UNIT-2

POLYNOMIALS

It is not once nor twice but times without number that the same ideas make their appearance in the world.

1. Find the value for K for which $x^4 + 10x^3 + 25x^2 + 15x + K$ exactly divisible by x + 7.

(Ans : K= - 91)

Ans: Let $P(x) = x^4 + 10x^4 + 25x^2 + 15x + K$ and g(x) = x + 7

Since P(x) exactly divisible by g(x)

$\therefore$ r (x) = 0

now

$$\begin{array}{r|l} & x^3+3x^2+4x-13 \\ x+7 & x^4+10x^3+25x^2+15x+K \\ & x^4+7x^3 \\ & \text{-------------} \\ & 3x^3+25x^2 \\ & 3x^3+21x^2 \\ & \text{-------------} \\ & 4x^2+15x \\ & 4x^2+28x \\ & \text{-------------} \\ & -13x+K \\ & -13x-91 \\ & \text{-------------} \\ & K+91 \\ & \text{-------------} \end{array}$$

$\therefore$ K + 91 = 0

K= -91

2. If two zeros of the polynomial $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$.Find the other zeros. (Ans:7, -5)

Ans: Let the two zeros are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

Sum of Zeros $= 2 + \sqrt{3} + 2 - \sqrt{3}$

$= 4$

Product of Zeros $= (2+ \sqrt{3})(2 - \sqrt{3})$

$= 4 - 3$

$= 1$

Quadratic polynomial is x^2 – (sum) x + Product

$$
\begin{array}{r}
x^2 - 2x - 35 \\
x^2 - 4x + 1 \overline{\smash{)}\, x^4 - 6x^3 - 26x^2 + 138x - 35} \\
x^4 - 4x^3 + x^2 \\
\text{-----------------} \\
-2x^3 - 27x^2 + 138x \\
-2x^3 + 8x^2 - 2x \\
\text{-----------------------} \\
-35x^2 + 140x - 35 \\
-35x^2 + 140x - 35 \\
\text{------------------------} \\
0 \\
\text{------------------------}
\end{array}
$$

$\therefore\ x^2 - 2x - 35 = 0$

$(x - 7)(x + 5) = 0$

$x = 7, -5$

other two Zeros are 7 and -5

3. Find the Quadratic polynomial whose sum and product of zeros are $\sqrt{2} + 1, \dfrac{1}{\sqrt{2}+1}$.

Ans: sum = $2\sqrt{2}$

Product = 1

Q.P =

X^2 – (sum) x + Product

$\therefore\ x^2 - (2\sqrt{2})\, x + 1$

4. If α,β are the zeros of the polynomial $2x^2 - 4x + 5$ find the value of a) $\alpha^2 + \beta^2$ b) $(\alpha - \beta)^2$.

(Ans: a) -1 , b) –6)

Ans: $p(x) = 2x^2 - 4x + 5$

$\alpha + \beta = \dfrac{-b}{a} = \dfrac{4}{2} = 2$

$\alpha\beta = \dfrac{c}{a} = \dfrac{5}{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Substitute then we get, $\alpha^2 + \beta^2 = -1$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Substitute, we get $= (\alpha - \beta)^2 = -6$

5. If α,β are the zeros of the polynomial $x^2 + 8x + 6$ frame a Quadratic polynomial whose zeros are a) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ b) $1 + \frac{\beta}{\alpha}$, $1 + \frac{\alpha}{\beta}$.

(Ans: $x^2+\frac{4}{3}x+\frac{1}{6}$, $x^2-\frac{32}{3}x+\frac{32}{3}$)

Ans: $p(x) = x^2 + 8x + 6$

$\alpha + \beta = -8$ and $\alpha\beta = 6$

a) Let two zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sum = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha.\beta} = \frac{-8}{6} = \frac{-4}{3}$

Product = $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha.\beta} = \frac{1}{6}$

Required Q.P is

$$x^2 + \frac{4}{3}x + \frac{1}{6}$$

b) Let two Zeros are $1+\frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$

sum = $1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}$

$= 2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= 2+ \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= 2+ \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ after solving this problem,

We get $= \frac{32}{3}$

Product = $(1 + \frac{\beta}{\alpha})(1+\frac{\alpha}{\beta})$

$= 1+ \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 1$

$= 2 + \frac{\alpha^2+\beta^2}{\alpha\beta}$

Substitute this sum,

We get = $\frac{32}{3}$

Required Q.P. is $x^2 - \frac{32}{3}x + \frac{32}{3}$

6. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial g(x) the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$.Find the polynomial g(x). (Ans:$4x^2+7x+2$)

Ans: p(x) = g (x) q (x) + r (x)

$g(x) = \frac{p(x) - r(x)}{q(x)}$

let $p(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2$

$q(x) = x^2 - 3x - 5$ and $r(x) = -5x + 8$

now $p(x) - r(x) = 4x^4 - 5x^3 - 39x^2 - 41x - 10$

when $\frac{p(x) - r(x)}{q(x)} = 4x^2 + 7x + 2$

$\therefore g(x) = 4x^2 + 7x + 2$

7. If the squared difference of the zeros of the quadratic polynomial $x2 + px + 45$ is equal to 144 , find the value of p. (Ans: ± 18).

Ans: Let two zeros are α and β where $\alpha > \beta$

According given condition

$(\alpha - \beta)^2 = 144$

Let $p(x) = x^2 + px + 45$

$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$

$\alpha\beta = \frac{c}{a} = \frac{45}{1} = 45$

now $(\alpha - \beta)^2 = 144$

$(\alpha + \beta)^2 - 4\alpha\beta = 144$

$(-p)^2 - 4(45) = 144$

Solving this we get $p = \pm 18$

8. If α,β are the zeros of a Quadratic polynomial such that $\alpha + \beta = 24$, $\alpha - \beta = 8$. Find a Quadratic polynomial having α and β as its zeros. (Ans: $k(x^2 - 24x + 128)$)

Ans: $\alpha+\beta = 24$

$\alpha - \beta = 8$

$2\alpha = 32$

$\alpha = \frac{32}{2} = 16, \therefore \alpha = 16$

Work the same way to $\alpha+\beta = 24$

So, $\beta = 8$

Q.P is x^2 – (sum) x + product

= $x^2 - (16+8)x + 16 \times 8$

Solve this,

it is $k\ (x^2 - 24x + 128)$

9. If α & β are the zeroes of the polynomial $2x^2 - 4x + 5$, then find the value of

a. $\alpha^2 + \beta^2$ b. $1/\alpha + 1/\beta$ c. $(\alpha - \beta)^2$ d. $1/\alpha^2 + 1/\beta^2$ e. $\alpha^3 + \beta^3$

(Ans:-1, $\frac{4}{5}$, -6, $\frac{-4}{25}$, -7)

Ans: Let $p(x) = 2x^2 - 4x + 5$

$\alpha+\beta = \frac{-b}{a} = \frac{4}{2} = 2$

$\alpha\beta = \frac{c}{a} = \frac{5}{2}$

a) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

Substitute to get = $\alpha^2+\beta^2 = -1$

b) $\frac{1}{a} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

substitute , then we get $= \frac{1}{a} + \frac{1}{\beta} = \frac{4}{5}$

b) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

Therefore we get, $(\alpha-\beta)^2 = -6$

d) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha\beta^2} = \frac{-1}{\left(\frac{5}{2}\right)^2}$

$\therefore \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{-4}{25}$

e) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2+\beta^2 - \alpha\beta)$

Substitute this,

to get, $\alpha^3+\beta^3 = -7$

10. Obtain all the zeros of the polynomial p(x) = $3x^4 - 15x^3 + 17x^2 + 5x - 6$ if two zeroes are $-1/\sqrt{3}$ and $1/\sqrt{3}$. (Ans:3,2)

11. Give examples of polynomials p(x), g(x), q(x) and r(x) which satisfy the division algorithm.
 a. deg p(x) = deg q(x) b. deg q(x) = deg r(x) c. deg q(x) = 0.

12. If the ratios of the polynomial $ax^3+3bx^2+3cx+d$ are in AP, Prove that $2b^3-3abc+a^2d=0$

Ans: Let $p(x) = ax^3 + 3bx^2 + 3cx + d$ and α , β , r are their three Zeros

but zero are in AP

let $\alpha = m - n$, $\beta = m$, $r = m + n$

sum = $\alpha+\beta+ r = \frac{-b}{a}$

substitute this sum , to get = $m= \frac{-b}{a}$

Now taking two zeros as sum $\alpha\beta +\beta r +\alpha r = \frac{c}{a}$

$(m-n)m + m(m+n) + (m + n)(m - n) = \frac{3c}{a}$

Solve this problem , then we get

$\frac{3b^2 - 3ac}{a^2} = n^2$

Product $\alpha\beta r = \frac{d}{a}$

$(m-n)m (m+n) = \frac{-d}{a}$

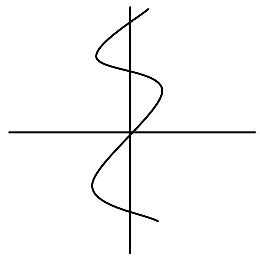
$(m^2 - n^2)m = \frac{-d}{a}$

$[(\frac{-b}{a})^2 - (\frac{3b^2 - 3ac}{a^2})] (\frac{-b}{a}) = \frac{-d}{a}$

Simplifying we get

$2b^3 - 3abc + a^2 d = 0$

13. Find the number of zeros of the polynomial from the graph given.



(Ans:1)

14. If one zero of the polynomial $3x^2 - 8x + 2k+1$ is seven times the other, find the zeros and the value of k (Ans k= 2/3)

Self Practice

14. If (n-k) is a factor of the polynomials x^2+px+q & $x^2 + mx+n$. Prove that

$$k = n + \frac{n-q}{m-p}$$

Ans : since (n – k) is a factor of $x^2 + px + q$

$\therefore (n-k)^2 + p(n-k) + q = 0$

And $(n-k)^2 + m(n-k) + n = 0$

Solve this problem by yourself,

$$\therefore k = n + \frac{n-q}{m-p}$$

<u>SELF PRACTICE</u>

16. If 2, ½ are the zeros of px^2+5x+r, prove that p= *r*.
17. If m, n are zeroes of ax^2-5x+c, find the value of a and c if m + n = m.n=10 (Ans: a=1/2 ,c=5)
18. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2+3x-2$. (Ans: 14x – 10)
19. What must be added to the polynomial $p(x)= x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by x^2+2x-3. (Ans: x-2)