GRAVITATION

MULTIPLE CHOICE QUESTIONS

1. A satellite is moving around the earth in a circular orbit. The following statements are given
(i)It is moving with a constant velocity.
(ii) It suffers no acceleration.
(iii) Its angular momentum w.r.t. the earth remains conserved.
(iv) Its distance from centre must be equal to $\sqrt{2}$ times of earth's radius.
The correct option is
(A) (i) and (ii) are true. (B) (i), (iii) and (iv) are true.
(C) only (iii) is true. (D) (i) and (iv) are true.

Sol. C
When the earth's satellite is moving around the earth in a circular orbit, it is moving with constant speed. It has centripetal acceleration and its angular momentum remains constant as no external torque is present. Thus, its radius need not be equal to the $\sqrt{2}R_e$

2. The fractional change in the value of free-fall acceleration 'g' for a particle when it is lifted from the surface to an

elevation h (h<<R) is

(A) h/R (B) –(2h/R)

(C) 2h/R (D) none of these.

Sol. B

$g = \frac{GM}{R^2}$ . . . .(i)

$\frac{dg}{dR} = \frac{-2GM}{R^3}$ putting dR = h we obtain

$\Rightarrow \frac{dg}{h} = \frac{-2GM}{R^2}.\frac{1}{R}$ . . . (ii)

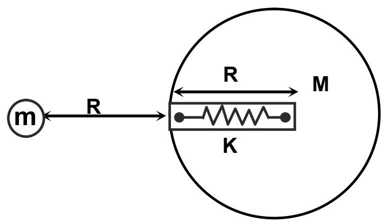
From (i) and (ii)

$\Rightarrow \frac{dg}{g} = -2\left(\frac{h}{R}\right)$

⇒ Change is –ve. That means g decreases.

Hence, (B) is correct.

3. A small ball of mass 'm' is released at a height 'R' above the earth surface, as shown in the given figure. The maximum depth of the ball to which it goes is R/2 inside the earth through a narrow groove before coming to rest momentarily. The groove, contains an ideal spring of spring constant K and natural length R. The value of K, if R is the radius of earth and M mass of earth, is



(A) $\frac{3GMm}{R^3}$ (B) $\frac{6GMm}{R^3}$

(C) $\frac{9GMm}{R^3}$ (D) none of above

Sol. D

4. A system consists of N identical particles of mass m placed rigidly on the vertices of a regular polygon with each side of length ℓ. If K_1 be the kinetic energy imparted to one of the particles so that it just escapes the gravitational pull of the system and thereafter kinetic energy K_2 is given by to the adjacent particle to escape, then the difference $(K_1 - K_2)$ is

(A) $\frac{nGm^2}{a}$ (B) $\frac{Gm^2}{na}$

(C) $\left(\frac{n}{n+1}\right)\frac{Gm^2}{a}$ (D) $\frac{Gm^2}{a}$

Sol. D

For the first particle, $K_1 + (-Gm^2)\left(\frac{1}{a_{12}}+\frac{1}{a_{13}}+\ldots+\frac{1}{a_{1n}}\right) = 0$

Similarly for the second particle, $K_2 + (-Gm^2)\left(\frac{1}{a_{12}}+\frac{1}{a_{13}}+\ldots+\frac{1}{a_{1,n-1}}\right)$ $= 0$

$\therefore K_1 - K_2 = Gm^2\frac{1}{a_{1,n}} = \frac{Gm^2}{a}$

5. A satellite is revolving around the earth at a height h. The satellite explodes into two equal parts, one part has zero velocity after the explosion, and the other moves in the

same direction with double the orbital speed of the original satellite. The maximum height reached by the part with double the velocity is (Neglect the gravitational effects between the fragments)

(A) 3h + 4R (B) 3R + 4h

(C) 3(R +h) (D) none of these

Sol. D

$v = \sqrt{\frac{GM}{R+h}}$

After the collision, the velocity becomes $2\sqrt{\frac{GM}{R+h}}$

Now $2\sqrt{\frac{GM}{R+h}} = \sqrt{2}\sqrt{\frac{2GM}{(R+h)}}$

$\therefore$ The velocity of satellite becomes greater than the escape velocity. Hence, it will escape to infinity.

6. Three spheres, each of radiusR are fixed at 3 points such that their centres form an equilateral triangle of side d. With what velocity should a particle of mass ‘m’ be projected from the mid-point of one of the sides of the triangle, so that it escapes the gravitational field of the system? Given that the mass of each sphere is M.

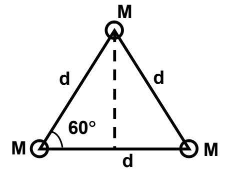
(A) $\sqrt{\frac{2GM\left(\sqrt{3}+1\right)}{d\sqrt{3}}}$ (B) $\sqrt{\frac{4GM\left(2\sqrt{3}+1\right)}{d\sqrt{3}}}$

(C) $\sqrt{\frac{3GM}{d}\left(\sqrt{3}+\frac{1}{2}\right)}$ (D) $\sqrt{\frac{GM}{d}\left(3\sqrt{3}+\frac{1}{2}\right)}$

Sol. B

$$\frac{1}{2}mv^2 - \frac{GMm}{d/2} - \frac{GMm}{d/2} - \frac{2GMm}{d\sqrt{3}} = 0$$

$$\frac{1}{2}mv^2 = \frac{GMm}{d}\left[4 + \frac{2}{\sqrt{3}}\right] = \frac{2GMm}{d}\left[\frac{2\sqrt{3}+1}{\sqrt{3}}\right] \Rightarrow v = \sqrt{\frac{4GM}{d}\left[\frac{2\sqrt{3}+1}{\sqrt{3}}\right]}$$



7. Consider two thin concentric shells of masses M and 2M and radii R_1 and R_2 ($R_1 < R_2$). Magnitude of the gravitational force experienced by a particle of mass m placed at a distance $\left(\frac{R_1+R_2}{2}\right)$ from the common centre will be

(A) $\frac{8GMm}{(R_1+R_2)^2}$ (B) $\frac{4GMm}{(R_1+R_2)^2}$

(C) $\frac{2GMm}{(R_1+R_2)^2}$ (D) $\frac{12GMm}{(R_1+R_2)^2}$

Sol. B

Sol The particle of mass m is located in between the shells

$F = \frac{GMm}{[R_1+R_2)/2]} = \frac{4GMm}{(R_1+R_2)^2}$

8. Two moving particles collide and stick together on a smooth horizontal surface. The following statements are given below

(a) Mechanical energy is conserved.

(b) Total energy is conserved.

(c) Work done by the system is positive

(d) Work done by the system is negative.

Choose the correct option.

(A) (a) and (c) are correct. (B) (b) and (c) are correct.

(C) (a) and (d) are correct. (D) (b) and (d) are correct.

Sol. D

It is obvious for 2 mass system, whether it is elastic, inelastic or perfectly elastic. Total energy of system will remain conserved. But final kinetic energy will be less than initial K.E. So work done by the system will be negative.

9. When a satellite has an elliptical orbit, the plane of the orbit

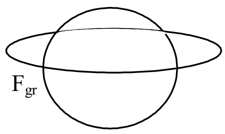
(A) Sometimes passes through the centre of earth.

(B) Does not pass through the centre of earth.

(C) Passes through the centre of earth always

(D) None of the above.

Sol. C



If the plane of the orbit does not pass through the centre of earth there will be a component of F_{gr} perpendicular to the plane which is not balanced by any force. The orbit would become unstable. Therefore, earth orbit must have to pass through the centre of earth.

10. If the radius of earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would

(A) Decrease (B) remains unchanged

(C) Increase. (D) None of these

Sol. C

$g = \frac{GM}{R^2} \quad \Rightarrow \quad \frac{dg}{g} = -\frac{2dR}{R}$

As $\frac{dR}{R} = -1\% \Rightarrow \frac{dg}{g}$ = –2 (–1) = +2 %. Thus acceleration due to gravity will increase.

11. Two isolated point masses m and M are separated by a distance ℓ. The moment of inertia of the system about an axis passing through a point where gravitational field is zero and perpendicular to the line joining the two masses, is

(A) $\frac{m^2 + M^2}{(\sqrt{m} + \sqrt{M})^2}\ell^2$ (B) $\frac{m + M}{(\sqrt{m} + \sqrt{M})}\ell^2$

(C) $\frac{m + M}{(\sqrt{m} + \sqrt{M})^2}\ell^2$ (D) none of the above

Sol. A

$\frac{Gm}{x^2} = \frac{GM}{(\ell - x)^2} \quad \Rightarrow \frac{\sqrt{m}}{x} = \frac{\sqrt{M}}{(\ell - x)}$

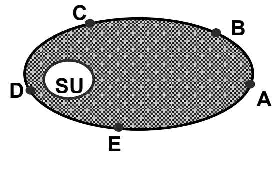
$\Rightarrow \sqrt{m}\ell - \sqrt{m}x = \sqrt{M}x$

$\Rightarrow x = \frac{\sqrt{m}}{\sqrt{m} + \sqrt{M}}\ell$

$I = mx^2 + M(\ell - x)^2$

$\Rightarrow \ I = \frac{m^2\ell^2}{(\sqrt{m} + \sqrt{M})^2} + M(\ell - \frac{\sqrt{m}}{\sqrt{m} + \sqrt{M}}\ell)^2 = \frac{(m^2 + M^2)\ell^2}{(\sqrt{m} + \sqrt{M})^2}$

12. The planet mercury is revolving in an elliptical orbit around Sun as shown.



The kinetic energy of mercury will be greatest at

(A) A (B) B

(C) C (D) D

Sol. D

13. The earth revolves round the sun in an elliptical orbit. Its speed is

(A) going on decreasing continuously

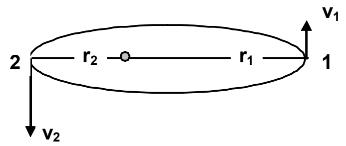
(B) greatest when it is closest to the sun

(C) greatest when it is farthest from the sun

(D) constant at all the points on the orbit.

Sol. B

Conservation of angular momentum of the planet yields,



$mv_1r_1 = mv_2r_2 \Rightarrow v_1 r_1 = v_2 r_2$

$\therefore$ At closest distance, speed is maximum.

14. Two massive particles of masses M & m (M > m) are separated by a distance ℓ. They rotate with equal angular velocity under their gravitational attraction. The linear speed of the particle of mass m is

(A) $\sqrt{\frac{GMm}{(M+m)\ell}}$ (B) $\sqrt{\frac{GM^2}{(M+m)\ell}}$

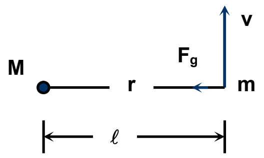
(C) $\sqrt{\frac{Gm}{\ell}}$ (D) $\sqrt{\frac{Gm^2}{(M+m)\ell}}$

Sol. B

The system rotates about the centre of mass. The gravitational force acting on the particle m accelerates it towards the centre of the circular path, which has the radius $R = \frac{M\ell}{M+m}$

$$\Rightarrow F = \frac{mv^2}{r} \Rightarrow \frac{GMm}{\ell^2} = \frac{mv^2}{\frac{M\ell}{M+m}}$$

$$\Rightarrow v = \sqrt{\frac{GM^2}{(M+m)\ell}}$$



Hence, (B) is correct.

15. A particle is projected from the mid-point of the line joining two fixed particles each of mass m. If the separation between the fixed particles is ℓ, the minimum velocity of projection of the particle so as to escape is equal to

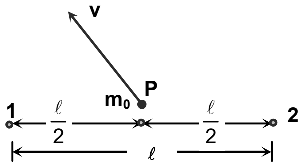
(A) $\sqrt{\frac{Gm}{\ell}}$ (B) $\sqrt{\frac{Gm}{2\ell}}$

(C) $\sqrt{\frac{2Gm}{\ell}}$ (D) $2\sqrt{\frac{2Gm}{\ell}}$

Sol. D

The gravitational potential at the mid-point P,

$$V = V_1 + V_2 = \frac{-Gm}{(\ell/2)} - \frac{Gm}{(\ell/2)} = -\frac{4Gm}{\ell}$$



$\Rightarrow$ The gravitational potential energy

$U = -\frac{4Gmm_0}{\ell}$, m_0 = mass of particle

When it is projected with a speed v, it just escapes to infinity, and the potential & kinetic energy will become zero.

$\Rightarrow \Delta KE + \Delta PE = 0$

$\Rightarrow \left(0 - \frac{1}{2}m_0 v^2\right) + \left\{0 - \left(-\frac{4Gmm_0}{\ell}\right)\right\} = 0$

$\Rightarrow v = 2\sqrt{\frac{2Gm}{\ell}}$.

Hence (D) is correct.

16. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of radius of earth R will be

(A) R/4 (B) R/2

(C) R/3 (D) R/8.

Sol. B

$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R^2} = \frac{4}{3}\pi G \rho R$

$\therefore \frac{g_p}{g_e} = \left(\frac{\rho_p}{\rho_e}\right)\left(\frac{R_p}{R_e}\right)$

$\Rightarrow 1 = (2)\frac{R_p}{R_e}$

$\Rightarrow \frac{R_p}{R_e} = ½ \Rightarrow R_p = \frac{R}{2}$

Hence (B) is correct.

17. A particle hanging from a spring stretches it by 1 cm at earth's surface. Radius of earth is 6400 km. At a place 800 km above the earth's surface, the same particle will stretch the spring by:

(A) 1 cm (B) 8 cm

(C) 0.1 cm (D) 0.79 cm.

Sol. D

$\frac{kx_1}{kx_2} = \frac{F_{gr_1}}{F_{gr_2}} = \left(\frac{r_2}{r_1}\right)^2$

$\Rightarrow \frac{x_1}{x_2} = \left(\frac{800+6400}{6400}\right)^2 = \left(\frac{72}{64}\right)^2 = \left(\frac{9}{8}\right)^2$

$\Rightarrow \frac{x_1}{x_2} = \frac{81}{64}$

$\Rightarrow x_2 = \frac{64}{81}x_1 = \frac{64}{81}$ cm = 0.79 cm.

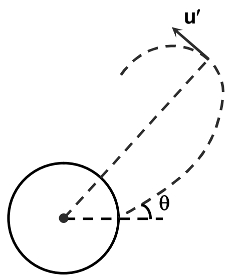
Hence (D) is correct.

18. A projectile is launched from the surface of the earth with a very high speed v at an angle θ with vertical. What is its velocity when it is at the farthest distance from the earth surface? Given that the maximum height reached by the projectile is equal to the height reached when it is launched perpendicular to earth with a velocity $= \sqrt{\frac{GM}{R}}$.

(A) $\frac{v\cos\theta}{2}$ (B) $\frac{v\sin\theta}{2}$

(C) $\sqrt{\frac{GM}{2R}}$ (D) $\sqrt{\frac{GM}{3R}}$

Sol. B



The maximum height reached by the projectile is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or, $$\frac{1}{2}m\frac{GM}{R} - \frac{GMm}{R} = -\frac{GMm}{R+h} = -\frac{GMm}{2R}$$

$\therefore \quad h = R$

Applying conservation of momentum

$$mu'(R+h) = mv\sin\theta R$$

$\therefore \quad u'(2R) = v\sin\theta R$

$\therefore \quad u' = \frac{v\sin\theta}{2}$

Hence, (B) is correct.

19. The mean radius of the earth is R, its angular speed about its own axis is ω and the acceleration due to gravity at the earth surface is g. The cube of radius of orbit of 'geostationary satellite' will be:

(A) (R^2g/ω) (B) $(R^2\omega/g)$

(C) (Rg/ω^2) (D) (R^2g/ω^2).

Sol. D

$mr\omega^2 = \frac{GMm}{r^2}$

$\Rightarrow r\omega^2 = \frac{GM}{r^2}$

$\Rightarrow r^3 = \frac{GM}{\omega^2} = \frac{GM}{R^2} \cdot \frac{R^2}{\omega^2}$

$\Rightarrow r^3 = g\frac{R^2}{\omega^2}.$

Hence, (D) is correct.

20. Two particles of masses M and m are initially at rest and infinitely separated. When they move towards each other due to gravitational attraction, their relative velocity at any instant in terms of distance 'd' between them at that instant is

(A) $\left(\frac{2Gd}{M+m}\right)^{1/2}$ (B) $\left[\frac{2G(M+m)}{d}\right]^{1/2}$

(C) $\frac{2G(M+m)}{d}$ (D) $\frac{2Gd}{M+m}$

Sol:

As the two particles moves in the influence of gravitational (conservative force) mechanical energy will be conserved.

Let p = momentum of mass m

So $U_i + K_i = U_f + K_f$

$$0 + 0 = -\frac{GMm}{d} + \frac{p^2}{2m} + \frac{(-p)^2}{2M}$$

Solving, $p = Mm\sqrt{\frac{2G}{d(m+M)}}$

Velocity of m, $v_1 = M\sqrt{\frac{2G}{d(m+M)}}$

Velocity of M, $v_2 = -m\sqrt{\frac{2G}{d(m+M)}}$

$\Rightarrow$ Relative velocity $= v_1 - v_2 = (M+m)\sqrt{\frac{2G}{d(m+M)}} = \sqrt{\frac{2G(m+M)}{d}}$

$\therefore$ (B)

21. The orbital velocity of an artificial satellite in a circular orbit just above earth's surface is v_0. For a satellite orbiting in a circular orbit at an altitude of half of earth's radius is

(A) $\sqrt{\frac{3}{2}}\,v_0$ (B) $\sqrt{\frac{2}{3}}\,v_0$

(C) $\frac{3}{2}v_0$ (D) $\frac{2}{3}v_0$

Sol. Orbital velocity = $\sqrt{\frac{g_0R^2}{R+h}}$ where R is radius of earth.

If h = 0, $v_0 = \sqrt{\frac{g_0R^2}{R}} = \sqrt{g_0R}$

If h = $\frac{R}{2}$, $v = \sqrt{\frac{g_0R^2}{R+\frac{R}{2}}} = \sqrt{\frac{2g_0R}{3}} = \sqrt{\frac{2}{3}}\,v_0$.

∴ (B)

22. The dimensional formula for gravitational constant is

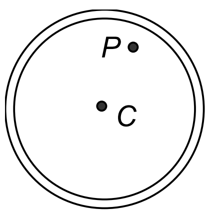
(A) $[M^{-1}\,L^3\,T^{-2}]$ (B) $[M^3\,L^{-1}T^{-2}]$

(C) $[M^{-1}L^2T^3]$ (D) $[M^2L^3T^{-1}]$

Sol: $G = \frac{Fr^2}{m_1m_2} = \frac{MLT^{-2}L^2}{M^2} = M^{-1}\,L^3T^{-2}$

∴ (A)

23. The force between a hollow sphere and a point mass at P inside it as shown in the figure



(A) is attractive and constant

(B) is attractive and depends on the position of the point with respect to centre C

(C) is zero

(D) is repulsive and constant.

Sol: Since gravitational field inside hollow sphere is zero. Therefore force acting on the particle P is zero.

$\therefore$ (C)

INTEGER TYPE QUESTIONS

24. Two satellites A and B, of equal mass, move in the equatorial plane of earth close to the earth's surface. Satellite A moves in the same direction as that of the rotation of the earth while satellite B moves in the opposite direction. Determine the ratio of the kinetic energy of B to that of A in the reference frame fixed to earth.

Sol. The orbital speed of a satellite very close to earth = V_0 = $\sqrt{\frac{GM}{R}} = \sqrt{g_0 R}$

The peripheral speed of earth = $V_e = R\omega_e = \frac{2\pi}{T_e} R$

$\Rightarrow$The velocities of the satellites relative to earth are given by

$V_{r_1} = \frac{2\pi}{T_e} R + \sqrt{g_o R}$ and $V_{r_2} = \sqrt{g_o R} - \frac{2\pi}{T_e} R$

Positive and Negative sign are for the satellites orbiting form east to west and west to east respectively because earth rotates from west to east

$$\Rightarrow \frac{KE_1}{KE_2} = \frac{\frac{1}{2} m V_{r_1}^2}{\frac{1}{2} m V_{r_2}^2} = \left(\frac{V_{r_1}}{V_{r_2}}\right)^2$$

$$= \frac{\left(\frac{2\pi}{T_e}R + \sqrt{g_o R}\right)^2}{\left(-\frac{2\pi}{T_e}R + \sqrt{g_o R}\right)^2}$$

Putting $R = 6.4 \times 10^6$ m, $g_0 = 9.8$ m/sec^2 and $T_e = 86400$ sec,

$$\frac{KE_1}{KE_2} = 1.265$$

25. If the radius and density of a planet are two times and half respectively of those of earth, find the intensity of gravitational field at planet surface and escape velocity from planet.

Sol. Acceleration due to gravity $g = \frac{GM}{R^2} = G\frac{4}{3}\pi R^3 \rho \frac{1}{R^2} = \frac{4}{3}\pi G R \rho$

Thus $\frac{g_2}{g_1} = \frac{R_2 \rho_2}{R_1 \rho_1} = (2)\left(\frac{1}{2}\right) = 1 \therefore g_2 = g_1 = g$

Escape velocity $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

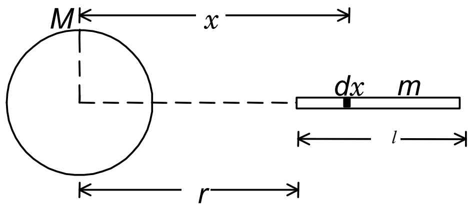
$\therefore$ Escape velocity at planet $= \sqrt{2gR'} = \sqrt{4gR}$

$= \left(\sqrt{2}\right)(11.2 \text{km/sec}) = 15.84$ km/sec.

26. A uniform sphere of mass M = 100 kg and a thin uniform rod of length l = 30cm and mass m = 300 kg oriented as shown in the figure. The distance of centre of sphere to the nearest end of the rod is r = 3 m. The gravitation force between them is given as x × 10^{-8} N. Find x

($G = 6.6 \times 10^{-11}\ Nm^2/kg^2$)

Sol: Since the sphere is uniform its entire mass may be considered to be concentrated at its centre. The force on the elementary mass dm is



$$dF = \frac{GMdm}{x^2}$$

But $dm = \frac{m}{l} dx$

$$F = \int_{r}^{r+\ell} \frac{GMm}{lx^2} dx = -\frac{GMm}{l}\left[\frac{1}{x}\right]_{r}^{r+\ell}$$

$$= -\frac{GMm}{l}\left[\frac{1}{r+l} - \frac{1}{r}\right]$$

$$F = \frac{GmM}{l}\left(\frac{l}{r(r+l)}\right) = \frac{GMm}{r(r+l)} = 2 \times 10^{-7}$$

N

x = 20

27. The distance between earth and moon is 4×10^5 km and the mass of earth is

81 times the mass of moon. Find the position (take 10^4 km as unit) of a point on the line joining the centres of earth and moon, where the gravitational field is zero.

Sol: Let x be the distance of the point of no net field from earth. The distance of this point from moon is

(r – x), where $r = 3.8 \times 10^5$ km.

The gravitational field due to earth $= \frac{GM_e}{x^2}$ and that due to moon $= \frac{GM_m}{(r-x)^2}$. For the net field to be zero these are equal and opposite.

$$\frac{GM_e}{x^2} = \frac{GM_m}{(r-x)^2}$$

$$\frac{M_e}{M_m} = \frac{x^2}{(r-x)^2}.\ \text{But}\ \frac{M_e}{M_m} = 81$$

$$\therefore \quad 81 = \frac{x^2}{(r-x)^2}$$

$$\frac{x}{r-x} = 9$$

$$9r - 9x = x$$

$$10x = 9\ r$$

$$x = \frac{9}{10} r = \frac{9}{10} \times 4 \times 10^5 = 3.6 \times 10^5\ \text{km} = 36 \times 10^4\ \text{km} \Rightarrow$$

$$x = 36\ \text{unit}$$

28. A person brings a mass of 1 kg from infinity to a point A. Initially the mass was at rest but it moves at a speed of 2 m/s as it reaches A. The work done by the person on the mass is –3J. The potential at infinity is –10 J. Then find the potential at A.

Sol: Work done by external agent

$$W_{ent}, = \Delta U + \Delta K$$

$$= U_A - U_\infty + K_A - K_\infty$$

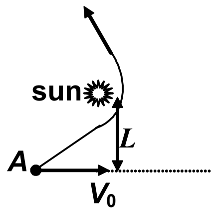
$= U_A - 10 + \frac{1}{2} \times mv^2 - 0$

$-3 = U_A - 10 + \frac{1}{2} \times 1 \times 4$

$+ 5\ J = U_A$

So $V_A = 5$ J/kg

29. A cosmic body A moves towards star with velocity v_0 (when far from the star) and aiming parameter L and arm of velocity vector v_0 relative to the centre of the star as shown in figure. Find the minimum distance (take 10^8 m as unit) by which this body will get to the star. Mass of the star is M.



[$M = 10^{20}$ kg, $G = \frac{2}{3} \times 10^{-10} Nm^2/kg^2$, $v_0 = \sqrt{\frac{200}{3}} m/s$, $L = \sqrt{15} \times 10^8 m$]

Sol. r = minimum distance

conservation of angular momentum about star $mv_0L = mrv_1$

$\frac{1}{2}mv_0^2 - 0 = \frac{1}{2}mv_1^2 - \frac{GMm}{r}$; $\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{v_0L}{r}\right)^2 - \frac{GMm}{r} \Rightarrow r^2 + \frac{3GM}{v_0^2}r - L^2 = 0$

Solving $r = \frac{GM}{v_0^2}\left[\sqrt{1+\left(\frac{rv_0^2}{GM}\right)} - 1\right]$

Putting the values $r = 3 \times 10^8$ m = 3 unit

30. The density inside an isolated large solid sphere of radius a = 4 km is given by $\rho = \rho_0 a/r$ where ρ_0 is the density at the surface and equals to 10^9 kg/m^3 and r denotes the distance from the centre. Find the gravitational field (in m/s^2) due to

this sphere at a distance 2a from its centre. Take G = 6.65 × 10^{-11} Nm^2/kg^2.

Sol: The gravitational field at the given point is $E = \frac{GM}{(2a)^2} = \frac{GM}{4a^2}$ … (i)

The mass M may be calculated as follows. Consider a concentric shell of radius r and thickness dr. Its volume is

$$dV = \left(4\pi r^2\right) dr$$

and its mass is $dM = \rho dV = \left(\rho_0 \frac{a}{r}\right)\left(4\pi r^2 dr\right) = 4\pi\rho_0 ar\ dr$.

The mass of the whose sphere is $M = \int_0^a 4\pi\rho_0 ar\ dr = 2\pi\rho_0 a^3$

Thus, by (i) the gravitational field is $E = \frac{2\pi G\rho_0 a^3}{4a^2} = \frac{1}{2}\pi G\rho_0 a$

$\therefore$ $E = \frac{1}{2} \times \frac{22}{7} \times 6.65 \times 10^{-11} \times 10^9 \times 4 \times 10^3$ $m/s^2 \Rightarrow$ $E = 418\ m/s^2$