CIRCLE

PRACTICE SHEET

1. An equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$ with one of the vertices at (a,0). What is the equation of the side opposite to this vertex?
(a) $2x - a = 0$ (b) $x + a = 0$
(c) $2x + a = 0$ (d) $3x - 2a = 0$

2. What is the radius of the circle passing through the points (0, 0) (a, 0) and (0, b)?
(a) $\sqrt{a^2 - b^2}$ (b) $\sqrt{a^2 + b^2}$
(c) $\frac{1}{2}\sqrt{a^2 + b^2}$ (d) $2\sqrt{a^2 + b^2}$

3. If two circles A, B of equal radii pass through the centres of each other, then what is the ratio of the length of the smaller arc to the circumference of the circle A cut off by the circle B?
(a) 1/2 (b) 1/4
(c) 1/3 (d) 2/3

4. What is the equation of a circle, whose centre lies on the x – axis at a distance h from the origin and the circle passes through the origin?
(a) $x^2 + y^2 - 2hx = 0$
(b) $x^2 + y^2 - 2hx + h^2 = 0$
(c) $x^2 + y^2 + 2hxy = 0$
(d) $x^2 + y^2 - h^2 = 0$

5. Consider a circle of radius R. What is the length of a chord which subtends an angle θ at the centre?
(a) $2R\sin\left(\frac{\theta}{2}\right)$ (b) $2R\sin\theta$
(c) $2R\tan\left(\frac{\theta}{2}\right)$ (d) $2R\tan\theta$

6. What is the equation of circle which touches the lines x = 0?
(a) $x^2 + y^2 + 2x + 2y + 1 = 0$
(b) $x^2 + y^2 - 4x - 4y + 1 = 0$
(c) $x^2 + y^2 - 2x - 2y + 1 = 0$
(d) None of these

7. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ $(c > 0)$ touches the y – axis, then which one of the following is correct?
(a) $g = -\sqrt{c}$ only (b) $g = \pm\sqrt{c}$
(c) $f = \sqrt{c}$ only (d) $f = \pm\sqrt{c}$

8. The circle $x^2 + y^2 + 4x - 4y + 4 = 0$ touches.
(a) Only the X-axis (b) Only the Y-axis
(c) Both the axis (d) Neither of the axis

9. The equation of the circle which touches the axes at a distance 5 from the origin is $y^2 + x^2 - 2\alpha x - 2\alpha y + \alpha^2 = 0$. What is the value of α?
(a) 4 (b) 5
(c) 6 (d) 7

10. The equation of the circle passing through the origin is $x^2 + y^2 - 6x + 2y = 0$. The equation of one of its diameters is:
(a) $x + 3y = 0$ (b) $x + y = 0$
(c) Both (a) and (b) (d) $3x + y = 0$

11. What is the equation to circle which touches both the axes and has centre on the line x+y = 4?
(a) $x^2+y^2 - 4x + 4y + 4 = 0$ (b) $x^2+y^2 - 4x - 4y + 4 = 0$
(c) $x^2+y^2 + 4x - 4y - 4 = 0$ (d) $x^2+y^2 + 4x + 4y - 4 = 0$

12. Consider the following statements in respect of circles $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 1$.
I. The radius of the first circle is twice that of the second circle.
II.Both the circles pass through the origin.
Which of the above statements is/are correct?
(a) Only 1 (b) Only II
(c) Both I and II (d) Neither I nor II

ANSWER KEY

1.	c	**2.**	c	**3.**	c	**4.**	a	**5.**	a	**6.**	c	**7.**	d	**8.**	c	**9.**	b	**10.**	a
11.	b	**12.**	d																

Solutions

Sol. 1. (c)
Since the equilateral triangle is inscribed in the circle with centre at the origin, centroid lies on the origin.

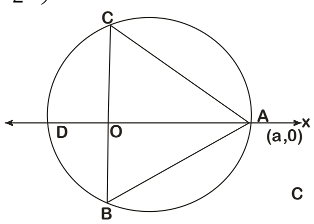
So, $\frac{AO}{OD} = \frac{2}{1}$

and $OD = \frac{1}{2}AO = \frac{a}{2}$

So, other verticles of triangle have coordinates,

$\left(-\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ and $\left[-\frac{a}{2}, \frac{\sqrt{3}}{2}a\right]$

$\left(-\frac{a}{2}\frac{\sqrt{3}a}{2}\right)$y



$\left(-\frac{a}{2}, \frac{-\sqrt{3}a}{2}\right)$

$\therefore$ Equation of line BC is:

$x = -\frac{a}{2}$

$\Rightarrow 2x + a = 0$

Sol. 2. (c)
Let (h, k) be the centre of the circle.
Since, circle is passing through (0,0), (a, 0) and (0, b), distance between centre and these points would be same and equal to radius.
Hence, $h^2 + k^2 = (h - a)^2 + k^2 = h^2 + (k - b)^2$

$\Rightarrow h^2 + k^2 = h^2 + k^2 + a^2 - 2ah = h^2 + k^2 + b^2 - 2bk$

$\Rightarrow h^2 + k^2 = h^2 + k^2 + a^2 - 2ah$

$\Rightarrow h = \frac{a}{2}$

Similarly, k = b/2

$\therefore$ Radius of circle =

$\sqrt{h^2 + k^2} = \frac{1}{2}\sqrt{a^2 + b^2}$

Sol. 3. (c)
When two circles A and B of equal radii pass through the centres of each other, the angle made by arc of B at the centre of B is 90°

So, length of small arc of $B = \frac{2\pi r 90°}{360°} = \frac{\pi r}{2}$

Hence, circumference of A cut off by the circle $B = 2\pi r - \frac{\pi r}{2} = \frac{3\pi r}{2}$

$\therefore$ Required ratio = $\frac{\pi r/2}{2\pi r/2} = \frac{1}{3}$

Sol. 4. (a)
Centre of the circle is (h, 0) and circle passes through the origin. In the general equation of circle.

$x^2 + y^2 + 2gx + 2fx + c = 0$

g = – h ad f = 0

so, $x^2 + y^2 - 2hx + 0 + c$

$= x^2 + y^2 - 2hx + c = 0$

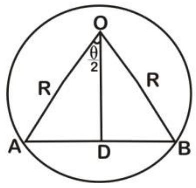
Since circle passes through origin (0, 0)

$0 + 0 - 0 + c = 0 \Rightarrow c = 0$

and equation (i) radius to $x^2 + y^2 - 2hx = 0$

Sol. 5. (a)
Let there be a circle of radius R and chord AB.

OD ⊥ AB and AD = DB



and AD = 2AD

$\angle AOB = \theta$

$\Rightarrow \angle AOD = \frac{\theta}{2}$

In $\angle AOD = \frac{\theta}{2}$

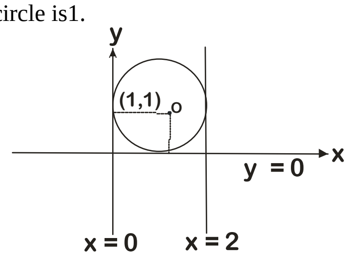
$\sin\frac{\theta}{2} = \frac{AD}{OA}$

$\sin\frac{\theta}{2} = \frac{AD}{R}$

AD = R sin θ/2

$\therefore$ length of chord AB = 2 AD = 2R sin θ□□

Sol. 6. (c)
Refer to the figure it is clear that coordinates of centre of circle are (1, 1) and diameter of circle = 2 and hence radius of circle is1.



$\therefore$ Equation of circle with centre (1, 1) and radius = 1 is $(x - 1)^2 + (y - 1)^2 = 1$

$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = 1$

$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

Sol. 7. (d)
As given, the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches y – axis.

Then r + ± g and $g^2 + f^2 - c = g^2$

$\Rightarrow f^2 - c = 0 \Rightarrow f^2 = c \Rightarrow f \pm \sqrt{c}$

Sol. 8. (c)
$x^2 + y^2 + 4x - 4y + 4 = 0$

$(x+2)^2 + (y-2)^2 = 4$

$(x+2)^2 + (y-2)^2 = 2^2$

$[x-(-2)^2 + (y-2)^2] = 2^2$

$\therefore$ Centre = (–2,2)

and radius = 2

So, circle touch has both axes.

Sol. 9. (b)
Coordinates of the centre of given circle = (α,α)

$\therefore$ Distance between point O(a,a) and A(5,0) = Radius and radius =

$\sqrt{(\alpha)^2 + (\alpha)^2 - \alpha^2} = \sqrt{\alpha^2} = \alpha$

$\therefore (\alpha - 5)^2 + (\alpha)^2 = (\alpha)^2$

$\Rightarrow \alpha^2 + 25 - 10\alpha = 0$

$\Rightarrow (\alpha - 5)^2 = 0 \Rightarrow \alpha = 5$

Sol. 10. (a)
The required line passes through centre (3,–1) and the origin (0,0)

Sol. 11. (b)
We know that the equation of circle, which touches both the axes, is

$x^2 + y^2 - 2rx - 2ry + r^2 = 0$ …(i)

The centre (r,r) of this circle lies on the line x + y = 4

$\therefore r + r = 4 \Rightarrow r = 2$

On putting the value of r in Eq. (i), we get

$x^2 + y^2 - 4x - 4y + 4 = 0$

which is required equation of circle.

Sol. 12. (d)
The equation of first circle is $x^2 + y^2 - 2x - 2y = 0$

Radius of this circle = $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

and equation of second circle is $x^2 + y^2 = 1$

Radius of this circle =1

From above, it is clear that the radius of first circle is no twice that of second circle.

Also, first circle passes through the origin while second circle does not pass through the origin.

Hence, neither I nor II statement is correct.

NDA PYQ

1. For the equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, where $a \neq 0$, to represent a circle, the condition will be:
(a) a=b and c=0 (b) f=g and h=0
(c) a=b and h=0 (d) f=g and c=0
[NDA-2011(1)]

2. What is the radius of the circle touching X-axis at (3,0) and Y-axis at (0,3)?
(a) 3units (b) 4 units
(c) 5units (d) 6 units
[NDA (II) - 2011]

3. What are the points on the curve $x^2+y^2-2x-3=0$ where the tangents are parallel to x-axis?
(a) (1,2) and (1,–2) (b) $(0,\sqrt{3})$ and $(0, -\sqrt{3})$
(c) (3,0) and (–3,0) (d) (2,1) and (2,–1)
[NDA-2011(2)]

4. Which one of the following points lies inside a circle of radius 6 and centre at (3,5)?
(a) (–2, –1) (b) (0,1)
(c) (–1, –2) (d) (2, –1)
[NDA (I) - 2013]

5. The radius of the circle $x^2 + y^2 + x + c = 0$ passing through the origin is:
(a) 1/4 (b)1/2
(c) 1 (d) 2
[NDA (II) - 2013]

Direction for the next two (2) items that follow:
Consider the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$

6. What is the distance between the centres of the two circles?
(a) $\sqrt{a^2+b^2}$ (b) $a^2 + b^2$
(c) a + b (d) 2(a + b)
[NDA (II) - 2014]

7. The two circles touch each other, if:
(a) $c=\sqrt{a^2+b^2}$ (b) $\frac{1}{c}=\frac{1}{a^2}+\frac{1}{b^2}$
(c) $c=\frac{1}{a^2}+\frac{1}{b^2}$ (d) $c=\frac{1}{a^2+b^2}$
[NDA (II) - 2014]

8. A straight line x = y + 2 touches the circle. $4(x^2 + y^2) = r^2$. The value of r is:
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) 2 (d) 1
[NDA (II) - 2015]

9. If the centre of the circle passing through the origin is (3,4), then the intercepts cut off by the circle on x-axis and y-axis respectively are:
(a) 3 unit and 4 unit (b) 6 unit and 4 unit
(c) 3 unit and 8 unit (d) 6 unit and 8 unit
[NDA (II) - 2015]

Direction for the next two (2) items that follow:
Consider the two circles
$(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$

10. What is the distance between the centres of the two circles?
(a) 5 units (b) 6 units
(c) 8 units (d) 10 units
[NDA (I) - 2016]

11. If the circles intersect at two distinct points, then which one of the following is correct?
(a) r = 1 (b)1 < r < 2
(c) r = 2 (d) 2 < r < 8
[NDA (I) - 2016]

12. If a circle of radius b units with centre of (0, b) touches the line $y = x - \sqrt{2}$, then what is the value of b?
(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$
(c) $2\sqrt{2}$ (d) $\sqrt{2}$
[NDA (I) - 2016]

Direction for the next two (2) items that follows:
Consider a circle passing through the origin and the points (a, b) and (–b, –a).

13. On which line does the centre of the circle lie?
(a) x + y = 0 (b) x – y = 0
(c) x + y = a + b (d) $x - y = a^2 - b^2$
[NDA (I) - 2016]

14. What is the sum of the squares of the intercepts cut off by the circle on the axis?
(a) $\left(\frac{a^2+b^2}{a^2-b^2}\right)^2$ (b) $2\left(\frac{a^2+b^2}{a-b}\right)^2$
(c) $4\left(\frac{a^2+b^2}{a-b}\right)^2$ (d) None of these
[NDA (I) - 2016]

15. What is the radius of the passing through the point (2, 4) and having centre at the intersection of the line x – y = 4 and 2x + 3y + 7 = 0?
(a) 3 units (b) 5 units
(c) $3\sqrt{3}$ units (d) $5\sqrt{2}$ units
[NDA (II) - 2016]

16. The two circle $x^2 + y^2 = r^2$ and $x^2 + y^2 - 10x + 16 = 0$ intersect at two distinct points. Then which one of the following is correct?
(a) 2 < r < 8 (b) r = 2 or r = 8
(c) r < 2 (d) r > 2
[NDA (I) - 2017]

17. What is the equation of the circle which passes through the points (3, –2) and (–2, 0) and having its centre on the line 2x + y – 3 = 0?
(a) $x^2 + y^2 + 3x + 2 = 0$ (b) $x^2 + y^2 + 3x + 12y + 2 = 0$
(c) $x^2 + y^2 + 2x = 0$ (d) $x^2 + y^2 = 5$
[NDA (I) - 2017]

18. The equation of the circle which passes through the points (1,0), (0, –6) and (3, 4) is
(a) $4x^2 + 4y^2 + 142x + 47y + 140 = 0$
(b) $4x^2 + 4y^2 - 142x - 47y + 138 = 0$
(c) $4x^2 + 4y^2 - 142x + 47y + 138 = 0$
(d) $4x^2 + 4y^2 + 150x - 49y + 138 = 0$
[NDA (II) - 2017]

19. If y-axis touches the circle $x^2 + y^2 + gx + fy + \frac{c}{4}=0$, then the normal at this point intersects the circle at the point.

(a) $\left(-\frac{g}{2}, -\frac{f}{2}\right)$ (b) $\left(-g, -\frac{f}{2}\right)$

(c) $\left(-\frac{g}{2}, f\right)$ (d) $(-g, -f)$

[NDA (II) - 2018]

20. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is

(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$

(b) $(x - x_1)^2 + (y - y_1)^2 = x_2 + y_2$

(c) $x^2 + y^2 + 2x_1x_2 + 2y_1y_2 = 0$

(d) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

[NDA (II) - 2018]

21. A circle is drawn on the chord of a circle $x^2 + y^2 = a^2$ as diameter. The chord lies on the line $x + y = a$. What is the equation of the circle?

(a) $x^2 + y^2 - ax - ay + a^2 = 0$

(b) $x^2 + y^2 - ax - ay = 0$

(c) $x^2 + y^2 + ax + ay = 0$

(d) $x^2 + y^2 + ax + ay - 2a^2 = 0$

[NDA-2019(1)]

22. The circle $x^2 + y^2 + 4x - 7y + 12 = 0$, cuts an intercept on y-axis equal to

(a) 1 (b) 3

(c) 4 (d) 7

[NDA (I) - 2019]

23. The centre of the circle

$(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$ is

(a) (2a, 2c) (b) (2b,2d)

(c) (a + b, c + d) (d) (a−b,c−d)

[NDA 2020]

24. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?

(a) 14 units (b) 10.5 units

(c) 7 units (d) 3.5 units

[NDA (I) 2021]

Directions: Consider the following for the next two (02) items that follow:

A circle is passing through the points (5, –8), (–2, 9) and (2, 1)

25. What are the coordinates of the centre of the circle?

(a) (–2, –50) (b) (–50, –20)

(c) (–24, –58) (d) (–58, –24)

[NDA (II) 2021]

26. If r is the radius of the circle, then which one of the following is correct?

(a) $r < 10$ (b) $10 < r < 30$

(c) $30 < r < 60$ (d) $r > 60$

[NDA (II) 2021]

27. What is the equation of the circle which touches both the axes in the first quadrant and the line $y - 2 = 0$?

(a) $x^2 + y^2 - 2x - 2y - 1 = 0$

(b) $x^2 + y^2 + 2x + 2y + 1 = 0$

(c) $x^2 + y^2 - 2x - 2y + 1 = 0$

(d) $x^2 + y^2 - 4x - 4y + 4 = 0$

[NDA (I) 2022]

28. The centre of the circle passing through origin and making positive intercepts 4 and 6 on the coordinate axes, lies on the line:

(a) $2x - y + 1 = 0$ (b) $3x - 2y - 1 = 0$

(c) $3x - 4y + 6 = 0$ (d) $2x + 3y - 26 = 0$

[NDA 2022 (II)]

29. If $3x + y - 5 = 0$ is the equation of a chord of the circle $x^2 + y^2 - 25 = 0$, then what are the coordinates of the mid point of the chord?

(a) $\left(\frac{3}{4}, \frac{1}{4}\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{3}{4}, -\frac{1}{4}\right)$ (d) $\left(\frac{3}{2}, -\frac{1}{2}\right)$

[NDA-2023 (2)]

30. Consider the following statements of the circle $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$

1.The circle touches both the axes

2.The diameter of the circle is 2a

3. The centre of the circle lies on the line $x + y = a$

How many of the statements given above are correct?

(a) only one (b) only two

(c) all three (d) none

[NDA-2023 (2)]

31. If (a, b) is the centre and c is the radius of the circle $x^2 + y^2 + 2x + 6y + 1 = 0$, then what is the value of $a^2 + b^2 + c^2$?

(a) 19 (b) 18

(c) 17 (d) 11

[NDA-2024 (1)]

Direction: Consider the following for the **two items** given below

A triangle ABC is inscribed in the circle $x^2 + y^2 = 100$. B and C have coordinates (6,8) and (–8, 6) respectively.

32. What is ∠BAC equal to ?

(a) $\pi/2$ (b) $\pi/3$ or $2\pi/3$

(c) $\pi/4$ or $3\pi/4$ (d) $\pi/6$ or $5\pi/6$

[NDA-2024 (2)]

33. What are the coordinates of A ?

(a) (–6, 8) (b) (–6, –8)

(c) $(5\sqrt{2}, 5\sqrt{2})$ (d) Can't determined

[NDA-2024 (2)]

34. The equation of the circle is $(x^2 - 4x + 3) + (y^2 - 6y + 8) = 0$

Which of the following statements are correct ?

I. The end points of a diameter of the circle are at (1,2) and (3,4).

II. The end points of a diameter of the circle are at (1,4) and (3,2).

III. The end points of a diameter of the circle are at (2,4) and (4,2).

Select the answer using the code given below.

(a) I and II only (b) II and III only

(c) I and III only (d) I, II and III

[NDA-2024 (2)]

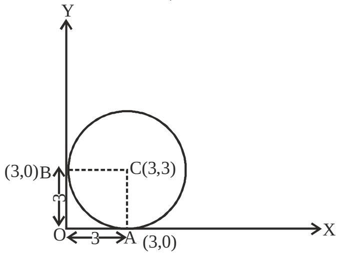
ANSWER KEY

1.	c	**2.**	a	**3.**	a	**4.**	b	**5.**	b	**6.**	a	**7.**	b	**8.**	b	**9.**	d	**10.**	a
11.	d	**12.**	a	**13.**	a	**14.**	b	**15.**	d	**16.**	a	**17.**	b	**18.**	c	**19.**	b	**20.**	d
21.	b	**22**	a	**23.**	c	**24.**	d	**25.**	d	**26.**	d	**27.**	c	**28.**	c	**29.**	b	**30.**	b
31.	a	**32.**	c	**33.**	d	**34.**	a												

Solutions

Sol. 1. (c)
Homogeneous equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, where $a \neq 0$, represents a circle when a = b and h = 0

Sol. 2. (a)
Radius of the circle, AC = BC



AC = OB = 3, and BC = OA = 3
$\therefore$ Radius = 3 units

Sol. 3. (a)
Centre of circle is at (1,0)
Put x = 1
$1 + y^2 - 2 - 3 = 0$
$y^2 = 4$
y = 2, - 2
(1,2) and (1,-2) are the end point of diameter that is parallel to Y axis.
At these points tangent will be parallel to x axis

Sol. 4. (b)
The equation of the circle of radius 6 and centre at (3,5) is $(x-3)^2 + (y-5)^2 = (6)^2$
Let $S \equiv (x-3)^2 + (y-5)^2 - 36 = 0$
At point (−2. −1),
$S \equiv (-2,-3)^2 + (-1,-5)^2 - 36 = 25 + 36 - 36 > 0$
Which represents outside the circle
At point (0,1)
$S \equiv (0-3)^2 + (1-5)^2 - 36 = 9 + 16 - 36 = -9 < 0$
Which represents inside the circle
At point (1, −2)
$S \equiv (-1-3)^2 + (-2-5)^2 - 36 \equiv 16 + 49 - 36 = 29 > 0$
Which represents outside the circle
At point (2, −1),
$S \equiv (2, -3)^2 + (-1-5)^2 - 36 = 1 + 36 - 36 = 1 > 0$
Which represent outside the circle
Hence, point (0,1) lies inside the circle S.

Sol. 5. (b)
Given equation of circle is
$x^2 + y^2 + x + c = 0$ …(i)
Since, the equation of circle passes through the origin.
$\therefore (0)^2 + (0)^2 + 0 + c = 0$
$\Rightarrow c = 0$
From Eq. (i),
$x^2 + y^2 + x = 0$
$\Rightarrow x^2 + y^2 + x + \frac{1}{4} = \frac{1}{4}$
$\Rightarrow \left(x + \frac{1}{2}\right)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$
So, the required radius of circle is 1/2

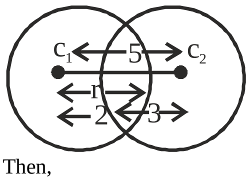
Sol. 6. (a)
Equations of circle are
$x^2 + y^2 + 2ax + c = 0$
and $x^2 + y^2 + 2by + c = 0$
Since, the centre of two circle are (−a,0) and (0, −b).
$\therefore$ Distance between two centre = $\sqrt{a^2 + b^2}$

Sol. 7. (b)
Two circles touch each other, if distance between two centre = sum of radius of two circles
$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$
On squaring both sides, we get
$a^2 + b^2 = a^2 - c + b^2 - c + 2\sqrt{(a^2 - c)(b^2 - c)}$
$\Rightarrow c = \sqrt{(a^2 - c)(b^2 - c)}$
Again, squaring both sides, we get
$c^2 = a^2 b^2 - a^2 c - b^2 c + c^2$
$\Rightarrow a^2 b^2 = (a^2 + b^2)c \Rightarrow \frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol. 8. (b)
y = x −2
$x^2 + y^2 = \left(\frac{r}{2}\right)^2$
for touches the circle
$a^2 (1+m^2) = c^2$
$\left(\frac{r}{2}\right)^2 (1+1) = 4$
$\frac{r^2}{4}.2 = 4$
$r^2 = 8$
$r = 2\sqrt{2}$

Sol. 9. (d)
If the centre of the circle passing through the origin, is (3,4), then the intercept cutoff on x-axis = 2 × 3 = 6 units and intercept cut off on y axis = 2 × 4 = 8 units.

Sol. 10. (a)
Circle $(x-1)^2 + (y-3)^2 = r^2$
…(i)
On comparing with
$(x-h)^2 + (y-k)^2 = r^2$
h= 1, k= 3 and $r_1 = r$



Then,
Radius of circle (i) $r_1 = r$
Centre of circle (i) = (h,k)
$c_1 = (1,3)$
Now,
$x^2 + y^2 - 8x + 2y + 8 = 0$ …(ii)
On comparing with
$x^2 + y^2 + 2gx + 2fy + c = 0$
g = −4, f = 1, c = 8
Radius of circle (ii)
$r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{15 + 1 - 8} = 3$
Centre point of circle (ii)
$c_2 = (-g, -f) = (4, -1)$
Distance between centre of circle
$c_1c_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(1-4)^2 + (3+1)^2}$
c_1c_2 = 5 units

Sol. 11. (d)
2 < r
2 < r < 8

Sol. 12. (a)
Radius for circle r = b unit
Centre of circle = (0, b)
Equation of line
$y = x - \sqrt{2}$
$\Rightarrow x - y - \sqrt{2} = 0$ …(i)
Let the given line touches the circle then radius of circle will be equal to the perpendicular to the line from the centre point. i.e.,
$\Rightarrow b = \frac{0 - b - \sqrt{2}}{\sqrt{1+1}}$
$\Rightarrow b = -\left(\frac{b + \sqrt{2}}{\sqrt{2}}\right)$
$\Rightarrow b = \frac{b + \sqrt{2}}{\sqrt{2}}$ (Neglect '−')
$\Rightarrow \sqrt{2b} - b = \sqrt{2}$
$b = \frac{\sqrt{2}}{\sqrt{2} - 1} \Rightarrow b = \frac{\sqrt{2}(\sqrt{2} + 1)}{1}$
$b = 2 + \sqrt{2}$

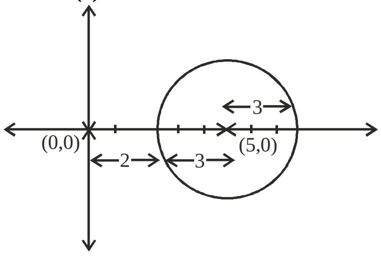
Sol. 13. (a)
Let the equation of circle
$x^2 + y^2 + 2gx + 2fy + c = 0$
This circle passing through the points
(0,0), (a,b), (−b, −a)
Then we get,
c = 0 …(i)
$a^2 + b^2 + 2ag + 2bf + c = 0$
$a^2 + b^2 + 2ag + 2bf = 0$ $\{\because c = 0\}$…(ii)
$b^2 + a^2 - 2bg - 2af = 0$ …(iii)
On solving equation (ii) and (iii)
$g = -\frac{(a^2 + b^2)}{2(a-b)}$
$f = \frac{a^2 + b^2}{2(a-b)}$
The centre of the circle = (−g, −f)
$= \left[\frac{a^2 + b^2}{2(a-b)}, \frac{(a^2 + b^2)}{2(a-b)}\right]$
The centre of the circle lies on the line x + y = 0

Sol. 14. (b)
Intercept cut on x axis = $2\sqrt{g^2 - c}$ = 2g $\{\because c = 0\}$
Intercept cut on y axis = 2f $\{\because c = 0\}$
The required value = $(2g)^2 + 2(f)^2$
$= \left[-\frac{(a^2 + b^2)}{a-b}\right]^2 + \left[\frac{a^2 + b^2}{a-b}\right]^2$

$$= \frac{\left(a^2+b^2\right)^2}{\left(a-b\right)^2} + \frac{\left(a^2+b^2\right)^2}{\left(a-b\right)^2} = 2\left[\frac{a^2+b^2}{a-b}\right]^2$$

Sol. 15.(d)
Given, point P(2,4)
$x - y = 4$ …(i)
and $2x + 3y + 7 = 0$ …(ii)
On solving equations (i) and (ii), we get $x = 1$ and $y = -3$
$\therefore$ centre of the circle is (1, –3)
$\therefore$ Radius of the circle is PO = $\sqrt{(2-1)^2 + (4+3)^2}$
$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$ units

Sol. 16.(a)

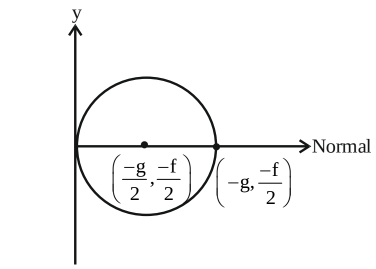


The two given equations of the circle are
$x^2 + y^2 = r^2$ …(i)
$\Rightarrow$Co-ordinates of the centre are (+5,0) and radius is 3. It is clear from the figure, that the two circles will intersect only if $2 < r < 8$

Sol. 17.(c)
Let the equation be $x^2 + y^2 + 2gx + 2fy + c = 0$
The circle passes through (1,0), (0, –6) and (3,4),
So, $2g + c = -1$ …(i)
$-12f + c = -36$ …(ii)
and $6g + 8f + c = -25$ …(iii)
Solving the above equations we get,
g= $\frac{-71}{4}$, f = $\frac{47}{8}$ and c = $\frac{69}{2}$
$\therefore$The required equation is:
$4x^2 + 4y^2 - 142x + 47y + 138 = 0$

Sol. 18. (b)
Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$
Then by the given condition,
$(3)^2 + (-2)^2 + 2 \times 3g + 2(-2)f + c = 0$
$6g - 4f + c = -13$ …(i)
and $(-2)^2 + (0)^2 + 2(-2)g + 0 + c = 0$
$\Rightarrow -4g + c = -4$ …(ii)
Since the centre of the circle lies on the line $2x - y - 3 = 0$
$\therefore -2g + f = 3$ …(iii)
Subtracting equation (ii) from equation (i),
$10g - 4f = -9$ …(iv)
Multiplying equation (iii) by 4 and then adding it to equation (iv)
$2g = +3 \Rightarrow g = +3/2$
$\therefore$ from equation (iii), f = 6
and from equation (ii), C = 2
$\therefore$The required equation is $x^2 + y^2 + 3x + 12y + 2 = 0$

Sol. 19.(b)



Sol. 20.(d)
Equation of circle is:
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Sol. 21.(b)
circle $x^2 + y^2 = a^2$ and line $x + y = a$ cut at (a,0) and (0,a)
by taking these points as diameter
circle is
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$(x - a)(x - 0) + (y - 0)(y - a) = 0$
$x^2 + y^2 - ax - ay = 0$

Sol. 22. (a)
Intercept on Y axis
$= 2\sqrt{f^2 - c} = 2\sqrt{\left(\frac{7}{2}\right)^2 - 12} = 1$

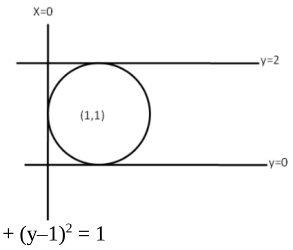
Sol. 23.(c)
given circle is
$(x-2a)(x-2b) + (y-2c)(y-2d) = 0$
this equation of circle is in diameter form
end points of diameter is (2a,2c) and (2b,2d)
centre will be at midpoint of diameter
so coordinates of centre will be
(a + b,c + d)

Sol. 24.(d)
circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?
$x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$
radius = $\sqrt{\left(\frac{-5}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \frac{15}{4}} = \frac{7}{2}$ units

Sol. 25.(d)
General ef of circle
$x^2 + y^2 + 2gx + 2fy + d = 0$
As (5, –8), (–2, 9) and (2, 1) passes through circle. So they will satisfy
$25 + 64 + 10g - 16f + d = 0$ …(1)
$4 + 84 - 4g + 18f + d = 0$ …(2)
$4 + 1 + 4g + 2f + d = 0$ …(3)
When we solve we get g = – 58
(–58, –24) f = – 24

Sol. 26.(d)
Radius $\sqrt{g^2 + f^2 - d}$
d = – 280
so, $\sqrt{(58)^2 + (24)^2 + 280} \approx 64 > 60$

Sol. 27.(c)



$(x-1)^2 + (y-1)^2 = 1$
$x^2 + y^2 - 2x - 2y + 1 = 0$

Sol. 28.(c)
General equation of circle
$x^2 + y^2 + 2gx + 2fy + c = 0$
Centre of the circle passing through origin so c = 0
Intercept on x axis = $2\sqrt{g^2 - c} = 4$
So g = 2 or – 2
Intercept on y axis = $2\sqrt{f^2 - c} = 6$
So f = 3 or – 3
Intercepts are positive i.e. centre will lie in 1st quadrant and value of g and f is negative because we know that coordinate of Centre are(-g,-f)
So coordinate of Centre (2,3)
That is lie on $3x - 4y + 6 = 0$

Sol. 29.(b)
If $3x + y - 5 = 0$ is the equation of a chord of the circle $x^2 + y^2 - 25 = 0$, then coordinates of end points of chord can be find by solving both equations
$3x + y - 5 = 0$ ……..(1)
$y = -3x + 5$
$x^2 + y^2 - 25 = 0$ ……(ii)
$x^2 + (-3x + 5)^2 - 25 = 0$
$10x^2 - 30x = 0$
x = 0,3
if x = 0 then y = 5 i.e. (0,5)
if x = 3 then y = - 4 i.e. (3,-4)
mid point of these points is $\left(\frac{3}{2}, \frac{1}{2}\right)$

Sol. 30.(b)
$4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$
$x^2 + y^2 - ax - ay + a^2/4 = 0$
Centre (a/2,a/2)
radius = a/2
coordinate of centre is same as radius so circle will touch both axes.
Diameter = a
Centre (a/2,a/2) lies on the line x + y = a
statement 1 and 3 is correct.

Sol. 31.(a)
circle $x^2 + y^2 + 2x + 6y + 1 = 0$
centre (–1, –3) and radius = $\sqrt{1+9-1} = 3$
a = –1, b = –3, c = 3
$a^2 + b^2 + c^2 = 1 + 9 + 9 = 19$

Sol. 32.(c)
centre of $x^2 + y^2 = 100$ is O(0,0)
slope of OB is 4/3 and OC is – 3/4 so OB and OC are perpendicular.
we know that angle subtended by an arc or a chord on centre is always double of angle subtended on circumference.
angle BOC is 90° than angle BAC = 45°(on major arc) and if A is on minor arc than angle BAC = 135°

Sol. 33.(d)
from given insufficient data we cannot determined exact position of vertex A.

Sol. 34.(a)
$(x^2 - 4x + 3) + (y^2 - 6y + 8) = 0$
$(x - 1)(x - 3) + (y - 2)(y - 4) = 0$
above equation is in diameter form so end point of diameter are (1,2) and (3,4) or (1,4) and (3,2)