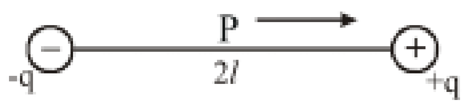
Electric Dipole

An arrangement of two equal and opposite charges separated by a small distance is known as an electric dipole.

Let q and –q be two charges separated by distance $2l$. The dipole moment of the dipole is :



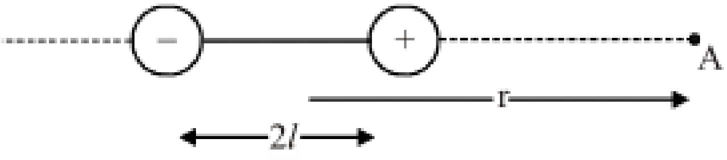
$\vec{p} = q\,2\vec{\ell}$

It is a vector quantity and is directed from –ve charge towards the +ve charge. The line joing -q to +q is known as *the axis of the dipole.*

Electric field due to a dipole

Electric field at axis : (Line joining the charges)

Electric field due to a short dipole on its axis at a point A at a distance r from dipole (ℓ <<r) :



$$E_A = \frac{q}{4\pi \epsilon_0 (r-\ell)^2} - \frac{q}{4\pi \epsilon_0 (r+\ell)^2}$$

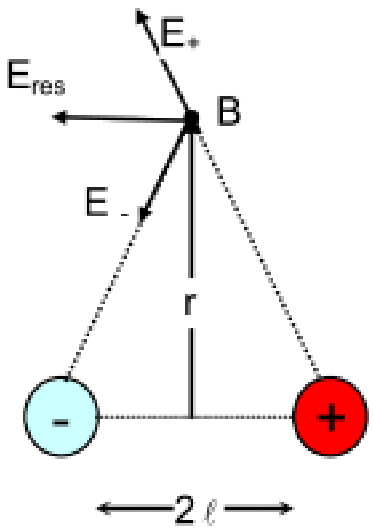
$$E_A = \frac{4q\ell r}{4\pi \epsilon_0 (r^2-\ell^2)^2}$$

after using dipole approximation $\ell << r$ we get

$$\vec{E}_A = \frac{2\vec{P}}{4\pi \epsilon_0 r^3}.$$

Electric field at equator : (Line perpendicular to axis passing through centre)

Electric field at a point distance r from the centre of the short dipole (ℓ << r)



$$E_B = 2\left[\left\{\frac{q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\right\}\cos\theta\right]$$

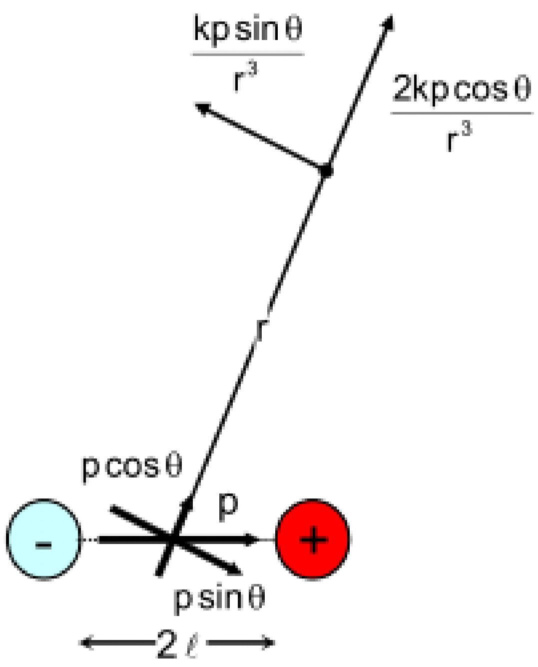
$$E_B = \frac{2q}{4\pi\epsilon_0\left(r+\ell\right)^2}\frac{\ell}{\sqrt{\ell^2+r^2}} = \frac{2q\ell}{4\pi\epsilon_0\left(r^2+\ell^2\right)^{3/2}}$$

after using dipole approximation $\ell << r$ we get

$$\vec{E}_B = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}$$ (-ve sign indicate that field is oppositly directed to dipole direction)

Electric field at any point A (r, θ) due to dipole

Let A be a point at a distance r from the mid-point O of the dipole. Let θ be the angle between OA and the dipole moment p. Since dipole moment is a vector so we can resolve its components $p\cos\theta$ and $p\sin\theta$ along and perpendicular to OA. Due to $p\cos\theta$ (axial point) electric field will be in the direction of $p\cos\theta$ and due to $p\sin\theta$ (equatorial position) electric field will be opposite to $p\sin\theta$



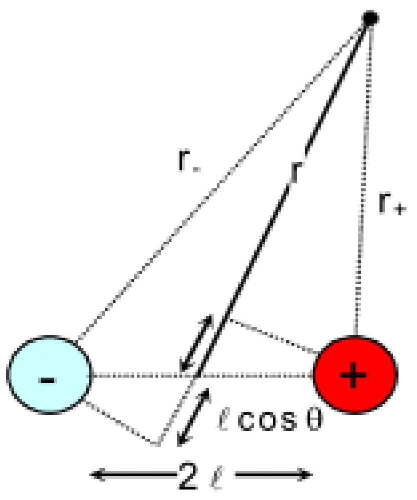
Electric field at A

$$E = \sqrt{\left(\frac{2kp\cos\theta}{r^3}\right)^2 + \left(\frac{kp\sin\theta}{r^3}\right)^2} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\& \quad \tan\alpha = \frac{E_\theta}{E_r} = \frac{\sin\theta}{2\cos\theta} = \frac{\tan\theta}{2}$$

Electric potential due to dipole

Electric field at any point A (r, θ) due to dipole :



$$V = V_+ + V_- = \frac{k(+q)}{r_+} + \frac{k(-q)}{r_-} = \frac{k(+q)}{r - l\cos\theta} + \frac{k(-q)}{r + l\cos\theta} = \frac{k(2lq)}{r^2 - l^2\cos^2\theta}$$

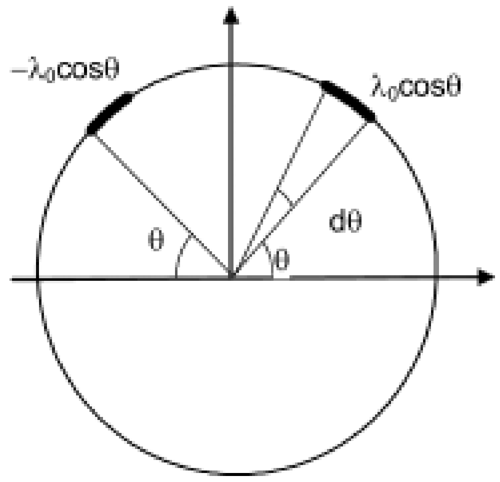
after using dipole approximation $\ell << r$ we get

$$V = \frac{kp}{r^2}$$

Illustration:

A nonunifom charge given on the ring according to the equation $\lambda = \lambda_0 \cos\theta$ (where θ is measured from x-axis).Find its dipole moment.

Sol.



In the figure shown the two symmetric elements are equal and opposite charge whose dipole moment will be

$$dp = \{(\lambda_0 \cos\theta)(Rd\theta)\}\{2R\cos\theta\} = 2\lambda_0 R^2 \cos^2\theta d\theta$$

$$\therefore p = \int dp = 2\lambda_0 R^2 \int_{-\pi/2}^{+\pi/2} \cos^2\theta d\theta = \pi\lambda_0 R^2$$

Illustration:

For a given dipole at a point (away from the center of dipole) intensity of the electric field is E. Charges of the dipole are brought closer such that distance between point charges is half, and magnitude of charges are also halved. Find the intensity of the field now at the same point

Sol. $P_i = 2ql$

$$P_f = 2\frac{q}{2}\frac{1}{2} = \frac{P_i}{4}$$

$r_f = r_i \quad \theta_f = \theta_i$

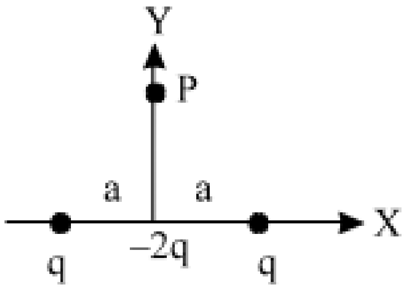
$$E = \frac{p}{4\pi \epsilon_0 r^3}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \qquad E_f = \frac{E_i}{4}.$$

Practice Exercise

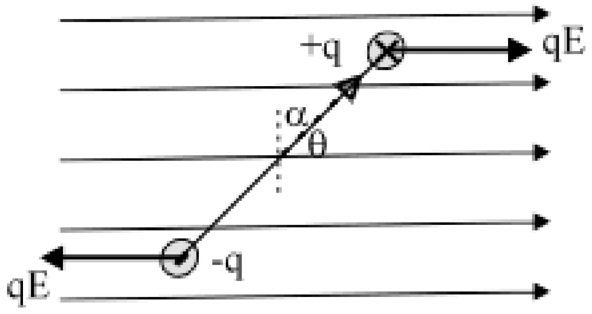
Q.1 Three point charges q, –2q and q are located along the x-axis as shown in figure. Show that the electric field at P (y >> a) along the y-xis is ,

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{3qa^2}{y^4}\hat{j}$$



Dipole in an external uniform electric field

If a dipole is placed in a uniform electric field E,
Force on the dipole is zero.
Torque on the dipole is given as



Here the net force is zero, but there can be a torque.

$$\tau = (qE)(2l \sin\theta) = pE \sin\theta$$

In vector form $\vec{\tau} = \vec{p} \times \vec{E}$

This torque has a tendency to orient dipole moment vector in the direction of field

Potential energy due to action of electric field

$$\tau = pE\cos\alpha$$

$$\therefore W_{fd} = \int dW = pE\int_0^{90-\theta}\cos\alpha\, d\alpha = pE[\sin\alpha]_0^{90-\theta} = pE\cos\theta$$

$$\therefore U_\theta = -pE\cos\theta = -\vec{p}.\vec{E} \quad (\text{taking } U = 0 \text{ at } \theta = 90^0)$$

When $\theta = 0^0$, the dipole moment p is in the direction of the field E and the dipole is in *stable equilibrium.* If it is slightly displaced, it performs oscillations.

When $\theta = 180^0$, the dipole moment p is opposite to the direction of the field E and the dipole is in *unstable equilibrium.*

Illustration:

A dipole of dipole moment P lies in a uniform electric field E such that dipole direction is along field. If dipole is rotated through 180° such that dipole direction becomes opposite to the field direction. Find the work done by the electrostatic field.

Sol.

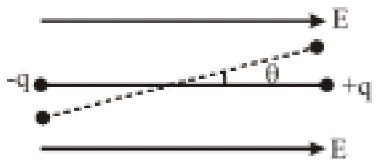
$$U_i = -\vec{P}.\vec{E} = -PE\cos O = -PE$$

$$U_f = -P.E.\cos(180^o) = PE$$

work done by the field $= -\Delta U = U_i - U_f = -2PE$

Illustration:

Figure shows an electric dipole formed by two particles fixed at the ends of a light rod of length l. The mass of each particle is m and the chargers are –q and +q. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular simple harmonic and find its time period.



Sol.

Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is

$$|\vec{\tau}| = |\vec{P} \times \vec{E}|$$

$$= ql\ E\sin\theta$$

This torque will be restoring & tend to rotate the elipole back towards the electric field. Also, for small angular displacement $\sin\theta = \theta$ so that

$$\tau = -ql\,E\theta$$

It the moment of inertia of the body about OA is I, the angular acceleration becomes.

$$\alpha = \frac{\tau}{I} = -\frac{qlE}{I}\theta \qquad \alpha = -\omega^2\theta$$

where $\omega^2 = \dfrac{qlE}{I}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{qlE}}$$

Now, moment of inertia of the system about the axis of rotation is

$$I = 2m\left(\frac{l}{2}\right)^2 - \frac{ml^2}{2}$$

So, $\quad T = 2\pi\sqrt{\dfrac{ml}{2qE}}$.

Practice Exercise

Q.1 A dipole of dipole moment p is placed at origin along x-axis. Another dipole of dipole moment p is kept at (0, 1, 0) along y-axis. Find the resultant potential and electric field at (1, 0, 0)

Q.2 Due to electric dipole, electric field at a distance r on axial position is $\vec{E}_1$ and at distance r on equatorial position is $\vec{E}_2$. What is the relation between $\vec{E}_2$ and $\vec{E}_1$.

Q.3 Three charges Q, Q and -2Q are placed at the three corners of an equilateral triangle of side a. Find the dipole moment of the combination.

Answers

Q.1 $v = kp(\frac{2\sqrt{2}-1}{2\sqrt{2}})$, $\vec{E} = kp[\frac{(8\sqrt{2}-3)}{4\sqrt{2}}\hat{i} + \frac{1}{4\sqrt{2}}\hat{j}]$ Q.2 $\vec{E}_1 = -2\vec{E}_2$

Q.3 $p = \sqrt{3}qa$

Dipole in an external non uniform electric field :

If a dipole is placed in non uniform electric field then let electric field at the location of positive charge will be $\vec{E}_1$ and at the location of –ve charge be $\vec{E}_2$. Usually for non uniform electric field $\vec{E}_1 + \vec{E}_2$ hence dipole will experience a net force (usually) which is equal to

$$\vec{F}_{net} = q\vec{E}_1 + (-q)\vec{E}_2$$

$$= q\left(\vec{E}_1 - \vec{E}_2\right)$$

$$= -\frac{q\left(\vec{E}_2 - \vec{E}_1\right)}{\ell} \qquad = -\, p\frac{\partial \vec{E}}{\partial \ell}$$

Where $\frac{\partial \vec{E}}{\partial \ell}$ = rate of change of electric field in the direction of dipole moment.

Torque of this electric field about geometric centre of dipole is still given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$