# Chapter 16

# Dual Nature of Matter and Radiation, Atoms and Nuclei

## DUAL NATURE OF MATTER AND RADIATION

### WAVE NATURE OF PARTICLES

The following points should be kept in mind :

1. Any particle in motion can act like a wave. Wave associated with a particle is called Matter wave or de-Broglie wave.
2. de-Broglie wavelength of a particle $\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$.

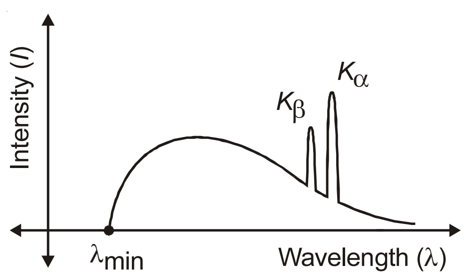
   where $p = mv$ is momentum of particle

   $E_k$ = kinetic energy.
3. For an electron accelerated through $V$ volts.

   $$E_k = eV \therefore \lambda = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}\text{Å} \quad \text{or} \quad \lambda = \sqrt{\frac{150}{V}}\ \text{Å}$$
4. For a proton accelerated through $V$ volts, $\lambda = \frac{0.286}{\sqrt{V}}\text{Å}$
5. For an $\alpha$-particle accelerated through $V$ volts, $\lambda = \frac{0.101}{\sqrt{V}}\text{Å}$
6. For an electron revolving in $n^{th}$ orbit of Bohr's Hydrogen atom, $mvr = \frac{nh}{2\pi}$, $\lambda = \frac{h}{mv} = \frac{2\pi r}{n}$.

### X-RAYS

**Variation of intensity ($I$) of X-rays with wavelength $\lambda$ :**



Important points related to the above curve :

1. At certain sharply defined wavelength, the intensity of X-rays is very large as marked $K_\alpha$ and $K_\beta$. These are known as **characteristic X-rays**.
2. At other wavelengths intensity varies continuously. These are known as **continuous X-rays**.
3. Minimum wavelength or cut off wavelength or threshold wavelength of continuous X-rays,

   $\lambda_{min} = \frac{hc}{eV} = \frac{12400 \text{Å}}{V}$, where $V$ is applied voltage in volts.

4. The minimum wavelength does not depend on the material of target. It depends only on the accelerating potential.
5. Continuous X-rays are due to continuous loss of energy of electrons striking the target through successive collisions.
6. Characteristic X-rays are due to the transition of electrons from higher energy level to the vacant space present in the lower energy level.
7. Wavelength of $K_\alpha$, $\lambda = \frac{hc}{E_L - E_K}$ (transition from $L$ to $K$)
8. Wavelength of $K_\beta$, $\lambda = \frac{hc}{E_M - E_K}$ (transition from $M$ to $K$)

**Moseley's law :** Applicable to characteristic X-rays only.

Mathematically $\boxed{\sqrt{\nu} = a(Z - b)}$ $a$ and $b$ are Moseley's constants, $\nu$ is frequency of X-rays.

$Z$ is atomic number of the target atom.

For $K_\alpha$ X-ray,

$$a = \sqrt{\frac{3Rc}{4}}$$

$$b = 1$$

**Diffraction of X-Rays**

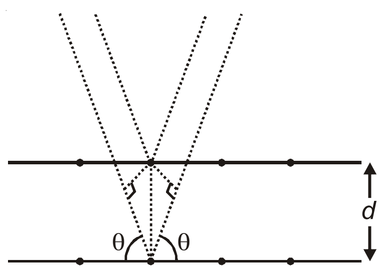
**Bragg's Law :** $2d \sin\theta = n\lambda$ [condition for constructive interference]

where, $\lambda$ = wavelength of X-ray.

$d$ = separation between crystal planes.

$\theta$ = angle between X-ray beam and crystal plane.

$2d \sin\theta$ : Path difference




Davisson and Germer's accidental discovery of the diffraction of electrons was the first direct evidence confirming de Broglie's hypothesis that particles have wave properties as well.

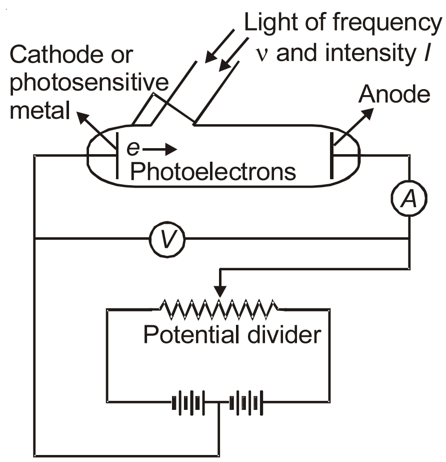
## PHOTOELECTRIC EFFECT

The emission of electrons from a metallic surface when illuminated with light of appropriate wavelength (or frequency) is known as photoelectric effect. It was discovered by Hertz in 1887.

### Einstein's Theory of Photoelectric Effect

Light of frequency $\nu$ consists of stream of packets or quanta of energy $E = h\nu$. These are called photons.

In the process of photoemission, a single photon gives up all its energy to a single electron. As a result, the electron can be ejected instantaneously.




**Exp. set up photoelectric effect**

**Work Function ($\phi$) :** It is the minimum energy of photon required to liberate an electron from a metal surface.

**Threshold Frequency ($\nu_0$) :** The frequency of incident radiation below which photoelectric effect does not take place. $h\nu_0 = \phi$.

**Stopping Potential ($V_0$) :** The smallest negative value of anode potential which just stops the photocurrent is called the stopping potential.

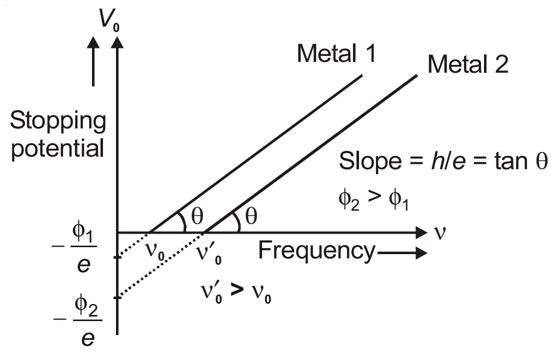
If the stopping potential is $V_0$ then $eV_0 = KE_{max}$ = Maximum kinetic energy of photoelectrons emitted.

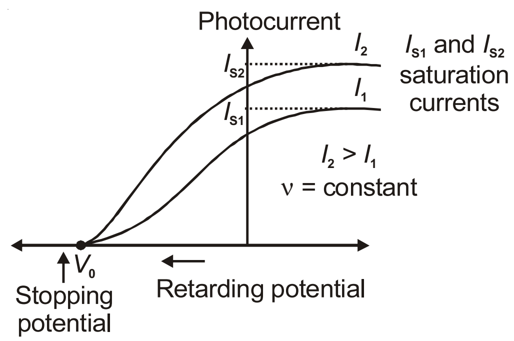
The following important points should be kept in mind :

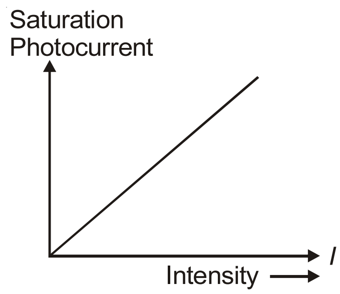
1. The kinetic energy of photoelectrons varies between zero to $KE_{max}$.
2. If $\nu$ ( > $\nu_0$) is frequency of incident photon, $h\nu_0$ is work function then $h\nu - h\nu_0 = KE_{max}$. This is Einstein's photoelectric equation. Here $h$ is Planck's constant.
3. Efficiency of photoelectric emission is less than 1%. It means it is not necessary that if the energy of incident photon is greater than work function electrons will definitely be ejected out.
4. If frequency of incident radiation ($\nu$) is doubled, stopping potential ($V_0$) or kinetic energy maximum ($K.E._{max}$) gets more than doubled.
5. If on a neutral ball made up of metal of work function $\phi$, radiation of frequency $\nu$ (greater than threshold frequency) is incident, number of photoelectrons emitted from the ball before the photoelectric emission stops is given by $n = \dfrac{(h\nu - \phi)4\pi\varepsilon_0 R}{e^2}$.
6. Saturation current depends upon intensity of incident light whereas stopping potential depends upon frequency of light as mentioned in graphs also.

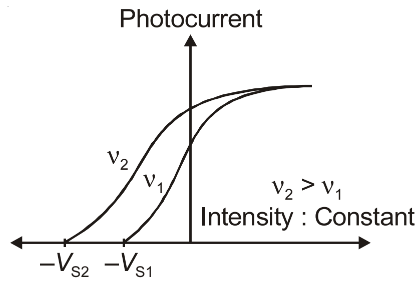
### Graphs for Photoelectric Effect (Lenard's Observations)

Following graphs are important :

1. 


2. 


3. 


4. 


## ATOMS AND NUCLEI

### BOHR'S ATOMIC MODEL

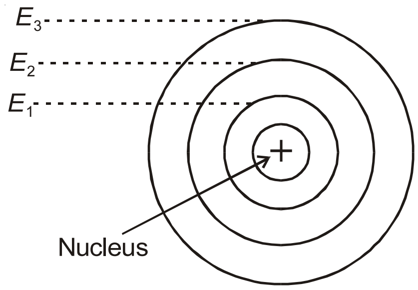
In 1913 Niels Bohr, a Danish physicist, introduced a revolutionary concept *i.e.*, the quantum concept to explain the stability of an atom. He made a simple but bold statement that "The old classical laws which are applicable to bigger bodies cannot be directly applied to the sub-atomic particles such as electrons or protons".

### Postulates of Bohr's Theory

1. Electron revolves round the nucleus in circular orbits.
2. Electron can revolve only in those orbits in which angular momentum of the electron about the nucleus is an integral multiple of $\frac{h}{2\pi}$

   *i.e.*, $mvr = \frac{nh}{2\pi}$

   $n$ = principal quantum number of the orbit in which electron is revolving.
3. Electrons in an atom can revolve only in discrete circular orbits called stationary energy levels (shells). An electron in such a shell is characterised by a definite energy, angular momentum and orbit number. While an electron is in any of these orbits it does not radiate energy although it is accelerated.
4. Electrons can jump from one stationary orbit to another stationary orbit. Electrons in outer orbits have greater energy than those in inner orbits. The orbiting electron emits energy when it jumps from a higher energy state to a lower energy state and absorbs energy when it makes a jump from lower orbits to higher orbits. This energy (emitted or absorbed) is in form of photons.



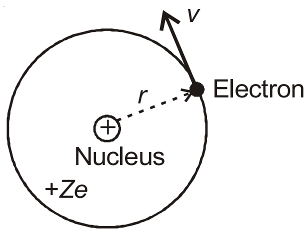

$$E_2 - E_1 = h\nu$$

where, $E_2$ = higher energy state

$E_1$ = lower energy state

and $\nu$ = frequency of photons of radiation/emitted absorbed.

**Mathematical Analysis of Bohr's Theory**




Electric force of attraction provides the centripetal force

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r} \qquad \text{...(i)}$$

where, $m$ = mass of electron

$v$ = velocity (linear) of electron

$r$ = radius of the orbit in which electron is revolving

$Z$ = atomic number of hydrogen like atom

Angular momentum about the nucleus, $mvr = \frac{nh}{2\pi}$ ...(ii)

**(a) Velocity of electron in *n*th orbit**

Putting value of *mvr* from equation (ii) into equation (i),

$$\frac{1}{4\pi\varepsilon_0}Ze^2 = \left(\frac{nh}{2\pi}\right)v$$

$$\Rightarrow \quad v = \frac{Z}{n}\left[\frac{e^2}{2\varepsilon_0 h}\right] = \frac{Z}{n}.v_0 \qquad \text{...(iii)}$$

where,

$$v_0 = \frac{c}{137} = 2.2 \times 10^6 \text{ m/s}$$

where $c = 3 \times 10^8$ m/s = speed of light in vacuum, $\frac{v_0}{c} = \frac{1}{137}$ = fine structure constant

**(b) Radius of the *n*th orbit**

Putting value of *v* from equation (iii) in equation (ii), we get,

$$m\left(\frac{Z}{n}\times\frac{e^2}{2\varepsilon_0 h}\right)r = \frac{nh}{2\pi}$$

$$\Rightarrow \quad r = \frac{n^2}{Z}\left[\frac{\varepsilon_0 h^2}{\pi m e^2}\right] = \frac{n^2}{Z}.r_0 \qquad \text{...(iv)}$$

where,

$r_0 = 0.53$ Å.

**(c) Total energy of electron in $n^{\text{th}}$ orbit**

From equation (i)

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

and $\text{P.E.} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -2\text{K.E.}$

P.E. = – 2 K.E.

Total energy, E = K.E. + P.E. = –K.E.

$$E = \frac{Z^2}{n^2}\left(-\frac{me^4}{8\varepsilon_0^2 h^2}\right) = \frac{Z^2}{n^2}.E_0$$

where, $E_0 = -13.6$ eV.

**(d) Time period of revolution of electron in *n*th orbit**

$$T = \frac{2\pi r}{v} = \frac{n^3}{Z^2}.T_0$$

where,

$T_0 = 1.51 \times 10^{-16}$ s.

**(e) Frequency of revolution in *n*th orbit**

$$f = \frac{1}{T} = \frac{Z^2}{n^3}.f_0$$

where,

$f_0 = 6.6 \times 10^{15}$ Hz.

**(f) Magnetic field at the centre due to revolution of electron**

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 e}{2r\,T} = \frac{\mu_0 e}{2r}\times\frac{v}{2\pi r}$$

$$B \propto \frac{v}{r^2} \quad \Rightarrow \quad B \propto \frac{Z}{n}\times\left(\frac{Z}{n^2}\right)^2 \quad \Rightarrow \quad B \propto \frac{Z^3}{n^5}$$

**(g) Wavelength of photon**

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2$$

where,

$\bar{\nu}$ **is called wave number.**

$R$ = Rydberg constant

= $1.09677 \times 10^7$ m$^{-1}$

= $1.09677 \times 10^{-3}$ Å$^{-1}$ (for stationary nucleus)

$$= \left(\frac{1}{912}\right)\text{Å}^{-1}$$

## BINDING ENERGY

The amount of energy needed to separate the constituent nucleons to large distances is called binding energy.

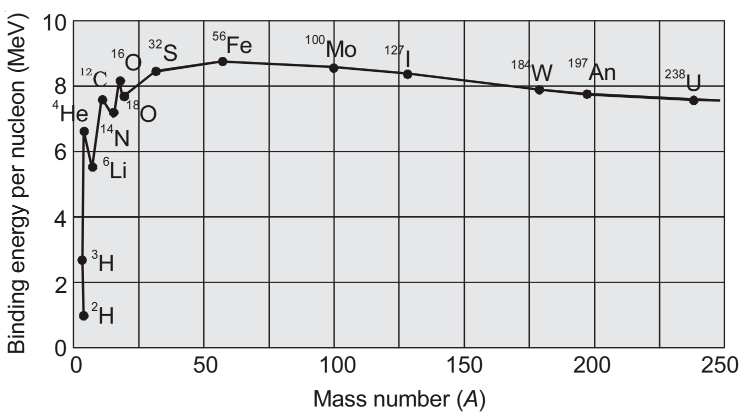
If the nucleons are initially well separated and are brought to form the nucleus, this much energy is released.

$\boxed{\text{BE} = (ZM_p + NM_n - M)c^2}$ (Where $M$ = mass of nucleus and $N = A - Z$)

$M_p$ = Mass of proton, $M_n$ = Mass of neutron.

### Binding Energy Curve

B.E./nucleon is very low for light nuclei. This means energy will be released if two nuclei combine to form a single middle mass nucleus. The release of energy in a fusion process is based on this fact.




Likewise, the low B.E. per nucleon for heavy nuclei indicates that if a single heavy nucleus breaks up into middle mass nuclei, energy will be released. Release of energy in fission process is based on this fact.

***Note :*** 1. *Binding energy per nucleon is practically constant for 30 < A < 170.*

2. *B.E. per nucleon is lower for both light nuclei (A < 30) and heavy nuclei (A > 170).*

## RADIOACTIVITY

### Law of Radioactive Disintegration

$$-\frac{dN}{dt} \propto N$$

$$-\frac{dN}{dt} = \lambda N \quad (\lambda \text{ is decay constant})$$

$$\Rightarrow \quad \boxed{N = N_0 e^{-\lambda t}}$$

Activity $A = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$

$$\boxed{A = A_0 e^{-\lambda t}}$$

**Half Life ($T_{1/2}$)**

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Let $N_0$ be the initial number of active nuclei and $N$ be the number of active nuclei remaining after $n$ half lives then $N = \frac{\mu_0}{2^n}$.

**Application :** Let $R_1$ be activity of radioactive substance at $t = T_1$ and $R_2$ be the activity at $t = T_2$, then $R_1 = \lambda N_1$ and $R_2 = \lambda N_2$

Number of nuclei disintegrated in $(T_2 - T_1)$ is

$$N_1 - N_2 = \frac{R_1 - R_2}{\lambda} = \frac{(R_1 - R_2)T}{\ln 2}$$

where $T$ is the half life of radioactive substance.

**Average Life ($T_{av}$)**

$$\boxed{T_{av} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}} \quad \text{or } T_{av} = 1.44\, T_{1/2} \quad \text{or} \quad T_{1/2} = 0.693\, T_{av}$$

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