Chapter 1

Introduction—Forces and Equilibrium

CHAPTER HIGHLIGHTS

- *Introduction*
- *Newtonian mechanics*
- *Deformation of body*
- *Force*
- *Resolution of a force into a force and a couple*
- *Resultant of a system of coplanar forces*
- *Resultant of multiple forces acting at a point*
- *Triangle law of forces*
- *Coplanar concurrent force system*
- *Coplanar non-concurrent, non-parallel force system*
- *Moment of a force*
- *Moment of a couple*
- *Equilibrium equations for different coplanar force systems*
- *Analysis of a system of forces in space*

Introduction

In physics, the branch which deals with the study of state of rest or motion caused by the action of forces on the bodies is called 'mechanics'.

Engineering mechanics applies the principles and laws of mechanics to solve the problems of common engineering elements.

Newtonian Mechanics

Newtonian mechanics or classical mechanics deals with the study of the motion of macroscopic objects under the action of a force or a system of forces.

Branches of Newtonian Mechanics

1. **Statics:** It is the study of forces and conditions of equilibrium of bodies at rest subjected to the action of forces.
2. **Dynamics:** It is the branch of mechanics which deals with the study of motion of rigid bodies and the co-relation with the forces causing and affecting their motion. Dynamics is divided into Kinematics and Kinetics.
3. **Kinematics:** Kinematics deals with space, time relationship of a given motion of body and not at all with the forces that cause the motion.
4. **Kinetics:** The study of the laws of motion of material bodies under the action of forces or kinetics is the study of the relationship between the forces and the resulting motion.

Some of the definitions of the idealizations used in engineering mechanics are as follows:

1. **Continuum:** It is defined as continuous nonspacial whole which has no empty spaces, and no part is distinct from the adjacent parts. Considering objects, in this way, ignores that the matter present in the object is made of atoms and molecules.
2. **Particle:** A particle is a body which has finite mass, but the dimensions can be neglected.
3. **System of particles:** When a group of particles which are inter-related are dealt together for studying the behaviour, it is called a system of particles.
4. **Rigid body:** A solid body which does not undergo any deformations under the application of forces is

called a rigid body. In reality, solid bodies are not rigid, but are assumed as rigid bodies.

5. **Matter:** It is anything which occupies space.
6. **Mass:** It is a measure of inertia. The mass of a body is the quantity of matter contained in it, and is the sum of the masses of its constituent mass points.

Deformation of Body

A body which changes its shape or size under the action of external forces is called deformable body.

Action and Reaction

Action and reaction occurs when one body exerts a force on another body, the later also exerts a force on the former. These forces are equal in magnitude and opposite in direction.

Tension

It is the pulling force which is acting through a string when it is tight. It acts in the outward direction.

Tension

Thrust

It is acting in the inward direction and it is the pushing force transferred through a light rod.

Thrust

Force

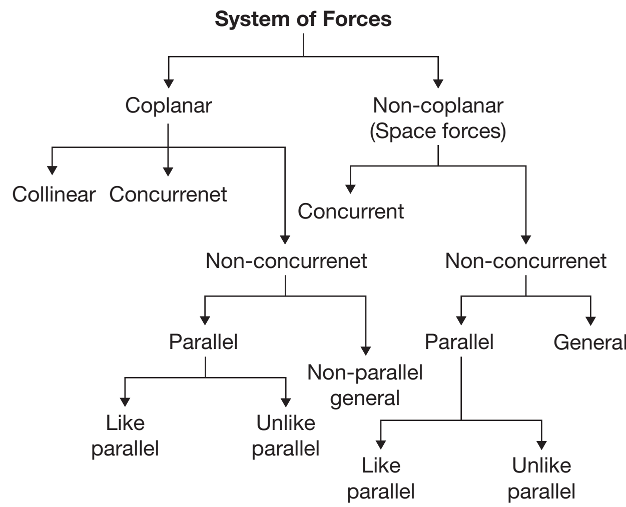
Force may be defined as any action that tends to change the state of rest or uniform motion of a body on which it is applied. The specifications or characteristics of a force are:

1. Magnitude
2. Point of application
3. Direction (force is a vector quantity)
4. Line of action

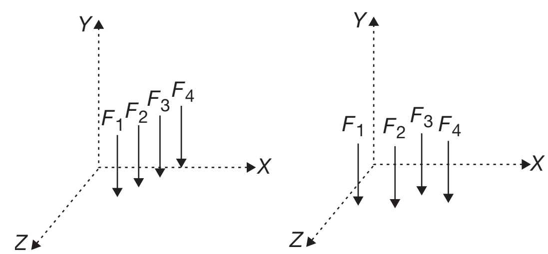
Force is a vector quantity since it has a magnitude and a direction (scalar quantities have only magnitudes and no directions).

The direction of a force is the direction, along a straight line passing through its point of application, in which the force tends to move the body on which it is applied. The straight line is called the line of action of the force. For the force of gravity, the direction of the force is vertically downward.

1. **System of forces:** A system of forces or a force system is the set of forces acting on the body or a group of bodies of interest. Force system can be classified according to the orientation of the lines of action of the constituting forces. It is shown as follows:

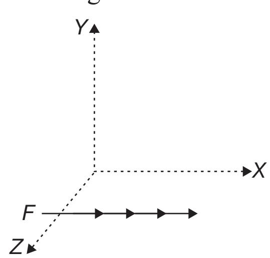


2. **Coplanar and non-coplanar (spatial) force systems:** In a coplanar force system (Figure (a)), the constituting forces have their lines of action lying in the same plane. If all the lines of action do not lie in the same plane, then the corresponding forces constitute a non-coplanar force system (Figure (b)).

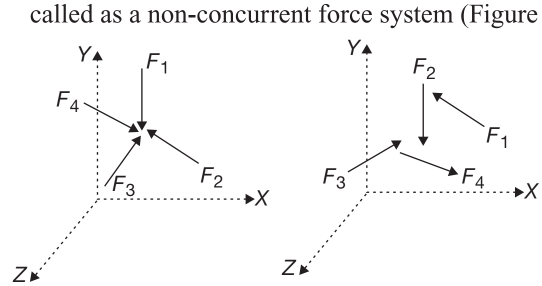


(a) (b)

3. **Collinear force system:** In a collinear force system (figure), the lines of action of the entire constituting forces will be along the same line.

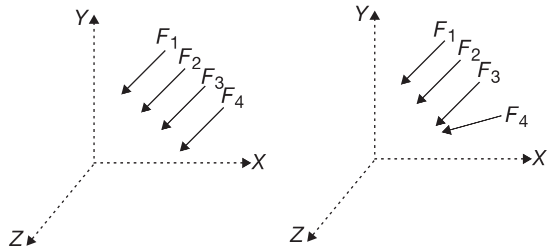


4. **Concurrent and non-concurrent force systems:** If the lines of action of all the forces in a force system pass through a single point, then the force system is called a concurrent force system (Figure (a)), else it is called as a non-concurrent force system (Figure (b)).



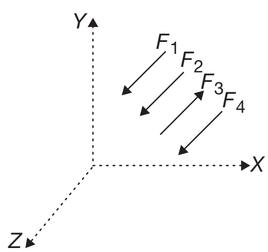
(a) (b)

5. **Parallel and non-parallel (general) force systems:** In a parallel force system (Figure (a)), the lines of action of the entire constituting forces are parallel to each other. If the line of action of at least one constituting force is not parallel to the line of action of another constituting force in a force system, then the force system is called 'non-parallel force system' (Figure (b)).



(a) (b)

6. **Like parallel and unlike parallel force systems:** In a like parallel force system (Figure (a)), the lines of action of the entire constituting forces are parallel to each other and act in the same direction. In an unlike parallel force system (figure), the lines of action of the entire constituting forces are parallel to each other, where some of them act in different directions.



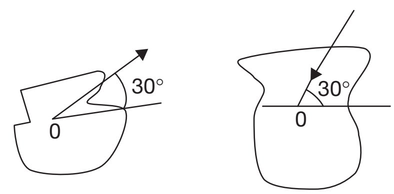
System of Forces

	Force System	Examples
1.	Collinear	Forces on a rope in a tug of war.
2.	Coplanar parallel	System of forces acting on a beam subjected to vertical loads including reactions.
3.	Coplanar like parallel	Weight of a stationary train on the rail when track straight.
4.	Coplanar concurrent	Forces of a rod resting against a wall.
5.	Coplanar, non-concurrent forces	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity.
6.	Non-coplanar parallel	The weight of the benches in a classroom.
7.	Non-coplanar concurrent forces	Forces on a tripod carrying a camera
8.	Non-coplanar non-concurrent forces	Forces acting on a moving bus.

NOTE

According to the magnitude of the constituting forces, force systems can also be classified as: (a) System of equal forces—all the constituting forces has the same magnitude; and (b) System of unequal forces—all the constituting forces do not have the same magnitude.

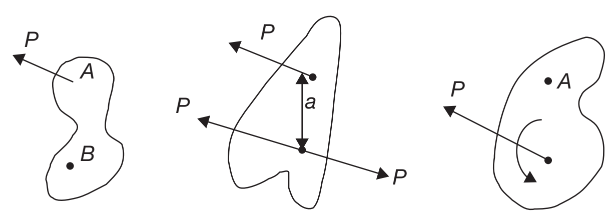
7. **Representation of a force:** Graphically, a force may be represented by the segment of a straight line with arrowhead at one end of the line segment. The straight line represents the line of action of the force, and its length represents its magnitude. The direction of force is indicated by placing an arrowhead on this straight line. The arrowhead at one end of the straight line segment indicate the direction of the force along the line segment. Either the head or the tail may be used to indicate the point of application of the force. Note that all the forces involved must be represented consistently as shown in figures below.



(a) (b)

Resolution of a Force into a Force and a Couple

A given force 'P' applied to a body at any point 'A' can always be replaced by an equal force applied at another point 'B' together with a couple which will be statically equivalent to the original force. To prove this, let the given force 'P' act at 'A' as shown below. Then at B, we introduce two oppositely directed collinear forces each of magnitude 'P' and parallel to the line of action of the given force 'P' at A.



(a) (b) (c)

It follows from the law of superposition that the system in Figure (b) is statically equivalent to that in Figure (a). However, we may now regard the original force 'P' at 'A', and the oppositely directed force 'P' at B as a couple of moment $M = Pa$. Since this couple may now be transformed in any manner in its plane of action as long as its moment remains unchanged, we may finally represent the system as shown in Figure (c), where the couple is simply indicated by a curved arrow and the magnitude of its moment. It will be noted that the moment of the couple introduced in the above manner will always be equal to the product of the original force 'P' and the arbitrary distance 'a' that we decide to move its line of action. This resolution of a force into a force and a couple is very useful in many problems of statics.

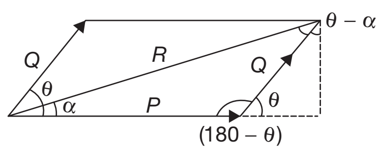
Resultant of a System of Coplanar Forces

Parallelogram Law of Forces

When two concurrent forces 'P' and 'Q' are represented in magnitude and direction by the two adjacent sides of a parallelogram as shown in the following figure, the diagonal of the parallelogram concurrent with the two forces, 'P' and 'Q' represents the resultant R of the forces in magnitude and direction.

If P and Q are two forces making an angle θ with each other, then

$$\boxed{R = \sqrt{P^2 + Q^2 + 2QP\cos\theta}}$$



$$\tan\alpha = \left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

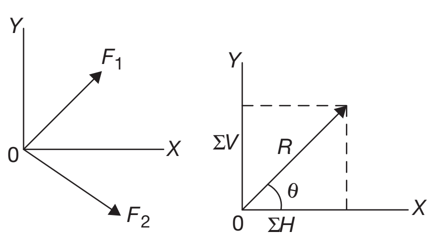
$$\boxed{\frac{Q}{\sin\alpha} = \frac{R}{\sin\theta} = \frac{P}{\sin(\theta - \alpha)}}$$

Resultant of Multiple Forces Acting at a Point

Let ΣH = Algebraic sum of resolved part of the forces along the X-axis.

ΣV = Algebraic sum of resolved part of the forces along the Y-axis

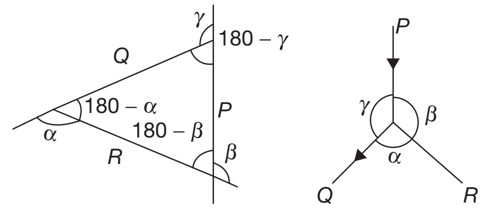
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$



$\tan\theta = \dfrac{\Sigma V}{\Sigma H}$ where θ is the angle which the resultant vector R makes with the X-axis.

Triangle Law of Forces

The resultant of two forces can be obtained by the triangle law of forces. The law states that if two forces acting at a point are represented by the two sides of a triangle, taken in order, the remaining side taken in an opposite order will give the resultant.



$$\boxed{\frac{Q}{\sin\alpha} = \frac{P}{\sin\gamma} = \frac{R}{\sin\beta}}$$

Coplanar Force System

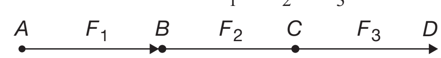
It can be classified into collinear, concurrent, parallel, non-concurrent, and non-parallel type of force system.

The resultant of a general coplanar system of forces may be: (a) single force, (b) a couple in the system's plane or in a parallel plane, or (c) zero.

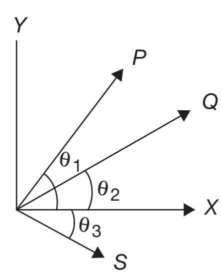
Collinear Forces

The resultant of a collinear force system (R) can be determined by algebraically adding the forces.

$$R = \Sigma F = F_1 + F_2 + F_3$$



Coplanar Concurrent Force System



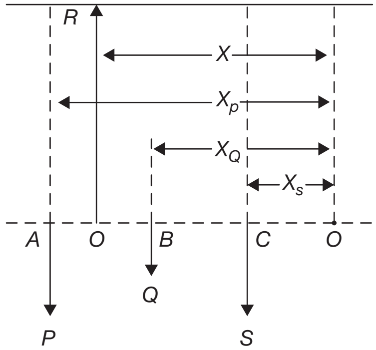
The analytical method consists of resolving the forces into components that coincide with the two arbitrarily chosen axes.

$$\Sigma F_x = P\cos\theta_1 + Q\cos\theta_2 + S\cos\theta_3$$

$$\Sigma F_y = P\sin Q_1 + Q\sin\theta_2 - S\sin\theta_3$$

and the resultant $\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$. Its angle with respect to the X-axis is given by $\alpha = \tan^{-1}\left(\dfrac{\Sigma Fy}{\Sigma Fx}\right)$

Coplanar Parallel Force System



Resultant of the parallel forces 'P', 'Q' and 'S' are $R = \Sigma F = P + Q + S$

$$\Sigma M_0 = Rx$$

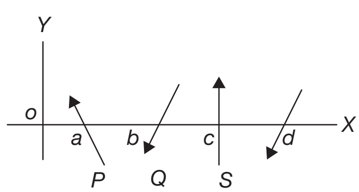
$\Sigma M_0 \rightarrow$ sum of the moments of the forces 'P', 'Q' and 'S' about point 0.

$$R_x = Px_p + Qx_Q + Sx_s$$

Coplanar Non-concurrent, Non-parallel Force System

As in the case of an unlike parallel force system, the resultant may be a single force, a couple in the plane of the system or zero. The resultant is given by $\boxed{R = \sqrt{(\Sigma fx)^2 + (\Sigma f_y)^2}}$,

and its angle α with the X-axis is given by $\boxed{\tan\alpha = \dfrac{\Sigma f_y}{\Sigma F_x}}$.



Distributed force system: Distributed forces (or loads) are those force that act over a length, area, or volume of a body. On the other hand, a concentrated force (point load) is a force which acts on a point.

SOLVED EXAMPLES

Example 1

The resultant of two concurrent forces '3P' and '2P' is R. If the first force is doubled, the resultant is also doubled. Determine the angle between the forces.

Solution

$$R = [(3P)^2 + (2P)^2 + 2\times 3P\times 2P\times\cos\alpha]^{1/2}$$
$$= P\times[13 + 12\cos\alpha]^{1/2} \quad (1)$$

α, being the angle between the forces.

$$2R = [(6P)^2 + (2P)^2 + 2\times 6P\times 2P\times\cos\alpha]^{1/2}$$
$$= [40 + 24\cos\alpha]^{1/2} \quad (2)$$

From Eqs. (1) and (2), we have

$$2P[13 + 12\cos\alpha]^{1/2} = P[40 + 24\cos\alpha]^{1/2}$$

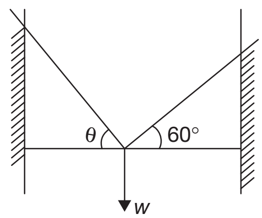
or

$$2[13 + 12\cos\alpha] = [40 + 24\cos\alpha]$$

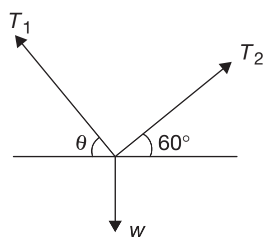
$$\cos\alpha = -\frac{1}{2},\ \alpha = 120°.$$

Example 2

A weight 'w' is supported by two cables. At what value of 'θ', the tension in the cable is minimum?



Solution



$$T_1\sin\theta + T_2\sin 60° = w$$

$$T_1\cos\theta = T_2\cos 60°$$

$$T_2 = \frac{T_1\cos\theta}{\cos 60°} = 2T_1\cos\theta$$

$$= T_1\sin\theta + 2T_1\cos\theta\cdot\sin 60° = w$$

$$= T_1\sin\theta + 2T_1\cos\theta\ .\ \sin 60° = w$$

$$T_1\sin\theta + \sqrt{3}\,T_1\cos\theta = w$$

$$\frac{dT_1}{d\theta} = 0 = T_1\cos\theta + \sqrt{3}T_1(-\sin\theta) = 0$$

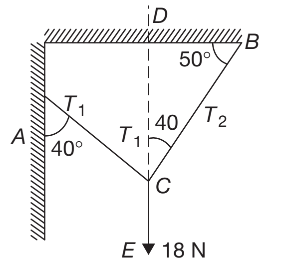
$$T_1\cos\theta = \sqrt{3}T_1\sin\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30°.$$

Example 3

An electric fixture weighing 18 N hangs from a point C by two strings AC and BC as shown in the following figure. The string AC is inclined to the vertical wall at 40° and BC is inclined to the horizontal ceiling at 50°. Determine the forces in the strings.



Solution

It can be deducted that $\angle DCA = 40$ and $\angle BCD = 40°$, so that $\angle ACB = 80°$.

$$\angle ACE = 180° - 40° = 140°$$

$$\angle BCE = 180° - 40° = 140°$$

Using sine rule:

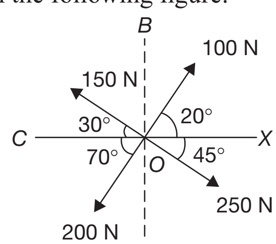
$$\frac{T_1}{\sin 40} = \frac{T_2}{\sin 40} = \frac{18}{\sin 80}$$

$$T_1 = \frac{18 \times \sin 40}{\sin 80} = 11.75 \text{ N}$$

and, $$T_2 = \frac{18 \times \sin 40}{\sin 80} = 11.75 \text{ N.}$$

Example 4

Determine the resultant of the coplanar concurrent force system shown in the following figure.



Solution

$$\Sigma F_x = 100 \cos 20 + 250 \cos 45 - 200 \cos 70 - 150 \cos 30$$
$$= 72.44 \text{ N}$$

$$\Sigma F_y = 100 \sin 20 - 250 \sin 45 - 200 \sin 70 + 150 \sin 30$$
$$= -255 \text{ N}$$

Resultant $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{72.44^2 + 255.5^2} = 265 \text{ N}$$

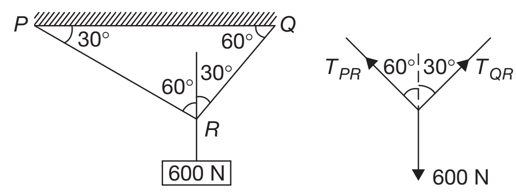
Its inclination $\alpha = \tan^{-1} \dfrac{\Sigma F_y}{\Sigma F_x}$

$$= \tan^{-1}\left(\frac{255.5}{72.44}\right) = 74°$$

Since Σy is negative, the angle falls in the fourth quadrant.

∴ Angle made with X-axis is 360° – 74° = 286° (Counter-clockwise).

Direction for solved examples 5 and 6:



Example 5

The tension in the wire QR will be:

(A) 519.6 N (B) 625 N
(C) 630 N (D) 735 N

Solution

$$\frac{T_{QR}}{\sin(180-60)} = \frac{T_{PR}}{\sin(180-30)} = \frac{600}{\sin 90}$$

$$\frac{T_{QR}}{\sin 60} = \frac{T_{PR}}{\sin 30} = \frac{600}{\sin 90}$$

The tension in the wire QR,

$$TQR = 300\sqrt{3} = 519.6 \text{ N}$$

Hence, the correct answer is option (A).

Example 6

The tension in the wire 'PR' will be

(A) 575 N (B) 300 N
(C) 275 N (D) 400 N

Solution

The tension in the wire 'PR',

$T_{PR} = 600 \sin 30 = 300$ N

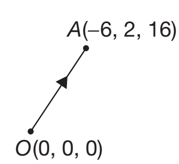
Hence, the correct answer is option (B).

Example 7

A point is located at (–6, 2, 16) with respect to the origin (0, 0, 0). Specify its position.

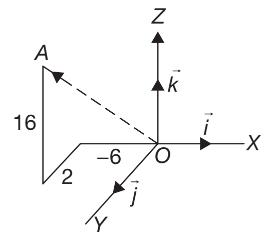
(i) In terms of the orthogonal components.
(ii) In terms of the direction cosines.
(iii) In terms of its unit vector.

Solution



The components of the vector OA are:

(–6 – 0) = –6 along the X-axis
(2 – 0) = 2 along the Y-axis
(16 – 0) = 16 along the Z-axis



Position vector, $r = -6i + 2j + 16k$

(In terms of the orthogonal components.)

Magnitude, $r = \sqrt{(-6)^2 + 2^2 + 16^2} = 17.2$

Direction cosines are:

$$l = \cos\theta_x = -\frac{6}{17.2} = -0.3488$$

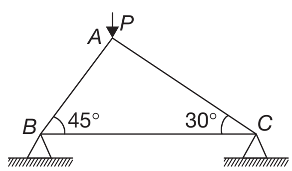
$$m = \cos\theta_y = \frac{2}{17.2} = 0.1163$$

$$n = \cos\theta_z = \frac{16}{17.2} = 0.9302$$

$r = (17.2l)i + (17.2m)j + (17.2n)$ (In terms of the direction of cosines.)

Example 8

Consider a truss ABC with a force 'P' at A as shown in the following figure.



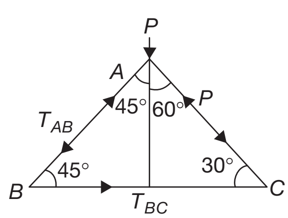
The tension in member CB is:

(A) $0.5P$ (B) $0.63P$

(C) $0.073P$ (D) $0.87P$

Solution

Consider point A. For equilibrium, resolving the forces:



$T_{AB}\cos 45° + T_{AC}\cos 60 - P = 0$

$T_{AB}\sin 45 = T_{AC}\sin 60$ solving.

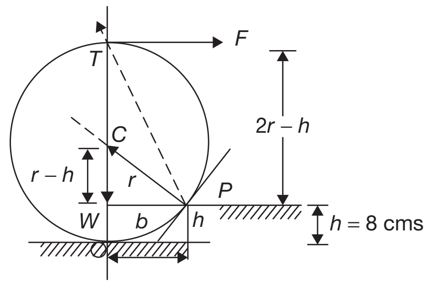
Hence, $T_{AB} = \dfrac{2\sqrt{3}P}{(\sqrt{6}+\sqrt{2})}$

Balancing of forces at point B gives $T_{AB}\cos 45 = T_{BC}$

$$T_{BC} = \left(\frac{\sqrt{6}}{\sqrt{6}+\sqrt{2}}\right)P = 0.633P.$$

Example 9

A man sitting in a wheelchair tries to roll up a step of 8 cm. The diameter of the wheel is 50 cm. The wheelchair together with the man weighs 1500 N. What force he will have to apply on the periphery of the wheel?



Solution

The free-body diagram (given above) shows the horizontal force applied by the man, the weight W acting at the centre of the wheel and the reaction R at the point P. (The reaction at O will be zero at the instant the wheel being lifted up).

From the geometry:

$$b^2 = r^2 - (r-h)^2 = 2rh - h^2$$
$$= 2 \times 0.25 \times 0.08 - (0.08)^2$$

$\therefore\ b = 0.1833$ m

Taking moments about the point P,

$$-F(2r-h) + wb = 0$$

$$F = \frac{Wb}{2r-h}$$

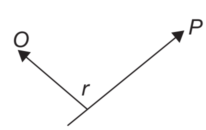
Where W = Load on one wheel $= \dfrac{1500}{2}$.

F = Force applied on one wheel.

$$\therefore\ F = \frac{\frac{1500}{2}\times 0.1833}{2\times 0.25 - 0.08} = \frac{137.475}{0.42} = 327.32 \text{ N.}$$

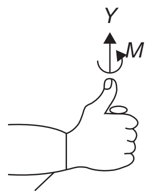
Moment of a Force

The product of a force and the perpendicular distance of the line of action of the force from a point or axis are defined as the moment of the force about that point or axis.

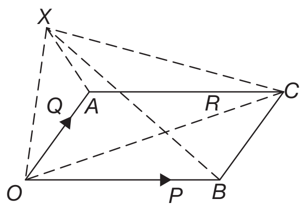


In the figure, the moment of force P about the point O or about the Y-axis is $P \times r$. Moment may be either clockwise or anti-clockwise. In the given figure, moment tends to rotate the body in anti-clockwise direction.

The right hand rule is a convenient tool to identify the direction of a moment. The moment M about an axis may be represented as a vector pointing towards the direction of the thumb of the right hand, while the other fingers show the direction of turn, the force offers about the axis (clockwise or anti-clockwise).



Varignon's Theorem of Moments



It can be proved that $\Delta OXA + \Delta OXB = \Delta OXC$. This illustrates Varignon's theorem of moments.

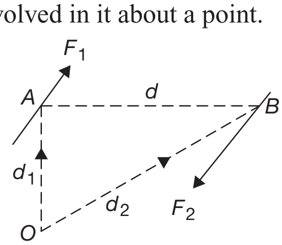
Moment of the force Q about X = Twice the area of ΔOXA. Moment of force P about X = Twice the area of ΔOXB. R is the resultant of P and Q. Moment of the resultant about X is = Twice the area of the triangle OXC.

The theorem may be stated as follows: The moment of a force about any point is equal to the sum of the moments of components about the same point.

Moment of a Couple

Two parallel forces having the same magnitude and acting in the opposite directions form a couple.

Moment of the couple is the algebraic sum of the moment of the forces involved in it about a point.



Moment of the force '$\vec{F_1}$' about $O = \overrightarrow{OA} \times \vec{F_1}$.

Moment of the force '$\vec{F_2}$' about $O = \overrightarrow{OB} \times (-\vec{F_2})$
$= -\overrightarrow{OB} \times \vec{F_2}$.

Algebraic sum of the moments,

$$\vec{M} = \overrightarrow{OA} \times \vec{F_1} - \overrightarrow{OB} \times F_2$$

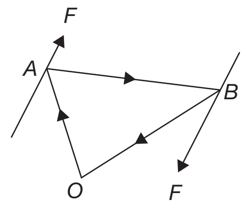
$$F_1 = F_2 = F$$

$$= (\overrightarrow{OA} - \overrightarrow{OB}) \times F \quad (1)$$

But $\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BO} = 0$

$\overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{AB}$

$\therefore$ Eq. (1), Becomes, $\vec{M} = \overrightarrow{AB} \times \vec{F}$.



The resultant force is zero, but the displacement 'd' of the force the couple creates a couple moment. Moment about some arbitrary point is 0.

$M = F_1d_1 + F_2d_2 = F_1d_1\ F_1d_2 = F_1(d_1 - d_2)$. If point O is placed in the line of action of force F_2 (or F_1), then $M = F_1d$ (or F_2d).

Orthogonal components (scalar components) of force 'F' along the rectangular axis, x, y and z axis are F_x, F_y and F_z, respectively.

$F_x = |F| \cos\theta_x$, $F_y = |F| \cos\theta_y$, $F_z = |F| \cos\theta_z$, where $\cos\theta_x$ (z_l), $\cos\theta_y (z_m)$ and $\cos\theta_z\ (z_n)$ are the direction cosines of the force 'F' and $|F|$ is the magnitude of the force 'F'.

$$|F| = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

$$F = F_x i + F_y j + F_z k = |F|(\cos\theta_n i + \cos\theta_y j + \cos\theta_z k)$$

$= |F|\ (l\hat{i} + m\hat{j} + n\hat{k})$, where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are vectors of unit length along the positive x, y and z directions.

Unit vector corresponding to the force vector F, $\hat{F} = \dfrac{F}{|F|}$.

If $\hat{n}$ is a unit vector in the direction of the force 'F', then $F = |F|\ \hat{n}$

Equilibrium of Force Systems

A body is said to be acted upon by a system of forces in equilibrium if the force system cannot change the body's stationary or constant velocity state. If the resultant is neither a force nor a couple. that is,

$$\Sigma F = 0 \quad (1)$$

$$\Sigma M = 0 \quad (2)$$

$\Sigma F \rightarrow$ Vector sum of all forces of the system

$\Sigma M \rightarrow$ Vector sum of the moments (relative to any point) of all the forces of the system.

Scalar equation equivalent to vector Eq. (1), in a rectangular coordinate system, are

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

Scalar equation equivalent to the vector Eq. (2), are

$$\Sigma M_x = 0$$
$$\Sigma M_y = 0$$
$$\Sigma M_z = 0$$

ΣF_x , ΣF_y and $\Sigma F_z \rightarrow$ Algebraic sum of forces in the x, y and z directions, respectively.

ΣM_x, ΣM_y and $\Sigma M_z \rightarrow$ Algebraic sum of moments in the x, y and z directions, respectively.

Equilibrium Equations for Different Coplanar Force Systems

1. **Concurrent coplanar force system:**

$$\Sigma F_x = 0, \Sigma F_y = 0$$

2. **Concurrent non-coplanar force system:**

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$

3. **Non-concurrent coplanar force system:**

$\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$ at any suitable point.

4. **Non-concurrent non-coplanar force system:**

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0 \text{ and } \Sigma M_x = 0, \Sigma M_y = 0, \Sigma M_z = 0$$

Analysis of a System of Forces in Space

A spatial force system may consist of a set of concurrent forces, parallel forces or non-concurrent non-parallel forces. The resultant of a spatial force system is a force 'R' and a couple C, where:

$$\boxed{R = \Sigma(\text{forces}) \text{ and } C = \Sigma(\text{moments})}$$

Concurrent Spatial Force System

Resultant R is given by

$$R = \boxed{\sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}}$$

with the direction cosines given by

$$\cos\theta_x = \frac{\Sigma F_x}{R}$$

$$\cos\theta_y = \frac{\Sigma F_y}{R}$$

$$\cos\theta_z = \frac{\Sigma F_z}{R}$$

ΣF_x, ΣF_y and ΣF_z are algebraic sums of the components of all the forces in the x, y, and z directions and θ_x, θ_y, θ_z are the angles which the resultant vector R makes with the x, y, and z axes, respectively.

Parallel Spatial Force System

The resultant,

$$\boxed{\begin{aligned} &R = \Sigma F_1 R_x = \Sigma M_x \\ &R_z = \Sigma M_z \end{aligned}}$$

where x and z are the perpendicular distances of the resultant vector from the xy and yz plane, respectively. ΣM_z, ΣM_x are the algebraic sums of the moments of forces of the force system about the x and z axes, respectively.

If $\Sigma F = 0$, the resultant couple can be evaluated as,

$$\boxed{C = \sqrt{(\Sigma M_x)^2 + (\Sigma M_z)^2}}$$

$$\boxed{\tan\phi = \frac{\Sigma M_z}{\Sigma M_x}}$$

Where ϕ is the angle made by the couple.

Non-concurrent, Non-parallel Force System

The resultant, $\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}}$ and corresponding direction cosines are:

$$\cos\theta_x = \frac{\Sigma F_x}{R}, \cos\theta_y = \frac{\Sigma F_y}{R}$$

$$\cos\theta_z = \frac{\Sigma F_z}{R}$$

$\boxed{C = \sqrt{(\Sigma M_x)^2 + (\Sigma M_y)^2 + (\Sigma M_z)^2}}$ and the corresponding direction cosines are:

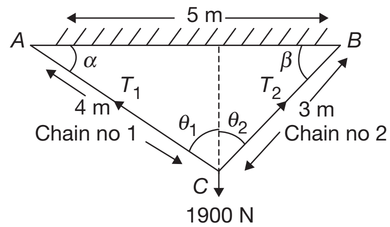
$$\boxed{\cos\theta_x = \Sigma\frac{M_x}{C}, \cos\theta_y = \Sigma\frac{M_y}{C}}$$

$$\boxed{\cos\theta_z = \frac{\Sigma M_z}{C_1}}$$

where θ_x, θ_y, and θ_z are the angles which the vector representing the couple C makes with the x, y and z axes, respectively.

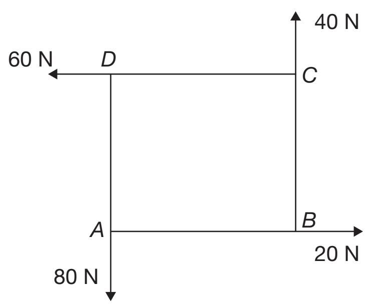
EXERCISES

1. A weight of 1900 N is supported by two chains of lengths of 4 m and 3 m as shown in figure. Determine the tension in each chain.



(A) 1200 N, 1300 N (B) 1100 N, 100 N
(C) 1100 N, 1200 N (D) 1520 N, 1140 N

2. Four forces of magnitudes 20 N, 40 N, 60 N and 80 N are acting respectively along the four sides of a square *ABCD* as shown in figure. Determine magnitude of resultant.



(A) $40\sqrt{2}$ N (B) $50\sqrt{2}$ N
(C) $45\sqrt{2}$ N (D) $60\sqrt{2}$ N

3. Match the following:

	List I		List II
a.	Two parallel forces acting on a body moving with uniform velocity	1.	Collision
b.	A moving particle	2.	Forces in equilibrium
c.	Two coplanar forces equal in magnitude but opposite in direction	3.	Kinetic energy
d.	Co-efficient of restitution	4.	Couple

Codes:

	a	b	c	d		a	b	c	d
(A)	4	3	2	1	(B)	1	2	3	4
(C)	2	3	4	1	(D)	None of these			

4. If the resulting torque act on a system is zero, then
(A) linear momentum is conserved.
(B) angular momentum is conserved.
(C) both momentums are conserved.
(D) None of these

5. The velocity-time graph of a body is passing through the velocity axis with intercept of 4. If the slope of the graph is 3, the distance travelled by the body in 6 seconds would be
(A) 40 m (B) 60 m
(C) 78 m (D) 80 m

6. Match the following:

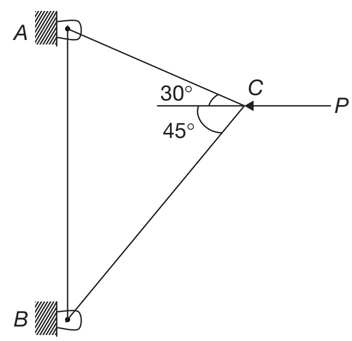
	List I		List II
a.	Two parallel forces acting on a body moving with uniform velocity	1.	Collision
b.	A moving particle	2.	Forces in equilibrium
c.	Two coplanar forces equal in magnitude but opposite in direction	3.	Kinetic energy
d.	Co-efficient of restitution	4.	Couple

Codes:

	a	b	c	d		a	b	c	d
(A)	4	3	2	1	(B)	1	2	3	4
(C)	2	3	4	1	(D)	None of these			

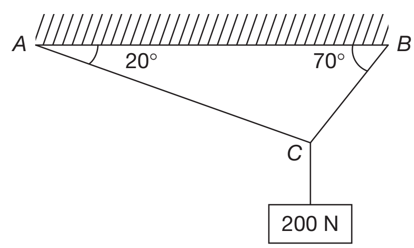
7. Two forces form a couple only when
(A) magnitude is same have parallel lines of action and same sense.
(B) magnitude is different, have parallel lines of action but same sense.
(C) magnitude is same have non parallel lines of action but same sense.
(D) magnitude is same and have parallel lines of action and opposite sense.

8.



Two steel truss members *AC* and *BC* with cross section area 100 mm^2 is subjected to a horizontal force *P* kN as shown in figure. Maximum value of *P* such that axial stress in any of the members does not exceed 50 MPa is
(A) 10.15 kN (B) 9.22 kN
(C) 7.92 kN (D) 6.83 kN

9.

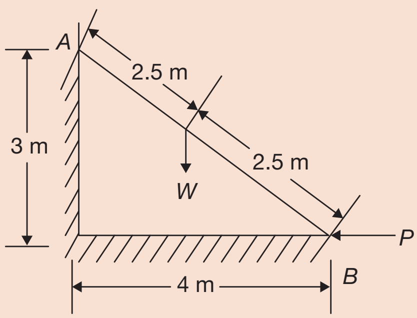


A weight of 200 N is hung using a cable as shown in the figure. Tensions in portions of cable AC and BC are respectively

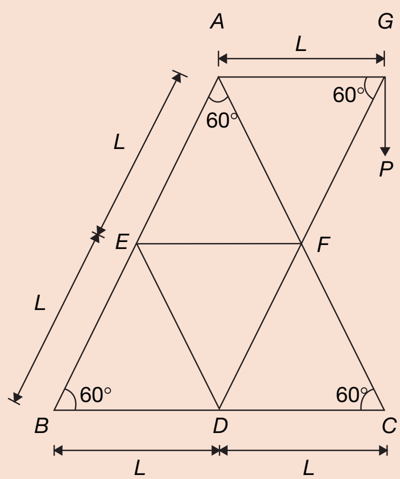
(A) 59.6, 171.7 N (B) 62.4, 176.8 N
(C) 62.5, 182.7 N (D) 68.4, 187.9 N

Previous Years' Questions

1. A ladder AB of length 5 m and weight (W) 600 N is resting against a wall. Assuming frictionless contact at the floor (B) and the wall (A), the magnitude of the force P (in newton) required to maintain equilibrium of the ladder is ____. **[GATE, 2014]**



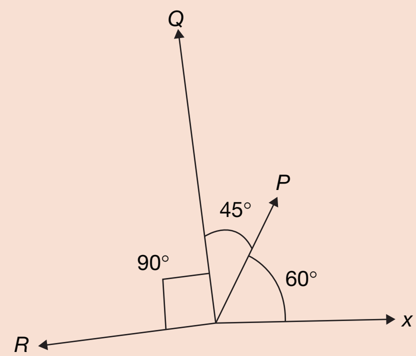
2. Consider the plane truss with load P as shown in the figure. Let the horizontal and vertical reactions at the joint B be H_B and V_B, respectively and V_C be the vertical reaction at the joint C. **[GATE, 2016]**



Which one of the following sets gives the correct values of V_B, H_B and V_C?

(A) $V_B = 0$; $H_B = 0$; $V_C = P$
(B) $V_B = P/2$; $H_B = 0$; $V_C = P/2$
(C) $V_B = P/2$; $H_B = P$ (sin 60°); $V_C = P/2$
(D) $V_B = P$; $H_B = P$ (cos 60°); $V_C = 0$

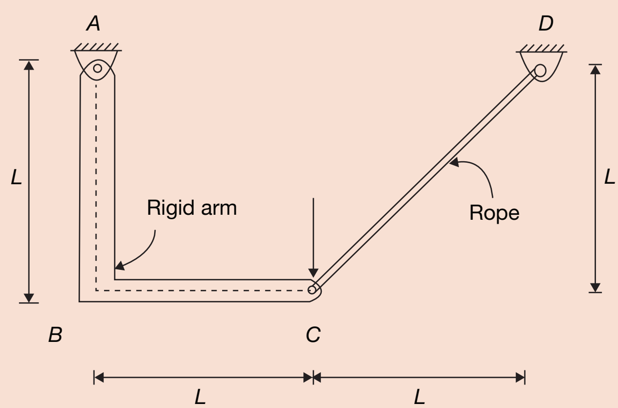
3. The magnitudes of vectors P, Q and R are 100 kN, 250 kN and 150 kN, respectively as shown in the figure. **[GATE, 2016]**



The respective values of the magnitude (in kN) and the direction (with respect to the X-axis) of the resultant vector are

(A) 290.9 and 96.0°
(B) 368.1 and 94.7°
(C) 330.4 and 118.9°
(D) 400.1 and 113.5°

4. An assembly made of a rigid arm A-B-C hinged at end A and supported by an elastic rope C-D at end C is shown in the figure. The members may be assumed to be weightless and the lengths of the respective members are as shown in the figure. **[GATE, 2016]**



Under the action of a concentrated load P and C as shown, the magnitude to tension developed in the rope is

(A) $\frac{3P}{\sqrt{2}}$ (B) $\frac{P}{\sqrt{2}}$

(C) $\frac{3P}{8}$ (D) $\sqrt{2}P$

Answer Keys

Exercises

1. D **2.** A **3.** C **4.** B **5.** C **6.** C **7.** D **8.** D **9.** D

Previous Years' Questions

1. 399 to 401 **2.** A **3.** C **4.** B