# Relation and Function

CHAPTER – 1

## Relation and Function

The topics and subtopics covered in relations and Functions for class 12 are:

- Cartesian product of sets
- Relation
- Types of Relations
- Types of Functions
- Composition of functions and invertible functions
- Binary operations

### Cartesian product of sets

Let A and B be two non empty sets. The set of all ordered pairs (a, b) such that a∈ A and b ∈ B is called the cartesian product of set A with set B and is denoted by A × B.
Thus , A × B= {( a ,b) : a∈ A and b∈ B }.

**Note**

B× A = { ( b , a) : b∈ B and a∈ A}

**Example**

If A = { 1, 2, 3} and B = { x ,y } then , A×B = { (1 ,x) ,( 1,y),( 2,x) , (2 ,y) , ( 3,x) ,( 3, y)}

### Relation

- Mathematically, "a relation R from a set A to a set B is a subset of the cartesian product A × B obtained by describing a relationship between the first element x and the second element y of the ordered pairs in A × B.
- **Representation :** If ( a, b) ∈ R ,then we write a R b which is read as " a is related to b by the relation R"
  If ( a , b) ∉ R then we say " a is not related to b under R"

**Note**

If A and B are finite sets consisting of m and n elements respectively ,then A × B has $mn$ ordered pairs .therefore , total number of relation from A to B is $2^{mn}$.

**Example**

Let A be the set of students of class XII of a school and B be the set of students of class XI of the same school. Then relation R = {(a , b) ∈A×B: a is brother of b}

**Domain :** Let R be a relation from a set A to a set B. Then the set of all first components of the ordered pairs belonging to R is called the domain of R.

Thus , Domain of R = { a : ( a,b) ∈ R }

**Example**

If A = { 1 ,3, 5, 7} , B = { 2 ,4,6,8,10} and R = {(1,8) ,( 3,6) ,(5,2), ( 1,4)} is a relation from A to B, then domain = { 1 ,3, 5}.

**Range :** Let be a relation from a set A to a set B. Then the set of all second components of ordered pairs belonging to R is called the range of R.

Thus , Range of R = { b : ( a,b) ∈ R }

**Example**

If A = { 1 ,3, 5, 7} , B = { 2 ,4,6,8,10} and R = {(1,8) ,( 3,6) ,(5,2),( 1,4)} is a relation from A to B, then range of R = { 2 ,4, 6, 8}.

## RELATION ON A SET

Let A be a non empty set. Then a relation from A to itself i.e., a subset of A × Ais called a relation on set A.

## INVERSE OF A RELATION

Let A ,B be two sets and let R be a relation from a set A to a set B .Then ,the inverse of R , denoted by $R^{-1}$ = { ( b ,a) : ( a, b) ∈ R}

Clearly , (a, b) ∈ R iff (b, a) ∈ $R^{-1}$

**Example**

Let A = { 1 ,2,3} , B = { a ,b ,c, d} be two sets and R = {( 1,a) ,( 1, c) ,( d,2),( c,2) } is a relation from B to A.

## Types of Relations

- A relation R from A to A is also stated as a relation on A, and it can be said that the relation in a set A is a subset of A × A. Thus, the empty set φ and A × A are two extreme relations. Below are the definitions of types of relations:

- **Identity Relation**
  Identity relation is the one in which every elements related to itself only. i.e., Let A be a set .Then ,the relation $I_A = \{(a, a): a \in A\}$ is called identity relation.

**Example**

Let A be a non empty set {1,2,3} then R= {(1,1),(2,2),(3,3)} is a identity relation.

- **Empty Relation**
  If no element of A is related to any element of A, i.e. R = φ ⊂ A **×** A**,** then the relation R in a set A is called empty relation.

**Example**

Let A be the set of all students of a boys school. Then the relation R = {(a, b):a is sister of b} is a empty relation.

**Universal Relation**

If each element of A is related to every element of A, i.e. R = A **×** A, then the relation R in set A is said to be universal relation.

**Example**

Let A be the set of all students of a boys school. Then the relation R = {(a, b): the difference between heights of a and b is less than 3 meters} is universal set.

- Both the empty relation and the universal relation are sometimes called trivial relations.
- A relation R in a set A is called-

**Reflexive-** if (a, a) ∈ R, for every a ∈ A

**Example**

Let A = {1, 2, 3} and R = {(1, 1), (2, 2)}
Then, R is not reflexive since 3 **ϵ** A but (3, 3) ∉ R.

**Note**

The identity relation on a non empty set A is always reflexive relation on A. However, a reflexive relation on A is not necessarily the identity relation on A.

**Example**

R = {(a, a),(b, b),(c, c),(a, b)} is a reflexive relation on set A ={ a, b, c} but it is not the identity relation on A.

**Symmetric-** if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$

**Example**

Let A = {2, 3, 4}
$R_1$ {(2, 3), (3, 2)}
$R_2 = \{(2, 3), (3, 2), (3, 4)\}$

Here $R_1$ is a symmetric relation on A but $R_2$ is not a symmetric relation on A because (3, 4) ϵ $R_2$ but (4, 3) ∉ $R_2$.

**Note**

The identity and the universal relation on a non empty set are symmetric relations.

A reflexive relation on a set A is not necessarily symmetric.

**Example**

The relation R = { (1 ,1) ,(2 ,2),(3,3) ,( 1,3) } is reflexive relation on set A= { 1 ,2,3} but it is not symmetric.

**Transitive-** if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$.

**Example**

Let A = {1, 2, 3}

R = {(1, 2), (2, 1), (1, 1)}

$R_1$= {(2, 2), (2, 3), (3, 2), (3, 3)}

Then, R is not transitive since (2, 1) ϵ R, (1, 2) ϵ R but (2, 2) ∉R. $R_1$ is transitive.

∵ (2, 3) ϵ

$R_1$, (3, 2)ϵ $R_1 \Rightarrow$ (2, 2)ϵ $R_1$

and (3, 2)ϵ $R_1$, (2, 3)ϵ $R_1 \Rightarrow$ (3, 3)ϵ $R_1$.

**Note**

The identity and universal relations on a non empty set are transitive.

**Equivalence Relation-** A relation R in a set A is an equivalence relation if R is reflexive, symmetric and transitive.

**Example**

Show that the relation R is an equivalence relation in the set A = { 1, 2, 3, 4, 5 } given by the relation R = { (a, b):|a-b| is even }.

**Solution:** R = {(a, b):|a-b| is even }. Where a, b belongs to A

**Note**

An equivalence relation on R defined on a set A partition the set A into pair wise disjoint subsets. These subsets are called equivalence classes determined by relation R.

The set of all elements of A related to an element a ∈ A is denoted by [a] = { x ∈ A : ( x , a) ∈ R}.This is an equivalence class.

The collection of all equivalence classes form a partition of set A.

**Antisymmetric Relation :** Let A any set .A relation R on set A is said to be an antisymmetric relation iff ( a ,b) ∈ R and ( b,a) ∈ R ⇒ a = b for all a , b A .

**Example**

Let S be a non empty set and R be a relation on power set P(S) defined by (A , B) ∈ R iff A subset B for all A ,B ∈ P(S).

Then , R is an antisymmetric relation on P(S) , because ( A,B) ∈ R and ( B ,A ) ∈ R ⇒ A is a subset B and B is a subset A ⇒ A = B.

**Note**

The identity relation on a set A is an antisymmetric relation.

### Reflexive Property

From the given relation,
|a – a| = | 0 |=0
And 0 is always even.
Thus, |a-a| is even
Therefore, (a, a) belongs to R
Hence R is Reflexive

### Symmetric Property

From the given relation,
|a – b| = |b – a|
We know that |a – b| = |-(b – a)|= |b – a|
Hence |a – b| is even,
Then |b – a| is also even.
Therefore, if (a, b) ∈ R, then (b, a) belongs to R
Hence R is symmetric.

### Transitive Property:

If |a-b| is even, then (a-b) is even.
Similarly, if |b-c| is even, then (b-c) is also even.
Sum of even number is also even
So, we can write it as a-b+ b-c is even
Then, a – c is also even

So,

|a – b| and |b – c| is even, then |a-c| is even.
Therefore, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) also belongs to R
Hence R is transitive.

### SOME USEFUL RESULTS ON RELATIONS

**Theorem:** If R and A are two equivalence relations on a set A, then R⋂ $S$ is also an equivalence relation on A.

**Theorem:** The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.

**Example**

Let A = { a, b , c} and let R and S be two relations on A, given by
R = {( a, a) ,(b, b) , (c ,c) ,( a, b) ,(b, a) } and S = {(a, a) ,( b ,b) , (c, c) ,(b ,c) ,( c, b)}

It can be easily seen that each one of R and S is ab equivalence relation on A. But ,R ⋃ $S$ is not transitive, because ( a, b) ∈ R ⋃ $S$ and ( b ,c) ∈ R ⋃ $S$ but ( a, c) ∉ R ⋃ $S$.

Hence , R ⋃ $S$ is not an equivalence relation on A.
Theorem: If R is an equivalence relation on set A, then $R^{-1}$ is also an equivalence relation on A.

### Functions

Mathematically, "a relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B".

In other words, Let A and B be two non – empty sets. A relation f from A to B i.e., a subset of A × B is called a function ( or a mapping or a map) from A to B , if

(i) For each a∈ A there exist b ∈ B such that ( a, b) ∈ f.

(ii) (a, b) ∈ f and ( a, c) ∈ f ⇒ b = c.

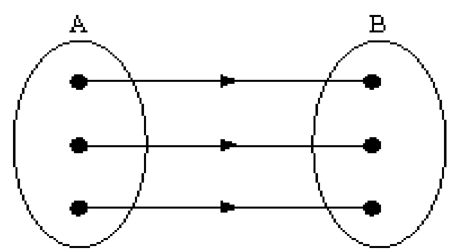
**Representation:** If f is a function from a set A to a set B, then we write f: A ⟶ B , which is read as f is a function from A to B or f maps A to B.

1. **Domain:** In function F: A ⟶ B, the set A is called domain and the elements of A are called pre-images.
   F: A ⟶ B

**Example**

f : {1,2,3,4} ⟶ {2,4,9,16} defined by f(x)=2x . Domain of function = {1,2,3,4}

2. **Co-domain** In function F: A ⟶ B, the set B is called co-domain and the element of B are called images.




**Example**

f : {1,2,3,4} ⟶ {1,2,3,...16}} defined by f(x)=2x. Co-domain = { 1,2,3 ,...,16} and range = {1,4,9,16}.

3. **Range :** The set of all f – images of A is known as the range of f or image set of A and is denoted by f(A).
   Thus , f(A) = { f(x) : x ∈ A} = Range of f.

**Example**

f : {1,2,3,4} ⟶ {1,2,3,...16}} defined by f(x)=2x, then range of f = {1,4,9,16}.

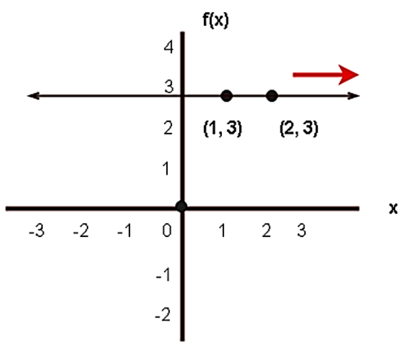
**GRAPH OF A FUNCTION:** The graph of a real function f consists of point whose coordinates ( x,y) satisfy y = f(x) , for all x ∈ Domain(f)

**vertical line test:** A curve in a plane represents the graph of a real function iff no vertical line intersect it more than once.

**constant function:** If k is a fixed real number, then a function f(x) = k is the set R for all x ∈ R is called a constant function.

The domain of the constant function f(x) = k is the set R of all real numbers and range of f is the singleton set {k}.

The graph of a constant function f(x) = k is a straight line parallel to x -axis which is above or below x -axis according as k is positive or negative .If k = 0 ,then the straight line is coincident to x-axis.

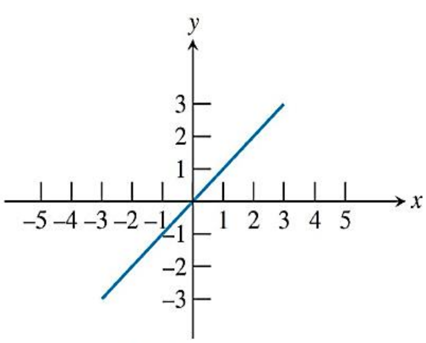



**IDENTITY FUNCTION:** The function that associates each real number to itself is called the identity function and is usually denoted by I.

Thus ,the function I : R → R defined by I(x) = x for all x ∈ R is called the identity function.

Domain = R and Range = R.

The graph of the identity function is a straight line passing through the origin and inclined at an angle of $45^o$ $with\ x-\ axis$.

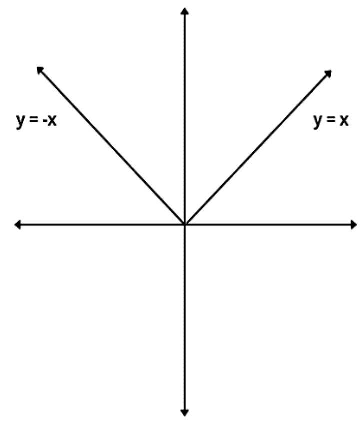



**MODULUS FUNCTION:** The function f(x) defined by

f(x) = |X| = $\begin{cases} x\ ,when\ x\ \geq 0 \\ -x\ ,when\ x < 0 \end{cases}$ is called the modulus function.

Domain = The set of all real numbers

Range = $R^+$ = { x ∈ R : x ≥ 0}




**Properties:** (i) For any real number x ,we have $\sqrt{x^2} = |x|$

(ii)If a , b are positive real numbers , then

$x^2 \leq a^2\ iff\ |x| \leq a\ iff - a\ \leq x \leq a$

$x^2 \geq a^2\ iff\ |x| \geq a\ iff\ x \leq -a\ , x\ \geq a$

$x^2 < a^2\ iff\ |x| < a\ iff - a < x < a$

$x^2 > a^2\ iff\ |x| > a\ iff\ x$ < -a or , x>a

$a^2 \leq x^2 \leq b^2\ iff\ a \leq |x| \leq b\ iff\ x \in$ [ −b , −a] ∪$[a\ , b]$

$a^2 < x^2 < b^2\ iff\ a < |x| < b\ iff\ x$ ( −b , −a) ∪$(a\ , b)$

(iii) For real numbers x and y , we have

|x + y| = |x| + |y| , if ( x ≥ 0 $and\ y \geq 0$ ) or ( x < 0 and y < 0)

| x -y| = |x| -|y| , if ( x≥ 0 $and\ |x| \geq |y|$) or , ( $x < 0\ and\ y <$ 0)

|x ± $y$ | ≤ $|x| + |y|$

|x ± $y$ | > $||x| - |y||$

**GREATEST INTEGER FUNCTION :** For any real number x ,we have use the symbol [x] to denote the greatest integer less than or equal to x.

**Example**

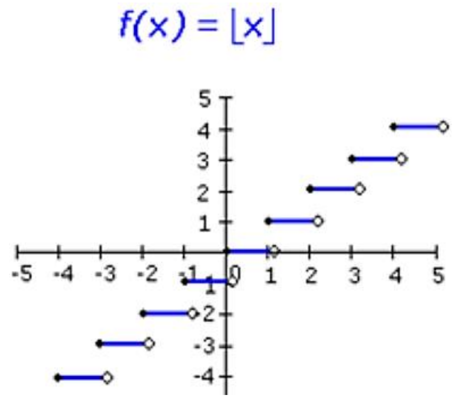
[ 2.75] = 2

**Note**

It is also called a step function.

Domain = The set of real numbers

Range = the set of integers.




**Properties:** If n is an integer and x is a real number between n and n+1 ,then

(i) [-n]=-[n]

(ii) [x+ k] =[x] +k for any integer k

(iii) [ -x] = -[x] – 1

(iv) [x]+[-x] = $\begin{cases} -1, if\ x \notin Z \\ 0, if\ x\ \in Z \end{cases}$

(v) [x] –[-x] = $\begin{cases} 2[x]+1, if\ x \notin Z \\ 2[x], if\ x \in Z \end{cases}$

(vi) [x] ≥ $k\ \Rightarrow x > k, ehere\ k\ \in Z$

(vii) [ x] < k ⇒ $x < k+1$ , where k ∈ Z

(viii) [x] > k ⇒ $x \geq k+1$ , where k ∈ Z

(ix) [x]<k ⇒ x < k ,where k ∈ Z

(x) [ x + y ] = [x] +[ y +x-[x]] for all x , y ∈ Z

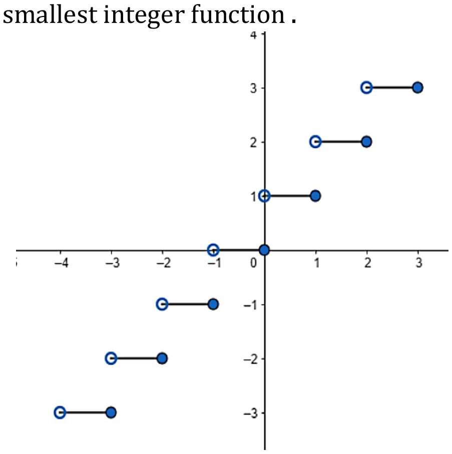
(xi) [x] + [ x + $\frac{1}{n}$]+ [ x + $\frac{2}{n}$] + …+ [ x + $\frac{n-1}{n}$ ] = [ nx] , n ∈ Z

**SMALLEST INTEGER FUNCTION :** For any real number x , we use the symbol ⌈$x$⌉ to denote the smallest integer greater than or equal to x.

**Example**

⌈4.7⌉ = 5

The function f: R →R defined by f(x) = ⌈$x$⌉ for all x ∈ R is called the smallest integer function .




Domain = Set of real numbers and Range = Set of integers.

**Properties :** (i) ⌈x⌉ + ⌈y⌉ – 1 ≤ ⌈x + y⌉ ≤ ⌈x⌉ + ⌈y⌉
(ii) ⌈x + a⌉ = ⌈x⌉ + a
(iii) ⌈x⌉ = a; iff x ≤ a < x + 1
(iv) ⌈x⌉ = a; iff x – 1 < a ≤ x
(v) a < ⌈x⌉ iff a < x
(vi) a ≤ ⌈x⌉ iff x < a

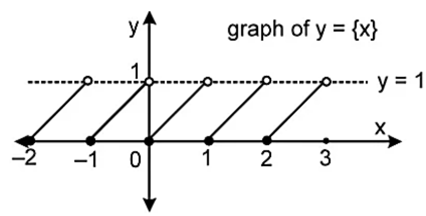
**FRACTIONAL PART FUNCTION :** f(x) is denoted by {f(x)}, is read as fractional part of f(x). Domain of Fractional Part function is same as Domain of function f(x) And Outcome always lie in 0 to 1 i.e 0≤{f(x)}<1.

Fractional part function is denoted by {x}, is read as fractional part of x. Let [x] be the greatest integer value of a real number x then fractional part of x is {x}=x-[x].

**Example**

{3.45} = 0.45

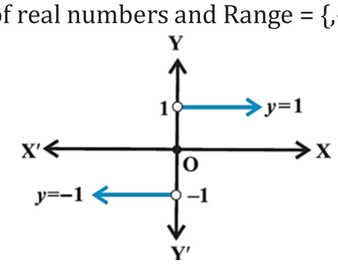
Domain = Set of real numbers , Range = [0 ,1).




**SINGUM FUNCTION:** The signum function of a real number $x$ is piecewise function which is defined as follows:

$$f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

Domain = Set of real numbers and Range = {-1 , 0 ,1}

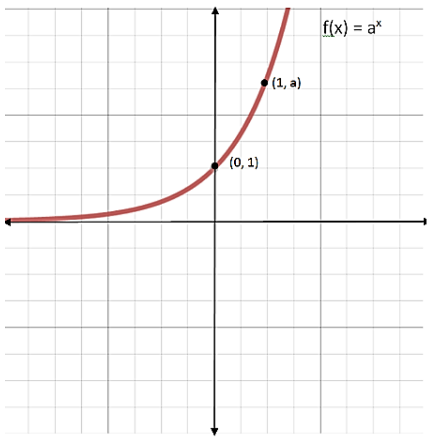



**EXPONENTIAL FUNCTION :** If a is a positive real number other than unity ,then a function that associates each x ∈ R to $a^x$ is called the exponential function.
In other word , a function f : R →R defined by f(x) = $a^x$ , where a > 0 and a ≠ 1 is called the exponential function.

Domain= Set of real numbers , Range = ( 0 ,∞)

**Case 1.** When a > 1
We observe that the value of y = f(x) = $a^x$ increase as the values of x increase.

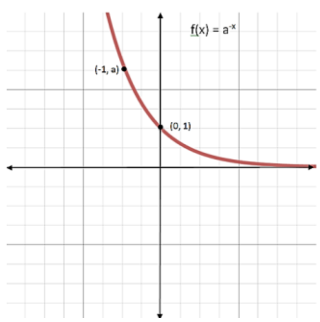
$$f(x) = a^x = \begin{cases} < 1, \text{for } x < 0 \\ = 1, \text{for } x = 0 \\ > 1, \text{for } x > 0 \end{cases}$$




**Case 2.** When 0 < a< 1

In this case , the values of y = f(x) = $a^x$ decrease with the increase in x and y > 0 for all x∈ R.

$$Y = f(x) = a^x = \begin{cases} > 1, \text{for } x < 0 \\ = 1, \text{for } x = 0 \\ < 1, \text{for } x > 0 \end{cases}$$




**LOGARITHIMIC FUNCTION:** If a > 0 and a ≠ 1, then the function defined by f(x) = $\log_a x$ , x > 0 is called the logarithmic function. i.e., $\log_a x = y$ iff $x = a^y$

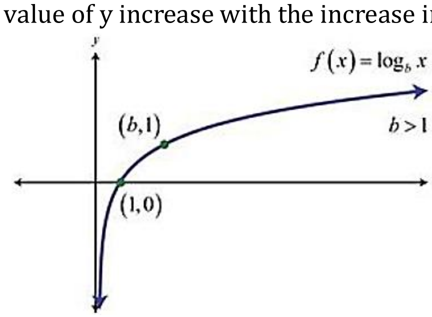
Domain = Set of all positive real numbers.

Range = Set of all real numbers.

Case 1. When a > 1

$$y = \log_a x = \begin{cases} < 0, \text{for } 0 < x < 1 \\ = 0, \text{for } x = 1 \\ > 0, \text{for } x > 1 \end{cases}$$

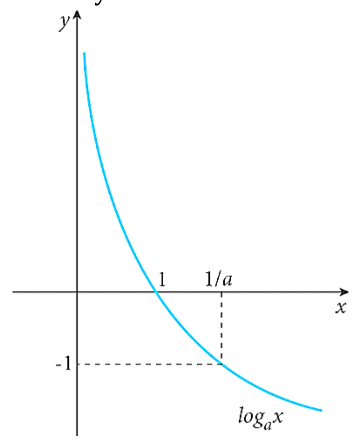
Also , the value of y increase with the increase in x.




**Case 2.** When 0 < a< 1

$$y = \log_a x = \begin{cases} > 0, \text{for } 0 < x < 1 \\ = 0, \text{for } x = 1 \\ < 0, \text{for } x > 1 \end{cases}$$

Also ,the value of y decrease with the increase in x.




**Remark**

Function f(x) = $\log_a x$ and g(x) = $a^x$ are inverse of each other, So, their graph are mirror images of each other in the line mirror y =x.

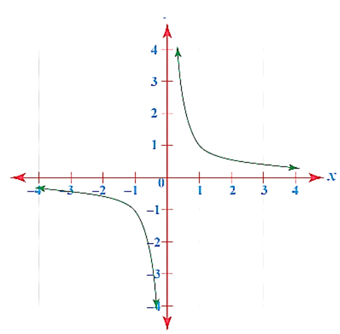
**RECIPORCAL FUNCTION:** The function that associates a real number x to its reciprocal $\frac{1}{x}$ is called the reciprocal function. Since $\frac{1}{x}$ is not defined for x = 0.So, we defined the reciprocal function as follows:

The function f : R – {0} →R defined by f(x) = $\frac{1}{x}$ is called the reciprocal function.

Domain = R – { 0} , Range = R – { 0}

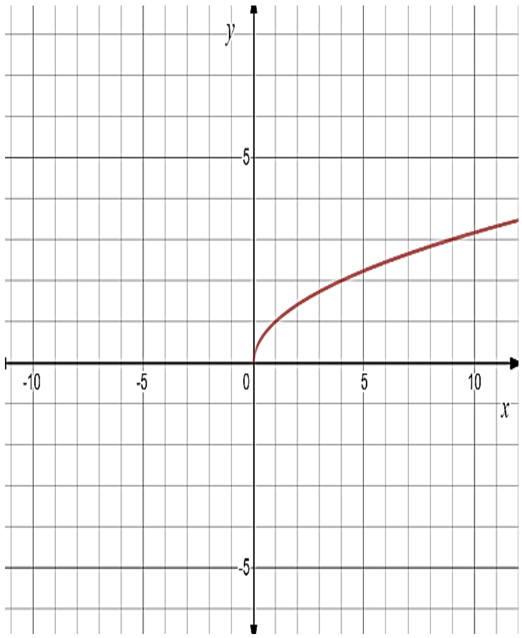
**Note**

We observe that the sign of $\frac{1}{x}$ is same as that of x and $\frac{1}{x}$ decreases with the increase in x.



$$f(x) = \frac{1}{x}$$

**SQUARE ROOT FUNCTION:** The function f : $R^+ \rightarrow$R defined by f(x) = $\sqrt{x}$ is called the square root function.
Domain = $R^+$ , Range = [0 , ∞)




## Types of Functions

**One to one Function:** A function f: X ⟶ Y is defined to be one-one (or injective), if the images of distinct elements of X under f are distinct, i.e., for every $x_1$ , $x_2$ ∈ X, f($x_1$ ) = f($x_2$ ) implies $x_1$ = $x_2$ .

**Algorithm**

**Step 1:** Take two arbitrary elements x ,y (say) in the domain of f :

**Step 2:** Put f(x) = f(y)

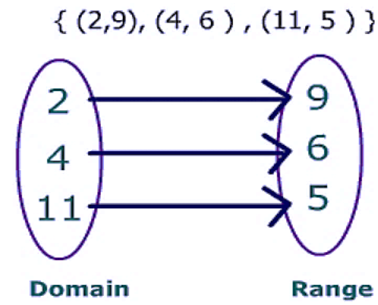
**Step 3:** Solve f(x) = f(y) .If it gives x = y only ,then f : A ⟶B is a one -one function ( or an injection) .Otherwise not.

**Note**

Let f : A ⟶ B and let x , y ∈ A. Then , x =y ⇒ f(x) =f(y) is always true from the definition. But f(x) =f(y) ⇒ x =y is true only when f is one – one.
If A and B are two sets having m and n elements respectively such that m ≤ n ,then total numbers of one – one functions from A to B is $n_{C_m} \times m!$

**Example**

The identity function X ⟶ X is always injective, if function f: R⟶ R, then f(x) = 2x is injective, If function f: R⟶ R, then f(x) = 2x+1 is injective.




- **Onto Function:** A function f: X ⟶ Y is said to be onto (or surjective), if every element of Y is the image of some element of X under f, i.e., for every y ∈ Y, there exists an element x in X such that f(x) = y.

**Thus , f: A** ⟶ B is a surjection iff for each b ∈B ,there exists a ∈A such that f(a) = b.

**Algorithm :** Let f : A ⟶ B be the given function

**Step 1** Choose an arbitrary element y in B

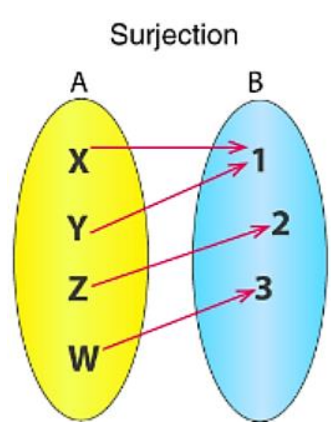
**Step 2** Put f(x) = y

**Step 3** Solve the equation f(x) =y for x and obtain x in terms of y .Let x = g(y)

**Step 4** If for all values of y ∈ B ,the values of x obtained from x = g(y) are In A, then f is onto.

**Example**

Let C = {1, 2, 3}, D = {4, 5} and let g = {(1, 4), (2, 5), (3, 5)}. Show that the function g is an onto function from C into D.
**Solution:** Domain = set C = {1, 2, 3}
We can see that the element from C,1 has an image 4, and both 2 and 3 have the same image 5. Thus, the Range of the function is {4, 5} which is equal to D. So we conclude that g: C ⟶D is an onto function.

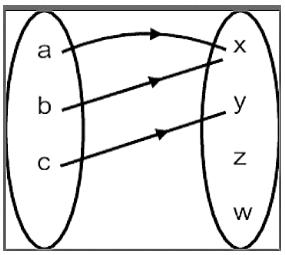


- **Many-one function:** If two or more elements of A have same image in B.

**Thus ,** f : A ⟶ B is a many one function if there exist x ,y ∈ A such that $x \neq y$ but f(x) = f(y)

**Example**

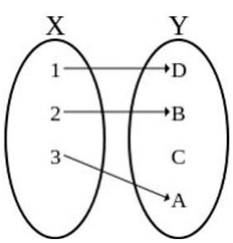
f: {a, b, c}⟶{x, y, z, w} defined by f(a)=x, f(b)=x and f(c) =y




- **Into function:** If there exists at least one element in B which does not have a pre-image in A.

**Example**

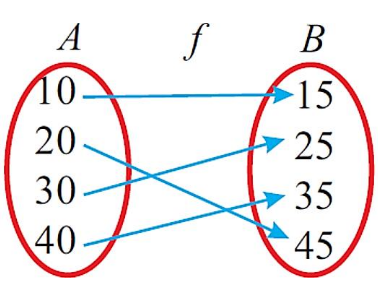
f: {1,2,3}⟶{A,B,C,D} defined by f(1) = D, f(2) = B and f(3) = A




- **One-one and Onto Function:** A function f: X ⟶ Y is said to be one-one and onto (or bijective), if f is both one-one and onto.

**Example**

The function *f*: **R** ⟶ **R**, $f(x) = 2x + 1$ is bijective, since for each $y$ there is a unique $x = (y - 1)/2$ such that $f(x) = y$.



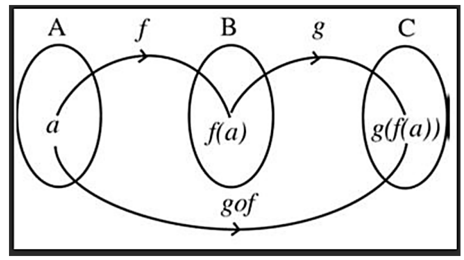

**Remark**

It follows from the above discussion that if A and B are two finite sets and f : A ⟶ B is a function, then

(i) f is an injection ⇒ n ( A) ≤ n( B)
(ii) f is a surjection ⇒ n ( B) ≤ n(A)
(iii) f is a bijection ⇒ n ( A) = n (B)

## Composition of Functions

Let f: A ⟶ B and g: B ⟶ C be two functions. Then the composition of f and g, denoted by **gof**, is defined as the function gof: A ⟶ C given by; (gof)(x) = g(f(x)) , for all x ∈A.




**Example**

Given the functions $f(x) = x^2 + 6$ and $g(x) = 2x - 1$, find (f ∘ g) (x).

**Solution:** Substitute x with 2x – 1 in the function $f(x) = x^2 + 6$.

$(f \circ g)(x) = (2x - 1)^2 + 6 = (2x - 1)(2x - 1) + 6$

## PROPERTIES

**Theorem 1:** The composition of functions is not commutative i.e. fog ≠ gof.

**Theorem 2:** The composition of functions is associative i.e., f ,g, h are three functions such that (fog)oh and fo(goh) exist , then (fog)oh = fo(goh)

**Theorem 3:** The composition of two bijections i.e., if f and g are two bijection , then gof is also a bijection.

**Theorem 4:** Let f : A ⟶ B. Then , $foI_A = I_Bof = f$ i.e. the composition of any function with the identity function is the function itself.

**Theorem 5:** Let f : A ⟶ B , g : B ⟶A be two functions such that gof = $I_A$.Then f is an injection and g is a surjection.

**Theorem 6:** Let f : A ⟶ B and g : B ⟶ A be two function such that fog = $I_B$.Then , f is a surjection and g is an injection.

**Theorem 7:** Let f : A ⟶B and g : B ⟶C be two function, Then ,

(i) gof : A ⟶ C is onto ⇒ g : B ⟶C is onto

(ii) gof : A ⟶ C is one - one ⇒ f : A ⟶B is one - one

(iii) gof : A ⟶ C is onto and g : B ⟶C is one - one ⇒ f : A ⟶B is onto

(iv) gof : A ⟶ C is one -one and f : A ⟶B is onto ⇒ g : B ⟶C is one -one.

## INVERSE OF AN ELEMENT

Let A and B be two sets and let f : A ⟶ B be a mapping. If a ∈A is associated to b ∈ B under the function f ,then b is called the f image of a and we write it as b = f(a).

We also say that a is the pre-image or inverse element of b under f and we write a = $f^{-1}(b)$

If f : A ⟶ B is a bijection ,we can define a new function from B to A which associates each element y ∈B to its pre-image $f^{-1}(y)$ ∈A. Such a function is known as the inverse of function f and is denoted by $f^{-1}$

Algorithm: Let A ⟶ B be a bijection. To find the inverse of f we follow the following steps:

**Step 1** Put f(x) = y ,where y ∈Y and x ∈A

**Step 2** Solve f(x) =y to obtain x in terms of y.

**Step 3** In the relation obtained in step 2 replace x by $f^{-1}(y)$ to obtain the required inverse of f.

**Example**

If A = { 1,2,3,4} , B = { 2 ,4,6,8} and f: A ⟶ B is given by f(x)=2x ,then write f and $f^{-1}$ as a set of ordered pairs.

**Solution:** f(1) =2 ,f(2) = 4 ,f(3) = 6 and f(4) =8

Therefore , f = {( 1 , 2) ,( 2, 4) ,( 3, 6) ,( 4,8) } which is clearly a bijection.

On interchanging the components of ordered pairs in f ,we obtain

$f^{-1} = \{(2,1), (2,4), (6,3), (8,4)\}$

## PROPERTIES

**Theorem 1:** The inverse of a bijection is unique.

**Theorem 2:** The inverse of a bijection is also a bijection

**Theorem 3:** If f : A ⟶ B is a bijection and g : B ⟶ A is the inverse of f, then fog = $I_B$ and gof = $I_A$ ,where $I_A$ and $I_B$ are the identity functions on the sets A and B respectively.

**Theorem 4:** If f : A ⟶ B and g : B ⟶ C are two bijection, then gof : A ⟶ C is a bijection and $(gof)^{-1} = f^{-1}og^{-1}$

**Theorem 5:** Let f: A ⟶ B and g : B ⟶ A be two functions such that gof = $I_A$ and fog = $I_B$.Then f and g are bijections and g= $f^{-1}$

**Theorem 6:** Let f : A ⟶ B be an invertible function, then $(f^{-1})^{-1} = f$

## INVERTIBLE FUNCTION

A function f: X ⟶ Y is defined to be invertible if there exists a function g: Y ⟶ X such that gof = $I_X$ and fog = $I_Y$. The function g is called the inverse of f and is denoted by $f^{-1}$.

**Example**

Find the inverse for the function f(x) = (3x+2)/(x-1)

**Solution:** First, replace f(x) with y and the function becomes,
y = (3x+2)/(x-1)
By replacing x with y we get,
x = (3y+2)/(y-1)
Now, solve y in terms of x :
x (y – 1) = 3y + 2
=> x.y – x = 3y +2
=> x.y – 3y = 2 + x
=> y (x – 3) = 2 + x
=> y = (2 + x) / (x – 3)
So, y = $f^{-1}(x)$ = (x+2)/(x-3)
An important note is that, if f is invertible, then f must be one-one and onto and conversely if f is one-one and onto, then f must be invertible.

## BINARY FUNCTION

A binary operation ∗ on a set A is a function ∗ : A × A ⟶ A. We denote ∗ (a, b) by a ∗ b

- **Closure Property:** A binary operation * on a non-empty set P has closure property, if a ∈ P, b ∈ P ⇒ a * b ∈ P.

**Example**

2 + 3 = 5, 5 ∈ R 6 + -9 = -3, -3 ∈ R This is true for all real umbers R

- **Associative Property:** The associative property of binary operations holds if, for a non-empty set S, we can write (a * b) *c = a*(b * c), where {a, b, c} ∈ S. Suppose Z be the set of integers and multiplication be the binary operation.

**Example**

Let, a = -3, b = 5, and c = -16. We can write (a × b) × c = 240 = a × (b × c). :-Please note that all binary operations are not associative, for example, subtraction denoted by '-'.

- **Commutative Property:** A binary operation * on a non-empty set S is commutative, if a * b = b * a, for all (a, b) ∈ S. Suppose addition be the binary operation and N be the set of natural numbers.

**Example**

Let, a = 4 and b = 5, a + b = 9 = b + a , where a , b belongs to set of real numbers

- **Distributive Property:** Let * and # be two binary operations defined on a non-empty set S. The binary operations are distributive if, a* (b # c) = (a * b) # (a * c), for all {a, b, c} ∈ S. Suppose * is the multiplication operation and # is the subtraction operation defined on Z (set of integers).

**Example**

Let, a = 3, b = 4, and c = 7. Then, a*(b # c) = a × (b – c) = 3 × (4 – 7) = -9. And, (a * b) # (a * c) = (a × b) – (a × c) = (3 × 4) – (3 × 7) = 12 – 21 = -9. Therefore, a*(b # c) = (a * b) # (a * c), for all {a, b, c} ∈ Z.

- **Identity Element:** A non-empty set P with a binary operation * is said to have an identity e ∈ P, if e*a = a*e= a, ∀ a ∈ P. Here, e is the identity element.

**Example**

'0' is additive identity for addition binary operation.

- **Inverse Property:** A non-empty set P with a binary operation * is said to have an inverse element, if a * b = b * a = e, ∀ {a, b, e} ∈ P. Here, a is the inverse of b, b is the inverse of a and e is the identity element.

**Example**

'-a' is the additive inverse of 'a' under the addition binary operation .Where a belongs to the set real number

**Example**

Show that subtraction is not binary operation on N, where N is the set of natural number.

**Solution:** – : N×N ⟶ N, given by (a, b) ⟶a-b , is not binary operation, as the image of (3,5) under ' - ' is 3 – 5 = -2 not belongs to N.

# QUESTIONS

## MCQ

**Q1.** Determine the binary operation on the set R a*b=1 for all a, b ∈R.
(a) $*$ $is\ only\ commutative$
(b) $*$ $is\ only\ associative$
(c) $*$ $is\ both\ commutative\ and\ associative$
(d) None of these

**Q2.** If f : R→ $R; f(x) = x^2\ amd\ g: R \to R; g(x) = 2x + 1, then\ fog\ is$
(a) $2x^2 + 1$
(b) $(2x + 1)^2$
(c) $4x^2 + 1$
(d) None of these

**Q3.** If $f: R \to R$ is a bijection given by $f(x) = x^3 + 3, then\ f^{-1}(x)$ is:
(a) $f^{-1}(x) = (x - 3)^{1/3}$
(b) $f^{-1}(x) = (x - 3)^{-1/3}$
(c) $f^{-1}(x) = (x + 3)^{1/3}$
(d) None of these

**Q4.** Let $A$ and $B$ be sets. Then the function $f: A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is :
(a) One-one only (b) Onto only
(c) Into only (d) Bijective

**Q5.** Let $f: \{1,3,4\} \to \{1,2,5\}$ and $g: \{1,2,5\} \to \{1,3\}$ be given by $f = \{(1,2), (3,5), (4,1)\}$ and $g = \{(1,3), (2,3), (5,1)\}$. Then gof =?
(a) $\{(1,3), (3,1), (4,3)\}$
(b) $\{(1,3), (3,1), (3,3)\}$
(c) $\{(1,3), (3,1), (4,3), (5,1)\}$
(d) None

**Q6.** What is the minimum value of the expression $x^2 + 8x + 10$
(a) -3 (b) -6
(c) 0 (d) 2

**Q7.** What is the maximum value of expression $5 - 6x - x^2$
(a) 10 (b) 12
(c) 14 (d) 16

**Q8.** Which of the following functions have inverse
$f: \{1,2,3,4\} \to \{10\}$ with $f = \{(1,10), (2,10), (3,10), (4,10)\}$
$g: \{5,6,7,8\} \to \{1,2,3,4\}$ with $g = \{(5,4), (6,3), (7,4), (8,2)\}$
$h: \{2,3,4,5\} \to \{7,9,11,13\}$ with $h = \{(2,7), (3,9), (4,11), (5,13)\}$
(a) $f, g$ but not $h$
(b) $g$ but not $h\ and\ f$
(c) $f$ but not $h$ and $g$
(d) $h$ but not $f$ and $g$

**Q9.** If f : R → R, defined by f(x) = $x^2$ + 1, then the values of $f^{-1}$ (17) and $f^{-1}$ (-3) respectively are
(a) Empty set, {4 , -4}
(b) {3 , -3} , Empty set
(c) Empty set, Empty set
(d) {4, -4}, Empty set

**Q10.** If $f: Z \to Z. f(x) = x^2 + x\ for\ all\ x \in Z, then\ f\ is$:
(a) Many one (b) One - One
(c) Onto (d) None of these

**Q11.** Let A={1,2,3}. Then number of equivalence relations containing (1,2) is
(a) 1 (b) 2
(c) 3 (d) 4

**Q12.** Let A={1,2,3}. Then number of relations containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is
(a) 1 (b) 2
(c) 3 (d) 4

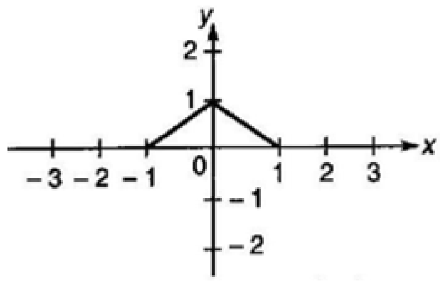
**Q13.** Number of binary operations on the set $\{a, b\}$ are
(a) 10 (b) 16
(c) 20 (d) 8

**Q14.** Find the number of all onto functions from the set $\{1,2,3, \dots \dots \dots .., n\}$ to itself.
(a) $2^n - n$
(b) $2^n$
(c) $n$
(d) $2^n$-1

**Q15.** If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$

(a) $x^4 - 6x^3 + 10x^2 - 3x + 1$
(b) $x^4 - 6x^3 + 10x^2 + 3x - 1$
(c) $x^4 - 6x^3 + 10x^2 - 3x$
(d) $x^4 - 6x^3 + 10x^2 + 3x$

**Q16.** In the following graph for $x \in [-1, 1] f(x)$ is defined by :




(a) $|x| + 1$
(b) $-|x| + 1$
(c) $-|x + 1|$
(d) $-|x| - 1$

**Q17.** Find the domain of the function, $f(x) = \frac{x^2+2x+3}{x^2-7x+10}$

(a) $\mathbb{R} - \{2,5\}$
(b) $\mathbb{R} - \{3,4\}$
(c) $\mathbb{R} - \{4,5\}$
(d) $\mathbb{R} - \{1,2\}$

**Q18.** What is the maximum possible value of xy, where |x + 5| =8 and y=9 – |x – 4|

(a) 401
(b) 124
(c) 104
(d) None

**Q19.** If A and B are finite sets such that n(A)=m and n(B)=k, find the number of relations from A to B

(a) $2^{mk}$
(b) $mk$
(c) $2^{mk} - 1$
(d) $mk + 1$

**Q20.** If $f(x) = x^3 - \frac{1}{x^3}$, then

(a) $f(x) + f\left(\frac{1}{x}\right) = 1$
(b) $f(x) + f\left(\frac{1}{x}\right) = 0$
(c) $f(x).f\left(\frac{1}{x}\right) = 0$
(d) $f(x).f\left(\frac{1}{x}\right) = 1$

**Q21.** Find the range of, $f(x) = |2x - 3| - 3$

(a) $[3, \infty)$ (b) $(2, \infty)$.
(c) $[-2, \infty)$. (d) $[-3, \infty)$.

**Q22.** If $f(x)$ is a one to one function, where $f(x) = x^2 - x + 1$, then find the inverse of the $f(x)$:

(a) $\left(x - \frac{1}{2}\right)$ (b) $\sqrt{x - \frac{3}{4}} + \frac{1}{2}$
(c) $\sqrt{x - \frac{3}{4}} - \frac{1}{2}$ (d) None of these

**Q23.** If $f(x)$ is a one to one function, where $f(x) = x^2 - x + 1$. *Find the value of* $f\left(\frac{3}{4}\right) + f^{-1}\left(\frac{3}{4}\right)$

(a) $\frac{21}{16}$ (b) $\frac{4}{5}$
(c) $\frac{16}{5}$ (d) None of these

**Q24.** If f(x)= $5^x$, then $f^{-1}(x)$ is :

(a) $x^5$ (b) $5^{-x}$
(c) $\log_5 x$ (d) None of these

**Q25.** Determine a quadratic function f defined by f(x)=$ax^2 + bx + c$ if $f(0) = 6, f(2) = 11$, and $f(-3) = 6$

(a) $f(x) = \frac{1}{10}x^2 + \frac{23}{10}x + 6$
(b) $f(x) = \frac{1}{10}x^2 - \frac{23}{10}x + 6$
(c) $f(x) = \frac{1}{10}x^2 + \frac{23}{10}x - 6$
(d) $f(x) = \frac{1}{10}x^2 - \frac{23}{10}x - 6$

**Q26.** Which of the following function is an even function

(a) $f(x) = \frac{a^x+1}{a^x-1}$ (b) $f(x) = \frac{a^x-a^{-x}}{a^x+a^{-x}}$
(c) $f(x) = x\frac{a^x+1}{a^x-1}$ (d) None of these

**Q27.** Find the domain and the range of the function f(x)=$\frac{x^2}{1+x^2}$.

(a) Domain=R Range= [0,1)
(b) Domain=R- {0} Range= [0,1)
(c) Domain=R- {0,1} Range= (0,1)
(d) Domain=R Range= (0,1)

**Q28.** Find domain of the function $f(x) = \frac{1}{\sqrt{x+[x]}}$.

(a) $(1, \infty)$ (b) $[0, \infty)$
(c) $(0, \infty)$ (d) $[1, \infty)$

**Q29.** If $A = \{1,2,3\}$, then the relation $R = \{(1,2), (2,3), (1,3)\}$ in $A$ is

(a) transitive only
(b) reflexive only
(c) symmetric only
(d) symmetric and transitive only

**Q30.** The domain of the relation $\{(x, y): y = |x - 1|, x \in Z, and\ |x| \leq 3\}$ is

(a) {– 3, – 2, – 1, 0, 1, 2, 3) (b) {4, 3, 2, 1, 0}
(c) {16, 1, 2, 3, 4} (d) None of these

## SUBJECTIVE QUESTIONS

**Q1.** Show that a relation R is defined on the set of positive integers N by a R b if $a^b = b^a$ then R is an equivalence relation.

**Q2.** For a real number 'x' and 'y' if x R y iff x -y +$\sqrt{2}$ is an irrational number, then what you say about relation R.

**Q3.** Let R be a relation on the set N of natural numbers defined by R = { ( x, y) : x + 2y = 8 }. What is domain of the R .

**Q4.** Show that a relation in the set of natural numbers N is defined as follows : a < b if a + x = b has a solution in N. Then the relation < is only transitive .

**Q5.** Find $R^{-1}$ if R = { ( a , b) : a= 2b for all a , b∈ $N$ }

## NUMERICAL TYPE QUESTIONS

**Q1.** If A = { 1 , 2,3,4} and B = { 5 , 6 , 7} , then number of relations from A to B is equal to _______.

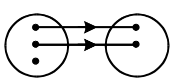
**Q2.** Let f(x) = cos x + x and g(x) = $x^2$, fog(0)____-.

**Q3.** The domain of f(x) = $\sqrt{x+3}$ is [ a , ∞ ) then a ____.

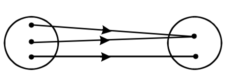
**Q4.** Range of gof (x) where f(x) = $\sqrt{x}$ , g(x) = $x^2 - 1$ is [α, ∞) then find the value of α ___.

**Q5.** Let g : R → R be a function such that , g(x) = 2x + 5 .Then , the value of $g^{-1}(5)$ ____-.

## TRUE AND FALSE

**Q1.**  pictorial diagrams represent the function .

**Q2.** A function whose domain and range are both subsets of real numbers is called **a real function**.

**Q3.**  many–one onto (surjective but not injective)

**Q4.** f (x) = cos x is an even function.

**Q5.** If f(x) has a period T, then f(ax + b) has a period - $\frac{T}{|a|}$.

## ASSERTION AND REASONING

**Q1.** **Assertion (A)** : { x ∈ $R|x^2 < 0$} is not a set .Here ,R is the set of real numbers.
**Reason (R) :** For every real number x , $x^2 \geq 0$
(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q2.** **Assertion (A) :** If A = { 1 ,2,3} ,B = { 2 ,4} ,then the number of relation from A to B is equal to 26
**Reason (R) :** The total number of relation from set A to set B is equal to {$2^{n(A).n(B)}$}
(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q3.** **Assertion (A)** : If A = { 1 ,2, 3} , then the relation R defined on A is { ( 1 ,1 ) , ( 2, 2) ,( 3,3) , ( 4, 4) } is transitive.
**Reason (R) :** A relation R defined on A is transitive iff aRb , bRc ⇒ cRa for all a , b , c ∈ A.
(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q4.** **Assertion (A)** : f(x) = cos x is even function.
**Reason (R) :** If f(-x) = - f(x) , for all x ∈ $R$ ,then f is odd function.
(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q5.** **Assertion (A)** : R is an equivalence relation.
**Reason (R) :** R is reflexive relation, R is symmetric relation and R is transitive relation.
(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

# HOME WORK

## MCQ

**Q1.** For any two real number, an operation defined by $a * b = 1 + ab$ then the value of $4 * 6$______.
(a) 25 (b) 26
(c) 40 (d) 45

**Q2.** A function f(x) is linear and has a value of 29 at x = -2 and 39 at x = 3,Then value of f(x) at x = 5 is _____.
(a) 34 (b) 43
(c) 21 (d) 50

**Q3.** If f(x) = $\frac{x+2}{x-3}$ , x ≠ 3, then $f^{-1}(-2)$ is equal to _____.
(a) 1.33 (b) 1.45
(c) 2.34 (d) 3.33

**Q4.** If solution of the equation 7cos x + 5 sin x = 2k + 1 is possible , then the number of integral values of k is_________.
(a) 4 (b) 6
(c) 8 (d) 10

**Q5.** The number of symmetric relation on A set with 5 elements is _____.
(a) $2^{10}$ (b) $2^{19}$
(c) $2^{11}$ (d) $2^{15}$

**Q6.** Number of equivalence relation defined in the set S = { a , b, c, d} is
(a) 15 (b) 12
(c) 20 (d) 29

**Q7.** If f(x) = $\frac{1}{1-x}$ , g(x) = f(f(x)) and h(x) = f(g(x)),then f(x).g(x).h(x) is equal to
(a) 2 (b) -1
(c) 3 (d) -3

**Q8.** If y = $4sin^2x - \cos 2x$ ,then y lies in the interval
(a) ( -1 , 5) (b) [ 1, 5]
(c) (-1, 5] (d) [ -1 , 5]

**Q9.** Let be the set of integers. Then the function f : Z → Z×Z defined by f(x) = ( x-1 ,1) ,for all $x \in Z$ is
(a) Only one- one (b) Only onto
(c) One – one and onto (d) None of these

**Q10.** Domain of the function f(x) = $\sqrt{2^x - 5^x}$ is x ≤ 0.
(a) x = 0 (b) x≥0
(c) x> 0 (d) x ≤ 0

## SUBJECTIVE QUESTIONS

**Q1.** Let A = { 1, 3, 5,7} and B = { 2, 4, 6, 8 } and R be the relation defined by { (x ,y) : x -y > 0 } ,then find the domain and range of R

**Q2.** Let f(1) =1 and f(n) = $2\sum_{r=1}^{n-1} f(r)$ .Then $\sum_{n=1}^{m} f(n)$ is equal to

**Q3.** If f : R → S ,defined by f(x) = sin x - $\sqrt{3}$ cos x + 1 , is onto ,then what is the interval of S .

**Q4.** The maximum value of f(x) =$\frac{3x^2+9x+17}{3x^2+9x+7}$ is 5k +1 then the value of k is

**Q5.** Let A and B be finite sets containing respectively 3 and 2 elements. Find the number of function that can be defined from A to B

## NUMERICAL TYPE QUESTIONS

**Q1.** The identity element of the set of real numbers under the binary operation '+' _________.

**Q2.** Determine the range of R = { (a ,b) : a∈ $N$ , a < 5 , b = 4 } is {a} then value of a is ______.

**Q3.** Let S { 1, 2, 3, 4} and ∗ be an operation on S defined by $a * b = r$ , where r is the least non-negative remainder when product is divided by 5 , ∗ is a binary operation then find the value of $2 * 3$ = ____.

**Q4.** The domain of the function f(x) = $\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is [ a , b ) then find the value of b = ______.

**Q5.** If f : R → R is given by f(x) = 3x -5 , the find the value of $f^{-1}(5)$ = _______.

## TRUE AND FALSE

**Q1.** $f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$ is exponential function.

**Q2.** A function f: X → Y is defined to be invertible if there exists a function g: Y → X such that gof = $I_X$ and fog = $I_Y$. The function g is called the inverse of f and is denoted by $f^{-1}$.

**Q3.** A non-empty set P with a binary operation * is said to have an identity e ∈ P, if e*a = a*e= a, ∀ a ∈ P. Here, e is the identity element

**Q4.** y = $x^2$ is symmetric about x-axis

**Q5.** If no element of A is related to any element of A, i.e. R = φ ⊂ A × A, then the relation R in a set A is called empty relation.

## ASSERTION AND REASONING

**Q1.** **Assertion (a) :** The function f : R → R given by f(x) =$x^3$ is injective.

**Reason (R) :** The function f : X→ Y is injective , if f(x) = f(y) ⇒ x = y for all x , y ∈ X.

(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q2.** Consider the function f : R→R defined as f(x) = $\frac{x}{x^2+1}$

**Assertion (A):** f(x) is not one-one.

**Reason (R):** f(x) is not onto.

(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q3.** Let W be the set of words in the English dictionary. A relation R is defined on W as R = {( x,y) ∈W× W such that x and y have at least one letter in common}

**Assertion (A):** R is reflexive

**Reason (R):** R is symmetric

(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**Q4.** Consider the set A = { 1, 3 ,5}

**Assertion (A):** The number of reflexive relations on set A is $2^9$

**Reason(R):** A relation is said to be reflexive if xRx , for all x ∈ $A$

(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true

**Q5.** Let R be any relation in the set A of human beings in a town at a particular time.

**Assertion (A):** If R = {( x ,y) : x is wife of y}, then R is reflexive.

**Reason (R):** If R = { ( x,y) : x is father of y } ,then R is neither reflexive nor symmetric nor transitive.

(a) Both A and R are individually true but R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

# SOLUTIONS

## MCQ

**S1.** **(c)** Clearly, by definition $a * b = b * a = 1, for\ all\ a, b \in R$. Also $(a * b) * c = (1 * c) = 1$ and $a * (b * c) = 1$ and $a * (b * c) = a * 1 = 1$, for all $a, b, c \in R$. Hence R is both associative and commutative.

**S2. (b)** $fog(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2$

**S3. (a)** Let $f(x) = y$
$\Rightarrow x^3 + 3 = y$
$\Rightarrow x = (y - 3)^{\frac{1}{3}}$
$\Rightarrow f^{-1}(y) = (y - 3)^{1/3}$
Thus $f^{-1}: R \to R\ is\ different\ as\ f^{-1}(x) = (x - 3)^{1/3}\ for\ all\ x \in R$.

**S4. (d)** Check for injectivity:
Let $(a_1, b_1)$ and $(a_2, b_2) \in A \times B$ such that
$f(a_1, b_1) = f(a_2, b_2)$
This implies, $(b_1, a_1) = (b_2, a_2)$
$\Rightarrow b_1 = b_2$ and $a_1 = a_2$
$(a_1, b_1) = (a_2, b_2)$ for all $(a_1, b_1)$ and $(a_2, b_2) \in A \times B$
Therefore, f is injective.
Check for Subjectivity:"
Let "(b,a)" be any element of " B×A". Then " a∈A" and b∈B
This implies "(a,b)∈A×B
For all "(b,a)∈B×A", their exists "(a,b)∈A×B
Therefore, $f: A \times B \to B \times A$ is bijective function.

**S5. (a)** Given function, f: $\{1,3,4\} \to \{1,2,5\}$ and g: $\{1,2,5\} \to \{1,3\}$ be given by
$f = \{(1,2), (3,5), (4,1)\}$ and $g = \{(1,3), (2,3), (5,1)\}$
At $f(1) = 2$ and $g(2) = 3, gof$ is
$gof(1) = g(f(1)) = g(2) = 3$
At $f(3) = 5$ and $g(5) = 1$, gof is
$g \circ f(3) = g(f(3)) = g(5) = 1$
At $f(4) = 1$ and $g(1) = 3, gof$ is
$g \circ f(4) = g(f(4)) = g(1) = 3$
Therefore, gof $= \{(1,3), (3,1), (4,3)\}$

**S6. (b)** $x^2 + 8x + 10 = x^2 + 8x + 16 - 6 \Rightarrow = (x + 4)^2 - 6$
Now the smallest possible value of the above expression can be –6
Since $(x + 4)^2$ can be minimum 0, when $x = -4$

**S7. (c)** $5 - 6x - x^2 = 14 - (3 + x)^2$
Thus the expression can have maximum value 14, when $x = -3$

**S8. (d)** $f: \{1,2,3,4\} \to \{10\}$ with $f = \{(1,10), (2,10), (3,10), (4,10)\}$
f has many-one function like f(1)=f(2)=f(3)=f(4)=10, therefore f has no inverse.
g: $\{5,6,7,8\} \to \{1,2,3,4\}$ with g = {(5,4), (6,3), (7,4), (8,2)} $g$ has many-one function like $g(5) = g(7) = 4$, therefore $g$ has no inverse.
$h: \{2,3,4,5\} \to \{7,9,11,13\}$ with $h = \{(2,7), (3,9), (4,11), (5,13)\}$
All elements have different images under h. So h is one-one onto function, therefore, h has an inverse.

**S9. (d)** $y = x^2 + 1$
$x^2 = y – 1$
$x = \sqrt{(y-1)}$
$f^{-1}(x) = \sqrt{(x-1)}$
$f^{-1}(17) = \sqrt{(17-1)} = 4,-4$
$f^{-1}(-3) = \sqrt{(-3-1)} = \sqrt{-4}$ which is not defined $= \phi$

**S10. (a)** Let $x, y \in Z\ (domain); then\ f(x) = f(y) \Rightarrow x^2 + x = y^2 + y$
$(x^2 - y^2) + (x - y) = 0 \Rightarrow (x - y)(x + y + 1) = 0 \Rightarrow x = y\ or\ y = -x - 1$
Since $f(x) = f(y)$ does not provide the unique solution $x = y$, but it also provides $y = -x - 1$. Putting $x = 1, we\ get\ y = 1\ and\ y = -2$. Thus $f$ provides 1 and –2 same image under $f$. Hence $f$ is a many-one function

**S11.** **(a)** $A = \{1,2,3\}$
For equivalence relation containing (1,2)
For symmetric, it must consists (1,2) and (2,1).
For transitivity, it must consists (1,3) and (3,2) and (1,2), (2,1), (2,3), (3,1)
For reflexivity, it must consists (1,1) and (2,2), (3,3)
⇒R={(1,1),(2,2),(2,3),(3,3),( 1,2) (1,3), (2,1), (3,1), (3,1)}
⇒ Only 1 such relation is possible.

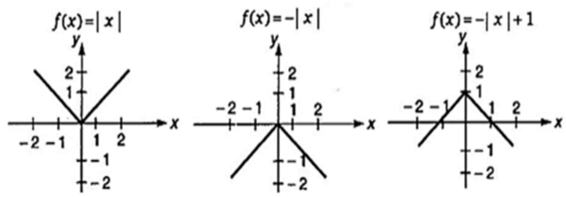
**S12. (a)** $A = \{1,2,3\}$
Relation containing (1,2) and (1,3)
For symmetricity, (1,2), (2,1), (1,3) and (3,1) must be included.
For reflexivity, (1,1), (2,2), (3,3) must be included.
For transitivity $(2,1), (1,3) \in R$ but (2,3)not belongs to R.
⇒ Only one set is there $R = \{(1,1), (1,2), (1,3), (2,2), (2,1), (3,3), (3,1)\}$

**S13. (b)** $A = \{a, b\}$ and $A \times A = \{(a,a),(a,b),(b,b),(b,a)\}$
Number of elements $= 4$
So, number of subsets $= 2^4 = 16$.

**S14. (a)** The number of onto functions that can be defined from a finite set A
containing n elements onto a finite set B containing elements $= 2^n - n$.

**S15. (c)** Given function:
$f(f(x)) = f(x)^2 - 3f(x) + 2$
$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$
By using the formula $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2ab$, we get
$= (x^2)^2 + (-3x)^2 + 2^2 - 2x^2(3x) + 2x^2(2) - 2 \times 2 \times (3x) - 3(x^2 - 3x + 2) + 2$
$= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$
$= x^4 - 6x^3 + 9x^2 + 4x^2 - 3x^2 - 12x + 9x - 6 + 2 + 4$
Simplifying the expression, we get,
$f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$

**S16. (b)**




**S17. (a)** The given function is $f(x) = \frac{x^2+2x+3}{x^2-5x+6}$
Now, $x^2 - 7x + 10 = (x-2)(x-5)$
Therefore, the given function can be written as
$f(x) = \frac{x^2+2x+3}{(x-2)(x-5)}$
So, clearly for $x =$
2,5, the function will be unbounded. Therefore, $f(x)$ exists for all real numbers except at $x = 2,5$.
Hence, the domain of the function $f(x)$ is $\mathbb{R} - \{2,5\}$

**S18. (c)** $|x + 5| = 8$
$\Rightarrow x = 3 \; and \; x = -13$
$and \; 9 - |x - 4| = y = f(x)$
$for \; x = 3, y = f(x) = 8$
$and \; for \; x = -13, y = f(x) = -8$
Max. $(x \cdot y) = (-13) \times (-8) = 104$

**S19. (a)** It is provided that, the number of elements in the set A and B is n(A)=m and $n(B) = k$ respectively.
Therefore, the number of relations from the set A to set B is $2^{n(A)m(B)} = 2^{mk}$.

**S20. (b)** The given function is $f(x) = x^3 - \frac{1}{x^3}$ Replacing $x$ by $\frac{1}{x}$ into the given function we obtain, $f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$
Therefore, adding these two functions, we get

$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$
$= 0$

**S21. (d)** The given function is $f(x) = |2x - 3| - 3$.
There does not exist any value of x for which f(x) is unbounded. So, domain
of the function $f(x)$ is the set of all real numbers $R$:
Observe that, $f(x) \geq -3$, since $|2x - 3| \geq 0$.
Therefore, the range of the function $f(x)$ is $[-3, \infty)$.

**S22.** (b) $y = x^2 - x + 1$
$\Rightarrow y = x^2 - x + \frac{1}{4} + \frac{3}{4} \Rightarrow y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$\Rightarrow y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2 \Rightarrow \left(x - \frac{1}{2}\right) = \sqrt{y - \frac{3}{4}}$
$\Rightarrow x = \sqrt{y - \frac{3}{4}} + \frac{1}{2} \Rightarrow y = \sqrt{x - \frac{3}{4}} + \frac{1}{2}$

**S23. (a)** $f(x) + f^{-1}(x) = (x^2 - x + 1) + \sqrt{x - \frac{3}{4}} + \frac{1}{2}$
$= \left(\frac{3}{4}\right)^2 - \frac{3}{4} + 1 + \sqrt{\frac{3}{4} - \frac{3}{4}} + \frac{1}{2}$
$= \frac{9}{16} + \frac{1}{4} + \frac{1}{2} = \frac{9+4+8}{16} = \frac{21}{16}$
Again $f(x) = x\frac{a^x+1}{a^x-1}$

$$f(-x) = (-x)\frac{a^{-x}+1}{a^{-x}-1} = (-x)\frac{\frac{(1+a^x)}{a^x}}{\frac{1-a^x}{a^x}}$$

$$= (-x)\frac{1+a^x}{1-a^x} = (-x)\frac{1+a^x}{-(a^x-1)} = x\frac{1+a^x}{a^x-1} = x\frac{a^x+1}{a^x-1}$$
$\therefore f(x) = f(-x)$

**S24. (c)** $y = 5^x \Rightarrow \log_5 y = x \Rightarrow f^{-1}(x) = \log_5 x$

**S25. (a)** The given quadratic function is $f(x) = ax^2 + bx + c$.
Since, $f(0) = 6, so$
$a(0)^2 + b(0) + c = 6$
$\Rightarrow c = 6$
Again, $f(2) = 11$

$\Rightarrow a(2)^2 + b(2) + 6 = 11$
$\Rightarrow 4a + 2b + 6 = 11$
$\Rightarrow 4a + 2b = 5 \ldots (i)$
Also, $f(-3) = 6$
$\Rightarrow a(-3)^2 + b(-3) + c = 0 \Rightarrow 9a - 3b = -6 \ldots (ii)$
Solving the equation (i) and (ii), we obtain
$a = \frac{1}{10}$ and $b = \frac{23}{10}$
Thus, the required quadratic function is $f(x) = \frac{1}{10}x^2 + \frac{23}{10}x + 6$

**S26. (c)** $f(x) = \frac{a^x+1}{a^x-1}$
$\therefore f(-x) = \frac{a^{-x}+1}{a^{-x}-1} = \frac{1+a^x}{1-a^x}$
$\Rightarrow f(x) \neq f(-x)$, hence $f(x)$ is not an even function
$\text{Again} f(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}}$
$\therefore f(-x) \frac{a^{-x} - a^x}{a^{-x} + a^x}$
Hence $f(x)$ is and even function.

**S27. (a)** The given function is $f(x) = \frac{x^2}{1+x^2}$ .Observe that, $1 + x^2 \neq 0$.
Therefore, the function is defined for all real numbers.

Thus, the domain of the function $f(x)$ is the set of all real numbers R.
Now, rewrite the given function in terms of x, taking f(x)=y.
$y = \frac{x^2}{1+x^2}$
$\Rightarrow y + x^2y = x^2$
$\Rightarrow x^2(1-y) = y$
$\Rightarrow x^2 = \frac{y}{1-y}$
$\Rightarrow x = \sqrt{\frac{y}{1-y}}$, which is valid if $\frac{y}{1-y} \geq 0$.
i.e., if $y(1-y) \geq 0$,
i.e., if $-y(y-1) \geq 0$,
i.e., if $y \geq 0$ and $y - 1 < 0$, since $y$ should not be 1.
i.e. if $0 \leq y < 1$.
Hence, the range of the function $f(x)$ is [0,1).

**28. (c)** The given function is $f(x) = \frac{1}{\sqrt{x+[x]}}$
Recall that, the greatest integer function [x] is defined as
$$[x] = \begin{cases} 0, \text{ if } 0 \leq x < 1 \\ 1 \text{ if } 1 \leq x < 2 \\ 2, \text{ if } 2 \leq x < 3 \\ \ldots, \end{cases}$$
$$\text{Now, } x + [x] = \begin{cases} x + [x] > 0 & \text{for all } x > 0 \\ 0, & \text{for all } x = 0 \\ x + [x] < 0 & \text{for all } x < 0 \end{cases}$$
Therefore, the function $f(x)$
$= \frac{1}{\sqrt{x+[x]}}$ is defined for all real values of $x$ such that $x + [x] > 0$
Thus, the domain of the function f(x) is $(0, \infty)$.

**S29. (a)** A relation $R$ on a non-empty set $A$ is said to be transitive if $xRy$ and $yRz \Rightarrow xRz$, for all $x \in R$. Here, (1,2) and (2,3) belongs to $R$ implies that (1,3) belongs to $R$.

**S30. (a)** R = {(- 3, 4), (- 2, 3), (- 1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}
Domain R Set of first entries of the ordered pairs in the relation R
{– 3, –2, –1, 0, 1, 2, 3}

## SUBJECTIVE QUESTIONS

**S1.** A relation R is defined on the set of positive N by a R b if $a^b = b^a$
Reflexive : Let any number ( positive) 2 ∈ $N$ such that 2 R 2 if $2^2 = 2^2$ (correct)
Therefore , R is reflexive relation.
Symmetric : Let a = 2 and b = 4 ∈ $N$ such that 2 R 4 $\Rightarrow 2^4 = 4^2$ ….(i)
And 4 R 2 $\Rightarrow 4^2 = 2^4$ ….(ii)
From (i) and (ii) , we have 2 R 4 = 4 R 2
Therefore , R is symmetric relation.
Transitivity : Let a = 2 , b = 4 and c = 2 ∈ $N$ such that 2 R 4 $\Rightarrow 2^4 = 4^2$ and 4 R 2 $\Rightarrow 4^2 = 2^4$
$\Rightarrow 2^4 = 2^4$
Therefore , the relation R is reflexive , symmetric and transitive. Hence the relation is an equivalence relation.

**S2.** Given relation R is defined as x R y iff x -y + $\sqrt{2}$ is an irrational number where x , y ∈ $R$.
For reflexive : x R x ⇒ x – x + $\sqrt{2} = \sqrt{2}$ which is an irrational number. Hence R is reflexive.
For symmetric : Let x R y ⇒ x -y + $\sqrt{2}$ ⇒ -y + x + $\sqrt{2}$ ⇒ $(-y) - (-x) + \sqrt{2}$ ⇒ y is not related to x
Hence R is not symmetric.
For transitive : Let x , y , z ∈ $R$ then x R y and y R z ⇒ x -y + $\sqrt{2}$ and y -z + $\sqrt{2}$
Adding we get x-y + $\sqrt{2}$ + y – z + $\sqrt{2}$ = x -z+ $2\sqrt{2}$
⇒ x is not related to z. Hence R is not transitive.

**S3.** Relation R is defined as R = { (x , y ) : x + 2y = 8}
Domain = { x : x, y ∈ $N$ and satisfying x + y+ y = 8}
For x = 2 ,4,6 , y is a natural number .Hence domain of R in { 2 , 4, 6 }

**S4.** a is not related to a { since a is not less than itself }
Hence not reflexive
Symmetric : 1 + x = 2

x = 1
2 + x = 1
x = -1
Not symmetric
Transitive : ( 1 ,2) ,( 2, 3) ⇒ 1 R 3
( 1 , 2) ∈ $R$ ⇒ 1 + x = 2 ⇒ x = 1∈ $N$
(2 , 3) ∈ $R$ ⇒ 2 + x= 3 ⇒ x = 1∈ $N$
Now 1 + x = 3 ⇒ x = 2∈ $N$
It is only transitive .

**S5.** if R = { ( a , b) : a= 2b for all a , b∈ $N$ }
R = { ( 1 , 2) ,( 2 , 4) ,( 3, 6) ,( 4, 8 ) ….}
$R^{-1}$ = { ( 2 ,1) , ( 4 ,2 ) ,( 6 ,3 ) ….}

## NUMERICAL TYPE QUESTIONS

**S1. ($2^{12}$)** Given , n(A) = 4 , n(B) =3
We know that , the total number of relations from two finite sets A and B is given by $2^{n(A).n(B)} = 2^{12}$

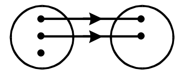
**S2. (1)** fog(x) = cos g(x) + g(x)
= cos $x^2$ + $x^2$
fog(0) = 1

**S3. (-3)** f(x) = $\sqrt{x+3}$, then Domain of f is [–3, ∞), then a = -3

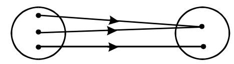
**S4. (-2)** f(x) = $\sqrt{x}$ , g(x) = $x^2$ – 1.
**For gof(x)**
Since range of f is a subset of the domain of g,
∴ domain of gof is [0, ∞) and g{f(x)}= g(√x) = x – 1. Range of gof is [–1, ∞)
Then α = -1 ⇒ $2\alpha = -2$

**S5. (0)** Let y = 2x + 5
⇒ y − 5 = 2x
$\Rightarrow x = \frac{y-5}{2} = g^{-1}(y)$
$\therefore g^{-1}(x) = \frac{x-5}{2}$
Then $g^{-1}(5) = 0$

## TRUE AND FALSE

**S1. (False)** In  one element of domain has no image, therefore it is not a function.

**S2. (True)** A function whose domain and range are both subsets of real numbers is called **a real function**.

**S3. (True)**  many –one onto (surjective but not injective)

**S4. (True)** If f (–x) = f (x) for all x in the domain of 'f', then f is said to be an even function.
**Example:** f (x) = cos x is even function.

**S5. (False)** If f(x) has a period T, then f(ax + b) has a period $\frac{T}{|a|}$.

## ASSERTION AND REASONING

**S1. (a)** Both A and R are true and R is the correct explanation of A.
Since , $x^2$ is never negative but it can be positive or zero.

**S2. (a)** We know by the property of relation ,the total number of relation from set A to set B is $2^{n(A).n(B)}$.
So, both A and R are true and R is the correct explanation of A.

**S3. (a)** Since , { (1,1),( 2, 2) ,( 3,3) ( 4,4)} is identity relation and identity relation is always equivalence relation.
So, it is transitive.
By definition of transitive relation
aRb , bRc iff cRa , for all a , b, c ∈ $A$
Therefore , A and R are both correct and R is the correct explanation of A.

**S4. (b)** Let f(x) = cos x then f( -x) = cos (-x) = cos x = f(x)
i.e., f(-x) = f(x) ,then f is even function.
Both A and R are individually true but R is not the correct explanation of A.

**S5. (a)** R is an equivalence relation ,if R is reflexive relation, R is symmetric relation and R is transitive relation.
So, both A and R are true and R is the correct explanation of A.

# HOMEWORK

## MCQ

**S1. (a)** For any two real number , an operation defined by $a * b = 1 + ab$ then the value of 4 ∗ 6 = 1 + 4×6= 25

**S2. (b)** Since the function f(x) is linear .So, let f(x) = ax + b
Therefore f( -2) = a (-2) + b = 29 …….(i)
And f(x) = a x+ b , f(3) = 3a + b = 39 ……..(ii)
⇒ a = 2 and b = 33
Sving (i) and (ii) ,we get f(x) = 2x+ 33
Therefore , f(5) = 2× 5 + 33 = 10 +33 = 43

**S3. (a)** If f(x) = $\frac{x+2}{x-3}$ , x ≠ 3, then $f^{-1}(-2)$ is equal to
Let f(x) = y ⇒ x = $f^{-1}(y)$
Therefore , y = $\frac{x+2}{x-3}$
⇒ xy -3y = x + 2

$\Rightarrow$ x = $\frac{3y+2}{y-1}$

$\Rightarrow$ $f^{-1}(y) = \frac{3y+2}{y-1}$

$\Rightarrow f^{-1}(x) = \frac{3x+2}{x-1}$

$\Rightarrow$ $f^{-1}(-2) = \frac{3(-2)+2}{(-2)-1} = \frac{-6+2}{-3} = \frac{4}{3}$ = 1.33

**S4.** **(c)** -1 ≤ $\frac{2k+1}{\sqrt{7^2+5^2}}$ ≤ 1 { since -1 ≤ $\cos(x-\alpha)$ ≤ 1 }

$\Rightarrow$ - $\sqrt{74} \leq 2k+1 \leq \sqrt{74}$

Therefore , k = -4 , -3 ,-2 ,-1 , 0 ,1, 2, 3

i.e., 8 value which will satisfy the above inequality.

**S5.** **(d)** We know that , If n( A) = n

Then , No. of symmetric relation = $2^{\frac{n^2+n}{2}}$

Since , n( A) = 5

Therefore , number of symmetric relation = $2^{\frac{n^2+n}{2}} = 2^{\frac{25+5}{2}} = 2^{\frac{30}{2}} = 2^{15}$

**S6.** **(a)** Number of equivalence relation defined in the set S = { a , b, c, d} is

If cardinality of the set S is n then number of equivalence relation = $n^2 - 1$

A.T.Q

Cardinality of set S = { a , b , c, d} = 4

Now , number of equivalence relation = $4^2 - 1 = 15$

**S7.** **(b)** f(x).g(x).h(x) = $\frac{1}{1-x}.\frac{1}{1-f(x)}.\frac{1}{1-g(x)} = \frac{1}{1-x}.\frac{1}{1-\frac{1}{1-x}}.\frac{1}{1-\frac{1}{1-f(x)}}$

$\Rightarrow \frac{-1}{x}.\frac{1}{1-\frac{1}{1-\frac{1}{1-x}}} = \frac{-1}{x}.\frac{1}{1-\frac{1-x}{-x}} = \frac{-1}{x}.x = -1$

**S8.** **(d)** Given that ,

y = $4sin^2x - \cos 2x$

y = $6sin^2x - (1-2sin^2x)$ [ Since -1 ≤ sin x ≤ 1 ]

$\Rightarrow 0 \leq sin^2x \leq 1$

$\Rightarrow 0 \leq 6sin^2x - 1 \leq 1 \times 6$

$\Rightarrow$ 0 -1≤ $6sin^2x - 1$ ≤ 6 -1

$\Rightarrow$ -1 ≤ $6sin^2x - 1 \leq 5$

$\Rightarrow$ -1≤ y≤ 5

$\Rightarrow$ y ∈ $[-1,5]$

**S9.** **(c)** Let f(x) = x -1

Now , $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

Therefore , f is one -one

Now , if y = x -1 ⇒ x = y +1

f( y + 1) = y +1 -1 = y

Therefore , f is onto.

Hence , f is bijective.

**S10.** **(d)** Let f(x) = $\sqrt{2^x - 5^x}$

$\Rightarrow 2^x - 5^x \geq 0$

$\Rightarrow 2^x \geq 5^x$

$\Rightarrow \frac{1}{2^x} \leq \frac{1}{5^x}$ …….(i)

Equation (i) is satisfied so that value of x i.e., x ≤ 0

## SUBJECTIVE QUESTIONS

**S1.** Let A = { 1, 3, 5,7} and B = { 2, 4, 6, 8 } and R be the relation defined by { (x ,y) : x -y > 0 } ,then find the domain and range of R

R = {( 3 ,2) ,(5,2) ,(5,4),(7,2),(7,4), (7 ,6)}

Therefore, domain of R = { 3 ,5,7} and Range = { 2,4,6}

**S2.** f(n) = 2{f(1)+f(2)+f(3)+….+f(n-1)}

Therefore , f(n+1)=2 { f(1) + f(2) +…+f(n)}

$\Rightarrow$ f(n+1) =3f(n) for n ≥ 2

Also f(2) =2f(1) =2

Therefore f(3) = 3f(2) =2.3 , etc.

Therefore , $\sum_{n=1}^{m} f(n)$ = f(1) +f(2) + …+f(m)

= 1 + 2+2.3 + $2.3^2$+….+$2.3^{m-2}$

= 1 + 2( 1 +3 +$3^2$ + …+$3^{m-2}$)

= $3^{m-1}$

**S3.** As f(x) is onto

Therefore S = Range

Minimum value of f(x) = - $\sqrt{1+\left(-\sqrt{3}\right)^2} + 1 = -1$

And maximum value of f(x) = $\sqrt{1+\left(-\sqrt{3}\right)^2} + 1 = 3$

$\Rightarrow$ S = [ -1 ,3]

**S4.** Let y = $\frac{3x^2+9x+17}{3x^2+9x+7} = 1 + \frac{10}{3x^2+9x+7}$

Now , $3x^2 + 9x + 7 = 3(x^2 + 3x)$+7 = $3\left(x+\frac{3}{2}\right)^2 + \frac{1}{4} \geq \frac{1}{4}$ for all x ∈ $R$

Maximum value of $\frac{10}{3x^2+9x+7}$ is 40

Maximum value of y is 1+ 40 = 41

Therefore 5k + 1 = 41

$\Rightarrow$ k = 8

**S5.** Sinve each of 3 elements of A can associated to an element of B in 2 ways. Therefore all the 3 elements can be associated with elements of B in $2^3$ ways.

## NUMERICAL TYPE QUESTIONS

**S1. (0)** The identity element of the set of real numbers under the binary operation '+'

Let e is the identity element then

X + e = x = e+ x for all $x \in R$.

$\Rightarrow e = 0$

**S2. (4)** Given

R = {(a , b) : a∈ $N$ , a < 5 , b =4}

Natural numbers less than 5 are 1 , 2, 3 and 4

a = { 1 ,2, 3, 4} and b = { 4 }

R = {( 1 , 4) , ( 2, 4) , ( 3, 4) , ( 4,4)}

So, Domain of relation R = { 1 , 2, 3, 4}

Range of function= { 4}

**S3. (1)** Let S { 1, 2, 3, 4} and $*$ be an operation on S defined by $a * b = r$ , where r is the least non-negative remainder when product is divided by 5 , $*$ is a binary operation then find the value of 2 $*$ 3 = (Remainder when 2 × 3 = 6 is divided by 5 ) = 1

**S4. (3)** Domain of f(x) = $\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$
Here -1 ≤ ( $x - 3$) ≤ 1 and $9 - x^2 > 0$
⇒ 2 ≤ $x$ ≤ 4 and -3 < x < 3
So , 2 ≤ $x$ < 3 , then b = 3

**S5. ($\frac{10}{3}$)** Clearly f : R → R is a one – one function. So, it is invertible .
Let f(x) = y . Then , 3 x -5 = y ⇒ $x = \frac{y+5}{3}$ ⇒ $f^{-1}(y) = \frac{y+5}{3}$
Hence $f^{-1}(x) = \frac{x+5}{3}$ , then value of $f^{-1}(5) = \frac{10}{3}$

## TRUE AND FALSE

**S1. (False)** The signum function of a real number $x$ is piecewise function which is defined as follows:

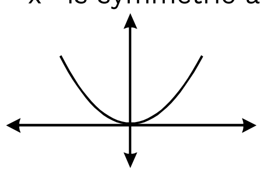
$$f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

Domain = Set of real numbers and Range = {,-1 , 0 ,1}

**S2. (True)** A function f: X → Y is defined to be invertible if there exists a function g: Y → X such that gof = $I_X$ and fog = $I_Y$. The function g is called the inverse of f and is denoted by $f^{-1}$.

**S3. (True)** A non-empty set P with a binary operation * is said to have an identity e ∈ P, if e*a = a*e= a, ∀ a ∈ P. Here, e is the identity element

**S 4. (False)** y = $x^2$ is symmetric about y-axis,



y = $x^2$

**S5. (True)** If no element of A is related to any element of A, i.e. R = φ ⊂ A × A, then the relation R in a set A is called empty relation.

## ASSERTION AND REASONING

**S1. (a)** Here , the function f : R → R given by f(x) =$x^3$
Suppose f(x) = f(y) where , x, y ∈ $R$
⇒ $x^3 = y^3$ ……..(i)
Now ,we try to show that x = y
Suppose $x \neq y$ ,their cubes will not be equal.
$x^3 \neq y^3$
However ,this will be a contradiction to equation (i)
Therefore , x = y .Hence ,f is injective. Hence both Assertion and reason are true and reason is the correct explanation of assertion.

**S2. (b)** Given f : R → R s.t .f(x) = $\frac{x}{1+x^2}$
Taking $x_1$= 4 , $x_2 = \frac{1}{4}$ ∈ R
f( $x_1$) = $f(4) = \frac{4}{17}$ , f( $x_2$) = $f\left(\frac{1}{4}\right) = \frac{4}{17}$
( $x_1 \neq x_2$)
Therefore , f is not one -one
A is true.
Let y ∈ $R$
f(x) =y
$\frac{x}{1+x^2} = y$
$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$
Since x ∈ $R$ , 1 - 4$y^2$ ≥ 0
⇒ $\frac{-1}{2} \leq y \leq \frac{1}{2}$
So, range (f) ∈ $\left[\frac{-1}{2}, \frac{1}{2}\right]$
Range (f) ≠ R
Therefore , f is not onto.
R is true
R is not the correct explanation for A.

**S3. (b)** For any word x ∈W
x and x have at least one ( all) letter in common
Therefore , ( x ,x) ∈ R , for all x ∈ W
Therefore R is reflexive
Symmetric : Let ( x,y) ∈ R , x, y ∈ W
⇒ x and y have at least one letter in common
⇒ y and x have at least one letter in common
⇒ ( y ,x) ∈ R
Therefore ,R is symmetric
Hence A is true ,R is true ,R is not a correct explanation for A.

**S4. (d)** By definition , a relation in A is said to be reflexive if x R x , for all x ∈ A. So R is true.
The number of reflexive relations on a set containing n elements is $2^{n^2-n}$
Here n = 3
The number of reflexive relations on a set A = $2^{9-3} = 2^6$
Hence A is false.

**S5. (d)** Assertion : Here R is not reflexive : as x cannot be wife of x.
Reason : Here , R is not reflexive ; as x cannot be father of y , then y cannot be father of x. Therefore , R is not symmetric. R is not transitive as if x is father of y and y is father of z ,then x is grandfather ( not father) of z.