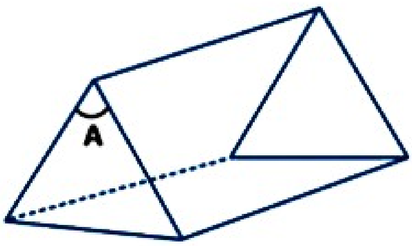
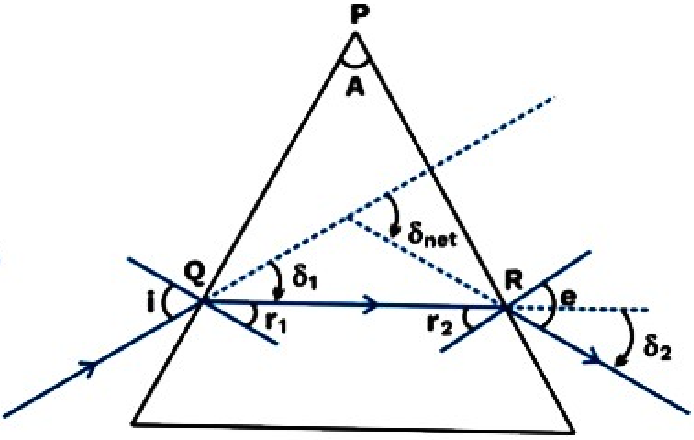
Prism

PRISM

Prism is a solid transparent object which has two refracting surfaces.

Angle between the two planes refracting surfaces is called the angle of a prism or refracting angle of a prism or apex angle.





$$\delta_1 = i - r_1$$

$$\delta_2 = e - r_2$$

$$\delta_{net} = i + e - (r_1 + r_2)$$

In ΔPQR,

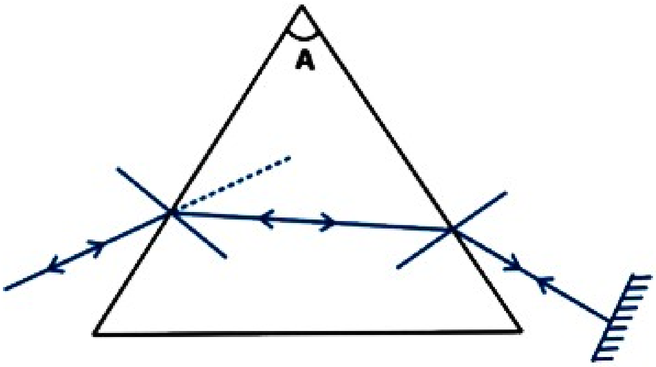
$$A + (90 - r_1) + (90 - r_2) = 180°$$

$$\boxed{r_1 + r_2 = A}$$

$$\boxed{\delta_{net} = i + e - A}$$

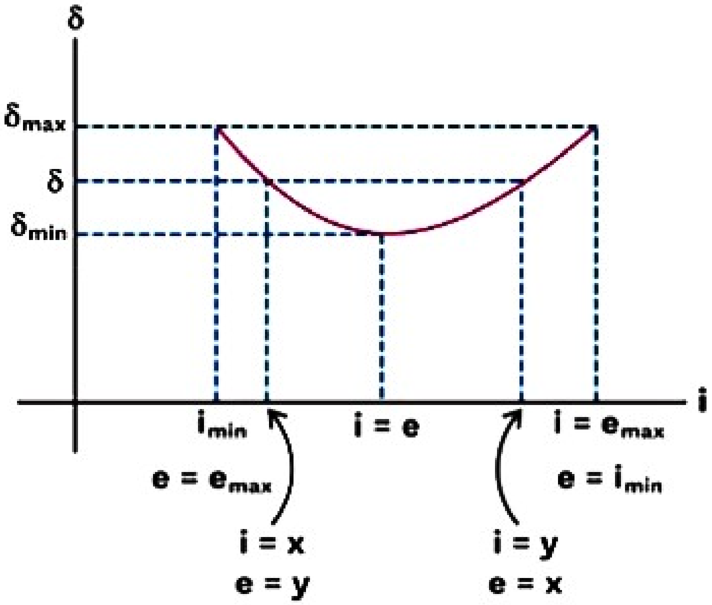
If we place a plane mirror normally to the emergent ray, the emergent ray will retrace its path.

Deviation will be the same as light ray retraces its path.



If we use the reversibility of a ray, we can conclude that the same deviation can be obtained for two different angles of incidence such that they are the angle of emergence for each other.

Variation of δ versus i



For the application of the above result medium on both sides of the prism must be the same. Based on the above graph, we can also derive the following result, which says that i and e can be interchanged for a particular deviation; in other words, there are two angles of incidence for a given deviation (except minimum deviation).

i	r_1	r_2	e	δ
θ_1	θ_2	θ_3	θ_4	θ_5
θ_4	θ_3	θ_2	θ_1	θ_5

For refraction at both the surfaces, deviation will be maximum for i = 90°.

There is one and only one angle of incidence for which the angle of deviation is minimum. When $\delta = \delta_{min}$, the angle of minimum deviation, then i = e and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces,

$$r_1 + r_2 = A$$

∴ In case of minimum deviation, $r_1 = r_2 = \frac{A}{2}$

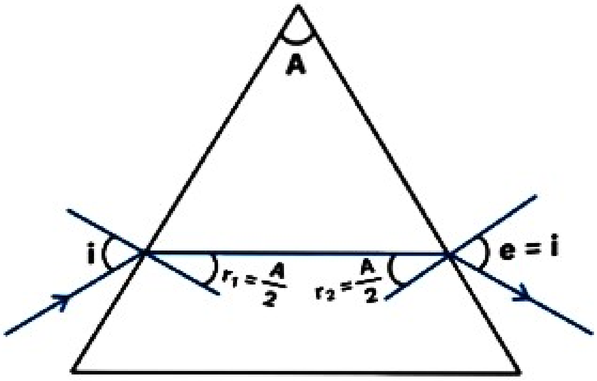
$$\delta = i + e - A$$

$\delta_{min} = i + i - A$ (∵ i = e for minimum deviation)

$\delta_{min} = 2i - A$

Q. **Prove that refractive index of the prism is given by** $\mu = \dfrac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\dfrac{A}{2}}$.

Sol: In case of minimum derivation



$$\delta_{min} = 2i - A \Rightarrow i = \frac{\delta_{min} + A}{2}$$

$$1 \sin i = \mu \sin r_1 = \mu \sin\frac{A}{2}$$

$$\sin i = \mu \sin\left(\frac{A}{2}\right) \quad ...(1)$$

$$\sin i = \sin\left(\frac{\delta_{min} + A}{2}\right) \quad ...(2)$$

From (1) and (2)

$$\mu \sin\left(\frac{A}{2}\right) = \sin\left(\frac{\delta_{min} + A}{2}\right)$$

$$\therefore \quad \mu = \frac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\dfrac{A}{2}}$$

When prism is placed in a medium

$$\mu_{rel} = \frac{\sin\left[\frac{A+\delta_{min}}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } \mu_{rel} = \frac{\mu_{prism}}{\mu_{surroundings}}$$

Q **Refracting angle of a prism A = 60° and its refractive index is, μ = 3/2, what is the angle of incidence i to get minimum deviation? Also, find the minimum deviation. Assume the surrounding medium to be air (μ = 1).**

Sol: For minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30°$

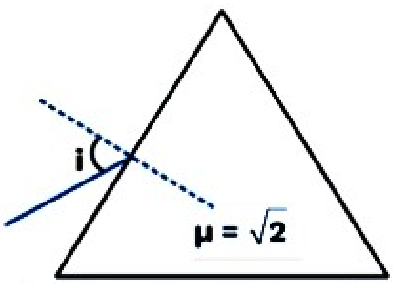
Applying Snell's law at 1st surface

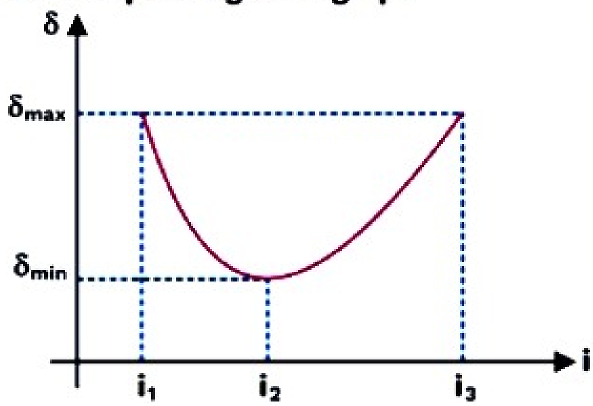
$$1 \times \sin i = \frac{3}{2}\sin 30°$$

$$\Rightarrow \; i = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\Rightarrow \; \delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - 60°$$

Q **Figure shows and equilateral prism and corresponding δ vs i graph**

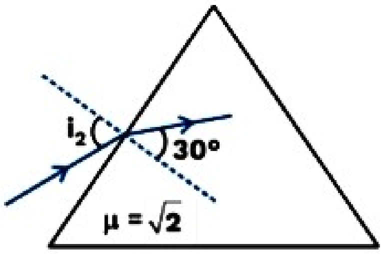




Find δ_{max}, δ_{min}, i_1, i_2 and i_3.

Sol: $\delta_{min} = 2i_2 - A = 2i_2 - 60°$ (A = 60° ← Equilateral prism)

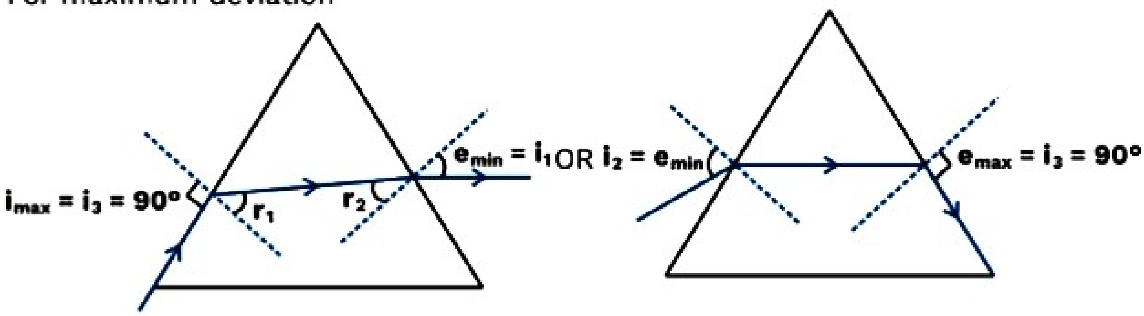
$$r_1 = r_2 = \frac{A}{2} = 30°$$



$1 \sin i_2 = \sqrt{2} \sin 30° \Rightarrow i_2 = 45°$

$\delta_{min} = 2 \times 45° - 60 = 30°$

For maximum deviation



$1 \sin 90° = \sqrt{2} \sin r_1$ and $\sqrt{2} \sin r_2 = 1 \sin i_1$

$r_1 = 45°$

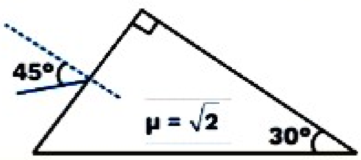
As $r_1 + r_2 = 60° \Rightarrow r_2 = 15°$

$\sqrt{2} \sin 15° = \sin i_1 \Rightarrow i_1 = \sin^{-1}\left(\sqrt{2} \times \frac{(\sqrt{3}-1)}{2\sqrt{2}}\right]$

$i_1 = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

$\delta_{max} = i_3 + i_1 - 60° = 90° + \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) - 60° = 30° + \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

Q **Find total deviation suffered by ray till it comes out of the prism.**



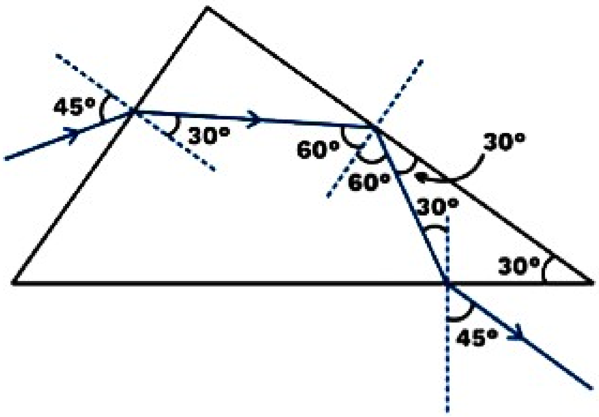
Sol: The critical angle for prism, $\sqrt{2}\sin i_c = 1 \Rightarrow i_c = 45°$

For 1st surface $\quad 1\sin 45° = \sqrt{2}\sin r \Rightarrow r = 30°$

For 2nd surface $\quad i = 60° \Rightarrow i > i_c$ (TIR)

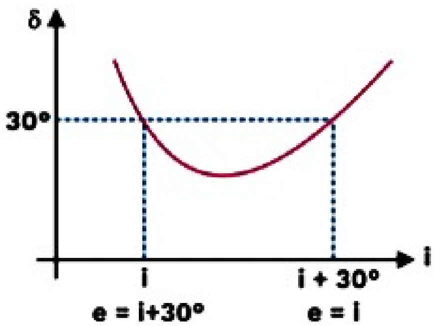
For 3rd surface $\quad \sqrt{2}\sin 30° = 1\sin r \Rightarrow r = 45°$

$\delta = 15° + 60° - 15° = 60°$ CW



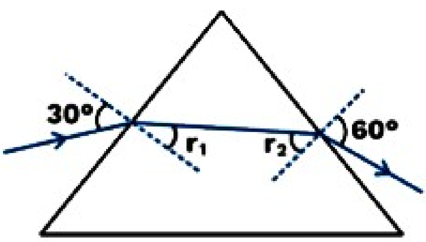
Q **A ray is incident on an equilateral prism such that the angle of deviation is 30°. It is seen that if the angle of incidence is increased by 30°, then the deviation is again 30°. Find the refractive index of the prism.**

Sol:



$\delta = i + e - A$

$30° = i + i + 30° - 60° \Rightarrow i = 30°$ & $e = 60°$



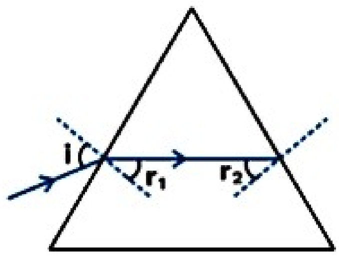
$$r_1 + r_2 = 60° \Rightarrow r_2 = 60° - r_1$$

$$1\sin 30° = \mu \sin r_1 \ \& \ \mu \sin r_2 = 1\sin 60°$$

$$\mu \sin r_1 = \frac{1}{2} \ \& \ \mu \sin(60° - r_1) = \frac{\sqrt{3}}{2}$$

$$\mu = \sqrt{\frac{4+\sqrt{3}}{2}}$$

Condition of Sure Emergence



For sure emergence on 2nd surface

$(r_2)_{max} \le i_e$

For r_2 to be maximum, r_1 must be minimum ($\because r_1 + r_2 = A$). For r_1 to be minimum, i must be minimum.

$\Rightarrow i_{min} = 0 \Rightarrow r_{1_{min}} = 0$

$\Rightarrow r_{2_{max}} = A$

$\boxed{A \le i_c}$

$\sin A \le \sin i_c$

$\sin A \le \dfrac{1}{\mu}$

$\boxed{\mu \le \operatorname{cosec} A}$

Condition of sure TIR

For sure TIR on 2rd surface

$(r_2)_{min} > i_c$

For $(r_2)_{min} \Rightarrow (r_1)_{max} \Rightarrow (i)_{max}$

$\Rightarrow i_{max} = 90 \Rightarrow r_{1_{max}} = \sin^{-1}\left(\dfrac{1}{\mu}\right) = i_c$

$r_1 + r_2 = A \Rightarrow r_2 = A - r_1$

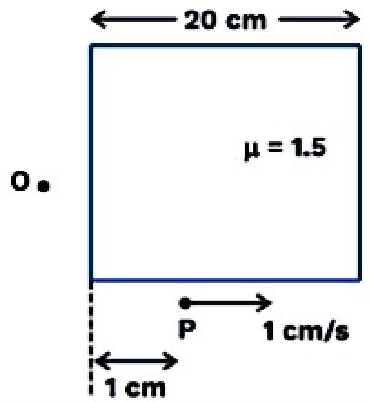
$r_{2_{min}} = A - r_{1_{max}} = A - i_c$

$A - i_c > i_c \Rightarrow A > 2i_c$

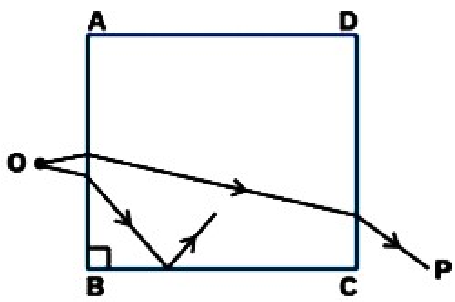
$\boxed{A > 2i_c}$

$\boxed{\mu > \text{cosec}\,\frac{A}{2}}$

Q **O and P both are very close to surface. At what time O will be visible to person (P)**



Sol:



$\text{cosec}\frac{A}{2} = \text{cosec} = 45° = \sqrt{2}$

$\mu > \text{cosec}\frac{A}{2}$

So, no light ray will emerge out from BC all light rays from O on BC will suffer TIR, but light ray on face DC will emerge out and P will see O. So P has to travel 19 cm to se 'O'.

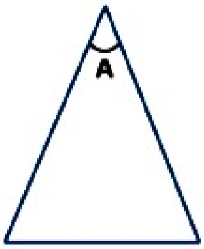
Thin Prism

$A \rightarrow$ small, $i \rightarrow$ small

At 1st surface,

$1 \sin i = \mu \sin r$

$i = \mu r_1$



For 2nd surface,

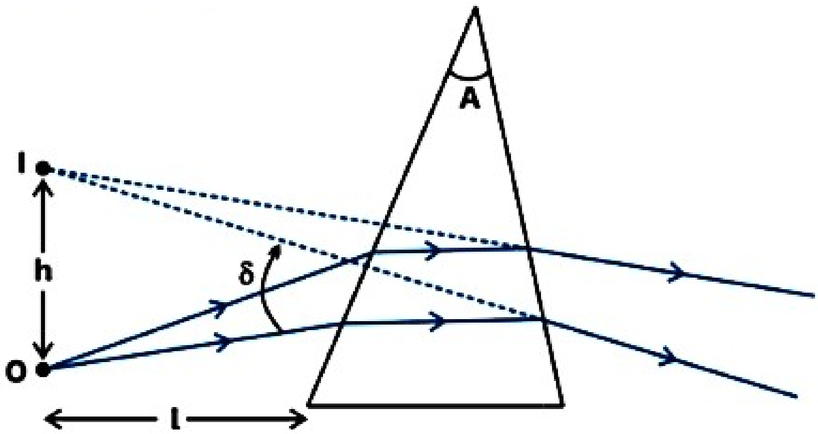
$e = \mu r_2$

$\delta = i + e - A$

$= \mu r_1 + \mu r_2 - A = \mu(r_1 + r_2) - A = \mu(A) - A$

$$\boxed{\delta = (\mu - 1)A}$$

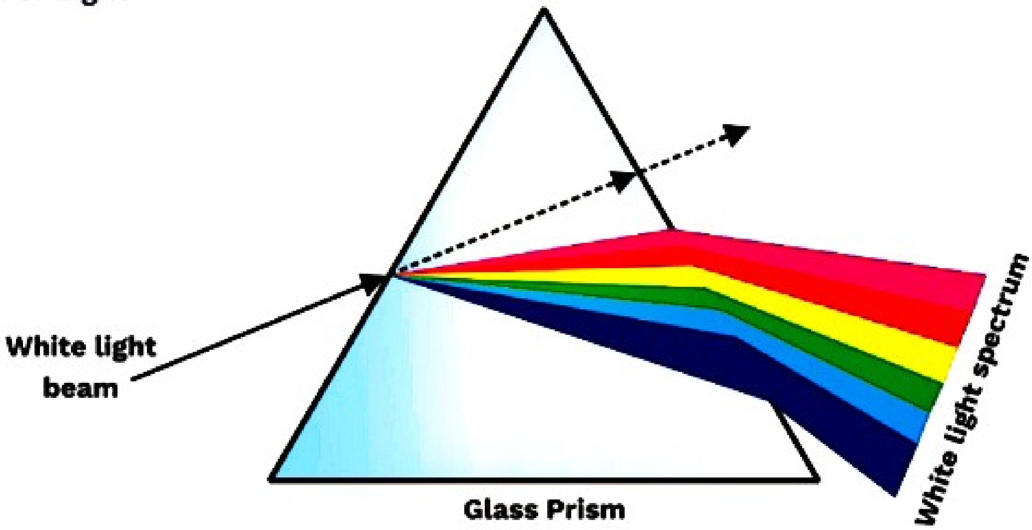
The shift in the Case of Thin Prism



$h = l\delta$

δ in radian

Dispersion of Light



Dispersion of sunlight or white light on passing through a glass prism.

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light. [It is for whole Electro Magnetic Wave in totality]. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's Formula

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \dots$$

Where A, B, C, ... are positive constants.

Note Such phenomenon is not exhibited by sound waves.

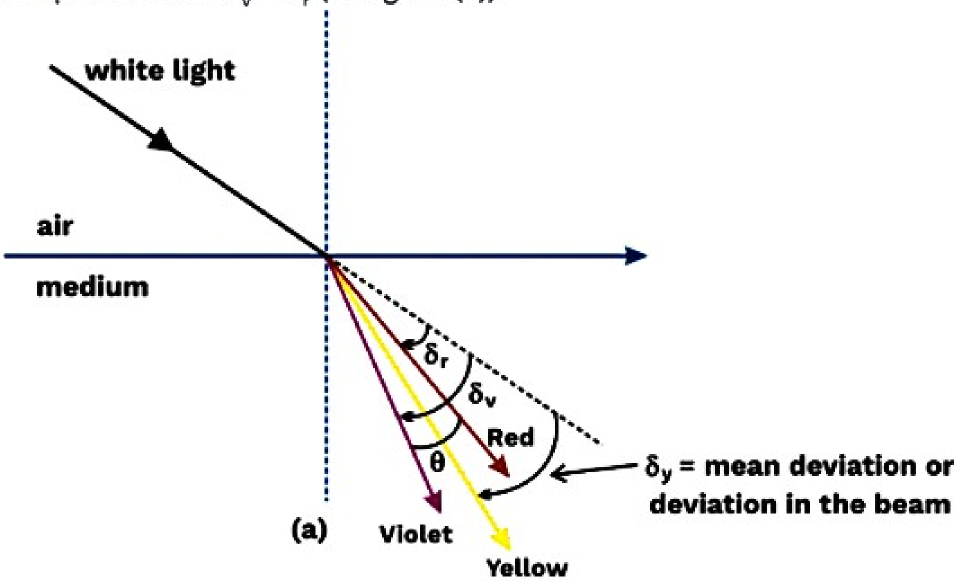
Colour	Wavelength (nm)	Crown glass	Flint glass
Violet	396.9	1.533	1.663
Blue	486.1	1.523	1.639
Yellow	589.3	1.517	1.627
Red	656.3	1.515	1.622

The variation of refractive index with wavelength may be more pronounced in some media than the others. In a vacuum, of course, the speed of light is independent of wavelength; thus, a vacuum (or air approximately) is a non-dispersive medium in which all colors travel with the same speed. This also follows from the fact that sunlight reaches us in the form of white light and not as its components. On the other hand, glass is a dispersive medium.

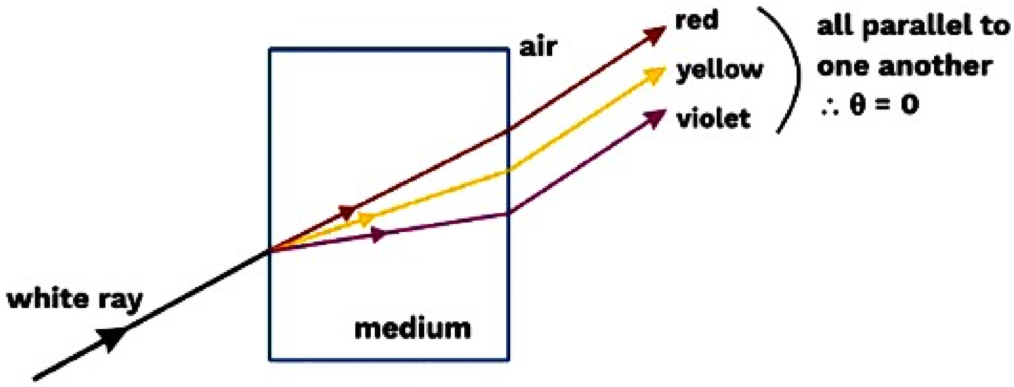
$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$$

$$\xrightarrow[\lambda \text{ increases}]{\text{V I B G Y O R}} \qquad \xrightarrow[\mu \text{ decreases}]{} \; ; \; \uparrow \mu = \frac{c_0}{c_{med} \downarrow}$$

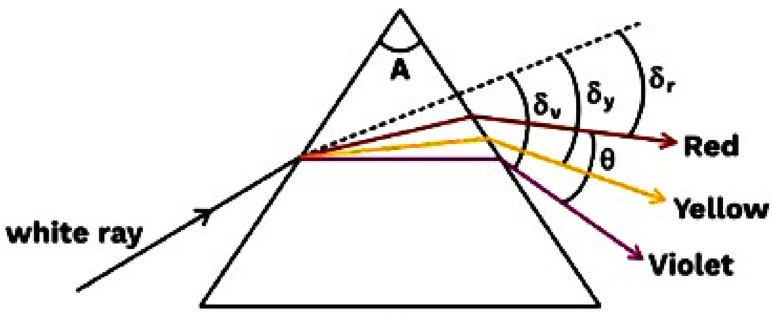
The angle between the rays of the extreme colours in the refracted (dispersed) light is called the angle of dispersion. $\theta = \delta_v - \delta_r$ (in figure (a))



(a)



(b)



(c)

For prism of small 'A' and with small 'i'

$$\theta = \delta_v - \delta_r = (\mu_v - 1)A - (\mu_r - 1)A = (\mu_v - \mu_r)A$$

Deviation of a beam (also called mean deviation)

$$\delta = \delta_y = (\mu_y - 1)A$$

μ_v, μ_r and μ_y are R.I. of material for violet, red and yellow colors, respectively. Figure (a) and (c) represents dispersion, whereas, in figure (b), there is no dispersion.

Q **The refractive indices of flint glass for red and violet light are 1.613 and 1.632, respectively. Find the angular dispersion produced by a thin prism of flint glass having a refracting angle 5°.**

Sol Deviation of the red light is $\delta_r = (\mu_r - 1)A$, and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion = $\delta_v - \delta_r = (\mu_v - \mu_r)A = (1.632 - 1.613) \times 5° = 0.095°$.

Note Numerical data reveals that if the average value of μ is small $\mu_v - \mu_r$ is also small and if the average value of μ is large $\mu_v - \mu_r$ is also large. Thus, the larger the mean deviation, larger will be the angular dispersion.

Dispersive Power

Dispersive power (ω) of the medium of the material of prims is given by

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

- ω is the property of a medium.

  For small angled prims ($A \le 10°$) with light incident at small angle i

  $$\frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y} = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

  [$\mu_y = \dfrac{\mu_v + \mu_r}{2}$ if μ_y is not given in the problem]

- $\mu - 1$ = refractivity of the medium for the corresponding color.

Q **The Refractive index of glass for red and violet colors are 1.50 and 1.60, respectively. Find**

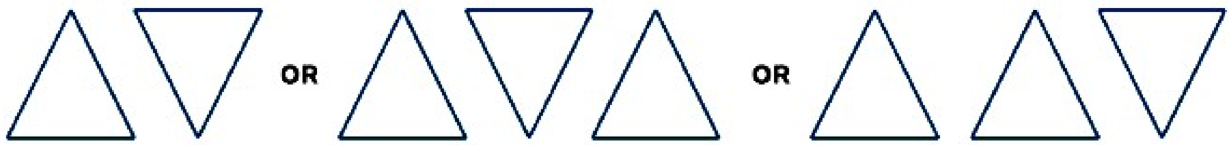
(a) The ref. index for yellow colour, approximately

(b) Dispersive power of the medium.

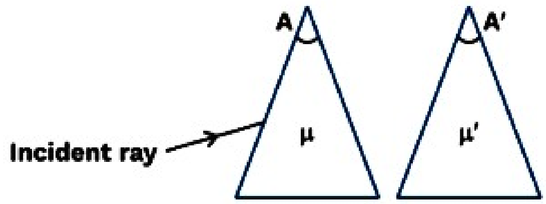
Sol: (a) $\mu_r = \dfrac{\mu_v + \mu_r}{2} = \dfrac{1.50 + 1.60}{2} = 1.55$

(b) $\omega = \dfrac{\mu_v - \mu_r}{\mu_r - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18$

Combination of Prisms



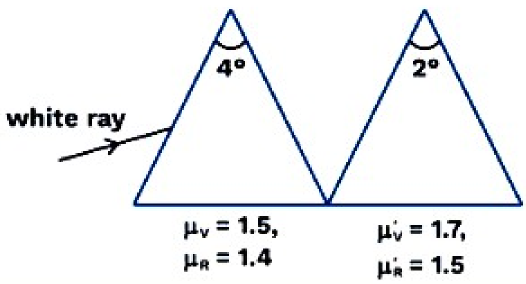
For two prisms placed as shown



$\delta_{net} = \delta + \delta'$; $\omega = \frac{\theta}{\delta}$; $\omega' = \frac{\theta'}{\delta'}$

$\theta_{net} = \theta + \theta'$

Q **If two prisms are combined, as shown in the figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.**



Sol: Both prisms will turn the light rays towards their bases and hence in the same direction. Therefore, turnings caused by both prisms are additive. Total angular dispersion

$= \theta + \theta' = (\mu_v - \mu_R)A + (\mu_v' - \mu_R')A'$

$= (1.5 - 1.4)4° + (1.7 - 1.5)2° = 0.8°$

Total deviation $= \delta + \delta'$

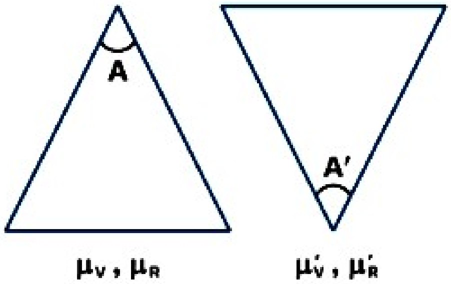
$= \left(\frac{\mu_v + \mu_R}{2} - 1\right)A + \left(\frac{\mu_v' + \mu_R'}{2} - 1\right)A' = \left(\frac{1.5 + 1.4}{2} - 1\right)4° + \left(\frac{1.7 + 1.5}{2} - 1\right)2°$

$= (1.45 - 1)4° + (1.6 - 1)2°$

$= 0.45 \times 4° + 0.6 \times 2°$

$= 1.80 + 1.2 = 3.0°$

Dispersion without deviation (Direct Vision Combination)

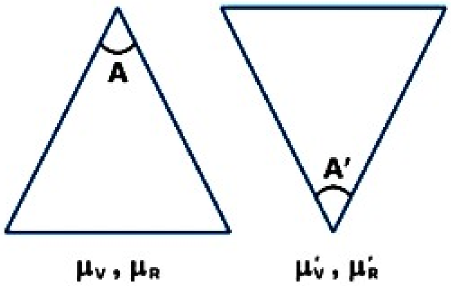


The condition for direct vision combination is $\delta_{net} = 0$

$$[\mu_y - 1]A - [\mu_y^{'} - 1]A' = 0$$

$$[\mu_y - 1]A = [\mu_y^{'} - 1]A' \Leftrightarrow \left[\frac{\mu_v + \mu_r}{2} - 1\right]A = \left[\frac{\mu_v^{'} + \mu_r^{'}}{2} - 1\right]A'$$

Deviation without dispersion (Achromatic Combination)



Condition for achromatic combination is $\theta_{net} = 0$

$$(\mu_v - \mu_r)A = (\mu_v^{'} - \mu_r^{'})A'$$

If $\theta_{net} = 0$ and $\delta_{net} = 0$

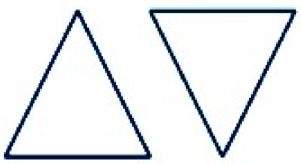
$$\delta_{net} = \delta + \delta' = 0$$

$$\delta' = -\delta$$

$$\theta_{net} = \theta + \theta'$$

$$= \omega\delta + \omega'\delta' = \delta(\omega - \omega')$$

If $\theta_{net} = 0$ and $\delta_{net} = 0$, then $\omega = \omega'$ (prisms with same material)



Q **Two thin prisms are combined to form an achromatic combination. For 1st prism A = 4°, μ_R = 1.35, μ_Y = 1.40, μ_V = 1.42. For 2nd prism $\mu'_R = 1.7$, $\mu'_Y = 1.8$ and $\mu'_R = 1.9$, find the prism angle of the 2nd prism and the net mean deviation.**

Sol Condition for achromatic combination.

$\theta = \theta'$

$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$

$\therefore \quad A' = \dfrac{(1.42 - 1.35)4^\circ}{1.9 - 1.7} = 1.4^\circ$

$\delta_{net} = \delta - \delta' = (\mu_Y - 1)A - (\mu'_Y - 1)A' = (1.40 - 1)4^\circ - (1.8 - 1)1.4^\circ = 0.48^\circ.$

Q **A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes un-deviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow, and violet light are 1.5, 1.6, and 1.7 respectively for crown glass and 1.8, 2.0 and 2.2 for flint glass.**

Sol: The deviation produced by the crown prism is

$\delta = (\mu - 1)A$

and by the flint prism is

$\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions.

Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A' \qquad ...(1)$$

(a) If the net deviation for the mean ray is zero,

$(\mu - 1)A = (\mu' - 1)A'$

or $\quad A' = \dfrac{(\mu - 1)}{(\mu' - 1)}A = \dfrac{1.6 - 1}{2.0 - 1} \times 5^\circ = 3^\circ$

(b) The angular dispersion produced by the crown prism is

$\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is,

$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

$= (1.7 - 1.5) \times 5^\circ - (2.2 - 1.8) \times 3^\circ$

$= -0.2^\circ$

The angular dispersion has magnitude 0.2°.

Chromatic Aberration

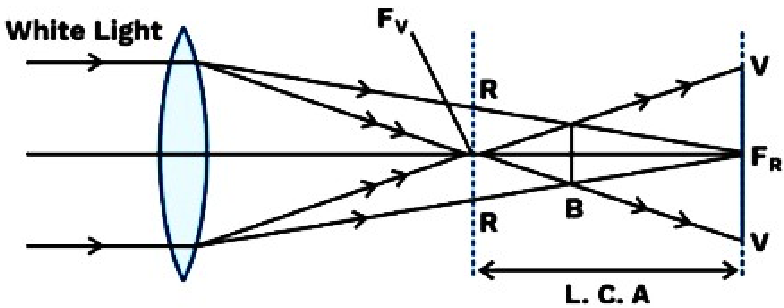
Thick lenses could be assumed as made of many prisms; therefore, thick lenses show chromatic aberration due to dispersion of light.

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

The image of a white object in white light formed by a lens is usually colored and blurred. This defect of the image is called chromatic aberration and arises due to the fact that the focal length of a lens is different for different colors. As R.I. μ of the lens is maximum for violet while the minimum for red, violet is focused nearest to the lens while red farthest from it as shown in the figure.

As a result of this, in the case of a convergent lens, if a screen is placed at F_V centre of the image will be violet and focused while sides are red and blurred. While at F_R, reverse is the case, i.e., centre will be red and focused while sides violet and blurred. The difference between f_v and f_R is a measure of the longitudinal chromatic aberration (L.C.A), i.e.,

$$L.C.A = f_R - f_V = -\,df \text{ with } df = f_V - f_R \qquad ...(1)$$



However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ...(2)$$

$$\Rightarrow \; -\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ...(3)$$

Dividing equation (3) by (2) ;

$$-\frac{df}{f} = \frac{d\mu}{(\mu - 1)} = \omega = \frac{\mu_v - \mu_r}{\mu_{y-1}}\left[\omega = \frac{d\mu}{(\mu - 1)}\right] = \text{dispersive power} \qquad ...(4)$$

And hence, from equations (1) and (4),

L.C.A. = $-df = \omega f$

Now, as for a single lens, neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism

In the case of two, this lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{i.e.,} \quad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if dF = 0

i.e., $\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$

Which, with the help of equation (d), reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0 \quad \text{i.e.,} \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \qquad \text{...(5)}$$

This condition is called the condition of achromatism (for two thin lenses in contact), and the lens combination which satisfies this condition is called an achromatic lens; from this condition, i.e., from equation (5), it is clear that in the case of achromatic doublet

(a) The two lenses must be of different materials. ($\mu_{\text{different}}$)

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$, i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(b) As ω_1 and ω_2 are positive quantities, for equation (5) to hold, f_1 and f_2 must be of opposite nature, i.e., if one of the lenses is converging the other must be diverging.

(c) If the achromatic combination is convergent,

$f_C < f_D$ and as $-\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}$, $\omega_C < \omega_D$

i.e., in a convergent achromatic doublet, the convex lens has a lesser focal length and dispersive power than the divergent one.