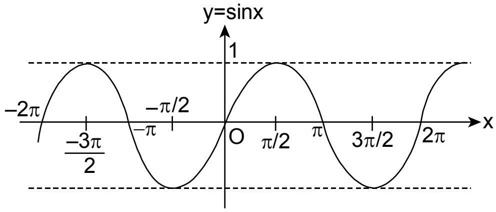
Properties:

P.1 The domain of cosx is R and the range is [–1, 1].

P.2 Principle domain is [0, π].

P.3 cosx is periodic with period 2π.

P.4 It is an even function so symmetric about the y-axis.



9.5.2.1 *y = tan x*

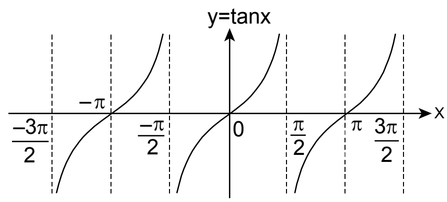
X	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	p
tan x	0	1/√3	1	√3	∞	–√3	–1	–1/√3	0

Properties:

P.1 The domain of tanx is R – {(2n + 1) π/2} and range R or (–∞, ∞). Principal domain is (–π/2, π/2).

P.2 It is periodic with period π.

P.3 It is discontinuous x = R – {(2n + 1) π/2} and it is strictly increasing function in its domain.



9.5.3 y = cot x

X	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	p
cot x	∞	√3	1	1/√3	0	–1/√3	–1	–√3	–∞

Properties:

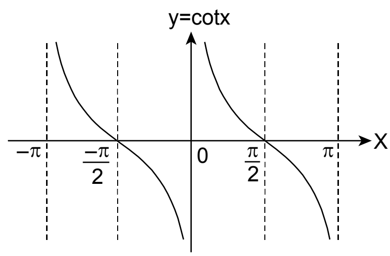
P.1 The domain of f(x) = cotx is domain ∈ R ~ {nπ}; Range ∈ ℝ.

P.2 It is periodic with period π and has x = nπ, n ∈ z as its asymptotes.

P.3 Principal domain is (0, π).

P.4 It is discontinuous at x = nπ.

P.5 It is strictly decreasing function in its domain.



9.5.4 y = cosec x

x	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	p
cosec x	∞	2	√2	2/√3	1	2/√3	√2	2	∞

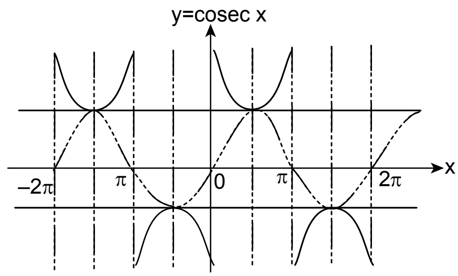
Properties:

P.1 The domain is R ~ {nπ | n ∈ z}.

P.2 Range of cosecx is R – (–1, 1).

P.3 Principal domain is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$.

P.4 The cosecx is periodic with period 2π.



9.5.5 y = sec x

X	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	p
sec x	1	2/√3	√2	2	∞	–2	–√2	–2/√3	–1

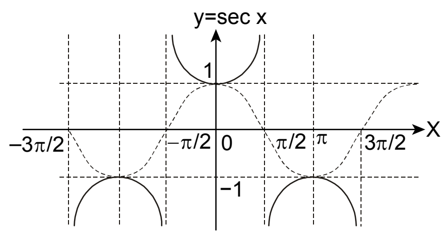
Properties:

P.1 The domain of sec x is $R-\left\{(2n+1)\frac{\pi}{2}\right\}n\in z$, and range is R – (–1, 1).

P.2 The sec x is periodic with period 2π.

P.3 Principal domain is [0, π] – {π/2}.

P.4 It is discontinuous at x = (2n + 1) π/2.



9.5.6 Trigonometric Identities

9.5.6.1 Pythagorean identities

The following three trigonometric identities are directly derived from the pythagoras theorem.

1. **$\sin^2 x + \cos^2 x = 1$**; $x \in \mathbb{R} \Rightarrow \cos^2 A = 1 - \sin^2 x$ or $\sin^2 x = 1 - \cos^2 x$ or $\frac{\cos x}{1-\sin x}=\frac{1+\sin x}{\cos x}$.

2. **$1+\tan^2 x = \sec^2 x$**; $x\in\mathbb{R}\sim\left\{(2n+1)\frac{\pi}{2};n\in\mathbb{Z}\right\} \Rightarrow \sec^2 x - \tan^2 x = 1$ or $\sec x-\tan x=\frac{1}{\sec x+\tan x}$.

3. **$\cot^2 x + 1 = \text{cosec}^2 x$**; $x \in \mathbb{R} \sim \{n\pi;\ n \in \mathbb{Z}\} \Rightarrow \text{cosec}^2 x - \cot^2 x = 1$ or $\text{cosec}\, x-\cot x=\frac{1}{\text{cosec}\, x+\cot x}$.

Note:

It is possible to express trigonometrical ratios in terms of any one of them as

$$\sin\theta=\frac{1}{\sqrt{1+\cot^2\theta}};\ \cos\theta=\frac{\cot\theta}{\sqrt{1+\cot^2\theta}};\ \tan\theta=\frac{1}{\cot\theta};\ \text{cosec}\,\theta=\sqrt{1+\cot^2\theta}$$

*** Remember sign of the dependent function will depend upon the location of angle in one or the other quadrant.*

9.5.7 Trigonometric Ratios of Compound Angles

An angle made up of the sum of the algebraic sum of the two or more angles is called a 'compound angle'. Some of the formulae on various trigonometric functions are given below:

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$

5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

9. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

10. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

9.5.8 Trigonometric Ratios of Multiples of Angles

1. $\sin^2 A = 2\sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$

2. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$; where $A \neq (2n+1)\dfrac{\pi}{4}$

3. $\dfrac{1-\cos A}{\sin A} = \tan\left(\dfrac{A}{2}\right)$; where $A \neq (2n+1)\pi$

4. $\dfrac{1+\cos A}{\sin A} = \cot\left(\dfrac{A}{2}\right)$; where $A \neq (2n\pi)$

5. $\dfrac{1-\cos A}{1+\cos A} = \tan^2\left(\dfrac{A}{2}\right)$; where $A \neq (2n+1)\pi$

6. $\dfrac{1+\cos A}{1-\cos A} = \cot^2\left(\dfrac{A}{2}\right)$; where $A \neq 2n\pi$

7. $\sin\dfrac{A}{2} + \cos\dfrac{A}{2} = \pm\sqrt{1+\sin A}$

8. $\sin\dfrac{A}{2} - \cos\dfrac{A}{2} = \pm\sqrt{1-\sin A}$

9. $\cos^2 A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

10. $1 + \cos 2A = 2\cos^2 A$, $1 - \cos^2 A = \sin^2 A$ or $\dfrac{1+\cos 2A}{2} = \cos^2 A$, $\dfrac{1-\cos 2A}{2} = \sin^2 A$

11. $\sin 3A = 3\sin A - 4\sin^3 A = 4\sin(60° - A).\sin A.\sin(60° + A)$

12. $\cos 3A = 4\cos^3 A - 3\cos A = 4\cos(60° - A)\cos A.\cos(60° + A)$

13. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = \tan(60° - A).\tan A.\tan(60° + A)$

14. $\sin A \pm \cos A = \sqrt{2}\sin\left(\dfrac{\pi}{2} \pm A\right) = \sqrt{2}\cos\left(A \mp \dfrac{\pi}{4}\right)$

9.5.9 Transformation Formulae

9.5.9.1 Expressing the product of trigonometric ratio sum or difference

(i) $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

(ii) $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(iii) $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

(iii) $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

9.5.9.2 Expressing the sum or difference of trigonometric ratios into product

1. $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right).\cos\left(\dfrac{C+D}{2}\right)$

2. $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right).\sin\left(\dfrac{C-D}{2}\right)$

3. $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right).\cos\left(\dfrac{C-D}{2}\right)$

4. $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

5. $\tan A+\tan B=\dfrac{\sin(A+B)}{\cos A.\cos B}$; where $A,B\neq n\pi+\dfrac{\pi}{2}$

6. $\tan A-\tan B=\dfrac{\sin(A-B)}{\cos A.\cos B}$; where A, B ≠ nπ + $A,B\neq n\pi+\dfrac{\pi}{2}$

7. $\cot A+\cot B=\dfrac{\sin(A+B)}{\sin A.\sin B}$; where A, B ≠ n, n ∈ z

8. $\cot A-\cot B=\dfrac{\sin(A-B)}{\sin A.\sin B}$; where A, B ≠ nπ n ∈ z

9.5.10 Conditional Identities

If A + B + C = π, then:

(i) sin2A + sin2b + sin2C = 4sinA sinB sinC

(ii) $\sin A+\sin B+\sin C=4\cos\dfrac{A}{22}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

(iii) cos2A + cos2B + cos2C = –1 – 4 cosA cosB cosC

(iv) $\cos A+\cos B+\cos C=1+4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

(v) tanA + tanB + tanC = tanA tanB tanC

(vi) $\tan\dfrac{A}{2}\tan\dfrac{B}{2}+\tan\dfrac{B}{2}\tan\dfrac{C}{2}+\dfrac{C}{2}\tan\dfrac{A}{2}=1$

(vii) $\cot\dfrac{A}{2}+\cot\dfrac{B}{2}+\cot\dfrac{C}{2}=\cot\dfrac{A}{2}.\cot\dfrac{B}{2}.\cot\dfrac{C}{2}$

(viii) cotA cotB + cotB cotC + cotC cotA = 1

(ix) $A+B+C=\dfrac{\pi}{2}$, then tanA tanB + tanB tanC + tanC tanA = 1

9.6 SOME OTHER USEFUL RESULTS

(i) sin α + sin (α + β) + sin (α + 2β) + … + … to n terms = $\dfrac{\sin\left[\alpha+\dfrac{(n-1)\beta}{2}\right]\sin\left(\dfrac{n\beta}{2}\right)}{\sin\left(\dfrac{\beta}{2}\right)}$

(ii) cos α + cos (α + β) + cos (α + 2β) + … + … to n term = $\dfrac{\cos\left[\alpha+\dfrac{(n-1)\beta}{2}\right]\sin\left(\dfrac{n\beta}{2}\right)}{\sin\left(\dfrac{\beta}{2}\right)}$

(iii) cos A cos 2A $\cos 2^3$ A … $\cos 2^{n-1}A=\dfrac{\sin 2^n A}{2^n \sin A}$; when $n\to\infty$, $\cos\dfrac{\theta}{2}.\cos\dfrac{\theta}{2^2}....\cos\dfrac{\theta}{2^{n-1}}...\infty=\dfrac{\sin\theta}{\theta}$

(iv) If A, B, C = π, then:

- cosA + cosB + cosC ≤ 3/2.
- sinA/2 . sinB/2 . sinC/2 ≤ 1/8, equality holds good if A = B = C = 60°.
- $\tan^2 A/2+\tan^2 B/2+\tan^2 C/2\geq 1$.

9.7 SOME OTHER IMPORTANT VALUES

S.No.	Angle	Value	S.No.	Angle	Value
1.	sin 15°	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	**2.**	cos 15°	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
3.	tan 15°	$2-\sqrt{3}$ = cot 75°	**4.**	cot 15°	$2+\sqrt{3}$ = tan 75°
5.	sin $22\frac{1}{2}°$	$\frac{1}{2}\left(\sqrt{2-\sqrt{2}}\right)$	**6.**	cos $22\frac{1}{2}°$	$\frac{1}{2}\left(\sqrt{2+\sqrt{2}}\right)$
7.	tan $22\frac{1}{2}°$	$\sqrt{2}-1$	**7.**	cot $22\frac{1}{2}°$	$\sqrt{2}+1$
9.	sin 18°	$\frac{\sqrt{5}-1}{4}$ = cos 72°	**10.**	cos 18°	$\frac{\sqrt{10+2\sqrt{5}}}{4}$ = sin 72°
11.	sin 36°	$\frac{\sqrt{10-2\sqrt{5}}}{4}$ = cos 54°	**12.**	cos 36°	$\frac{\sqrt{5}+1}{4}$ = sin 54°
13.	sin 9°	$\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{4}$ or cos 81°	**14.**	cos 9°	$\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$ or sin 81°
15.	cos 36° – cos 72°	1/2	**16.**	cos 36°.cos 72°	1/4

9.8 MAXIMUM AND MINIMUM VALUES OF A COS θ + B SIN θ

Consider a point (a, b) on the cartesian plane. Let its distance from origin be r and the line joining the point and the origin make an angle a with the positive direction of x axis. Then a = r cos α and b = r sin α.

Squaring and adding $r=\sqrt{a^2+b^2}$; So, a cos θ + b sin θ = r [cos α cos θ + sin α sin θ] = r cos (α – θ),

but, – 1 ≤ cos (α – θ) ≤ 1; ⇒ – r ≤ a cos θ + b sin θ ≤ r.

So maximum value is $\sqrt{a^2+b^2}$ and minimum value is $-\sqrt{a^2+b^2}$.

9.9 TIPS AND TRICS

1. if x = secθ + tanθ. Then 1/x = secθ – tanθ
2. if x = cosecθ + cotθ. Then 1/x = cosecθ – cotθ
3. cos A. cos2A.cos2^2A ... $\cos 2^{n-1}A=\frac{\sin 2^n A}{2^n \sin A}$; if A ≠ n π = 1; if A = 2n π = $(-1)^n$; if A = (2n + 1) π
4. sinA/2 ± cosA/2 = √2 = sin[π /4 ± A] = √2cos [A ∓ π/4].
5. cos α + cos β + cos γ + cos (α + β +γ) = $4\cos\frac{(\alpha+\beta)}{2}\cos\frac{(\beta+\gamma)}{2}\cos\frac{(\gamma+\alpha)}{2}$.
6. sin α + sin β + sin γ – sin (α + β +γ) = $4\sin\frac{(\alpha+\beta)}{2}\sin\frac{(\beta+\gamma)}{2}\sin\frac{(\gamma+\alpha)}{2}$.