SAMPLE QUESTION PAPER (BASIC) - 01

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

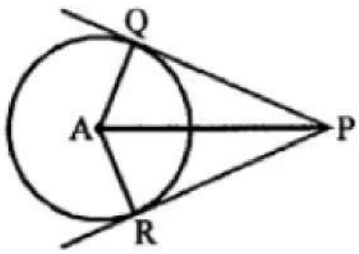
Section A

1. If the line segment joining the points A (x_1, y_1) and $B(x_2, y_2)$ is divided by a point P in the ratio 1 : k internally, then the coordinates of the point P are **[1]**

a) $\left(\frac{x_2-kx_1}{1+k}, \frac{y_2-ky_1}{1+k}\right)$ b) $\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

c) $\left(\frac{x_2+kx_1}{1-k}, \frac{y_2+ky_1}{1-k}\right)$ d) $\left(\frac{x_1+kx_2}{1+k}, \frac{y_1+ky_2}{1+k}\right)$

2. In the given figure, PQ and PR are tangents to a circle with centre A. If ∠QPA = 27° then ∠QAR equals **[1]**



a) 153° b) 117°

c) 63° d) 126°

3. A die is thrown once. The probability of getting a prime number is **[1]**

a) $\frac{1}{3}$ b) $\frac{1}{6}$

c) $\frac{1}{2}$ d) $\frac{2}{3}$

4. The distance of the point (4, 7) from the x-axis is **[1]**

a) 7

b) 4

c) 11

d) $\sqrt{65}$

5. The value of k so that the system of equations 3x - 4y - 7 = 0 and 6x - ky - 5 = 0 have a unique solution is **[1]**

a) k $\neq$ -8

b) k $\neq$ 4

c) k $\neq$ -4

d) k $\neq$ 8

6. The distance between the points (3, -2) and (-3, 2) is: **[1]**

a) 40

b) $4\sqrt{10}$

c) $2\sqrt{10}$

d) $\sqrt{52}$

7. A letter is chosen at random from the word ASSASSINATION. The probability that it is a vowel is **[1]**

a) $\frac{6}{13}$

b) $\frac{7}{13}$

c) $\frac{6}{31}$

d) $\frac{3}{13}$

8. How many bricks each measuring (25 cm $\times$ 11.25 cm $\times$ 6 cm) will be required to construct a wall (8 m $\times$ 6 m $\times$ 22.5 cm)? **[1]**

a) 7200

b) 4800

c) 8000

d) 6400

9. A card is selected from a deck of 52 cards. The probability of its being a red face card is **[1]**

a) $\frac{3}{13}$

b) $\frac{1}{2}$

c) $\frac{2}{12}$

d) $\frac{3}{26}$

10. x^2 - 30x + 225 = 0 have **[1]**

a) Real roots

b) No real roots

c) Real and Equal roots

d) Real and Distinct roots

11. If one root of the equation x^2 + ax + 3 = 0 is 1, then its other root is **[1]**

a) 3

b) -3

c) 2

d) -2

12. If in a $\Delta ABC, \angle C = 90^\circ$ and $\angle B = 45^\circ$, then state which of the following is true? **[1]**

a) Perpendicular = Hypotenuse

b) Base = Hypotenuse

c) Base = Hypotenuse + Perpendicular

d) Base = Perpendicular

13. The HCF of 135 and 225 is: **[1]**

a) 5

b) 15

c) 45

d) 75

14. If the coordinates of a point are (-5, 11), then its abscissa is **[1]**

a) -5

b) 11

c) 5

d) -11

15. The ratio between the height and the length of the shadow of a pole is $\sqrt{3}$: 1, then the sun's altitude is **[1]**

a) 45°

b) 30°

c) 75°

d) 60°

16. The arithmetic mean of 1, 2, 3, 4, ..., n is: **[1]**

a) $\frac{n-1}{2}$

b) $\frac{n(n+1)}{2}$

c) $\frac{n}{2}$

d) $\frac{n+1}{2}$

17. The least positive integer divisible by 20 and 24 is **[1]**

a) 480

b) 240

c) 360

d) 120

18. Graphically, the pair of equations 6x - 3y + 10 = 0, 2x - y + 9 = 0 represents two lines which are **[1]**

a) parallel

b) Intersect at two points

c) coincident

d) intersect at a point

19. **Assertion (A):** No two positive numbers can have 18 as their H.C.F and 380 as their L.C.M. **[1]**

Reason (R): L.C.M. is always completely divisible by H.C.F.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two similar triangles are always congruent. **[1]**

Reason (R): If the areas of two similar triangles are equal then the triangles are congruent.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Is the given statement correct or not correct? **[2]**

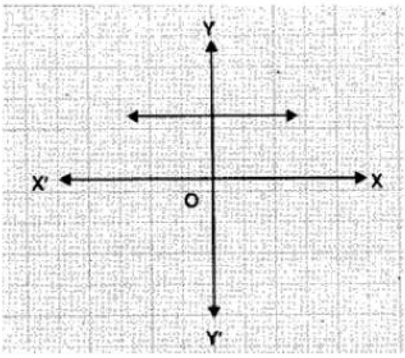
If two coins are tossed simultaneously there are three possible outcomes- two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

22. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the pair of linear equations are consistent, or inconsistent: $\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12. **[2]**

OR

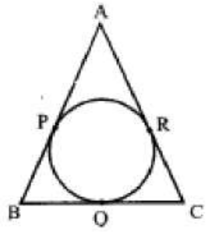
Half the perimeter of a rectangle garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

23. The graph of y = p(x) in a figure given below, for some polynomial p(x). Find the number of zeroes of p(x). **[2]**



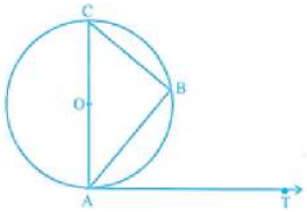
24. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4). **[2]**

25. In the adjoining figure, sides AB, BC and CA of a triangle ABC, touch a circle at P, Q, and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then find the length of BC. **[2]**



OR

If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB.



Section C

26. Write all the other trigonometric ratios of ∠A in terms of sec A. **[3]**

27. Form the pair of linear equations for the problem and find their solution by substitution method. The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball. **[3]**

28. Show that $5-\sqrt{3}$ is irrational. **[3]**

OR

Find the LCM and HCF of 336 and 54 and verify that LCM × HCF = product of two numbers.

29. Diagonal AC and BD of a trapezium ABCD with $AB \| DC$ intersect each other at point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC}=\frac{OB}{OD}$. **[3]**

30. O is the centre of a circle of radius 5 cm. T is a point such that OT =13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E. Find length of AB. **[3]**

OR

Prove that parallelogram circumscribing a circle is a rhombus.

31. From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45^{o} and 60^{o}. Find the distance between the objects. **[3]**

Section D

32. A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed? **[5]**

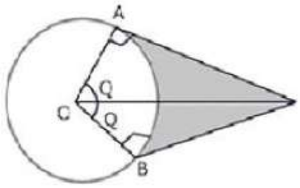
OR

The difference of the squares of two numbers is 45. The square of the smaller number is 4 times the larger number. Determine the numbers.

33. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$ **[5]**

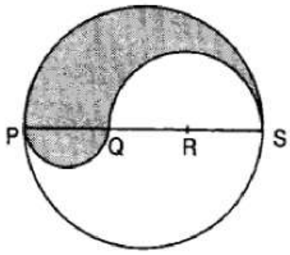
34. An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the **[5]**

rim of the pulley. Also, find the shaded area.



OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region



35` If the median of the distribution given below is 28.5, then find the values of x and y. **[5]**

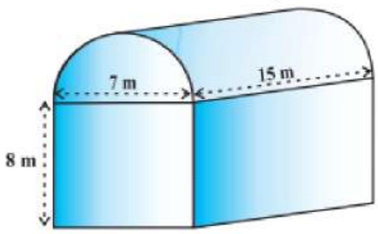
Class Interval	**frequency**
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.

The diameter of the half cylinder was 7 m and length was 15 m.



(i) Find the volume of the air that the shed can hold.

(ii) If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space on an average, how much air would be in the shed when it is working?

(iii) Find the surface area of the cuboidal part.

OR

Find the surface area of the cylindrical part.

37. **Read the text carefully and answer the questions:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

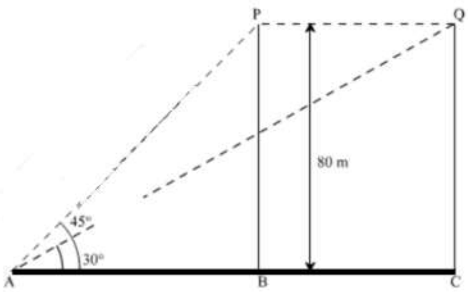
(iii) How much money Akshar saves in 10 days?

OR

How many coins are there in piggy bank on 15th day?

38. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^{o}. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^{o}. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

(iii) Find the distance between second position of bird and observer?

OR

Find the distance between initial position of bird and observer?

Solution

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Class 10 - Mathematics

Section A

1. **(b)** $\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

   Explanation: Let coordinates of P be (x, y) which divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio 1 : k

   $m_1 : m_2 = 1 : k$

   $\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

   $= \frac{1\times x_2+k\times x_1}{1+k} = \frac{x_2+kx_1}{1+k}$

   And $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

   $= \frac{1\times y_2+k\times y_1}{1+k}$

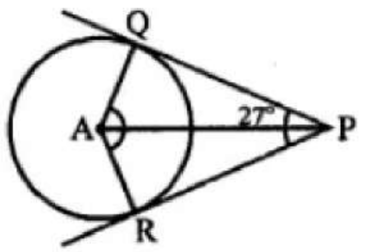
   $= \frac{y_2+ky_1}{1+k}$

   $\therefore P\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

2. **(d)** 126°

   Explanation:

   In the given figure, PQ and PR are tangents drawn from an external point P to a circle with centre A.



   $\angle QPA = 27°, \angle QAR = ?$

   AP bisects $\angle QPR$ and $\angle QPA$ =27° given

   $\angle QPR = 2 \times 27° = 54°$

   But $\angle QPR + \angle QAR = 180°$ (QARP is a cyclic quadrilateral)

   $\Rightarrow 54° + \angle QAR = 180°$

   $\Rightarrow \angle QAR = 180° – 54° = 126°$

3. **(c)** $\frac{1}{2}$

   Explanation: Prime number on a die are 2, 3, 5

   $\therefore$ Probability of getting a prime number on the face of the die $= \frac{3}{6} = \frac{1}{2}$

4. **(a)** 7

   Explanation: The distance of the point (4, 7) from x-axis = 7

5. **(d)** $k \neq 8$

   Explanation: Given: $a_1 = 3$, $a_2 = 6$, $b_1 = -4$, $b_2 = -k$, $c_1 = -7$ and $c_2 = -5$

   If there is a unique solution, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   $\Rightarrow \frac{3}{6} \neq \frac{-4}{-k}$

   $\Rightarrow -3k \neq -4 \times 6$

   $\Rightarrow k \neq 8$

6. **(d)** $\sqrt{52}$

   Explanation: Let us take (3, – 2) and (– 3, 2) as (x_1, y_1) and (x_2, y_2)

   Using distance formjula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $d = \sqrt{(-3-3)^2 + (2-(-2))^2}$

   $d = \sqrt{(-6)^2 + (2+2)^2}$

   $d = \sqrt{36 + (4)^2}$

d = $\sqrt{36+16}$
d = $\sqrt{52}$

7. **(a)** $\frac{6}{13}$

Explanation: Vowels present in the given word are A, A, I, A, I, O = 6

Number of possible outcomes = {A, A, I, A, I, O} = 6

Number of total outcomes = 13

Required Probability = $\frac{6}{13}$

8. **(d)** 6400

Explanation: Volume of the wall = (800 × 600 × 22.5) cm^3,

Number of bricks = $\frac{volume\ of\ the\ wall}{volume\ of\ 1\ brick}$

= $\left(\frac{800\times600\times22.5}{25\times11.25\times6}\right)$ = 6400

9. **(d)** $\frac{3}{26}$

Explanation: In a deck of 52 cards, there are 12 face cards i.e. 6 red (3 hearts and 3 diamonds) and 6 black cards (3 spade and 3 clubs)

So, probability of getting a red face card = 6/52 = 3/26

10. **(c)** Real and Equal roots

Explanation: D = $(-30)^2$ - 4 × 1 × 225

D = 900 - 900

D = 0. Hence Real and Equal roots.

11. **(a)** 3

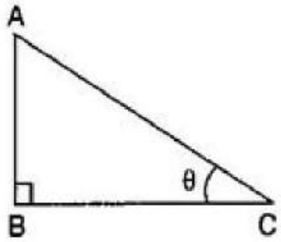
Explanation: The given equation is $x^2 + ax + 3 = 0$

One root = 1

and product of roots = $\frac{c}{a} = \frac{3}{1}$ = 3

Second root = $\frac{3}{1}$ = 3

12. **(d)** Base = Perpendicular

Explanation: 

Given: in triangle ABC, $\angle C = 45^\circ$, and $\angle B = 90^\circ$,

Since, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan 45^\circ = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow 1 = \frac{\text{Perpendicular}}{\text{Base}}$

⇒ Base = perpendicular

13. **(c)** 45

Explanation: We have,

135 = 3 × 45

= 3 × 3 × 15

= 3 × 3 × 3 × 5

= $3^3 \times 5$

Now, for 225 will be

225 = 3 × 75

= 3 × 3 × 5×5

= $3^2 \times 5^2$

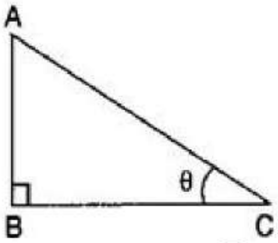
The HCF will be $3^2 \times 5$ = 45

14. **(a)** -5

Explanation: Since x-coordinate of a point is called abscissa.

Therefore, the abscissa is -5.

15. **(d)** 60°

Explanation: 

Let the height of the pole be AB = $\sqrt{3}x$ meters and the length of the shadow be BC = x meters and angle of elevation = θ

$\therefore \tan\theta = \frac{\sqrt{3}x}{x}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60^{\circ}$

$\Rightarrow \theta = 60^{\circ}$

16. **(d)** $\frac{n+1}{2}$

Explanation: According to question,

Arithmetic Mean = $\frac{1+2+3+\ldots+n}{n}$

= $\frac{\frac{n(n+1)}{2}}{n}$

= $\frac{n+1}{2}$

17. **(d)** 120

Explanation: Least positive integer divisible by 20 and 24 is

LCM of (20, 24).

20 = $2^2 \times 5$

24 = $2^3 \times 3$

$\therefore$ LCM (20, 24) = $2^3 \times 3 \times 5$ = 120

Thus 120 is divisible by 20 and 24.

18. **(a)** parallel

Explanation: Given: $a_1 = 6$, $a_2 = 2$, $b_1 = -3$, $b_2 = -1$, $c_1 = 10$ and $c_2 = 9$

$a_1 = 6, a_2 = 2, b_1 = -3, b_2 = -1, c_1 = 10$ and $c_2 = 9$

Here $\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$

but $\frac{c_1}{c_2} = \frac{10}{9}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the lines are parallel.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: 380 is not divisible by 18.

20. **(d)** A is false but R is true.

Explanation: Two similar triangles are not congruent generally. So, A is false but R is true.

Section B

21. All possible outcomes = (H, H), (H, T), (T, H), (T, T)

Probability of an event = $\frac{\text{Favourabe outcomes}}{\text{Total outcomes}}$

Probability (2 heads) = $\frac{1}{4}$

Probability (2 tails) = $\frac{1}{4}$

It is not correct.

If we want to get the probability of them we should categorize the outcomes like this but they are not equally likely because one of each can result in two ways from a head-on first coin and tail on second or from the tail on first and head on second.

22. Given equations are:

$\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12

Compare equation $\frac{4}{3}x + 2y = 8$ with $a_1x + b_1y + c_1 = 0$ and 2x + 3y = 12

with $a_2x + b_2y + c_2 = 0$, We get, $a_1 = \frac{4}{3}$, $a_1 = \frac{4}{3}$, $b_1 = 2$, $c_1 = -8$, $a_2 = 2$, $b_2 = 3$, $c_2 = -12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinitely many solutions.

Hence, they are consistent.

OR

Let length of rectangular garden = x metres

and width of rectangular garden = y metres

According to given conditions, perimeter = 36 m

$\frac{1}{2}(2x + 2y) = 36$

⇒ x + y = 36 ....(i)

and x = y+4

⇒ x - y =4 ....(ii)

Adding eq. (i) and (ii),

2x = 40 ⇒ x = 20 m

Subtracting eq. (ii) from eq. (i),

2y = 32 ⇒ y = 16 m

Hence, length = 20 m and width = 16 m

23. There is no zero as the graph does not intersect the x-axis at any point.
24. We want to find coordinates of point A. AB is the diameter and coordinates of centerare (2, -3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$2 = \frac{x+1}{2}$

⇒ 4 = x + 1

⇒ x = 3

Using section formula, we get

$-3 = \frac{4+y}{2}$

⇒ 4 + y = −6

⇒ y =− 6 − 4 = −10

Therefore, Coordinates of point A are (3, −10).

25. In triangle ABC, we have

BP = BQ = 3cm

AP = AR = 4cm

(tangents drawn from an external point to the circle are equal).

So, RC = AC - AR

= 11 - 4

= 7 cm

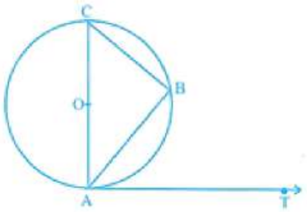
Hence RC = CQ = 7 cm

Then, BC = BQ + QC

= 7 + 3

= 10 cm

OR



Given: Chord AB, diameter AOC and tangents at A of a circle with centre O.

To prove: ∠BAT = ∠ACB

Proof: Radius OA and tangent AT at A are perpendicular.

$\therefore \angle OAT = 90^o$ (radius at the point of contact of tangent is perpendicular)

$\Rightarrow \angle BAT = 90^o - \angle BAC$ .....(i)

AOC is diameter.

$\therefore \angle B = 90$

$\Rightarrow \angle C + \angle BAC = 90^o$

$\Rightarrow \angle C = 90^o - \angle BAC$ ...(ii)

From (i) and (ii), we get

$\angle BAT = \angle ACB$. Hence, proved.

Section C

26. $\sin A$ can be expressed in terms of sec A as:

$\sin A = \sqrt{\sin^2 A}$

$sinA = \sqrt{(1-\cos^2 A)}$

$\sin A = \sqrt{1-\frac{1}{\sec^2 A}}$

$\sin A = \sqrt{\frac{\sec^2 A-1}{\sec^2 A}}$

$\sin A = \frac{1}{\sec A}\sqrt{\sec^2 A - 1}$

Now,

$\cos A$ can be expressed in terms of secA as:

$\cos A = \frac{1}{\sec A}$

tanA can be expressed in the form of sec A as:

As, $1 + \tan^2 A = \sec^2 A$

$\Rightarrow \tan A = \pm\sqrt{(\sec^2 A - 1)}$

since A is an acute angle, and tan A is positive when A is acute, So, tan A = $\sqrt{(\sec^2 A - 1)}$

Now $cosecA$ can be expressed in the form of sec A as:

$cosecA = \frac{1}{\sin A}$

$cosecA = \frac{1}{\frac{\sec A}{\sqrt{1-\sec^2 A}}}$

$cosecA = \frac{\sqrt{1-\sec^2 A}}{\sec A}$

Now, cot A can be expressed in terms of sec A as:

cot A $= \frac{1}{\tan A}$

as, $1 + \tan^2 A = \sec^2 A$

$cotA = \frac{1}{\sqrt{\sec^2 A-1}}$

27. Let the cost of each bat and each ball be Rs.x and Rs. y respectively. Then, according to the equation, The pair of linear equations formed is

7x + 6y = 3800 .... (1)

3x + 5y = 1750 ...... (2)

From equation (2), 5y = 1750 - 3x

$y = \frac{1750-3x}{5}$ .... (3)

Substitute this value of y in equation (1), we get

$7x + 6\left(\frac{1750-3x}{5}\right) = 3800$

$\Rightarrow \quad 35x + 10500 - 18x = 19000$

$\Rightarrow \quad 17x + 10500 = 19000$

$\Rightarrow \quad 17x = 19000 - 10500$

$\Rightarrow \quad 17x = 8500$

$\Rightarrow \quad x = \frac{8500}{17} = 500$

Substituting this value of x in equation (3), we get

$y = \frac{1750-3(500)}{5} = \frac{1750-1500}{5} = \frac{250}{5} = 50$

Hence, the cost of each bat and each ball is Rs.500 and Rs.50 respectively.

Verification,

Substituting x = 500 and y = 50, we find that both the equations (1) and (2) are satisfied as shown below:

$7x + 6y = 7(500) + 6(50)$

$= 3500 + 300 = 3800$

$3x + 5y = 3(500) + 5(50)$

$= 1500 + 250 = 1750$. This verifies the solution.

28. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b ≠ 0) such that $5 - \sqrt{3} = \frac{a}{b}$

Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$

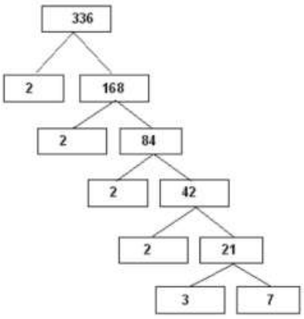
Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

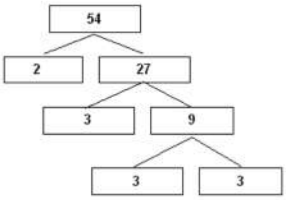
This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

OR



So, 336 = 2 × 2 × 2 × 2 × 3 × 7 = $2^4 \times 3 \times 7$



So, 54 = $2 \times 3 \times 3 \times 3 = 2 \times 3^3$

Therefore,

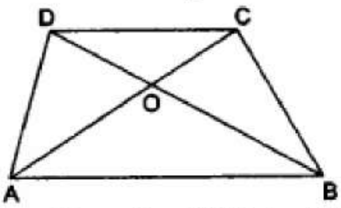
LCM (336, 54) = $2^4 \times 3^3 \times 7 = 3024$

HCM (336, 54) = $2 \times 3 = 6$.

Verification:

LCM × HCF = 3024 × 6 = 1814 and 336 × 54 = 18144

i.e. LCM × HCF = product of two numbers

29. 

Given A trapezium ABCD in which $AB \| DC$. The diagonals AC and BD intersect at O.

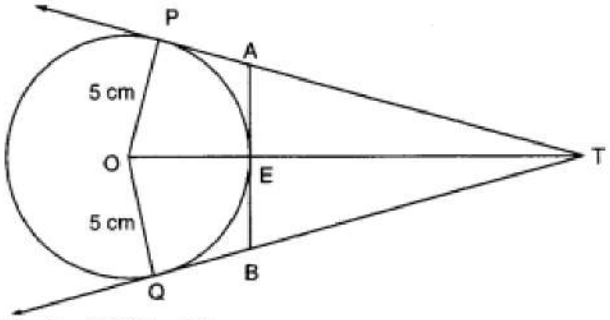
To Prove In $\triangle OAB$ and $\triangle OCD$, we have

$\angle OAB = \angle OCD$ [alternate angles, since $AB \| DC$]

and $\angle OBA = \angle ODC$ [alternate angles, since $AB \| DC$]

$\therefore \quad \triangle OAB \sim \Delta OCD$ [by AA-similarity].

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

30. 

Clearly $\angle OPT$ = 90°

Applying Pythagoras in ΔOPT, we have

$OT^2 = OP^2 + PT^2$

$\Rightarrow 13^2 = 5^2 + PT^2$

$\Rightarrow PT^2 = 169 - 25 = 144$

$\Rightarrow$ PT = 12 cm

Since lengths of tangents drawn from a point to a circle are equal. Therefore,

AP = AE = x(say)

$\Rightarrow$ AT = PT - AP = (12 - x)cm

Since AB is the tangent to the circle E. Therefore, $OE \perp AB$

$\Rightarrow \quad \angle OEA = 90^\circ$

$\Rightarrow \quad \angle AET = 90^\circ$ [Applyng Pythagoras Theorem in ΔAET]

$\Rightarrow (12 - x)^2 = x^2 + (13 - 5)^2$

$\Rightarrow 144 - 24x + x^2 = x^2 + 64$

$\Rightarrow$ 24x = 80

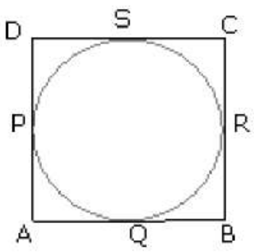
$\Rightarrow x = \frac{10}{3}$cm

Similarly, BE $= \frac{10}{3}$cm

$\therefore$ AB = AE + BE = $\left(\frac{10}{3} + \frac{10}{3}\right)$ cm

$= \frac{20}{3}$cm

OR



Given ABCD is a parallelogram in which all the sides touch a given circle

To prove:- ABCD is a rhombus

Proof:-

$\because$ ABCD is a parallelogram

$\therefore$ AB = DC and AD = BC

Again AP, AQ are tangents to the circle from the point A

$\therefore$ AP = AQ

Similarly, BR = BQ

CR = CS

DP = DS

$\therefore$(AP + DP) + (BR + CR) = AQ + DS + BQ + CS = (AQ + BQ) + (CS + DS)

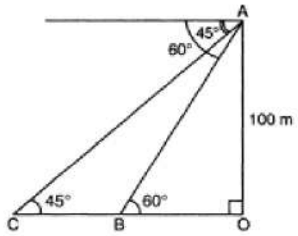
$\Rightarrow$ AD + BC = AB + DC

$\Rightarrow$ BC + BC = AB + AB [$\because$ AB = DC, AD = BC]

$\Rightarrow$ 2BC = 2AB

$\Rightarrow$ BC = AB

Hence, parallelogram ABCD is a rhombus

31. 

In the given figure,

$\angle ACO = \angle CAX = 45^\circ$

and $\angle ABO = \angle XAB = 60^\circ$

Let A be a point and B, C be two objects.

In $\triangle$ AOC, $\frac{AO}{CO} = \tan 45^\circ$

$\Rightarrow \quad \frac{100}{\text{CO}} = 1$

$\Rightarrow CO = 100m$

Also in $\triangle$ ABO, $\frac{AO}{OB} = \tan 60^\circ$

$\Rightarrow \quad \frac{100}{OB} = \sqrt{3}$

$\Rightarrow \quad OB = \frac{100}{\sqrt{3}}$

$\therefore \quad BC = CO - OB = 100 - \frac{100}{\sqrt{3}}$

$= 100\left(1 - \frac{1}{\sqrt{3}}\right)$ m

$100\frac{(\sqrt{3}-1)}{\sqrt{3}} = 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{100(3-\sqrt{3})}{3}$m

Section D

32. Let the original average speed of the train be x km/hr.

Time taken to cover 63 km $= \frac{63}{x}$ hours

Time taken to cover 72 km when the speed is increased by 6 km/hr $= \frac{72}{x+6}$ hours

By the question,we have,

$\frac{63}{x} + \frac{72}{x+6} = 3$

$\Rightarrow \frac{21}{x} + \frac{24}{x+6} = 1$

$\Rightarrow \frac{21x+126+24x}{x^2+6x} = 1$

$\Rightarrow$ 45x + 126 = x^2 + 6x

$\Rightarrow$ x^2 - 39x - 126 = 0

$\Rightarrow$ x^2 - 42x + 3x - 126 = 0

$\Rightarrow$ x(x - 42) + 3(x - 42) = 0

$\Rightarrow$ (x - 42)(x + 3) = 0

$\Rightarrow$ x - 42 = 0 or x + 3 = 0

$\Rightarrow$ x = 42 or x = -3

Since the speed cannot be negative, $x \neq -3$.

Thus, the original average speed of the train is 42 km/hr.

OR

Let the larger number be x. Then,

Square of the smaller number = 4x

Also, Square of the larger number = x^2

It is given that the difference of the squares of the numbers is 45.

$\therefore$ x^2 - 4x = 45

$\Rightarrow$$x^2$ - 4x - 45 = 0

$\Rightarrow$ x^2 - 9x + 5x - 45 = 0

$\Rightarrow$ x (x - 9) + 5 (x - 9) = 0

$\Rightarrow$ x - 9 = 0 or, x + 5 = 0 $\Rightarrow$ x = 9, -5

Case I When x = 9: In this case, we have

Square of the smaller number = 4x = 36

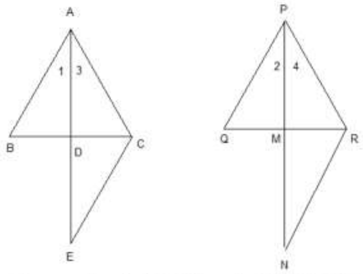
$\therefore$ Smaller number = ± 6.

Thus, the numbers are 9, 6 or 9, - 6

CASE II When x = -5: In this case, we have

Square of the smaller number = 4x = - 20. But, square of a number is always positive. Therefore, x = - 5 is not possible.

Hence, the numbers are 9, 6 or 9, - 6.

33. 

Given : In ΔABC and ΔPQR The AD and PM are their medians,

such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$

To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN. Join CE and RN.

Proof : In ΔABD and ΔEDC

$AD = DE$

$\angle ADB = \angle EDC$ (vertically opposite angles)

$BD = DC$(as AD is a median)

$\therefore \quad \Delta ABD \equiv \Delta EDC$ (By SAS congruency)

or, $AB = CE$ (By CPCT)

Similarly, PQ = RN

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ (Given)

or, $\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$

or $\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$

So $\Delta ACE \sim \Delta PRN$

$\angle 3 = \angle 4$

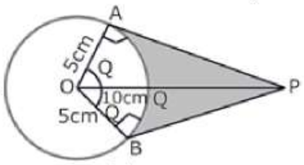
Similarly $\angle 1 = \angle 2$

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

So $\angle A = \angle P$ and

$\frac{AB}{PQ} = \frac{AC}{PR}$(given)

Hence $\Delta ABC \sim \Delta PQR$

34. 

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 60^o$

$\Rightarrow \angle AOB = 2 \times \theta = 120^o$

$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times 5}{360} cm = \frac{10\pi}{3} cm \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm

= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = $\frac{120\times\pi\times 5\times 5}{360} cm^2 = \frac{25\pi}{3} cm^2 \left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2

$[\because AP = \sqrt{100\ - 25} = \sqrt{75}\ = 5\sqrt{3}\ cm]$

Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3}\ - \pi]\ cm^2$

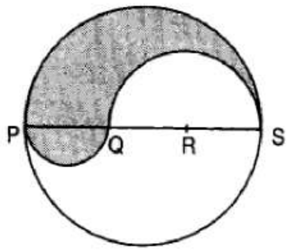
OR

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2$ = 37.71 cm^2

35.

Monthly Consumption	**Number of consumers** (f_i)	**Cumulative Frequency**
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
Total	$\sum f_i = n = 60$	

Here, $\sum f_i = n = 60$, then $\frac{n}{2} = \frac{60}{2} = 30$, also, median of the distribution is 28.5, which lies in interval 20 – 30.

$\therefore$ Median class = 20 – 30

So, l = 20, n = 60, f = 20, cf = 5 + x and h = 10

$\because 45 + x + y = 60$

$\Rightarrow x + y = 15$ ………..........(i)

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$\Rightarrow 28.5 = 20 + \left[\frac{30-(5+x)}{20}\right] \times 10$

$\Rightarrow 28.5 = 20 + \frac{30-5-x}{2}$

$\Rightarrow 28.5 = \frac{40+25-x}{2}$

$\Rightarrow 57.0 = 65 - x$

$\Rightarrow x = 65 - 57 = 8$

$\Rightarrow$ x = 8

Putting the value of x in eq. (i), we get,

8 + y = 15

$\Rightarrow$ y = 7

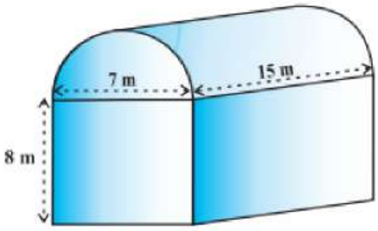
Hence the value of x and y are 8 and 7 respectively.

Section E

36. **Read the text carefully and answer the questions:**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m $\times$ 7 m $\times$ 8 m.

The diameter of the half cylinder was 7 m and length was 15 m.



(i) Total volume = volume of cuboid + $\frac{1}{2}$ × volume of cylinder.

For cuboidal part we have

length = 15 m, breadth = 7 m and height = 8 m

$\therefore$ Volume of cuboidal part = l × b × h $= 15 \times 7 \times 8\text{m}^2 = 840\text{m}^3$

Clearly,

r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2}$m

and, h = Height (length) of half-cylinder = Length of cuboid = 15 m

$\therefore$ Volume of half-cylinder = $\frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15\text{m}^3 = \frac{1155}{4}\text{m}^3 = 288.75\text{m}^3$

Thus the volume of the air that the shed can hold = (840 + 288.75) m^3 = 1128.75 m^3

(ii) Total space occupied by 20 workers = $20 \times 0.08\text{m}^3$ = = 1.6 m^3

Total space occupied by the machinery = 300 m^3

$\therefore$ Volume of the air inside the shed when there are machine and workers inside it

= (1128.75 - 1.6 - 300) m^3

= (1128.75 - 301.6) m^3 = 827.15 m^3

Hence, volume of air when there are machinery and workers is 827.15

(iii) Given for the cuboidal part

length L = 15 m, Width B = 7 m, Height = 8 m

Surface area of the cuboidal part

= 2(LB + BH + HL)

= 2(15 × 7 + 7 × 8 + 8 × 15)

= 2(105 + 56 + 120) = 2 × 281 = 562 m^2

OR

For the cylindrical part r = 3.5 cm and l = 15 m

Thus the surface area of the cylindrical part

$= \frac{1}{2}(2\pi rl) = 3.14 \times 3.5 \times 15$

= 164.85 m^2

37. **Read the text carefully and answer the questions:**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1)1]$ = 190

$\Rightarrow$ n(n + 1) = 380 $\Rightarrow$ n^2 + n - 380 = 0

$\Rightarrow$ (n + 20)(n − 19) = 0 $\Rightarrow$ (n + 20)(n - 19) = 0

$\Rightarrow$ n = −20 or n = 19 $\Rightarrow$ n = -20 or n = 19

But number of coins cannot be negative

$\therefore$ n = 19 (rejecting n = -20)

So, number of days = 19

(ii) Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

$= \frac{19}{2}[2 \times 5 + (19 - 1)5]$

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹950

(iii) Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10-1) \times 5]$

$\Rightarrow S_{10} = 5[10 + 45\}]$

$\Rightarrow S_{10} = 275$

Money saved in 10 days = ₹275

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 5]$

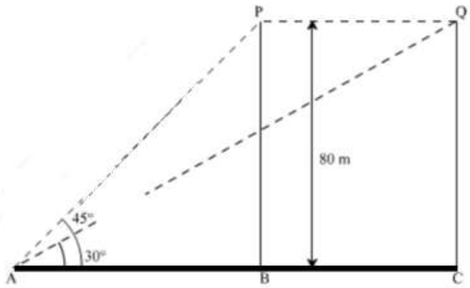
$\Rightarrow S_{15} = \frac{15}{2}$ [2 + 14]

$\Rightarrow S_{15} = 120$

So, there are 120 coins on 15th day.

38. **Read the text carefully and answer the questions:**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^o. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^o. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle$ABP

tan $45^o = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

$\Rightarrow$ AB = 80 m

(ii) The speed of the bird

In $\triangle$AQC

tan $30^o = \frac{QC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow$ AC = $80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow$ BC = $80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec

(iii) The distance between second position of bird and observer.

In $\triangle$AQC

sin $30^o = \frac{QC}{AQ}$

$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$

$\Rightarrow$ AQ = 160 m

OR

The distance between initial position of bird and observer.

In $\triangle$ABP

sin 45^o = $\frac{BP}{AP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$

$\Rightarrow$ AP = $80\sqrt{2}m$