Pairs of Linear Equations in Two Variables

Exercise-3.1

Question :

Obtain a pair of linear equations in two variables from the following information :

Question 1:

Father tells his son, “Five years ago, I was seven times as old as you were. After five years, I will be three times as old as you will be”.

Solution :

Let the present age of the father be x years and that of the son be y years.

∴ Before 5 years,

Age of father = $(x – 5)$ years and

Age of son = $(y – 5)$ years.

But, before 5 years, the father’s age was seven times the age of the son.

$\therefore\ x - 5 = 7(y - 5)$

$\therefore\ x - 5 = 7y - 35$

$\therefore\ x - 7y + 30 = 0$ … … (1)

After 5 years,

Age of father = $(x + 5)$ years

Age of son will be $(y + 5)$ years.

But after 5 years, the father’s age will be three times the son’s age,

$\therefore\ x + 5 = 3(y + 5)$

$\therefore\ x + 5 = 3y + 15$

$\therefore\ x - 3y - 10 = 0$ … … (2)

Hence eq. (1) and (2) represent a pair of linear equations in two variables.

Question 2:

The sum of the cost of 1kg apple and 1kg pine-apple is ₹ 150 and the cost of 1kg apple is twice the cost of 1kg pine-apple.

Solution :

Let the cost of 1 kg of apples be Rs. x and the cost of 1 kg of pineapples be Rs. y.

∴ The total cost of 1 kg of apples and 1 kg of pineapples is Rs. $(x + y)$.

But, the total cost is given to us as Rs. 150.

∴ $x + y = 150$ … (1)

The cost of 1 kg of apples is twice the cost of 1 kg of pineapples.

∴ $x = 2y$ i.e., $x - 2y = 0$ … (2)

Hence eq. (1) and (2) represent a pair of linear equation in two variables.

Question 3:

Nilesh got twice the marks as obtained by Ilesh, in the annual examination of mathematics of standard 10. The sum of the marks as obtained by them is 135.

Solution :

Let the marks obtained by Ilesh be x and those obtained by Nilesh be y.

But, Nilesh's marks are twice Ilesh's marks.

∴ $y = 2x$ i.e., $2x - y = 0$ …(1)

The sum of the marks obtained by Nilesh and Ilesh, i.e. $(x + y)$ is 135.

∴ $x + y = 135$ … … (2)

Hence eq. (1) and (2) represent a pair of linear equations in two variables.

Question 4:

Length of a rectangle is four less than the thrice of its breadth. The perimeter of the rectangle is 110.

Solution :

Let the length of rectangle be x units and its breadth be y units.

But, the length of rectangle (x) is five less than thrice its breadth (y).

∴ $x = 3y - 5$

∴ $x - 3y + 5 = 0$ … … (1)

The perimeter of a rectangle having length x and breadth y is $2(x + y)$.

But, this perimeter is given as 110.

∴ $2(x + y) = 110$

∴ $x + y = 55$ … … (2)

Hence eq. (1) and (2) represent a pair of linear equations in two variables.

Question 5:

The sum of the weights of a father and a son is 85 kg. The weight of the son is $frac14$ of the weight of his father.

Solution :

Suppose, the present weights of the father and son are x kg and y kg respectively.
$\therefore$ The sum of the weights of the father and son is $(x + y)$ kg.
But, this is given as 85 kg.
$\therefore \; x + y = 85 \quad \ldots \quad \ldots(1)$

The weight of the son (y) is $\frac{1}{4}$ of the weight of his father (x).

$\therefore \; y = \frac{1}{4}x$

$\therefore \; 4y = x$

$\therefore \; x - 4y = 0 \quad \ldots \quad \ldots(2)$

Hence eq.(1) and (2) represent a pair of linear equations in two variables.

Question 6:

In a cricket match, Sachin Tendulkar makes his score thrice the Sehwag's score. Both of them together make a total score of 200 runs.

Solution :

Assume that in the cricket match, Sachin scored x runs and Sehwag scored y runs.

Also, in this match Sachin's scored thrice the runs scored by Sehwag.

$\therefore x = 3y$

$\therefore x – 3y = 0$ … … (1)

Sachin's runs are x and Sehwag's runs are y. So the total runs are $(x + y)$.

But both of them together make a total score of 200 runs.

$\therefore x + y = 200$ … … (2)

Hence eq. (1) and (2) represent a pair of linear equations in two variables.

Question 7:

In tossing a balanced coin, the probability' of getting head on its face is twice to the probability of getting tail on its face. The sum of both probabilities (head and tail) is 1.

Solution :

Assume that on tossing a coin the probabilities of getting heads is x and getting tails is y.

But, the probability of getting heads (x) is twice the probability of getting taisl (y).

$\therefore x = 2y$

$\therefore x – 2y = 0$

The sum of the probabilities of getting heads (x) and tails (y) is $(x + y)$.

This sum is 1.

$\therefore x + y = 1$

Hence eq. (1) and (2) represent a pair of linear equations in two variables.

Exercise-3.2

Question 1:

Solve the following pair of Linear equations in two variable (by graph) :

1. $2x + y = 8, x + 6y = 15$
2. $X + y = 1, 3x + 3y = 2$
3. $2x + 3y = 5, x + y = 2$
4. $X – y = 6, 3x - 3y = 18$
5. $(x + 2)(y – 1) = xy, (x – 1)(y + 1) = xy$

Question 1(1):

Solution :

Here $2x + y = 8$,

$\Rightarrow y = 8 – 2x$

For $x = 4$, $y = 8 – 2(4) = 8 – 8 = 0$

For $x = 0$, $y = 8 – 2(0) = 8 – 0 = 8$

x	4	0
y	0	8

∴ Plot the ordered pairs (4, 0) and (0, 8) of the equation $2x + y = 8$ on the graph paper and draw a line joining them.

Next, $x + 6y = 15$

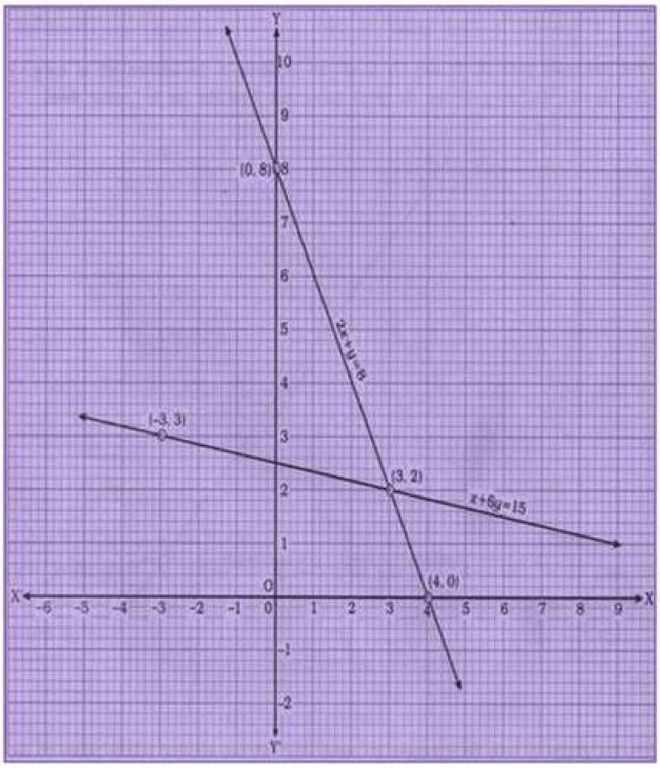
$$\Rightarrow 6y = 15 - x \qquad \therefore y = \frac{15 - x}{6}$$

$$\text{For } x = 3,\ y = \frac{15-3}{6} = \frac{12}{6} = 2$$

$$\text{For } x = -3,\ y = \frac{15-(-3)}{6} = \frac{18}{6} = 3$$

x	3	-3
y	2	3

∴ Plot the ordered pairs (3, 2) and (-3, 3) of the equation $x + 6y = 15$ on the graph paper and draw a line joining them.



The intersection point (common point) of these two lines is (3, 2) which satisfies both the equations.

Hence, the solution set of the given pair of linear equations is {(3, 2)}.

Question 1(2):

Solution :

Here x + y = 1

⇒ y = 1 – x

For x = 0, y = 0 – 0 = 1

For x = 1, y = 1 – 1 = 0

X	0	1
Y	1	0

Plot the ordered pair (0, 1) and (1, 0) for equation x + y = 1 on the graph paper and draw a line joining them.

Next, 3x + 3y = 2

⇒ 3y = 2 – 3x

$\therefore y = \frac{2-3x}{3}$

For $x = 0$, $y = \frac{2-3(0)}{3} = \frac{2}{3}$

For $x = \frac{2}{3}$, $y = \frac{2-3\left(\frac{2}{3}\right)}{3} = \frac{2-2}{3} = 0$

X	0	$\frac{2}{3}$
Y	$\frac{2}{3}$	0

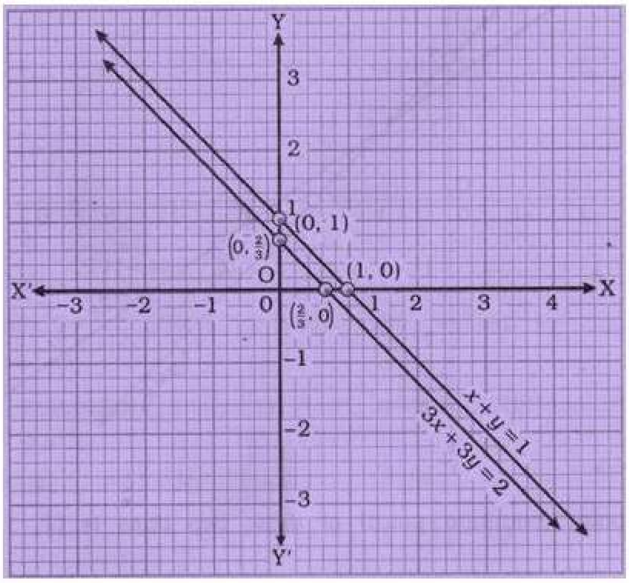
[Note: here, $3x + 3y = 2$ i.e., $x + y = \frac{2}{3}$,

Also the sum of two integers cannot be a fraction $\left(\frac{2}{3}\right)$.

So either x or y is taken as fraction.

$\therefore$ Plot the ordered pair $\left(0, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, 0\right)$ of equation $3x+3y=2$

on the graph paper and draw a line joining them.



Here the lines do not intersect each other, i.e. the lines are parallel. So the given pair of linear equation does not have a solution.

Hence, the solution set of the given pair of linear equations is Ø.

Question 1(3):

Solution :

Here $2x + 3y = 5$

$\Rightarrow 3y = 5 - 2x$

$\therefore y = \frac{5-2x}{3}$

For $x = -2$, $y = \frac{5-2(-2)}{3} = \frac{5+4}{3} = \frac{9}{3} = 3$

For $x = 4$, $y = \frac{5-2(4)}{3} = \frac{5-8}{3} = \frac{-3}{3} = -1$

X	-2	4
y	3	-1

∴ Plot the ordered pair (-2, 3) and (4, -1) of the equation 2x + 3y = 5 on the graph paper and draw a line joining them.

Next, x + y = 2

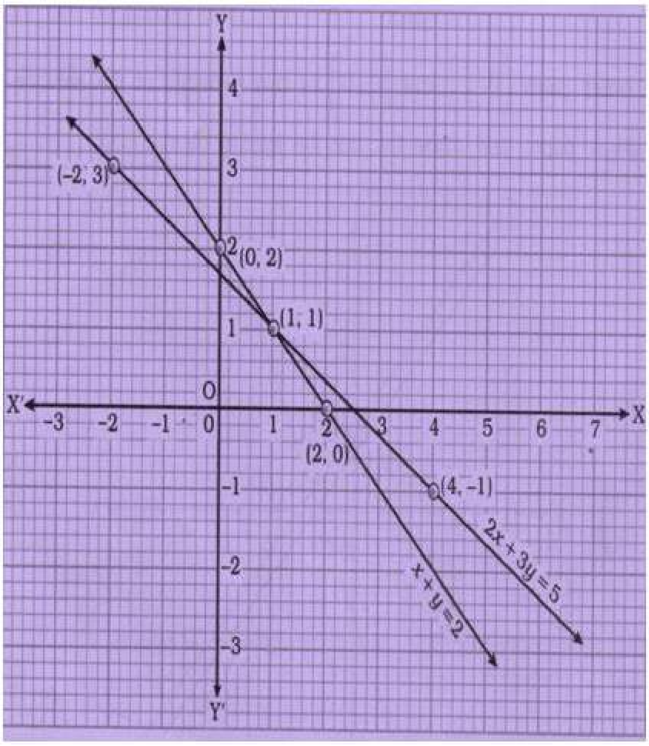
∴ y = 2 – x

For x = 2, y = 2 – 2 = 0

For x = 0, y = 2 – 0 = 2

X	2	0
y	0	0

∴ Plot the ordered pairs (2, 0) and (0, 2) of the equation x + y = 2 on the graph paper and draw a line by joining them.



The intersection point of these two lines is (1, 1) which satisfies both the equations.

Hence, the solution set of the given pair of linear equations is {(1, 1)}.

Question 1(4):

Solution :

Dividing each term of 3x – 3y = 18 by 3, we get the equation x – y = 6.

Thus, the given pair of equations is identical.

∴ As both the lines are same or coincident, they have infinitely many solutions.

x – y = 6

⇒ x – y – 6 = 0

For x = 6, y = 6 – 6 = 0

For x = 0, y = 0 – 6 = -6

Next,

$$3x - 3y = 18$$

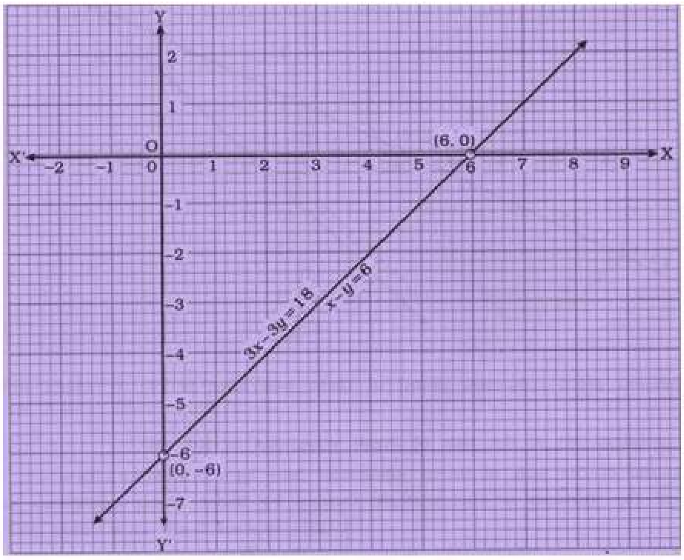
$$\therefore 3y = 3x - 18$$

$$\therefore y = \frac{3x - 18}{3}$$

$$y = x - 6$$

x	6	0
y	0	-6

Plot the ordered pair (6, 0) and (0, -6) of equation x – y = 6 (or 3x – 3y = 18, i.e., x – y = 6) on the graph paper and draw a line joining them.



Here, the graph of both the equations is same. Also, we can see that infinite points on the line make the solution set.

Thus, the lines are coincident and have infinite solutions. So the solutions set is {(x, y)| x – y = 6, x, y ∈ R}.

Question 1(5):

Solution :

Here, (x + 2)(y – 1) = xy

∴ xy – x + 2y – 2 = xy

∴ -x + 2y – 2 = 0

∴ 2y = x + 2

∴ y = c

$$\text{For } x = 0, y = \frac{0+2}{2} = \frac{2}{2} = 1$$

$$\text{For } x = -2, y = \frac{-2+2}{2} = \frac{0}{2} = 0$$

x	0	-2
Y	1	0

∴ Plot the ordered pair (0, 1) and (-2, 0) of equation x – 2y = 2 on the graph paper and draw a line joining them.

(x – 1)(y + 1) = xy

∴ xy + x – y – 1 = xy

∴ x – y – 1 = 0

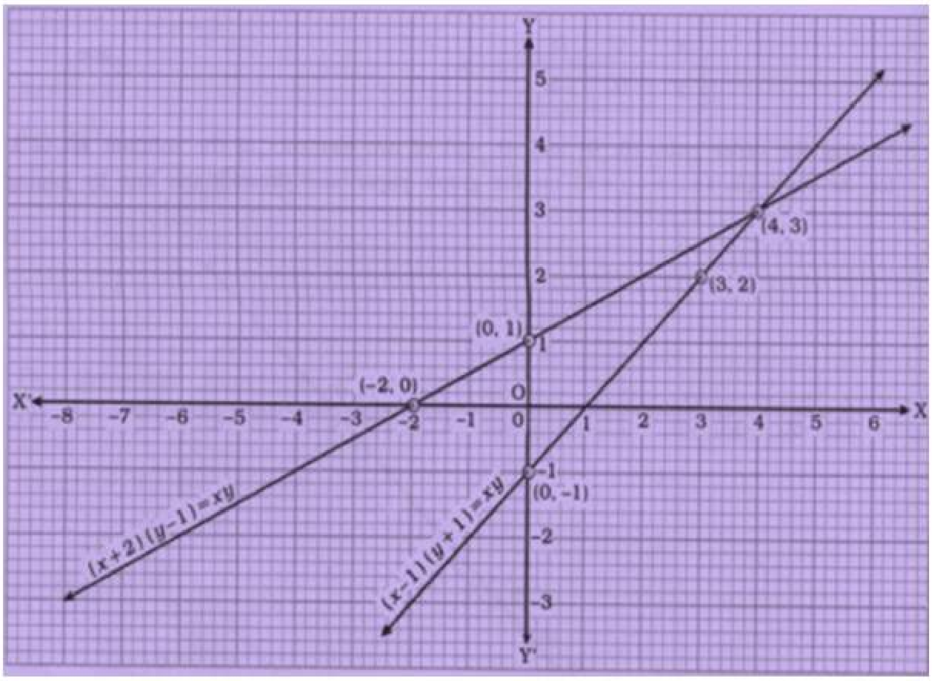
∴ y = x – 1

For x = 0, y = 0 – 1 = -1

For x = 3, y = 3 – 1 = 2

x	0	3
y	-1	2

∴ Plot the ordered pair (0, 1) and (-2, 0) of the equation x – y = 1 on the graph paper and draw a line joining them.



Plot the ordered pair (0, 1) and (-2, 0) of the two elements of the solution set of the equation x – y = 1 on the graph paper and draw a line joining them.
The intersection point of these two lines is (4, 3) which satisfies both the equations.
Hence, the solution see of the given pair of linear equations is {(4, 3)}.

Question 2:

Draw the graphs of the pair of linear equations 3x + 2y = 5 and 2x – 3y = -1. Determine the coordinates of the vertices of the triangle formed by these linear equations and the X-axis.

Solution :

Here $3x + 2y = 5$

$\therefore 2y = 5 - 3x$

$$y = \frac{5 - 3x}{2}$$

For $x = 1,\ y = \dfrac{5 - 3(1)}{2} = \dfrac{2}{2} = 1$

For $x = -1,\ y = \dfrac{5 - 3(-1)}{2} = \dfrac{8}{2} = 4$

x	1	-1
y	1	4

∴ Plot the ordered pair (1, 1) and (1, 4) of the equation
3x – 2y = 5 on the graph paper and draw a line joining them.

Next,

$2x - 3y = -1$

$\therefore 3y = 2x + 1$

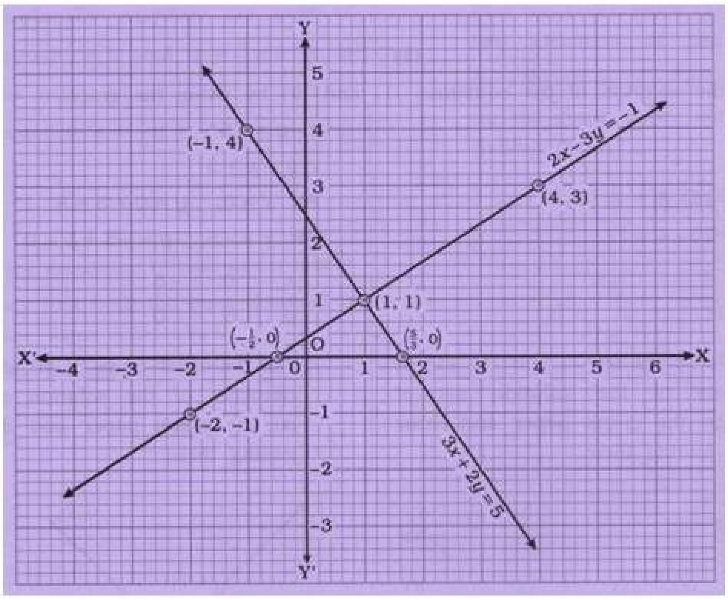
$\therefore y = \frac{2x + 1}{3}$

For $x = 4$, $y = \frac{2(4)+1}{3} = \frac{9}{3} = 3$

For $x = -2$, $y = \frac{2(-2)+1}{3} = \frac{-3}{3} = -1$

x	4	-2
y	3	-1

$\therefore$ Plot the ordered pair (4, 3) and (-2, -1) of the equation $2x - 3y = -1$ on the graph paper and draw a line joining them.



The intersection point of these two lines is (1, 1).

Also, if the line $3x + 2y = 5$ intersects the x-axis, then taking $y = 0$.

$3x + 2(0) = 5$

$\therefore 3x = 5$

$\therefore x = \frac{5}{3}$

$\therefore$ Intersection point with x-axis is $\left(\frac{5}{3}, 0\right)$

Also, if the line $2x-3y=-1$ intersect the x-axis, then taking $y=0$,

$2x - 3(0) = -1$

$\therefore 2x = -1$

$\therefore x = -\frac{1}{2}$

$\therefore$ Intersection point with the x-axis is $\left(-\frac{1}{2}, 0\right)$

Hence the co-ordinates of the vertices of the triangles formed by these linear equations on x-axis are $(1, 1)$, $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{5}{3}, 0\right)$.

Question 3:

15 students of class X took part in the examination of Indian mathematics olympiad. The number of boys participants is 5 less than the number of girls participants. Find the number

of boys and girls (using a graph) who took part in the examination of Indian mathematics olympiad.

Solution :

Let the number of boys participating in the Math's Olympiad be = x

Number of girls participating in the Math's Olympiad be = y Total number of participants who took part in the examination is (x + y).

But, this number is given as 15.

∴ x + y = 15 … … (1)

The number of boys participating in the Math's Olympiad is 5 less than the number of girls.

∴ x = y – 5

∴ x – y = -5 … … (2)

We will find the solution of the equations (1) and (2) by drawing a graph.

x + y = 15

∴ y = 15 – x

For x = 8, y = 15 – 8 = 7

For x = 4,y = 15 – 4 = 11

x	8	4
y	7	11

∴ Plot the ordered pairs (8, 7) and (4, 11) of the equation x – y = 15 on the graph paper and draw a line joining them.

x – y = -5

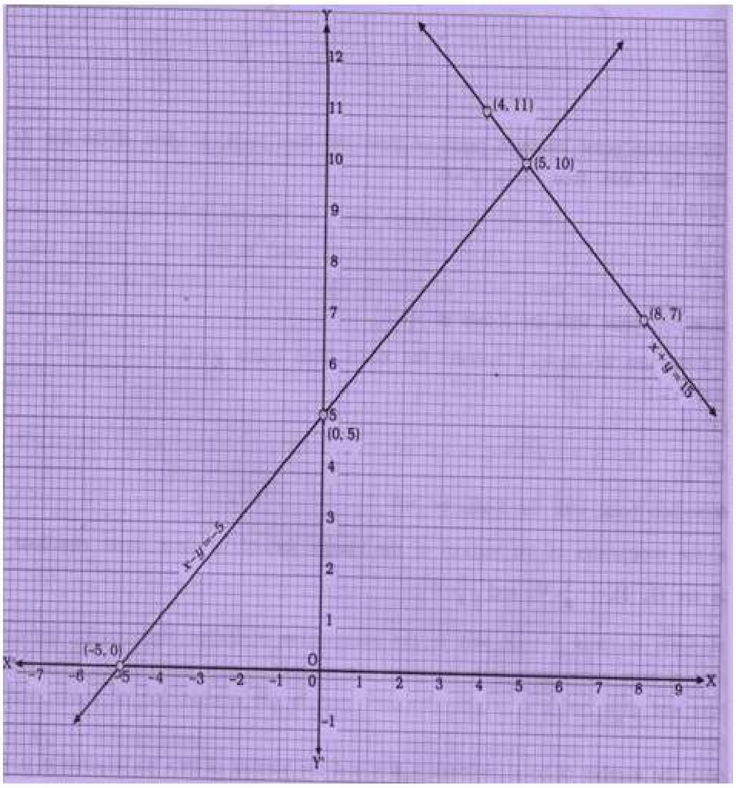
∴ y = x + 5

For x = 0, y = 0 + 5 = 5

For x = -5, y = -5 + 5 = 0

x	0	-5
y	5	0

∴ Plot the ordered pair (0, 5) and (-5, 0) of two elements of the solution set of x – y = -5 on the graph paper and draw a line joining them.



The intersection point of these two lines is (5, 10).

∴ x = 5 and y = 10, i.e., the number of boys participating is 5 and the number of girls participating is 10.

Question 4:

Examine graphically whether the pair of equations 2x + 3y = 5 and $x + \frac{9}{6}y = \frac{5}{2}$ is consistent.

Solution :

For equation $x + \frac{9}{6}y = \frac{5}{2}$

$\therefore x + \frac{3}{2}y = \frac{5}{2}$

$\therefore 2x + 3y = 5$

∴ The pair of equations are identical.

Hence, we say that both the lines obtained in the graph are same (coincident).

Now, to draw the graph,

$2x + 3y = 5$

$\therefore 3y = 5 - 2x$

$\therefore y = \frac{5 - 2x}{3}$

For x = 1.

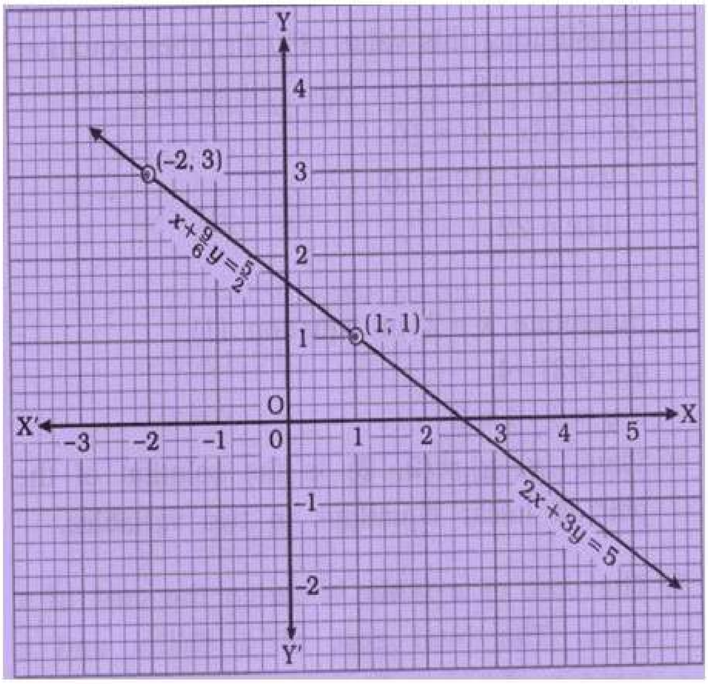
$y = \frac{5 - 2(1)}{3} = \frac{3}{3} = 1$

For $x = -2$, $y = \frac{5 - 2(-2)}{3} = \frac{9}{3} = 3$

x	1	-2
y	1	3

∴ Plot the ordered pair (1, 1) and (-2, 3) of equation 2x − 3y = 5

(or $x + \frac{9}{6}y = \frac{5}{2}$, i.e., 2x − 3y = 5) on the graph paper and draw a line joining them.



Here, the graph of both the equations is the same. Also, we can see that there are infinite points on the graph, hence, the pair of linear equations of two variables is consistent and has infinite solutions.

Exercise-3.3

Question 1:

Solve the following pairs of linear equations by the method of substitution :

1. x + y = 7, 3x – y = 1
2. 3x – y = 0, x – y + 6 = 0
3. 2x + 3y = 5, 2x + 3y = 7
4. x – y = 3, 3x – 3y = 9
5. $\frac{3x}{2} - \frac{5y}{3} = -2, \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Question 1(1):

Solution :
Here, 3x – y = 1
∴ y = 3x – 1 … … (1)
Substituting this value of y in the equation x + y = 7
x + (3x – 1) = 7.
∴ 4x = 8
∴ x = 2
Substituting x = 2 in equation (1).
y = 3(2) – 1
∴ y = 6 – 1 = 5
Hence (x, y) = (2, 5) is the solution of the given pair of linear equations.

Question 1(2):

Solution :
Here, x – y + 6 = 0

∴ $y = x + 6$ … … (1)

Substituting this value of y in the equation $3x – y = 0$,

$3x – (x + 6) = 0$

∴ $2x = 6$

∴ $x = 3$

Substituting $x = 3$ in equation (1),

$y = 3 + 6$

∴ $y = 9$

Hence $(x, y) = (3, 9)$ is the solution of the given pair of linear equations.

Question 1(3):

Solution :

Here, the equations are

$2x + 3y = 5$ … … (1)

And $2x + 3y = 7$ … … (2)

Hence, from equations (1) and (2),

$2x + 3y = 5 = 7$

∴ $5 = 7$, which is not true.

So, there is no real solution of the given pair of linear equations.

∴ The solution set of the given pair of linear equations is $\varnothing$.

Question 1(4):

Solution :

Here, the equations are

$x – y = 3$ … … (1)

and $3x – 3y = 9$

i.e., $x – y = 3$ (Dividing by 3) … … (2)

Hence, from equations (1) and (2), it is clear that both the equations are similar.

∴ The solution of one equation always satisfies the other equation.

∴ The given pair of linear equations do not have a unique solution.

Hence, the solution set is infinite and can be given by

$\{(x, y) | x – y = 3, x, y \in R\}$

Question 1(5):

Solution :

Here, multiplying $\frac{3x}{2} - \frac{5y}{3} = -2$ by 6 which is the l.c.m. of 2 and 3.

$9x - 10y = -12$ … … (1)

Next, multiplying $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ by 6 which is the l.c.m. of 3 and 2.

$\therefore 2x + 3y = 13$ … … (2)

$\therefore 3y = 13 - 2x$

$\therefore y = \frac{13 - 2x}{3}$

Substituting the value of y in equation (1).

$9x - 10\left(\frac{13 - 2x}{3}\right) = -12$

$\therefore 27x - 130 + 20x = -36$

$\therefore 47x = 94$

$\therefore x = 2$

Substituting $x = 2$ in equation $y = 13 - 2x$,

$y = \frac{13 - 2(2)}{3}$

$\therefore y = \frac{9}{3} = 3$

Hence, (x, y) = (2, 3) is the solution of the given pair of linear equations in two variablese .

Question 2:

Solve the pair of linear equations x – y = 28 and x – 3y = 0 and if the solution satisfies, y = mx + 5, then find m.

Solution :

For x – 3y = 0

∴ x = 3y … … (1)

Substituting the value of x in the other equation x – y = 28

3y – y = 28

⇒ 2y = 28

⇒ y = 14

Substituting y = 14 in equation (1).

x = 3(14) ⇒ x = 42

Hence (x, y) = (42, 14) is the solution of the pair of linear equation in two variables.

Now, substituting x = 42 and y = 14 in the equation y = mx + 5,

14 = m(42) + 5 ⇒ 42m=9

$\therefore m = \frac{9}{42} = \frac{3}{14}$

The required solution is (x, y) = (42, 14) and $m = \frac{3}{14}$

Question 3:

A fraction becomes $\frac{4}{5}$ if 3 is added to both the numerator and the denominator. If 5 is added to the numerator and the denominator, it becomes $\frac{5}{6}$. Find the fraction.

Solution :

Let the numerator of the fraction = x
Denominator of the fraction = y.
If 3 is added to both the numerator and the denominator,
then the new fraction formed is $\frac{x+3}{y+3}$.
According to the given conditions,
$\therefore \frac{x+3}{y+3} = \frac{4}{5}$
$\therefore 5x + 15 = 4y + 12$
$\therefore 5x - 4y = -3 \quad \ldots \quad \ldots (1)$
If 5 is added to both the numerator and the denominator,
then the new fraction is $\frac{x+5}{y+5}$,
According to the given conditions, the new fraction formed is $\frac{5}{6}$.
$\therefore \frac{x+5}{y+5} = \frac{5}{6}$
$\therefore 6x + 30 = 5y + 25$
$\therefore 6x - 5y = -5 \quad \ldots \quad \ldots (2)$
Now, from the equation (1),
$5x = 4y - 3 \quad \therefore x = \frac{4y-3}{5}$
Substituting $x = \frac{4y-3}{5}$ in equation (2)
$6\left(\frac{4y-3}{5}\right) - 5y = -5$
$\therefore 24y - 28 - 35y = -25$
$\therefore y = -7$
$\therefore y = 7$

Now, substituting $y = 7$ in $x = \frac{4y-3}{5}$,
Thus, $x = 5$, $y = 7$
Original fraction$= \frac{x}{y} = \frac{5}{7}$

Question 4:

The sum of present ages of a father and his son is 50 years. After 5 years, the age of the father becomes thrice the age of his son. Find their present ages.

Solution :
Suppose, the present age of the father is x years and the son is y years.
The sum of the present ages of the father (x) and son (y) is (x + y) years.
But, this sum is given to be 50 years.
$\therefore x + y = 50$
After 5 years, the father's will become (x + 5) years and the son's age will become (y + 5) years.
But after 5 years, the father's age will be three times the son's age.
$\therefore x + 5 = 3(y + 5)$
$\therefore x + 5 = 3y + 15$
$\therefore x - 3y = 10 \quad \ldots \ldots (2)$
From eq. (1), substituting $x = 50 - y$ in eq. (2).
Now, substituting $y = 10$ in $x = 50 - y$.
$x = 50 - 10 \quad x = 40$
Thus, the present age of the father is 40 years and that of the son is 10 years.

Question 5:

A bus traveller travelling with some of his relatives buys 5 tickets from Ahmedabad to Anand and 10 tickets from Ahmedabad to Vadodara for ₹ 1100. The total cost of one ticket from Ahmedabad to Anand and one ticket from Ahmedabad to Vadodara is ₹ 140. Find the cost of a ticket from Ahmedabad to Anand as well as the cost of a ticket from Ahmedabad to Vadodara.

Solution :

Let the cost of one ticket from Ahmedabad to Anand be Rs. x and the cost of one ticket from Ahmedabad to Vadodara be Rs. y

∴ Cost of 5 tickets from Ahmedabad to Anand is Rs. 5x

And cost of 10 tickets from Ahmedabad to Vadodara is Rs. 10y.

∴The total cost 5 tickets from Ahmedabad to Anand and 10 tickets

from Ahmedabad to Vadodara is Rs. (5x + 10y).

But, this cost is given as Rs. 1100.

∴ 5x + 10y = 1100

∴ x + 2y = 220 (dividing by 5) … … (1)

The total cost of 1 ticket from Ahmedabad to Anand and 1 ticket

from Ahmedabad to Vadodara is Rs. (x + y)

But, this cost is Rs. 140

∴ x + y = 140 … … (2)

From eq. (2), substituting y = 140 – x in eq. (1),

x + 2(140 – x) = 220

∴ x + 280 – 2x = 220

∴ – x = -60

∴ x = 60

Now, substituting x = 60 in eq. (2)

⇒ y = 140 – x

⇒ y = 140 – 60 = 80

Thus, the cost of one ticket from Ahmedabad to Anand is Rs. 60 and the cost of one ticket from Ahmedabad to Vadodara is Rs. 80.

Exercise-3.4

Question 1:

Solve the following pair of linear equations by elimination method

1. $\frac{x}{5}-\frac{y}{3}=\frac{4}{15}, \frac{x}{2}-\frac{y}{9}=\frac{7}{18}$
2. 4x – 19y + 13 = 0, 13x – 23y = -19
3. x + y = a + b, ax – by = $a^2 - b^2$
4. 5ax + 6by = 28; 3ax + 4by = 18

Question 1(1):

Solution :

For $\frac{x}{5} - \frac{y}{3} = \frac{4}{15}$

Multiplying both sides of the equation by 15 we get,

$3x - 5y = 4 \quad \ldots \quad \ldots (1)$

For $\frac{x}{2} - \frac{y}{9} = \frac{7}{18}$

Multiplying both sides of the equation by 18 we get,

$9x - 2y = 7 \quad \ldots \quad \ldots (2)$

Multiplying eq.(1) by 2 and eq.(2) by 5 and subtracting eq. (3) from eq.(4), we get

$6x - 10y = 8 \quad \ldots \quad \ldots (3)$

$\underline{45x - 10y = 35} \quad \ldots \quad \ldots (4)$

$39x = 27$

$\Rightarrow 13x = 9$

$\Rightarrow x = \frac{9}{13}$

Substituting $x = \frac{9}{13}$ in eq.(1)

$3\left(\frac{9}{13}\right) - 5y = 4$

$\therefore 27 - 65y = 52$

$\therefore 65y = -25$

$y = -\frac{25}{65} = -\frac{5}{13}$

Hence, $(x, y) = \left(\frac{9}{13}, -\frac{5}{13}\right)$ is the solution of the given pair of linear equations.

Question 1(2):

Solution :

Here, $4x - 19y + 13 = 0, \ldots \ldots (1)$

$13x - 23y = 19 \ldots \ldots (2)$

Multiplying equation (1) by 13 and equation (2) by 4,

$52x - 247y = -169 \ldots \ldots (3)$

$52x - 92y = -76 \ldots \ldots (4)$

Subtracting equation (3) from equation (4), we get

$155y = 93 \quad \therefore y = \frac{93}{155} = \frac{3}{5}$

Substituting $y = \frac{3}{5}$ in equation (1),

$4x - 19\left(\frac{3}{5}\right) = -13$

$\therefore 20x - 57 = -65$

$\therefore 20x = -8$

$x = -\frac{8}{20} = -\frac{2}{5}$

Hence, $(x, y) = \left(-\frac{2}{5}, \frac{3}{5}\right)$ is the solution of the given pair of linear equations.

Question 1(3):

Solution :

Here, $x + y = a + b \ldots \ldots (1)$

$ac - by = a^2 - b^2 \ldots \ldots (2)$

Multiplying eq. (1) by b we get,

$bx + by = ab + b^2$

Adding eq. (2) and (3),

$(a + b)x = a^2 + ab$

$\therefore (a + b)x = a(a + b)$

$\therefore x = a$

Substituting x = a in eq. (1),

$a + y = a + b \therefore y = b$

Hence (x, y) = (a, b) is the solution of the given pair of linear equations.

Question 1(4):

Solution :

Here, $5ax + 6by = 28$ … … (1)

and $3ax + 4by = 18$ … … (2)

Multiplying eq.(1) by 2 and eq. (2) by 3,

$10ax + 12by = 56$ … … (3)

$9ax + 12by = 54$ … … (4)

Subtracting eq.(4) from eq.(3) we get

$ax = 2 \quad \Rightarrow x = \frac{2}{a}$

Substituting $x = \frac{2}{a}$ in eq. (2)

$3a\left(\frac{2}{a}\right) + 4by = 18$

$\therefore 6 + 4by = 18$

$\therefore 4by = 12$

$\therefore y = \frac{3}{b}$

Thus, $(x, y) = \left(\frac{2}{a}, \frac{3}{b}\right)$ is the solution of the given pair of linear equations.

Question 2:

The sum of two numbers is 35. Four times the larger number is 5 more than 5 times the smaller number. Find these numbers.

Solution :

Let the larger number be x and the smaller number be y.

$\therefore$ The sum of both the numbers = x + y.

But, the sum is given as 35.

$\therefore x + y = 35$ … … (1)

Four times the larger number (x) is 5 more than 5 times the smaller number (y)

$\therefore 4x = 5y + 5$

$\therefore 4x - 5y = 5$ … … (2)

Multiplying eq. (1) by 5 we get,

$5x + 5y = 175$ … … (3)

Adding eq. (2) and (3) we get,

$9x = 180$

$\therefore x = 20$

Substituting x = 20 in eq. (1),

$20 + y = 35$

Hence the larger number is 20 and smaller number is 15.

Question 3:

There are some 25 paise coins and some 50 paise coins in a bag. The total number of coins is 140 and the amount in the bag is ₹ 50. Find the number of coins of each value in the bag.

Solution :

Let the number of 50 paise coins in the bag be x.

Number of 25 paise coins in the bag be y.

Total number of 50 paise and 25 paise coins in the bag = x + y

But this sum is given as 140.

$\therefore x + y = 140$ ..... (1)

Now, the amount of x coins of 50 paise is 50x paise and

the amount of y coins of 25 paise is 25y paise.

∴ The total amount is (50x + 25y) paise.

But, this amount is given as Rs. 50, i.e., 5000 paise.

$\therefore 50x + 25y = 5000$

$\therefore 2x + y = 200$ ......(2)

Subtracting eq. (1) from eq. (2),

We get x = 60.

Substituting x = 60 in eq. (1),

$60 + y = 140$

$\therefore y = 80$

Thus, the number of 50 paise coins is 60 and the its total

amount (in Rs.) is $\frac{50 \times 60}{100}$ = Rs. 30

The number of 25 paise coins in the bag is 80

and its total amount (in Rs.) is $\frac{25 \times 80}{100}$ =Rs. 20.

Question 4:

The sum of the digits of two digit number is 3. The number obtained by interchanging the digits is 9 less than the original number. Find the original number.

Solution :

For a two digit number,

Let the digit in the ten's place be y and the digit in the units place be x.

∴ The number is 10y + x.

Now, the sum of the digits = x + y.

But, this sum is given 3.

$\therefore x + y = 3$ … … (1)

On interchanging the digits,

Digit at ten's place = x and

Digit at units place = y.

∴ The new number formed is 10x + y.

According to the given conditions, this number is 9 less than the original number 10y+x.

∴ Original number – 9 = New number

$10y + x – 9 = 10x + y$

$\therefore -9x + 9y = 9$

$\therefore x – y = -1$ (∵ taking – 9 common) … … (2)

Adding eq. (1) and eq. (2),

$2x = 2 \therefore x = 1$

Substituting x = 1 in eq. (1),

$1 + y – 3 \therefore y = 2$

Thus, for x = 1 and y = 2, the original number

$10y + x = 1 – (2) + 1 = 21$.

Question 5:

The length of a rectangle is twice its breadth. The perimeter of the rectangle is 120 cm. Find the length and breadth of this rectangle. Also find its area.

Solution :

Let the length of the rectangle be x cm and the its breadth be y cm.

Now, according to the given conditions, the length of a rectangle (x) is twice the breadth (y).

$\therefore x = 2y$

$\therefore x - 2y = 0$ … … (1)

The perimeter of the rectangle, i.e. $(2x + 2y)$ cm is given as 120cm.

$\therefore 2x + 2y = 120$ … … (2)

Adding eq. (1) and eq. (2).

$3x = 120 \therefore x = 40$

Substituting $x = 40$ in eq. (1),

$40 - 2y = 0$

$\therefore 2y = 40$

$\therefore y = 20$

Thus, the length(x) of the rectangle is 40 cm and the breadth (y) is 20 cm.

Now, area of a rectangle

= length × breadth

$= (x) \times (y)$

$= 40 \times 20\ cm^2$

$= 800\ cm^2$

Thus, area of the rectangle is $800\ cm^2$.

Question 6:

An employee deposits certain amount at the rate of 8% per annum and a certain amount at the rate of 6% per annum at simple interest. He earns ₹ 500 as annual interest. If he interchanges the amount at the same rates, he earns ₹ 50 more. Find the amounts deposited by him at different rates.

Solution :

Suppose the employee deposits Rs. x at the rate of 8% per annum and Rs. y at the rate of 6% per annum.

We know that the formula for simple interest is $I = \frac{PRN}{100}$.
Now, at the rate of 8%, P = Rs. x, R = 8%, N = 1 year
and I = Interest

$\therefore I = \frac{x \times 8 \times 1}{100}$

$\therefore I = \frac{8x}{100}$

Next, at the rate of 6%, P = y, R = 6%, N = 1 year
and I = interest

$\therefore I = \frac{y \times 6 \times 1}{100}$

$\therefore I = \frac{6y}{100}$

But, total interest for 1 year is Rs.500.

$\therefore \frac{8x}{100} + \frac{6y}{100} = 500$

$\therefore 8x + 6y = 50000$

$\therefore 4x + 3y = 25000 \qquad \ldots \quad \ldots (1)$

By, interchanging the parts of amount, *Rs.* y is at the rate of 8% and Rs. x is at the rate of 6% are arranged.
By interchanging the amount he earns Rs.50 more,
i.e., Rs. 500 + Rs.50 = Rs. 550.

$\therefore \frac{8x}{100} + \frac{6y}{100} = 550$

$\therefore 8x + 6y = 55000$

$\therefore 3x + 4y = 27500 \qquad \ldots \quad \ldots (2)$

Now, adding and subtracting eq. (1) and eq. (2) respectively,
$7x + 7y = 52500$ i.e.,
$x + y = 7500 \qquad \ldots.. (3)$
and $x - y = -2500 \qquad \ldots.. (4)$
adding equation (3) and equation (4),
$2x = 5000$
$\therefore x = 2500$
Substituting x = 2500 in equation (3),
$2500 + y = 7500$
$\therefore y = 5000$
Thus, the employee deposited Rs. 2500 at the rate of 8% and Rs. 5000 at the rate of 6%.

Exercise-3.5

Question 1:

Solve the following pairs of equations by cross multiplication method :

1. 0.3x + 0.4y = 2.5 and 0.5x – 0.3y = 0.3
2. 5x + 8y= 18, 2x – 3y = 1
3. $\frac{x}{3} + \frac{y}{5} = 1$, 7x – 15y = 21
4. 3x + y = 5, 5x + 3y = 3

Question 1(1):

Solution :

Converting the given equation in the standard form,

$0.3x + 0.4y = 2.5$ and $0.5x - 0.3y = 0.3$

$\therefore \frac{3}{10}x + \frac{4}{10}y = \frac{25}{10}$

$\therefore 3x + 4y - 25 = 0 \qquad \ldots \quad \ldots (1)$

Next,

$\therefore \frac{5}{10}x - \frac{3}{10}y = \frac{3}{10}$

$\therefore 5x - 3y - 3 = 0 \qquad \ldots \quad \ldots (2)$

Comparing with the general form,

$a_1 = 3,\ b_1 = 4,\ c_1 = -25$

$a_2 = 5,\ b_2 = -3,\ c_2 = -3$

Applying cross multiplication method to solve the equations,

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\therefore \frac{x}{\begin{matrix} 4 & -25 \\ -3 & -3 \end{matrix}} = \frac{y}{\begin{matrix} -25 & 3 \\ -3 & 5 \end{matrix}} = \frac{1}{\begin{matrix} 3 & 4 \\ 5 & -3 \end{matrix}}$$

$$\therefore \frac{x}{(4)(-3)-(-3)(-25)} = \frac{y}{(-25)(5)-(-3)(3)} = \frac{1}{(3)(-3)-(5)(4)}$$

$$\therefore \frac{x}{-12-75} = \frac{y}{-125+9} = \frac{1}{-9-20}$$

$$\therefore \frac{x}{-87} = \frac{y}{-116} = \frac{1}{-29}$$

$$\therefore x = \frac{-87}{-29} = 3 \text{ and } y = \frac{-116}{-29} = 4$$

Thus, solution of the given pair of linear equations is $(x, y) = (3, 4)$

Question 1(2):

Solution :

First we write the given equations in standard form,

$5x + 8y = 18 \Rightarrow 5x + 8y - 18 = 0 \qquad \ldots\ldots(1)$

$2x - 3y = 1 \Rightarrow 2x - 3y - 1 = 0 \qquad \ldots\ldots(2)$

Comparing with the general form,

$a_1 = 5,\ b_1 = 8,\ c_1 = -18$

$a_2 = 2,\ b_2 = -3,\ c_2 = -1$

Applying cross multiplication method to solve the equations,

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\therefore \frac{x}{\begin{matrix} 8 & -18 \\ -3 & -1 \end{matrix}} = \frac{y}{\begin{matrix} -18 & 5 \\ -1 & 2 \end{matrix}} = \frac{1}{\begin{matrix} 5 & 8 \\ 2 & -3 \end{matrix}}$$

$$\therefore \frac{x}{(8)(-1)-(-3)(-18)} = \frac{y}{(-18)(2)-(-1)(5)} = \frac{1}{(5)(-3)-(-2)(8)}$$

$$\therefore \frac{x}{-8-54} = \frac{y}{-36-5} = \frac{1}{-15-16}$$

$$\therefore \frac{x}{-62} = \frac{y}{-31} = \frac{1}{-31}$$

$$\therefore x = \frac{-62}{-31} = 2 \text{ and } \frac{-31}{-31} = 1$$

Thus, solution of the given pair of linear equations is $(x, y) = (2, 1)$

Question 1(3):

Solution :

First we convert the given equations in the standard form,

$\therefore\ 5x + 3y = 15$

$\therefore\ 5x + 3y - 15 = 0$ & & (1)

and $7x - 15y - 21 = 0$ & & (2)

Comparing with the general form,

$a_1 = 5,\ b_1 = 3,\ c_1 = -15$

$a_2 = 7,\ b_2 = -15,\ c_2 = -21$

Applying cross multiplication method to solve the equations,

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\therefore \frac{x}{\begin{vmatrix} 3 & -15 \\ -15 & -21 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -15 & 5 \\ -21 & 7 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 5 & 3 \\ 7 & -15 \end{vmatrix}}$$

$$\therefore \frac{x}{(3)(-21)-(-15)(-15)} = \frac{y}{(-15)(7)-(-21)(5)} = \frac{1}{(5)(-15)-(7)(3)}$$

$$\therefore \frac{x}{-63-225} = \frac{y}{-105+105} = \frac{1}{-75-21}$$

$$\therefore \frac{x}{-288} = \frac{y}{0} = \frac{1}{-96}$$

$$\therefore x = \frac{-288}{-96} = 3 \text{ and } y = \frac{0}{-96} = 0$$

Thus, solution of the given pair of linear equations is $(x, y) = (3, 0)$

Question 1(4):

Solution :

First we convert the given equations in the standard form,

$3x + y = 5,\ \therefore\ 3x + y - 5 = 0$ ..... (1)

$5x + 3y = 3\ \therefore\ 5x + 3y - 3 = 0$ ..... (2)

Comparing with the general form,

$a_1 = 3,\ b_1 = 1,\ c_1 = -5$

$a_2 = 5,\ b_2 = 3,\ c_2 = -3$

Applying cross multiplication method to solve the equations,

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\therefore \frac{x}{\begin{vmatrix} 1 & -5 \\ 3 & -3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -5 & 3 \\ -3 & 5 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & 1 \\ 5 & 3 \end{vmatrix}}$$

$$\therefore \frac{x}{(1)(-3)-(3)(-5)} = \frac{y}{(-5)(5)-(-3)(3)} = \frac{1}{(3)(3)-(5)(1)}$$

$$\therefore \frac{x}{-3+15} = \frac{y}{-25+9} = \frac{1}{9-5}$$

$$\therefore \frac{x}{12} = \frac{y}{-16} = \frac{1}{4}$$

$$\therefore x = \frac{12}{4} = 3 \text{ and } y = \frac{-16}{4} = -4$$

Thus, solution of the given pair of linear equations is $(x, y) = (3, -4)$

Question 2:

By cross multiplication method, find such a two digit number such that, the digit at unit's place is twice the digit at tens place and the number obtained by interchanging the digits of the number is 36 more than the original number.

Solution :

Let the digit at the tens place of the two digit number be y and the digit in the units place be x.

$\therefore$ The number is $10y + x$.

Now, the digit at units place(x) is twice the digit in the tens place(y)

$\therefore x = 2y$ ..... (1)

By interchanging the digits, the digit in the tens place becomes x and the digit at units place becomes y.

So the new number formed is $10x + y$.

The new number formed is 36 more than the original number.

$\therefore$ Original number + 36 = New number

$\therefore \ 10y + x + 36 = 10x + y$

$\therefore \ -9x + 9y + 36 = 0$

$\therefore \ x - y - 4 = 0 \quad (\because \text{dividing by } -9)$ .....(2)

Converting eq.(1) and eq.(2) in standard form,

$x - 2y = 0$ ..... (3)

and $x - y - 4 = 0$ ...... (4)

Comparing with the general form,

$a_1 = 1,\ b_1 = -2,\ c_1 = 0$

$a_2 = 1,\ b_2 = -1,\ c_2 = -4$

Applying the cross multiplication methods,

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\therefore \frac{x}{\begin{matrix} -2 & 0 \\ -1 & -4 \end{matrix}} = \frac{y}{\begin{matrix} 0 & 1 \\ -4 & 1 \end{matrix}} = \frac{1}{\begin{matrix} 1 & -2 \\ 1 & -1 \end{matrix}}$$

$$\therefore \frac{x}{(-2)(-4)-(-1)(0)} = \frac{y}{(0)(1)-(-4)(1)} = \frac{1}{(1)(-1)-(1)(-2)}$$

$$\therefore \frac{x}{8+0} = \frac{y}{0+4} = \frac{1}{-1+2}$$

$$\therefore \frac{x}{8} = \frac{y}{0} = \frac{1}{1}$$

$$\therefore x = \frac{8}{1} = 8 \text{ and } y = \frac{4}{1} = 4$$

Thus, for $x = 8$ and $y = 4$,

Original number $= 10y + x = 10(4) + 8 = 48$.

Question 3:

The sum of two numbers is 70 and their difference is 6. Find these numbers by cross-multiplication method.

Solution :

Let the larger number be x and the smaller number be y.

$\therefore$ The sum of the larger and the smaller number is $x + y$.

But this sum is given as 70.

$\therefore$ $x + y = 70$ ..... (1)

Also, their difference (subtraction of smaller no. from the larger no.) is given as 6.

$\therefore$ $x - y = 6$ ...... (2)

Converting eq. (1) and eq. (2) in the standard form,

$x + y - 70 = 0$ .... (3)

and $x - y - 6 = 0$ ..... (4)

Comparing with the general from,

$a_1 = 1, b_1 = 1, c_1 = -70$

$a_2 = 1, b_2 = -1, c_2 = -6$

Using cross multiplication method to solve the equations,

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\therefore \frac{x}{\begin{matrix} 1 & -70 \\ -1 & -6 \end{matrix}} = \frac{y}{\begin{matrix} -70 & 1 \\ -6 & 1 \end{matrix}} = \frac{1}{\begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix}}$$

$$\therefore \frac{x}{(1)(-6)-(-1)(-70)} = \frac{y}{(-70)(1)-(-6)(1)} = \frac{1}{(1)(-1)-(1)(1)}$$

$$\therefore \frac{x}{-6-70} = \frac{y}{-70+6} = \frac{1}{-1-1}$$

$$\therefore \frac{x}{-76} = \frac{y}{-64} = \frac{1}{-2}$$

$$\therefore x = \frac{-76}{-2} = 38 \text{ and } y = \frac{-64}{-2} = 32$$

Thus, the required two numbers are 38 and 32.

Question 4:

While arranging certain students of a school in rows containing equal number of students; if three rows are reduced, then three more students have to be arranged in each of the remaining rows. If three more rows are formed, then two students have to be taken off from each previously arranged rows. Find the number of students arranged.

Solution :

Let the number of rows be x and the number of students arranged in each row be y.

$\therefore$ Total number of students is xy.

If three rows are reduced, we have $x – 3$ rows, then three more students i.e. $(y + 3)$ have to be arranged in each of the remaining rows.

Now, the total number of student is the product of the number of new rows and the number of students arranged in each new row.

$\therefore$Total number of students = No. of new rows × No. of students arranged in each new row

$\therefore xy = (x – 3)(y + 3)$

$\therefore xy = xy + 3x – 3y – 9$

$\therefore 3x – 3y – 9 = 0$

$\therefore x – y – 3 = 0$ … … (1)

Three more rows are formed $= (x + 3)$

Also, two students have to be taken off from each row arranged earlier $= (y – 2)$

$\therefore$ Now,

Total number of students = No. of new rows × No. of students arranged in each new row

$\therefore xy = (x – 3)(y – 2)$

$\therefore xy = xy + 2x – 3y – 6$

$\therefore -2x – 3y – 6 = 0$

$\therefore 2x - 3y + 6 = 0 \ \ldots \ldots (2)$

Now, comparing eq.(1) and eq.(2) with the general form,

$a_1 = 1, b_1 = -1, c_1 = -3$

$a_2 = 2, b_2 = -3, c_2 = -6$

Applying cross multiplication method to solve the equations,

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\therefore \frac{x}{\begin{vmatrix} -1 & -3 \\ -3 & 6 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -3 & 1 \\ 6 & 2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}}$$

$$\therefore \frac{x}{(-1)(6)-(-3)(-3)} = \frac{y}{(-3)(2)-(6)(1)} = \frac{1}{(1)(-3)-(2)(-1)}$$

$$\therefore \frac{x}{-6-9} = \frac{y}{-6-6} = \frac{1}{-3+2}$$

$$\therefore \frac{x}{-15} = \frac{y}{-12} = \frac{1}{-1}$$

$$\therefore x = \frac{-15}{-1} = 15 \quad \text{and} \quad y = \frac{-12}{-1} = 12$$

Thus, the total number of students = xy = 15 × 12 = 180.

Question 5:

In ΔABC, the measure of ∠B is thrice to the measure of ∠C and the measure of ∠A is $\frac{1}{2}$ the sum of the measures of ∠B and ∠C. Find the measures of all the angles of ΔABC and also state the type of this triangle.

Solution :

In ΔABC, let m∠A = x and m∠B = y

But, for Δ ABC, m∠A + m∠B + m∠C = 180°

∴ x + y + m∠C = 180°

∴ m∠C = 180 – x – y

Now, the measure of ∠B is thrice the measure of ∠C.

∴ y = 3(180 – x – y)

∴ y = 540 – 3x – 3y

∴ 3x + 4y = 540 … … (1)

Also, measure of ∠A is half the sum of the measures of ∠B and ∠C.

$$\therefore m\angle A = \frac{1}{2}(m\angle B + m\angle C)$$

$$\therefore x = \frac{1}{2}$$

∴ 2x = 180 – x

∴ 3x = 180

∴ x = 60

Substituting x = 60 in eq.(1),

3(60) + 4y = 540

∴ 180 + 4y = 540

∴ 4y = 540 – 180

∴ 4y = 360

∴ y = 90

Now, m∠A + m∠B + m∠C = 180°

∴ x + y + m∠C = 180

∴ 60 + 90 + m∠C = 180

∴ m∠C = 180 – 150 = 30

Thus, m∠A = x = 60°, m∠B = y = 90° and also m∠C = 30° and m∠B = 90°

∴ ΔABC is a right angled triangle.

Exercise-3.6

Question 1:

Solve the following pairs of linear equations :

1. $\frac{5}{2x}+\frac{2}{3y}=7, \frac{3}{x}+\frac{2}{y}=12$, X ≠ 0, y ≠ 0
2. 2x + 3y = 2xy, 6x + 12y = 7xy
3. $\frac{4}{x-1}+\frac{5}{y-1}=2, \frac{8}{x-1}+\frac{15}{y-1}=3$, X ≠ 1, y ≠ 1
4. $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}, \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$, 3x + y ≠ 0, 3x – y ≠ 0
5. $\frac{3}{\sqrt{x}}+\frac{4}{\sqrt{y}}=2, \frac{5}{\sqrt{x}}+\frac{7}{\sqrt{y}}=\frac{41}{12},$ x > 0, y > 0

Question 1(1):

Solution :

Here, $\frac{5}{2x}+\frac{2}{3y}=7$ and $\frac{3}{x}+\frac{2}{y}=12$ are not in the general form.

Substituting $\frac{1}{x}$ = a and $\frac{1}{y}$ = b in both the equations we get,

15a + 4b = 42 … … (1)

3a + 2b = 12 … … (2)

Multiplying equation (2) by 2,

6a + 4b = 24

Now, subtracting eq.(3) from eq.(1) we get,

9a = 18

∴ a = 2

Substituting a = 2 in equation (2),

3(2) + 2b = 12

∴ 6 + 2b = 12 ∴ 2b = 6 ∴ b = 3

Hence, $\frac{1}{x}$ = a = 2 and $\frac{1}{y}$ = b = 3

∴ x = $\frac{1}{2}$ and y = $\frac{1}{3}$

Thus, the solution of the given pair of equations is (x, y) = $\left(\frac{1}{2}, \frac{1}{3}\right)$.

Question 1(2):

Solution :

Here, both $2x + 3y = 2xy$ and $6x + 12y = 7xy$ are satisfied by substituting $x = 0$ and $y = 0$.
$\therefore$ (0, 0) is the solution of the given pair of equations.

Suppose, $x \neq 0$, $y \neq \sqrt{a^2 + b^2}0$
$\therefore xy \neq 0$
Dividing each term of the equations by $xy \neq 0$, we get

$\frac{2x}{xy} + \frac{3y}{xy} = \frac{2xy}{xy}$ i.e., $\frac{2}{y} + \frac{3}{x} = 2$ ... ... (1)

$\frac{6x}{xy} + \frac{12y}{xy} = \frac{7xy}{xy}$ i.e., $\frac{6}{y} + \frac{12}{x} = 7$ ... ... (2)

The given pair of equations are not linear.

Substituting $\frac{1}{x} = a$ and $\frac{1}{y} = b$ in both the equations we get,

$2b + 3a = 2$ i.e., $3a + 2b = 2$ ... ... (3)
and $6b + 12a = 7$ i.e.,
$12a + 6b = 7$ ... ... (4)
Multiplying eq. (3) by 3 and then subtracting it from eq. (4),
$3a = 1$

$a = \frac{1}{3}$

Substituting $a = \frac{1}{3}$ in eq. (3),

$3\left(\frac{1}{3}\right) + 2b = 2$

$\therefore 2b = 2 - 1 = 1$

$\therefore b = \frac{1}{2}$

Hence, $\frac{1}{x} = a = \frac{1}{3}$ and $\frac{1}{y} = b = \frac{1}{2}$

$\therefore x = 3$ and $y = 2$
Thus, the solutions of the given pair of equations are $(x, y) = (0, 0)$ and $(x, y) = (3, 2)$.

$\therefore$ Solution set: {(0, 0), (3, 2)}.

Question 1(3):

Solution :

Here, $\frac{4}{x-1}+\frac{5}{y-1} = 2$

and, $\frac{8}{x-1}+\frac{15}{y-1} = 3$ are not linear equations.

Taking $x - 1 = a$ and $y - 1 = b$ and subs. in eq. (1) and (2) we get,

$4a + 5b = 2$ ... ...(3)

$8a + 15b = 3$ ... ...(4)

Multiplying equation (3) by 2 and then subtracting it from equation (4),

$5b = -1$

$\therefore b = -\frac{1}{5}$

Substituting $b = -\frac{1}{5}$ in equation (1),

$4a + 5\left(-\frac{1}{5}\right) = 2$

$\therefore 4a - 1 = 2$

$\therefore 4a = 3$

$\therefore a = \frac{3}{4}$

Now, $\frac{1}{x-1} = a = \frac{3}{4}$ and $\frac{1}{y-1} = b = -\frac{1}{5}$

$\therefore x - 1 = \frac{4}{3}$ and $y - 1 = -5$

$\therefore 3x - 3 = 4$ and $y = -5 + 1$

$\therefore 3x = 7$ and $y = -4$

$\therefore x = \frac{7}{3}$ and $y = -4$

Thus, the solution set is $(x, y) = \left(\frac{7}{3}, -4\right)$.

Question 1(4):

Solution :

Here, $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ and, $\frac{1}{2(3x+y)}+\frac{1}{2(3x-y)}=\frac{-1}{8}$

are not the pair of linear equations.

Substituting $\frac{1}{3x+y}$ = a and $\frac{1}{3x-y}$ = b in both the equations we get,

$a+b=\frac{3}{4}$ and $\frac{a}{2}+\frac{a}{2}=-\frac{1}{8}$ which are linear equations.

Now, $a+b=\frac{3}{4}$ $\quad\therefore 4a+4b=3 \quad \ldots \quad \ldots (1)$

and $\frac{a}{2}+\frac{a}{2}=-\frac{1}{8}$ $\quad\therefore 4a-4b=-1 \quad \ldots \quad \ldots (2)$

Adding eq.(1) and eq.(2) we get,

$8a=2$

$\therefore a=\frac{1}{4}$

Substituting $a=\frac{1}{4}$ in equation (1),

$4\left(\frac{1}{4}\right)+4ab=3$

$\therefore 1+4b=3$

$\therefore 4b=2$

$\therefore b=\frac{1}{2}$

Now, $\frac{1}{3x+y}=a=\frac{1}{4}$ and $\frac{1}{3x-y}=b=\frac{1}{2}$

$3x+y=4$ and $3x-y=2$

Thus, $3x+y=4 \quad \ldots \quad \ldots (3)$

and $3x-y=2 \quad \ldots \quad \ldots (4)$

Adding eq.(3) and eq.(4) we get,

$6x=6$

$x=1$

Now, substituting $x=1$ in equation (3),

$3(1)+y=4$

$y=1$

Thus, the solution set is $(x, y) = (1, 1)$.

Question 1(5):

Solution :

Here, $\dfrac{3}{\sqrt{x}}+\dfrac{4}{\sqrt{y}}=2$ and $\dfrac{5}{\sqrt{x}}+\dfrac{7}{\sqrt{y}}=\dfrac{41}{12}$

are not a pair of linear equations.

Substituting $\dfrac{1}{\sqrt{x}}=a$ and $\dfrac{1}{\sqrt{y}}=b$ in both the equations we get,

$3a+4b=2 \quad \ldots \quad \ldots (1)$

and $5a+7b=\dfrac{41}{12}$

$60a+84b=41 \quad \ldots \quad \ldots (2)$

Multiplying eq.(1) by 20 and then subtracting it from eq.(2),

$4b=1$

$\therefore b=\dfrac{1}{4}$

Substituting $b=\dfrac{1}{4}$ in equation (1),

$3a+4\left(\dfrac{1}{4}\right)=2$

$\therefore 3b+1=2$

$\therefore a=\dfrac{1}{3}$

Now, $\dfrac{1}{\sqrt{x}}=a=\dfrac{1}{3}$ and $\dfrac{1}{\sqrt{y}}=b=\dfrac{1}{4}$

$\therefore \sqrt{x}=3$ and $\sqrt{y}=4$

$\therefore x=9$ and $y=16$

Thus, the solution set is $(x, y)=(9, 16)$.

Question 2:

5 women and 2 men together can finish an embroidary work in 4 days, while 6 women and 3 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work. Also find the time taken by 1 man alone to finish the work.

Solution :

Suppose x days are taken to finish the work by a woman
and y days are taken to finish the work by a man independently.

$\therefore$ A woman can finish $\frac{1}{x}$ of the work in a day and a man can finish $\frac{1}{y}$ of the work in a day.

So 5 women can finish $\frac{5}{x}$ of the work in a day and
2 men can finish $\frac{2}{y}$ of the work in a day.

$\therefore$ 5 women and 2 men can finish total $\left(\frac{5}{x}+\frac{2}{y}\right)$ of the work in a day. But 5 woman and 2 men together can finish an embroidery work in 4 days.

$$\therefore 4\times\left(\begin{array}{c}\text{work done by 5 woman}\\ \text{and 2 man in a day}\end{array}\right) = \text{Complete work}$$

$$\therefore 4\times\left(\frac{5}{x}+\frac{2}{y}\right)=1$$

$$\therefore \frac{5}{x}+\frac{2}{y}=\frac{1}{4} \qquad \ldots\ \ldots (1)$$

Also, 6 women and 3 men can finish the same work in 3 days.

$$\therefore 3\times\left(\frac{6}{x}+\frac{3}{y}\right) = 1 \qquad (\because \text{According to above discussion})$$

$$\therefore \frac{6}{x} + \frac{3}{y} = \frac{1}{3} \qquad \ldots\ \ldots (2)$$

But eq. (1) and eq. (2) are not linear equations.

Substituting $\frac{1}{x}$ = a and $\frac{1}{y}$ = b in both the equations we get,

$$5a + 2b = \frac{1}{4} \quad 20a + 8b = 1 \qquad \ldots\ \ldots (3)$$

$$\text{and } 6a + 3b = \frac{1}{3} \quad 18a + 9b = 1 \qquad \ldots\ \ldots (4)$$

which are linear equations.
Comparing eq. (3) and eq.(4) with the general form,we get,
$a_1 = 20,\ b_1 = 8,\ c_1 = -1$
$a_2 = 18,\ b_2 = 9,\ c_2 = -1$
Applying cross multiplication method,

$$\frac{a}{\begin{array}{cc} b_1 & c_1 \\ b_2 & c_2 \end{array}} = \frac{b}{\begin{array}{cc} c_1 & a_1 \\ c_2 & a_2 \end{array}} = \frac{1}{\begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array}}$$

$$\therefore \frac{a}{\begin{array}{cc} 8 & -1 \\ 9 & -1 \end{array}} = \frac{b}{\begin{array}{cc} -1 & 20 \\ -1 & 18 \end{array}} = \frac{1}{\begin{array}{cc} 20 & 8 \\ 18 & 9 \end{array}}$$

$$\therefore \frac{a}{(8)(-1)-(9)(-1)} = \frac{b}{(-1)(18)-(-1)(20)} = \frac{1}{(20)(9)-(18)(8)}$$

$$\therefore \frac{a}{-8+9} = \frac{b}{-18+20} = \frac{1}{180-144}$$

$$\therefore \frac{a}{1} = \frac{b}{2} = \frac{1}{36}$$

$$\therefore a = \frac{1}{36} \text{ and } b = \frac{1}{18}$$

Hence $\frac{1}{x} = a = \frac{1}{36}$ and $\frac{1}{y} = b = \frac{1}{18}$
$\therefore$ x = 36 and y = 18

Thus, the time taken by 1 woman to finish the given work is 36 days and the time taken by 1 man to finish the same work is 18 days.

Question 3:

A boat goes 21 km upstream and 18 km downstream in 9 hours. In 13 hours, it can go 30 km upstream and 27 km downstream. Determine the speed of the stream and that of the boat in still water. (Speed of boat in still water is more than the speed of the stream of river.)

Solution :

Suppose the speed of the boat in still water is x km/hr and the speed of the stream is y km/hr, where $x > y$.
The speed of the boat downstream becomes $(x + y)$ km/hr.
And the speed of the boat upstream becomes $(x - y)$ km/hr.

Now, $\text{time} = \dfrac{\text{distance}}{\text{speed}} \quad \left(\because \text{speed} = \dfrac{\text{distance}}{\text{time}}\right)$

So, suppose, time taken by the boat when it goes 21 km upstream is t_1 (in hours).

$\therefore t_1 = \dfrac{21}{x-y}$ hours

Suppose, time taken by the boat when it goes 18 km downstream is t_2(in hours).

$\therefore t_2 = \dfrac{18}{x+y}$ hours

Also, it is given that the total time taken is 9 hours

$\therefore t_1 + t_2 = 9.$

$\therefore \dfrac{21}{x-y} + \dfrac{18}{x+y} = 9 \qquad \ldots \ldots (1)$

Similarly, in 13 hours, the boat goes 30 km upstream and 27 km downstream.

$\therefore \dfrac{30}{x-y} + \dfrac{27}{x+y} = 13 \qquad \ldots \ldots (2)$

Substituting $\dfrac{1}{x-y} = a$ and $\dfrac{1}{x+y} = b$ in both the equations, we get,

$21a + 18b = 9$ i.e., $7a + 6b = 3 \qquad \ldots \ldots (3)$

and $30a + 27b = 13 \qquad \ldots \ldots (4)$

Comparing eq.(3) and eq.(4) with the general form, we get

$a_1 = 7,\ b_1 = 6,\ c_1 = -3$

$a_2 = 30,\ b_2 = 27,\ c_2 = -13$

$$\frac{a}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{b}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\therefore \frac{a}{\begin{vmatrix} 6 & -3 \\ 27 & -13 \end{vmatrix}} = \frac{b}{\begin{vmatrix} -3 & 7 \\ -13 & 30 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 7 & 6 \\ 30 & 27 \end{vmatrix}}$$

$$\therefore \frac{a}{(6)(-13)-(27)(-3)} = \frac{b}{(-3)(30)-(-13)(7)} = \frac{1}{(7)(27)-(30)(6)}$$

$$\therefore \frac{a}{-78+81} = \frac{b}{-90+91} = \frac{1}{189-180}$$

$$\therefore \frac{a}{3} = \frac{b}{1} = \frac{1}{9}$$

$$\therefore a = \frac{3}{9} = \frac{1}{3} \text{ and } b = \frac{1}{9}$$

$$\therefore \frac{1}{x-y} = a = \frac{1}{3} \text{ and } \frac{1}{x+y} = b = \frac{1}{9}$$

$x - y = 3 \qquad \ldots \ldots (5)$

and $x + y = 9 \qquad \ldots \ldots (6)$

Adding eq.(5) and eq.(6), we get,

$2x = 12$

$\therefore x = 6$

Substituting $x = 6$ in eq.(6),

$6 + y = 9$

$\therefore y = 3$

Thus, the speed of the boat in still water (x) is 6 km/hr and speed of the stream (y) is 3 km/hr.

Question 4:

Solve the following pair of equations by cross multiplication method :

$$\frac{4x+7y}{xy} = 16, \frac{10x+3y}{xy} = 11, x \neq 0, y \neq 0$$

Solution :

$\frac{4x}{xy} + \frac{7y}{xy} = 16$ i.e., $\frac{4}{y} + \frac{7}{x} = 16$ and

$\frac{10x}{xy} + \frac{3y}{xy} = 11$ i.e., $\frac{10}{y} + \frac{3}{x} = 11$

It can be seen that the above equations are not linear equations.

Substituting $\frac{1}{x} = a$ and $\frac{1}{y} = b$ in both the equations, we get

$4b + 7a = 16 \quad \therefore 7a + 4b - 16 = 0 \quad \ldots \ldots (1)$

and $10b + 3a = 11 \quad \therefore 3a + 10b - 11 = 0 \quad \ldots \ldots (2)$

Comparing with the general from, we get

$a_1 = 7, b_1 = 4, c_1 = -16$

$a_2 = 3, b_2 = 10, c_2 = -11$

Applying cross multiplication method,

$$\frac{a}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{b}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\therefore \frac{a}{\begin{matrix} 4 & -16 \\ 10 & -11 \end{matrix}} = \frac{b}{\begin{matrix} -16 & 7 \\ -11 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 7 & 4 \\ 3 & 10 \end{matrix}}$$

$$\therefore \frac{a}{(4)(-11)-(10)(-16)} = \frac{b}{(-16)(3)-(-11)(7)} = \frac{1}{(7)(10)-(3)(4)}$$

$$\therefore \frac{a}{-44+160} = \frac{b}{-48+77} = \frac{1}{70-12}$$

$$\therefore \frac{a}{116} = \frac{b}{29} = \frac{1}{58}$$

$$\therefore a = \frac{116}{58} = 2 \text{ and } b = \frac{29}{58} = \frac{1}{2}$$

Hence, $\therefore \frac{1}{x} = a = 2$ and $\frac{1}{y} = b = \frac{1}{2}$

$\therefore x = \frac{1}{2}$ and $y = 2$

Thus, the solution set is $(x, y) = \left(\frac{1}{2}, 2\right)$

Question 5:

Mahesh travels 250 km to his home partly by train and partly by bus. He takes 6 hours if he travels 50 km by train and remaining distance by bus. If he travels 100 km by train and remaining distance by bus, he takes 7 hours. Find the speed of the train and the bus separately.

Solution :

Suppose, the average speed of the train is x km/hr and the average speed of the bus is y km/hr.

Case 1:

If Mahesh travels 50 km by train and the remaining (250 – 50) km, i.e., 200 km by bus, then he takes 6 hours to reach home.

Time taken by the train to travel 50 km is $\frac{50}{x}$ hours. $\left(\because t = \frac{d}{s}\right)$

Time taken by the bus to travel 200km is $\frac{200}{y}$ hours. $\left(\because t = \frac{d}{s}\right)$

But, in this case, time taken to reach home is 6 hours.

$\therefore \frac{50}{x} + \frac{200}{y} = 6 \qquad \ldots \quad \ldots (1)$

Case 2 :

If Mahesh travels 100 km by train and the remaining (250 – 100) km, i.e. 150 km by bus, then he takes 7 hours to reach home.

Time taken by the train to travel 100 km is $\frac{100}{x}$ hours. $\left(\because t = \frac{d}{s}\right)$

Time taken by the bus to travel 150 km is $\frac{150}{y}$ hours. $\left(\because t = \frac{d}{s}\right)$

But, in this case time taken to reach home is 7 hours.

$\therefore \frac{100}{x} + \frac{150}{y} = 7 \qquad \ldots \quad \ldots (2)$

Here eq.(1) and (2) are not linear.

Let $\frac{1}{x}$ = a and $\frac{1}{y}$ = b,

Here eq.(1) and (2) are not linear.

Let $\frac{1}{x}$ = a and $\frac{1}{y}$ = b,

$50a + 200b = 6 \qquad \ldots \quad \ldots (3)$

and $100a + 150b = 7 \qquad \ldots \quad \ldots (4)$

Multiplying eq.(3) by 2 and subtracting eq.(4) from it,

$250b = 5$

$\therefore b = \frac{1}{50}$

Substituting $b = \frac{1}{50}$ in eq.(3)

$50a + 200\left(\frac{1}{50}\right) = 6$

$\therefore 50a + 4 = 6$

$\therefore 50a = 2$

$\therefore a = \frac{1}{25}$

Now, $\frac{1}{x} = a = \frac{1}{25}$ and $\frac{1}{y} = b = \frac{1}{50}$

$\therefore x = 25$ and $y = 50$

Thus, average speed of the train is 25 km/hour and average speed of the bus is 50 km/hour.

Exercise-3

Question 1:

Obtain a pair of linear equations from the following information :

“The rate of tea per kg is seven times the rate of sugar per kg. The total cost of 2 kg tea and 5 kg sugar is ₹ 570.”

Solution :

Suppose, the rate of tea per kg is Rs. x and the rate of sugar per kg is Rs.y.

The rate per kg of tea (x) is seven times the rate per kg of sugar (y).

∴ $x = 7y$ i.e., $x - 7y = 0$ … … (1)

The cost of 2 kg of tea is Rs. 2x and the cost of 5 kg of sugar is Rs.5y.

∴ Total cost becomes Rs. (2x+5y).

But, this cost is given to be Rs. 570.

∴ $2x + 5y = 570$ … … (2)

Thus, equations (1) and (2) represent a pair of linear equation in two variables.

Question 2:

Draw the graphs of the pair of linear equations in two variables. $x + 3y = 6$, $2x - y = 5$. Find its solution set.

Solution :

$x + 3y = 6$

∴ $3y = 6 - x$

$$\therefore y = \frac{6-x}{3}$$

$$\text{For } x = 0,\ y = \frac{6-0}{3} = 2$$

$$\text{For } x = 6,\ y = \frac{6-6}{3} = 0$$

x	0	6
y	2	0

∴ Plot the ordered pair (0, 2) and (6, 0) of the equation $x + 3y = 6$ on the graph paper and draw a line joining them.

$2x - y = 5$

∴ $y - 2x - 5$

For $x = 0$, $y = 2(0) - 5 = -5$

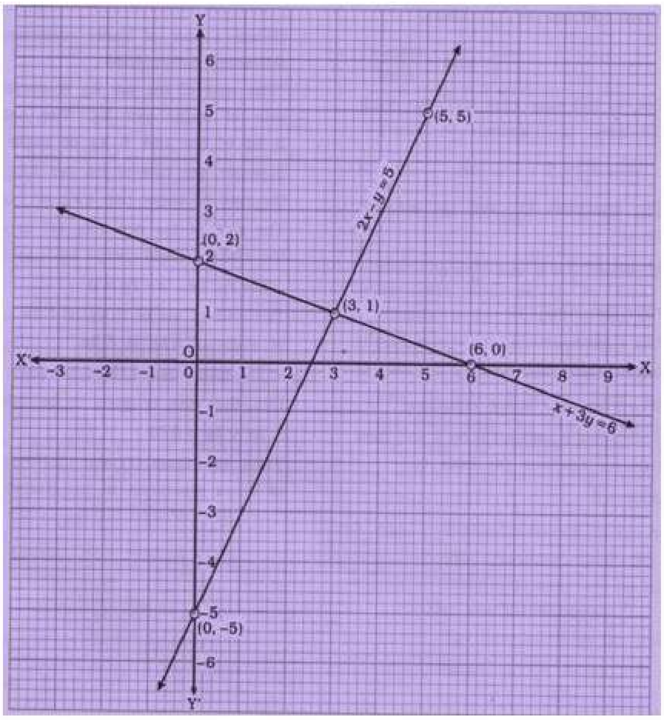
For $x = 5$, $y = 2(5) - 5 = 10 - 5 = 5$

x	0	5
y	-5	5

∴ Plot the ordered pairs (0, -5) and (5, 5) of equation $2x - y = 5$ on the graph and draw a line joining them.

The intersection point a (common point) of these two lines is (3, 1) which satisfies both the equations.

Thus, the solution set of the given pair of linear equations is {(3, 1)}.



Question 3:

Solve the following pair of equations by the method of elimination :

$$\frac{4}{x}+\frac{5}{y}=7, \quad \frac{5}{x}+\frac{4}{y}=\frac{13}{2}$$

Solution :

Here, $\frac{4}{x}+\frac{5}{y}=7$ and $\frac{5}{x}+\frac{4}{y}=\frac{13}{2}$ do not form a pair of linear equations.

Substituting $\frac{1}{x}=a$ and $\frac{1}{y}=b$ in both the equations, we get

$4a + 5b = 7 \quad \dots \dots (1)$

and $5a + 4b = \frac{13}{2}$

$\therefore$ 10a + 8b = 13 … … (2)

Multiplying eq.(1) by 5 and eq.(2) by 2,

20a + 25b = 35 … … (3)

20a + 16b = 26 … … (4)

Subtracting eq.(4) from eq.(3),

9b = 9 $\therefore$ b = 1

Substituting b = 1 in eq. (1),

4a + 5(1) = 7

$\therefore 4a = 2 \quad \therefore a = \frac{1}{2}$

Hence, $\frac{1}{x} = a = \frac{1}{2}$ and $\frac{1}{y} = b = 1$

$\therefore$ x = 2 and y = 1

Thus, the solution set is (x, y) = (2, 1).

Question 4:

Solve the following pair of linear equations by the method of cross-multiplication :

$(a + b)x + (a – b)y = a^2 + 2ab – b^2$, $a \neq b$

$(a – b)(x + y) = a^2 – b^2$, $a \neq b$

Solution :

Converting in the standard from,

$(a + b)x + (a - b)y + b^2 - 2ab - a^2 = 0$ and

$(a - b)(x + y) = a^2 - b^2$

$\therefore (a - b)(x + y) = (a - b)(a + b)$

$\therefore x + y = a + b$ ($\because a \neq b$ so dividing by $a - b \neq 0$)

Hence, $(a + b)x + (a - b)y + b^2 - 2ab - a^2 = 0$ … … (1)

$x + y - (a + b) = 0$ … … (2)

Comparing the equations with the general form,

$a_1 = a + b$, $b_1 = a - b$, $c_1 = b^2 - 2ab - a^2$, $a_2 = 1$, $b_2 = 1$, $c_2 = (a + b)$

Applying cross multiplication method,

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

$$\therefore \frac{x}{\begin{matrix} a-b & b^2-2ab-a^2 \\ 1 & -(a+b) \end{matrix}} = \frac{y}{\begin{matrix} b^2-2ab-a^2 & a+b \\ -(a+b) & 1 \end{matrix}} = \frac{1}{\begin{matrix} a+b & a-b \\ 1 & 1 \end{matrix}}$$

$$\therefore \frac{x}{-(a-b)(a+b)-(1)(b^2-2ab-a^2)} = \frac{y}{(b^2-2ab-a^2)(1)+(a+b)(a+b)} = \frac{1}{(a+b)(1)-(1)(a-b)}$$

$$\therefore \frac{x}{-(a^2-b^2)-b^2+2ab+a^2} = \frac{y}{b^2-2ab-a^2+b^2+2ab+a^2} = \frac{1}{a+b-a+b}$$

$$\therefore \frac{x}{2ab} = \frac{y}{2ab^2} = \frac{1}{2b}$$

$$\therefore x = \frac{2ab}{2b} \text{ and } y = \frac{2b^2}{2b}$$

$\therefore x = a$ and $y = b$

Thus, solution of the given pair of linear equations is $(x, y) = (a, b)$.

Question 5:

Solve the following pair of equations :

$\frac{4}{x+1} + \frac{5}{y+2} = 2, \quad \frac{10}{x+1} + \frac{14}{y+2} = \frac{9}{2}$, $x \neq -1$, $y \neq -2$

Solution :

Here, $\frac{4}{x+1} + \frac{7}{y+2} = 2$ and $\frac{10}{x+1} + \frac{14}{y+2} = \frac{9}{2}$ is not a pair linear equations.

Substituting $\frac{1}{x+1} = a$ and $\frac{1}{y+2} = b$ in both the equations, we get

$4a+7b=2$ … … (1)

and $10a + 14b = \frac{9}{2}$ i.e.,

$20a + 35b = 10$ … … (2)

Multiply equation (1) by 5,

$20a + 35b = 10$ … … (3)

Subtracting equation (2) from equation (3),

$7b = 1 \quad \therefore b = \frac{1}{7}$

Substituting $b = \frac{1}{7}$ in equation (1),

$4a + 7\left(\frac{1}{7}\right) = 2$

$\therefore 4a + 1 = 2$

$\therefore a = \frac{1}{4}$

Hence $\frac{1}{x+1} = a = \frac{1}{4}$ and $\frac{1}{y+2} = b = \frac{1}{7}$

$\therefore x + 1 = 4$ and $y + 2 = 7$

$\therefore x = 3$ and $y = 5$

Thus, the solution of the given pair of equations is $(x, y) = (3, 5)$.

Question 6:

The difference between two natural numbers is 6. Adding 10 to the twice of the larger number, we get 2 less than 3 times of the smaller number. Find these numbers.

Solution :

Let x be the larger natural number and y be the smaller number from the two natural numbers. The difference between the two natural number is 6.

$\therefore x – y = 6$ … … (1)

Adding 10 to twice the larger number(x), we get 2x+10; and 3 times the smaller number is 3y.

But, from the given data, adding 10 to twice the larger number, we get 2 less than 3 times the smaller number.

$\therefore 2x + 10 = 3y – 2$

$\therefore 2x – 3y = -12$ … … (2)

Multiplying eq. (2) from eq.(3),

$2x – 2y = 12$

Substituting y = 24 in eq. (1),

$x – 24 = 6 \therefore x = 30$

Thus, the required larger natural number(x) is 30 and the smaller natural number is 24.

Question 7:

The area of a rectangle gets increased by 30 square units, if its length is reduced by 3 units and breadth is increased by 5 units. If we increase the length by 5 units and reduce the breadth by 3 units then the area of a rectangle reduces by 10 square units. Find the length and breadth of the rectangle.

Solution :

Let the length of the rectangle = x units

Breath of the rectangle = y units.

Now,

Area of the rectangle = length × breadth = xy $(\text{unit})^2$

If the length of the rectangle is reduced by 3 units, then the length becomes (x – 3) units and if the breadth of the rectangle is increased by 5 units, then the breadth become (y + 5) units.

Now the area of rectangle increases by 30 square units,

$\therefore (x – 3)(y + 5) = xy + 30$

$\therefore xy + 5x – 3y – 15 = xy + 30$

$\therefore 5x – 3y = 45$ … … (1)

Now, if the length of the rectangle is increased by 5 units, then the length becomes (x + 5) units; and if the breadth of the rectangle is reduced by 3 units, then the breadth becomes (y – 3) units.

Now the area of the rectangle decreases by 10 square units,

$\therefore (x + 5)(y – 3) = xy – 10$

$\therefore xy – 3x + 5y – 15 = xy – 10$

$\therefore -3 + 5y = 5$

$\therefore 3x – 5y = -5$ … … (2)

Now, multiplying eq.(1) by 3 and eq.(2) by 5 and then subtracting them,

$16y = 160$

$\therefore y = 10$

Substituting y – 10 in eq.(1),

$5x – 3(10) = 45$

$\therefore 5x – 30 = 45$

$\therefore 5x = 75$

$\therefore x = 15$

Thus, the length (x) of the rectangle is 15 units and its breadth (y) is 10 units.

Question 8:

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. Yash takes food for 25 days. He has to pay ₹ 2200 as hostel charges where as Niyati takes food for 20 days. She has to pay ₹ 1800 as hostel charges. Find the fixed charges and the cost of food per day.

Solution :

Suppose the fixed (constant) hostel charge of a student is Rs. x and food-charge per day is Rs. y.

Yash ate food for 25 days and his total charge is Rs.2200.

∴ Fixed hostel charge + 25 × food – charge per day= Rs. 2200.

$\therefore x + 25y = 2200$ … … (1)

Similarly, Niyati ate food for 20 days and her total charge is Rs. 1800.

$\therefore x + 20y = 1800$ … … (2)

Subtracting eq. (2) from eq.(1),

$5y = 400$ $y = 80$

Substituting $y = 80$ in eq.(2),

$x + 20(80) = 1800$

$\therefore x + 1600 = 1800$

$\therefore x = 200$

Thus, the fixed charges (x) is Rs. 200 and the food-charge per day is Rs. 80.

Question 9:

A fraction becomes when 2 is subtracted from the numerator and denominator it becomes when 5 is added to its denominator and numerator, find the fraction.

Solution :

Suppose, the numerator of the fraction is x and its denominator is y.

If 2 is subtracted from both the numerator and the denominator,

New denominator $= \dfrac{x-2}{y-2}$.

But, reduced from of this fraction is $\dfrac{2}{5}$

$$\therefore \frac{x-2}{y-2} = \frac{2}{5}$$

$\therefore 5x - 10 = 2y - 4$

$\therefore 5x - 2y = 6$ … … (1)

If 5 is added to both the numerator and the denominator, then the new fraction becomes,

$\frac{x+5}{y+5}$

But, the reduced from of this fraction is $\frac{3}{4}$,

$\therefore \frac{x+5}{y+5} = \frac{3}{4}$

$\therefore 4x + 20 = 3y + 15$

$\therefore 4x - 3y = -5$ ... ... (2)

Multiplying eq.(1) by 3 and eq.(2) by 2,

$15x - 6y = 18$ ... ... (3)

and $8x - 6y = -10$ ... ... (4)

Subtracting eq. (4) from eq.(3),

$7x = 28$

$\therefore x = 4$

Substituting $x = 4$ in eq.(1),

$5(4) - 2y = 6$

$\therefore 20 - 2y = 6$

$\therefore 2y = 14$

$\therefore y = 7$

Thus, $x = 4$, $y = 7$ and the original fraction= $\frac{x}{y} = \frac{4}{7}$.

Question 10:

Select a proper option (a), (b), (c) or (d) from given options :

Question 10(1):

The solution set of x – 3y = 1 and 3x + y = 3 is …..

Solution :

c. {(1, 0)}

Here, it is clear that, $\frac{a_1}{a_2}\left(=\frac{1}{3}\right) \neq \frac{b_1}{b_2}\left(=\frac{-3}{1}\right)$.

So, we can find the unique solution by substituting each option till we get the one which satisfies both the equations,

(1, 0) satisfies both the equations.

Question 10(2):

The solution set of 2x + y = 6 and 4x + 2y = 5 is ……

Solution :

c. Ø

Comparing $2x + y = 6$ and $4x + 2y = 5$ i.e., $2x + y = \frac{5}{2}$,

i.e. $6 = \frac{5}{2}$ which is not possible,

$\therefore$ The solution is a Ø set.

Question 10(3):

To eliminate x, from 3x + y = 7 and -x + 2y = 2 second equation is multiplied by……

Solution :

c. 3

The second equation is multiplied by 3. Thus in both the equations, the coefficient of x becomes the same. (without taking sign into consideration)

Question 10(4):

If 2x + 3y = 7 and 3x + 2y = 3, then x – y = …..

Solution :

b. -4

Here, $2x + 3y = 7$ … … (1)

$3x + 2y = 3$ … … (2)

Subtracting equation (1) from equation (2),

$x - y = -4$

Question 10(5):

If the pair of linear equations ax + 2y = 7 and 2x + 3y = 8 has a unique solution, then a ≠ …….

Solution :

c. $\frac{4}{3}$

Here, $ax + 2y = 7$ i.e.,

$ax + 2y - 7 = 0$ … … (1)

and $2x + 3y = 8$ i.e.,

$2x + 3y - 8 = 0$ … … (2)

Comparing with the general from,

$a_1 = a,\ b_1 = 2,\ c_1 = -7$

$a_2 = 2,\ b_2 = 3,\ c_2 = -8.$

If the equation has a unique solution,

Then

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\therefore \frac{a}{2} \neq \frac{2}{3}$$

$$\therefore a \neq \frac{4}{3}$$

Question 10(6):

The pair of linear equations 2x + y – 3 = 0 and 6x + 3y = 9 has………

Solution :

d. infinitely many solutions

For $2x + y - 3 = 0$ … … (1)

and $6x + 3y - 9$ i.e.,

$2x + y - 3 = 0$ (dividing by 3) … … (2)

∴The equations are same and hence have infinitely many solutions.

Question 10(7):

If in a two digit number, the digit at unit place is x and the digit at tens place is 5, then the number is……

Solution :

c. x + 50

Let the digit in the unit's place be x and the digit in the ten's place be 5. So, the number is

$10(5) + x = 50 + x = x + 50$.

Question 10(8):

In a two digit number, the digit at tens place is 7 and the sum of the digits is 8 times the digit at unit place. Then the number is……….

Solution :

d. 71

In the two digit number, the digit in the unit's place is x and the digit in the ten's place is 7.

∴ The number is $10(7) + x = 70 + x$.

Sum of digits = 8 times the digit in the unit's place.

∴ Sum of both the digits

= 8 × digit at unit's place

∴ $7 + x = 8 \times x$

∴ $7 = 7x$

∴ $x = 1$

Now, substituting $x = 1$ in required number $70 + x$,

$70 + x = 70 + 1 = 71$

Question 10(9):

The sum of two numbers is 10 and the difference of them is 2. Then the greater number of these two is…….

Solution :

c. 6

Let the two numbers be x and y such that $x > y$.

The sum of the digits is 10.

∴$x + y = 10$ … … (1)

Difference between the digits is 2.

∴ $x - y = 2$ (∵ $x > y$) … … (2)

Adding eq. (1) and eq.(2),

$2x = 12$

∴ $x = 6$

Also, $x > y$ so $x = 6$ is the required number.

Question 10(10):

3 years ago, the sum of ages of a father and his son was 40 years. After 2 years the sum of ages of the father and his son will be……..

Solution :

c. 50

Let the present age of the father be x years and the son's be y years.

Before 3 years,

Age of the father = $(x - 3)$ years

Age of the son = $(y - 3)$ years.

Also, the sum of their ages is 40.

∴ $(x - 3) + (y + 3) = 40$

∴ $x + y = 46$ … … (1)

After 2 years,