2.1 CHAPTER

Surds, Indices & Logarithms

LAWS OF INDICES

(1) $a^m \times a^n = a^{m+n}$

Example:

$\rightarrow$ $2^4 \times 2^{13} = 2^{17}$

$\rightarrow$ $3^8 \times 3^7 = 3^{15}$

(2) $\dfrac{a^m}{a^n} = a^{m-n}$

Example:

(i) $\dfrac{7^5}{7^2} = 7^{5-2} = 7^3$

(ii) $\dfrac{27^5}{27^2} = 27^3$

(3) $(a^m)^n = a^{mn}$

Example:

(i) $(3^2)^4 = 3^{2\times4} = 3^8$

(ii) $(5^3)^9 = 5^{3\times9} = 5^{27}$

(4) $(a \times b)^n = a^n \times b^n$

Example:

(i) $(3\times5)^2 = 3^2 \times 5^2$

(ii) $(4\times5)^3 = 4^3 \times 5^3$

(5) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Example:

(i) $\left(\dfrac{3}{4}\right)^2 = \dfrac{3^2}{4^2} = \dfrac{9}{16}$

(ii) $\left(\dfrac{5}{8}\right)^3 = \dfrac{5^3}{8^3} = \dfrac{125}{512}$

(6) $a^0 = 1$

Here a may be any number

Example:

(i) $5^0 = 1$

(ii) $1000^0 = 1$

(7) $a^x = a^y$

If and only if Condition

Example:

(i) $x = y$ or

(ii) $x = 0,\ y = 0$

(8) $a^{-m} = \dfrac{1}{a^m}$

Example:

(i) $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

(ii) $5^{-3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

(9) $a^{-1} = \dfrac{1}{a}$

Example: $10^{-1} = \dfrac{1}{10}$

Laws of Surds

We write $\sqrt[n]{a} = a^{1/n}$ and it is called a surd of order 'n'

(1) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a$

(2) $\sqrt[n]{ab} = \sqrt[n]{a}\ .\ \sqrt[n]{b}$

Example:

(i) $\sqrt[3]{2\times5} = \sqrt[3]{2} \times \sqrt[3]{5}$

(ii) $\sqrt[5]{7\times13} = \sqrt[5]{7} \times \sqrt[5]{13}$

(3) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example:

(i) $\sqrt[3]{\dfrac{2}{5}} = \dfrac{\sqrt[3]{2}}{\sqrt[3]{5}}$

(ii) $\sqrt[5]{\dfrac{7}{13}} = \dfrac{\sqrt[5]{7}}{\sqrt[5]{13}}$

(4) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Example:

(i) $\left(\sqrt[3]{2}\right)^5 = \sqrt[3]{2^5} = \sqrt[3]{32}$

(ii) $\left(\sqrt[5]{4}\right)^3 = \sqrt[5]{4^3} = \sqrt[5]{64}$

(5) $\sqrt[m]{\sqrt[n]{\sqrt[p]{a}}} = \sqrt[mnp]{a} = a^{\frac{1}{mnp}}$

Example:

(i) $\sqrt[2]{\sqrt[3]{\sqrt[4]{10}}} = 10^{\frac{1}{2\times3\times4}} = 10^{\frac{1}{24}}$

(ii) $\sqrt[3]{\sqrt[5]{25}} = 25^{\frac{1}{3\times5}} = 25^{\frac{1}{15}}$

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Solved Examples

1. Find the value of $\left(\frac{32}{343}\right)^{-3/5}$

Sol.: $\left(\frac{32}{343}\right)^{-3/5} = \left(\frac{2^5}{3^5}\right)^{-3/5} = \left(\frac{2}{3}\right)^{5\times\frac{-3}{5}}$

$= \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{27}{8}$

2. Solve $4^{3x-2} \cdot 8 = 1$

Sol.: $4^{3x-2} \times 8 = \left(2^2\right)^{3x-2} \times 2^3 = 1$

$= 2^{6x-4+3} = 1$

$= 2^{6x-1} = 2^{\circ} \quad [\because 2^{\circ} = 1]$

$= 2^{6x-1} = 2^{\circ} \Rightarrow x = \frac{1}{6}$

3. Find the value of x if $\left(\frac{a}{b}\right)^{3x-5} = \left(\frac{b}{a}\right)^{x+3}$

Sol.: $= \left(\frac{a}{b}\right)^{3x-5} = \left(\left(\frac{a}{b}\right)^{-1}\right)^{x+3}$

$= \left(\frac{a}{b}\right)^{3x-5} = \left(\frac{a}{b}\right)^{-x-3}$

$= 3x - 5 = -x - 3$

$= 4x = 2 \Rightarrow x = \frac{1}{2}$

4. Which is greater $\sqrt[4]{4}$ or $\sqrt[5]{5}$

Sol.: $\sqrt[4]{4} = 4^{\frac{1}{4}}$ and $\sqrt[5]{5} = 5^{\frac{1}{5}}$

Now the LCM of 4 & 5 is 20, so to compare two surds raise the power by 20.

$\left(4^{1/4}\right)^{20} = 4^5 = 1024$

$\left(5^{1/5}\right)^{20} = 5^4 = 625$

Hence, $\sqrt[4]{4} > \sqrt[5]{5}$

5. Simplify, $\frac{\sqrt{8}+\sqrt{2}}{\sqrt{8}-\sqrt{2}}$

Sol.: $\frac{\sqrt{8}+\sqrt{2}}{\sqrt{8}-\sqrt{2}} \times \frac{\sqrt{8}+\sqrt{2}}{\sqrt{8}+\sqrt{2}}$

(Multiply numerator & denominator by conjugate of denominator)

$= \frac{\left(\sqrt{8}+\sqrt{2}\right)^2}{\left(\sqrt{8}\right)^2 - \left(\sqrt{2}\right)^2} = \frac{8+2+2\sqrt{16}}{8-2} = \frac{10+8}{6} = 3$ Ans.

6. Simplify $\frac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}-\sqrt{a^2-1}}$

Sol.:

$\frac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}-\sqrt{a^2-1}} \times \frac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}+\sqrt{a^2-1}}$

$= \frac{a^2+1+a^2-1+2\left(\sqrt{a^2+1}\right)\left(\sqrt{a^2-1}\right)}{a^2+1-\left(a^2-1\right)}$

$= \frac{2a^2+2\sqrt{a^4-1}}{2} = a^2 + \sqrt{a^4-1}$

7. Which is greater $\sqrt[3]{4}$ or $\sqrt[4]{5}$

Sol.: $\sqrt[3]{4} = 4^{\frac{1}{3}}$ and $\sqrt[4]{5} = 5^{\frac{1}{4}}$

raise the power of both the surds by 12(LCM of 3 and 4)

$\left(4^{\frac{1}{3}}\right)^{12} = 4^4 = 256$

$\left(5^{\frac{1}{4}}\right)^{12} = 5^3 = 125$

Hence, $\sqrt[3]{4} > \sqrt[4]{5}$

8. If $2^{2x-1} = \frac{1}{8^{x-3}}$ then find value of x

Sol.: $2^{2x-1} = \frac{1}{8^{x-3}}$

$= 2^{2x-1} = \frac{1}{(2^3)^{x-3}} = \frac{1}{2^{3x-9}}$

$= 2^{2x-1} = 2^{-3x+9} \quad \left[\because \frac{1}{a^m} = a^{-m}\right]$

$= 5x = 10, \ x = 2$

9. If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to?

Sol.: let $2^x = 3^y = 6^{-z} = k$

So, $2 = k^{1/x}$, $3 = k^{1/y}$, $6 = k^{-1/z}$

We know that 2 × 3 = 6 this gives

$k^{1/x} \times k^{1/y} = k^{-1/z} = \frac{1}{x}+\frac{1}{y} = -\frac{1}{z}$

$= \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

10. Arrange $\sqrt{2}$, $\sqrt[3]{4}$ and $\sqrt[4]{6}$ in **ascending order**

Sol.: $\sqrt{2} = 2^{1/2}$

$\sqrt[3]{4} = 4^{1/3}$, $\sqrt[4]{6} = 6^{1/4}$

Now raise the power of given surds by LCM of 2, 3, 4 that is 12

$\left(2^{1/2}\right)^{12} = 2^6 = 64$

$\left(4^{1/3}\right)^{12} = 4^4 = 256$

$\left(6^{1/4}\right)^{12} = 6^3 = 216$

Ans: $\sqrt{2}$, $\sqrt[4]{6}$, $\sqrt[3]{4}$

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Logarithms

Definition

The logarithm of any number of a given base is equal to the index to which the base should be raised to obtain the given number.

For example if

$a^x = c$, then x is called logarithm of a number c to the base a. It is written as $\log_a c = x$

Similarly

$2^3 = 8$ is similar as $\log_2 8 = 3$

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ is written

as $\log_4 \frac{1}{16} = -2$

$10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

$3^5 = 243 \Rightarrow \log_3 243 = 5$

Properties of Logarithms

(a) $\log_a(m \times n) = \log_a m + \log_a n$

Example 1.

$\log_{10}(15) = \log_{10} 3 + \log_{10} 5$

(b) $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$

Example 2.

$\log_2\left(\frac{6}{5}\right) = \log_2 6 - \log_2 5$

(c) $\log_a(m^n) = n\log_a m$

Example 3.

$\log_5 625 = \log_5 5^4 = 4\log_5 5$

(d) $\log_{a^n}(m) = \frac{1}{n}\log_a(m)$

Example 4.

$\log_{2^4}(8) = \frac{1}{4}\log_2 8$

(2) $\log_{10^2} 1000 = \frac{1}{2}\log_{10} 1000$

(e) $\log_a b = \frac{\log_n b}{\log_n a}$

[here n may be any natural number)

Example 5.

$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2}$

$\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$

(f) $\log_a x = \frac{\log_n x}{\log_n a} = \frac{1}{\log_x a}$

Example 6.

$\log_2 5 = \frac{1}{\log_5 2}$

$\log_3 8 = \frac{1}{\log_8 3}$

(g) $\log_a b \times \log_b a = \dfrac{\log_n b}{\log_n a} \times \dfrac{\log_n a}{\log_n b} = 1$

Remark:

(1) When base is not mentioned it is taken as 10.

(2) Logarithms to the base 10 is known as common logarithms.

(3) $\log_a a = 1$

Example 7.

$\log_5 5 = 1$

$\log_2 2 = 1$

(i) $\log_a 1 = 0$

[a may be any natural number.]

(j) $a^{\log_a x} = x$

$5^{\log_5 3} = 3$

$3^{\log_3 7} = 7$

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Solved Examples

1. Find the logarithm of 32 to the base 2

Sol.: $\log_2 32 = \log_2 (2^5)$

$= 5\log_2 2 = 5 \times 1 = 5$

2. Find the value of $2^{\log_2 5}$

Sol.: $a^{\log_a x} = x$

So $2^{\log_2 5} = 5$

3. Find the value of $3^{2+\log_3 5}$

Sol.: $3^{2+\log_3 5} = 3^2 \times 3^{\log_3 5}$

$= 3^2 \times 5 = 45$

4. Find the value of $3^{2-\log_3 5}$

Sol.: $3^{2-\log_3 5} = \dfrac{3^2}{3^{\log_3 5}} = \dfrac{9}{5}$

5. Find the value of $\log x^3 \div \log x$

Sol.: $\log x^3 \div \log x = 3\log x \div \log x = 3$

6. If $\log_{16} a = \dfrac{1}{2}$, Find a

Sol.: $\log_{16} a = \dfrac{1}{2}$

$a = 16^{1/2}$

$a = 4$

7. $\log_a \sqrt{3} = \dfrac{1}{6}$, find the value of a

Sol.: $\log_a \sqrt{3} = \dfrac{1}{6}$

$= \log_a 3^{1/2} = \dfrac{1}{6}$

or $a^{1/6} = 3^{1/2}$, $a = 3^3 = 27$

8. If $\log_{27} x + \log_9 x + \log_3 x = 11$, find the value of x

Sol.: $\log_{27} x + \log_9 x + \log_3 x = 11$

$= \dfrac{1}{3}\log_3 x + \dfrac{1}{2}\log_3 x + \log_3 x = 11$

$= \dfrac{11}{6}\log_3 x = 11$

$= \dfrac{1}{6}\log_3 x = 1$

$= \log_3 x = 6$

$= x = 3^6 = 729$

9. Solve $\log_3 n - \log_3 4 = 2$

Sol.: $\log_3 n - \log_3 4 = \log_3 \dfrac{n}{4} = 2$

$\log_3 \dfrac{n}{4} = 2$

or $\dfrac{n}{4} = 3^2$, $n = 3^2 \times 4 = 36$

10. Find the value of $\log\dfrac{x^2}{yz} + \log\dfrac{y^2}{zx} + \log\dfrac{z^2}{xy}$

Sol.: $\log\dfrac{x^2}{yz} + \log\dfrac{y^2}{zx} + \log\dfrac{z^2}{xy}$

$= \log\dfrac{x^2}{yz} \times \dfrac{y^2}{zx} \times \dfrac{z^2}{xy}$

$= \log\dfrac{x^2y^2z^2}{y^2z^2x^2} = \log 1 = 0$

11. Evaluate $\log_7\left(\dfrac{1}{343}\right)$

Sol.: $\log_7\left(\dfrac{1}{343}\right) = \log_7 \dfrac{1}{7^3}$

$= \log_7 7^{-3} = -3\log_7 7 = -3$

12. Evaluate $= \log_{100}(0.01)$

Sol.: $\log_{100}(100)^{-1} = -1\log_{100}100 = -1$

13. If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x

Sol.: $\log_{\sqrt{8}} x = \frac{10}{3}$, $x = \left(2^{3/2}\right)^{10/3}$

$x = 2^{3/2 \times 10/3} = 2^5 = 32$

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Surds, Indices & Logarithms

I Practice Exercise: I

1. The value of $\left(\sqrt{8}\right)^{1/3}$ is:

(a) 2 (b) 4
(c) $\sqrt{2}$ (d) 8

2. $\left(\frac{1}{216}\right)^{-2/3} \div \left(\frac{1}{27}\right)^{-4/3} = ?$

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$
(c) $\frac{4}{9}$ (d) $\frac{1}{8}$

3. If $\sqrt{2^n} = 64$, then the value of n is:

(a) 8 (b) 4
(c) 6 (d) 12

4. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then n equals:

(a) 0 (b) 2
(c) 3 (d) 4

5. If $\left(\sqrt{3}\right)^5 \times 9^2 = 3^\alpha \times 3\sqrt{3}$, then α equals:

(a) 2 (b) 3
(c) 4 (d) 5

6. If x, y, z are real numbers, then the value of: $\sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x}$ is

(a) xyz (b) $\sqrt{xyz}$
(c) $\frac{1}{xyz}$ (d) 1

7. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} . \left(\frac{x^c}{x^a}\right)^{(c+a-b)} . \left(\frac{x^a}{x^b}\right)^{(a+b-c)} = ?$

(a) x^{abc} (b) x^{a+b+c}
(c) $x^{ab+bc+ca}$ (d) 1

8. If $2^{x+4} - 2^{x+2} = 3$, then x is equal to:

(a) 0 (b) 2
(c) −1 (d) −2

9. If $2^{2x-1} = \frac{1}{8^{x-3}}$, the value of x is:

(a) 3 (b) 2
(c) 0 (d) −2

10. If $\sqrt{5 + \sqrt[3]{x}} = 3$, then x is equal to:

(a) 125 (b) 64
(c) 27 (d) 9

11. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals:

(a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$
(c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

12. If $x = y^a$, $y = z^b$ and $z = x^c$, then the value of abc is:

(a) 4 (b) 3
(c) 2 (d) 1

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Solutions

1. Ans. (c)

$$\left(\sqrt{8}\right)^{1/3} = \left(8^{1/2}\right)^{1/3} = 8^{\left(\frac{1}{2}\times\frac{1}{3}\right)} = 8^{1/6}$$

$$= \left(2^3\right)^{1/6} = 2^{\left(3\times\frac{1}{6}\right)} = 2^{1/2} = \sqrt{2}.$$

2. Ans. (c)

$$\left(\frac{1}{216}\right)^{-2/3} \div \left(\frac{1}{27}\right)^{-4/3}$$

$$= (216)^{2/3} \div (27)^{4/3} = \left(6^3\right)^{2/3} \div \left(3^3\right)^{4/3}$$

$$= 6^{\left(3\times\frac{2}{3}\right)} \div 3^{\left(3\times\frac{4}{3}\right)} = 6^2 \div 3^4 = \frac{36}{81} = \frac{4}{9}.$$

3. **Ans. (d)**

$\sqrt{2^n} = 64 \quad \Rightarrow 2^{n/2} = 64 = 2^6.$

$\therefore \quad \frac{n}{2} = 6$ or $n = 12.$

4. **Ans. (c)**

$$\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$$

$$\Rightarrow \quad \frac{3^{2n} \times 3^5 \times \left(3^3\right)^3}{3 \times \left(3^4\right)^4} = 3^3.$$

or $\frac{3^{2n} \times 3^5 \times 3^9}{3^1 \times 3^{16}} 3^3$

or $3^{2n+5+9} = 3^3 \times 3^1 \times 3^{16}$

or $3^{2n+14} = 3^{20}$

$\therefore \quad 2n + 14 = 20$

or $2n = 6$ or $n = 3.$

5. **Ans. (d)**

$$\left(\sqrt{3}\right)^5 \times 9^2 = 3^\alpha \times 3\sqrt{3}$$

$$\Rightarrow \left(3^{1/2}\right)^5 \times \left(3^2\right)^2 = 3^\alpha \times 3^1 \times 3^{1/2}$$

$$\therefore \quad 3^{5/2} \times 3^4 = 3^\alpha \times 3^1 \times 3^{1/2}$$

or $3^{\left(\frac{5}{2}+4\right)} = 3^{\left(\alpha+1+\frac{1}{2}\right)}$

or, $3^{13/2} = 3^{\alpha+\frac{3}{2}} \qquad$ So, $\alpha + \frac{3}{2} = \frac{13}{2}$

or $\alpha = \left(\frac{13}{2} - \frac{3}{2}\right) = 5.$

6. **Ans. (d)**

$$\sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x}$$

$$= \sqrt{\frac{y}{x}}.\sqrt{\frac{z}{y}}.\sqrt{\frac{x}{z}} = \frac{\sqrt{y}}{\sqrt{x}} \times \frac{\sqrt{z}}{\sqrt{y}} \times \frac{\sqrt{x}}{\sqrt{z}} = 1.$$

7. **Ans. (d)**

Given Exp.

$$= x^{(b-c)(b+c-a)}.x^{(c-a)(c+a-b)}.x^{(a-b)(a+b-c)}$$

$$= x^{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)}.x^{-a(b-c)-b(c-a)-c(a-b)}$$

$$= x^0.x^0 = 1$$

8. **Ans. (d)**

$$2^{x+4} - 2^{x+2} = 3$$

$$\Rightarrow 2^{x+2}\left(2^2 - 1\right) = 3$$

$$\Rightarrow 2^{x+2} = 1 = 2^\circ.$$

$\therefore \quad x + 2 = 0$ or $x = -2.$

9. **Ans. (b)**

$$2^{2x-1} = \frac{1}{8^{x-3}} \qquad \Rightarrow 2^{2x-1} = \frac{1}{(2^3)^{(x-3)}}$$

$$\Rightarrow 2^{2x-1} = \frac{1}{2^{3x-9}} \qquad \Rightarrow 2^{2x-1} = 2^{9-3x}$$

$\therefore \quad 2x - 1 = 9 - 3x$ or $5x - 10$ or $x = 2.$

10. **Ans. (b)**

On squaring both sides, we get :

$5 + \sqrt[3]{x} = 9$ or $\sqrt[3]{x} = 4.$

Cubing both sides, we get

$x = (4 \times 4 \times 4) = 64.$

11. **Ans. (d)**

Let $a^x = b^y = c^z = k.$

Then $a = k^{1/x}$, $b = k^{1/y}$ and $c = k^{1/z}.$

$b^2 = ac \Rightarrow k^{2/y} = k^{1/x}.k^{1/z}$

$$\Rightarrow \quad k^{2/y} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)}.$$

12. **Ans. (d)**

$x = y^a = \left(z^b\right)^a = z^{ab} = \left(x^c\right)^{ab} = x^{abc}. \quad \therefore \quad abc = 1.$

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Surds, Indices & Logarithms

II Practice Exercise: II

1. The value of $\log_{343}7$ is:

(a) $\frac{1}{3}$ (b) -3

(c) $-\frac{1}{3}$ (d) 3

2. The value of $\log_{10}(.0001)$ is:

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) -4 (d) 4

3. If $\log_8 x = \frac{2}{3}$, then the value of x is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) 4 (d) 3

4. $\log_{32} x = 0.8$, then x is equal to:

(a) 25.6 (b) 16

(c) 10 (d) 12.8

5. If $\log_4 x + \log_2 x = 6$, then x is equal to

(a) 2 (b) 4

(c) 8 (d) 16

6. If log 2 = 0.30103, then the number of digits in 5^{20} is:

(a) 14 (b) 16

(c) 18 (d) 25

7. The value of $\log_2(\log_5 625)$ is:

(a) 2 (b) 5

(c) 10 (d) 15

8. $(\log_b a \times \log_c b \times \log_a c)$ is equal to:

(a) 0 (b) 1

(c) abc (d) a+b+c

9. $\left[\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ca)+1} + \frac{1}{(\log_c ab)+1}\right]$ is equal to:

(a) 1 (b) 2

(c) 3 (d) $\frac{3}{2}$

10. If $\log_2[\log_3(\log_2 x)] = 1$, then x is equal to:

(a) 512 (b) 128

(c) 12 (d) 0

11. If $\log_{10}2 = 0.3010$, then $\log_2 10$ is :

(a) .3322 (b) 3.2320

(c) 3.3222 (d) 5

12. If $\log_{10}2 = 0.3010$, the value of $\log_{10}5$ is:

(a) 0.3241 (b) 0.6911

(c) 0.6990 (d) 0.7525

13. The value of $(\log_9 27 + \log_8 32)$ is :

(a) 4 (b) 7

(c) $\frac{7}{2}$ (d) $\frac{19}{6}$

14. If $\log_5(x^2 + x) - \log_5(x + 1) = 2$, then the value of x is:

(a) 5 (b) 32

(c) 25 (d) 10

15. If $\log x + \log y = \log(x + y)$, then:

(a) $x = y$ (b) $xy = 1$

(c) $y = \frac{x-1}{x}$ (d) $y = \frac{x}{x-1}$

16. The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right]$ is:

(a) 3 (b) 2

(c) 1 (d) 0

17. If log 2 = 0.3010 and log 3 = 0.4771 then the value of log 4.5 is:

(a) 0.6532 (b) 0.7727

(c) 0.3266 (d) None of these

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Solutions

1. **Ans. (a)**

Let $\log_{343}7 = m$.

Then, $(343)^m = 7 \Rightarrow (7^3)^m = 7$

$\Rightarrow 7^{3m} = 7$

$\Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}.$

$\therefore \quad \log_{343} 7 = \frac{1}{3}.$

2. **Ans. (c)**

Let $\log_{10}(.0001) = m$. Then,

$10^m = .0001 \Rightarrow 10^m = \frac{1}{10000} \Rightarrow 10^m$

$= \frac{1}{10^4} \Rightarrow 10^m = 10^{-4} \Rightarrow m = -4$

$\therefore \quad \log_{10}(.0001) = -4.$

3. **Ans. (c)**

$\log_8 x = \frac{2}{3} \Rightarrow x = 8^{2/3} = \left(2^3\right)^{2/3}$

$= 2^{\left(3\times\frac{2}{3}\right)} = 2^2 = 4.$

4. **Ans. (b)**

$\log_{32} x = 0.8 \Rightarrow x = (32)^{0.8}$

$= \left(2^5\right)^{4/5} = 2^4 = 16.$

5. **Ans. (d)**

$\log_4 x + \log_2 x = 6 \Rightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6$

$\therefore \quad \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 6 \Rightarrow 3\log x = 12\log 2$

or $\log x = 4\log 2 \Rightarrow \log x$

$= \log 2^4$ or $x = 2^4 = 16.$

6. **Ans. (a)**

$\log 5^{20} = 20\log 5 = 20 \times \left[\log\left(\frac{10}{2}\right)\right]$

$= 20 \times [\log 10 - \log 2] = 20 \times [1 - 0.3010]$

$= 20 \times 0.6990 = 13.9800$

$\therefore$ Characteristic $= 13.$

$\therefore$ Number of digits in 5^{20} is 14.

7. **Ans. (a)**

Let $\log_5 625 = x$. Then, 5^x
$= 625 = 5^4$ or $x = 4$

Let $\log_2(\log_5 625) = y$. Then, $\log_2(4) = y$

or $2^y = 4 = 2^2$. So, $y = 2$.

$\therefore \quad \log_2(\log_5 625) = 2.$

8. **Ans. (b)**

Given Exp. $= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

9. **Ans. (a)**

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b}$

$+ \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc}(abc) = 1.$

10. **Ans. (a)**

$\log_2[\log_3(\log_2 x)] = 1$

$\Rightarrow \quad \log_3(\log_2 x) = 2$

$\Rightarrow \log_2 x = 3^2 = 9$

$\Rightarrow x = 2^9 = 512$

11. **Ans. (c)**

$\log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = 3.3222$

12. **Ans. (c)**

$\log_{10} 5 = \log_{10}\left(\frac{10}{2}\right) = \log_{10} 10 - \log_{10} 2$

$= 1 - \log_{10} 2 = (1 - 0.3010) = 0.6990.$

13. **Ans. (d)**

$\log_9 27 + \log_8 32$

$\log_{3^2} 3^3 + \log_{2^3} 2^5$

$\frac{3}{2}\log_3 3 + \frac{5}{3}\log_2 2$

So, $\frac{3}{2} + \frac{5}{3} = \frac{19}{6}$

14. **Ans. (c)**

$\log_5\left(x^2 + x\right) - \log_5(x + 1) = 2$

$$\Rightarrow \log_5\left(\frac{x^2+x}{x+1}\right)=2$$

$$\therefore \log_5\left[\frac{x(x+1)}{x+1}\right]=2$$

or $\log_5 x = 2$ or $x=5^2 = 25.$

15. Ans. (d)

$$\log x + \log y = \log(x+y)$$

$$\Rightarrow \log(x+y) = \log(xy)$$

$$\therefore x + y = xy \text{ or } x = y(x-1)$$

$$\Rightarrow y = \frac{x}{x-1}$$

16. Ans. (d)

Given Expression

$$= \log_x\left(\frac{p}{q}\right) + \log_x\left(\frac{q}{r}\right) + \log_x\left(\frac{r}{p}\right)$$

$$\log_x\left(\frac{p}{q}\times\frac{q}{r}\times\frac{r}{p}\right) = \log 1 = 0.$$

17. Ans. (a)

$$\log 4.5 = \log\left(\frac{9}{2}\right) = \log 9 - \log 2$$

$$= \log\left(3^2\right) - \log 2 = 2\log 3 - \log 2$$

$$= (2\times 0.4771 = 0.3010) = 0.6532.$$

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