Chapter 1

Number and Letter Series

CHAPTER HIGHLIGHTS

- *Number Series*
- *Difference Series*
- *Product Series*
- *Squares/Cubes Series*
- *Miscellaneous Series*
- *Combination Series*
- *Letter Series*

Introduction

Number and Letter Series form an important part of the Reasoning section in various competitive examinations. There are two or three broad categories of questions that appear in various exams from this particular chapter.

In the first category of questions, a series of numbers/letters is given with one number/letter (or two numbers/letters) missing, represented by a blank or a question mark. The given series of numbers/letters will be such that each one follows its predecessor in a certain way, i.e. according to a definite pattern. Students are required to find out the way in which the series is formed and, hence, work out the missing number/numbers or letter/letters to complete the series. For the purpose of our discussion, we will refer to this category of questions as Number Series Type I or Letter Series Type I questions. Under Type I questions, there are a large variety of patterns that are possible, and the student requires a proper understanding of various patterns to be able to do well in these types of questions.

In the second category of questions, a series of numbers/letters is given, and the student is required to count how many numbers/letters in that series satisfy a given condition and mark that as the answer. For the purpose of our understanding, we will refer to this category of questions as Number Series Type II or Letter Series Type II questions. These questions will mainly involve counting of numbers/letters satisfying a given condition.

Number Series

For better understanding, we will classify this into the following broad categories.

1. Difference series
2. Product series
3. Squares/cubes series
4. Miscellaneous series
5. Combination series

Difference Series

The difference series can be further classified as follows.

1. Number series with a constant difference.
2. Number series with an increasing or decreasing difference.

In the number series with a constant difference, there is always a constant difference between two consecutive numbers. For example, the numbers of the series 1, 4, 7, 10, 13, … are such that any number is obtained by adding a constant figure of 3 to the preceding term of the series.

If we have to find the next number in the aforementioned series, we need to add a 3 to the last term 13. Thus, 16 is the next term of the series.

Under the series with constant difference, we can have series of odd numbers or series of even numbers also.

In the series with increasing/decreasing difference, the difference between consecutive terms keeps increasing (or decreasing, as the case may be). For example, let us try to find out the next number in the series 2, 3, 5, 8, 12, 17, 23, …

Here, the difference between the first two terms of the series is 1; the difference between the second and third terms is 2; the difference between the third and the fourth terms is 3; and so on. That is, the difference between any pair of consecutive terms is one more than the difference between the first number of this pair and the number immediately preceding this number. Here, since the difference between 17 and 23 is 6, the next difference should be 7. So, the number that comes after 23 should be (23 + 7) = 30.

We can also have a number series where the difference is in decreasing order (unlike in the previous example where the difference is increasing). For example, let us find out the next term of the series 10, 15, 19, 22, 24, …

$$\underbrace{10, \quad 15,}_{+5} \quad \underbrace{15, \quad 19,}_{+4} \quad \underbrace{19, \quad 22,}_{+3} \quad \underbrace{22, \quad 24}_{+2}$$

Here, the differences between 1st and 2nd, 2nd and 3rd, 3rd and 4th numbers, etc., are 5, 4, 3, 2, and so on. Since the difference between 22 and 24 is 2, the next difference should be 1. So, the number that comes after 24 should be 25.

Product Series

A product series is usually a number series where the terms are obtained by a process of multiplication. Here also, there can be different types of series. We will look at these through examples.

Consider the series 2, 4, 8, 16, 32, 64, …

$$2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} 16 \xrightarrow{\times 2} 32 \xrightarrow{\times 2} 64$$

Here, each number in the series is multiplied by 2 to get the next term. So, the term that comes after 64 is 128. So, each term is multiplied by a fixed number to get the next term. Similarly, we can have a series where we have numbers obtained by **dividing** the previous term with a constant number. For example, in the series 64, 32, 16, 8, …, each number is obtained by dividing the previous number by 2 (or in other words, by multiplying the previous term by $\frac{1}{2}$). So, here, the next term will be 4 (obtained by dividing 8 with 2).

Consider the series 4, 20, 80, 240, …

$$4 \xrightarrow{+5} 20, \xrightarrow{+4} 80, \xrightarrow{+3} 240,$$

Here, the first term is multiplied by 5 to get the second term; the second term is multiplied by 4 to get the third term; the third term is multiplied by 3 to get the fourth term. Hence, to get the fifth term, we have to multiply the fourth term by 2, i.e. the fifth term is 480. So, each term is multiplied by a decreasing factor (or it could also be an increasing factor) to get the next term. That is, with whatever number a particular term is multiplied to get the next term, this latest term is multiplied by a number different from the previous multiplying factor to get the next term of the series. All the multiplying factors follow a certain pattern (normally of increasing or decreasing order).

Consider the series 2, 6, 12, 20, 30, …

$$2, \xrightarrow{+4} 6, \xrightarrow{+6} 12, \xrightarrow{+8} 20, \xrightarrow{+10} 30$$

This can be looked at a series of increasing differences. The differences of consecutive pairs of terms are 4 (between 2 and 6), 6 (between 6 and 12), 8 (between 12 and 20), 10 (between 20 and 30), and so on. Hence, the difference between 30 and the next term should be 12, and, so, the next term will be 42. But, this series can also be looked at as a product series.

$$\begin{array}{ccccc} 2, & 6, & 12, & 20, & 30 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1\times 2 & 2\times 3 & 3\times 4 & 4\times 5 & 5\times 6 \end{array}$$

The first term is the product of 1 and 2; the second term is the product of 2 and 3; the third term is the product of 3 and 4; the fourth term is the product of 4 and 5; the fifth term is the product of 5 and 6. Hence, the next term will be the product of 6 and 7, that is 42.

Squares/Cubes Series

There can be series where all the terms are related to the squares of numbers or cubes of numbers. With squares/cubes of numbers as the basis, there can be many variations in the pattern of the series. Let us look at various possibilities of series based on squares/cubes.

Each term of the series may be the square of a natural number, such as 1, 4, 9, 16, …

$$\begin{array}{cccc} 1, & 4, & 9, & 16 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1^2 & 2^2 & 3^2 & 4^2 \end{array}$$

The numbers are squares of 1, 2, 3, 4 …, respectively. The number which follows 16 (which is the square of 4) will be 25 (which is the square of 5).

The terms of the series may be the squares of odd numbers (e.g. 1, 9, 25, 49, …) or even numbers (e.g. 4, 16, 36, 64, …).

The terms of the series could be such that a number and its square are given one after the other and such pairs are given in some specific pattern. For example, take the series 2, 4, 3, 9, 4, 16, …

$$\underbrace{2, \quad 4, \quad 3,}_{+1} \quad \underbrace{3, \quad 9, \quad 4,}_{+1} \quad 16,$$

Here, 2 is followed by its square 4; then comes the number 3 (which is one more than 2) followed by its square 9 and so on. Hence, the next number in the series is 5, and the one after that is its square, i.e. 25.

Similarly, each term could be the square root of its predecessor. For example, in the series 81, 9, 64, 8, 49, 7, 36, …, 81 is the square of 9, 64 the square of 8, and so on. Therefore, the next number which follows in the series should be the square root of 36, i.e. 6.

The terms of the series could be the squares of natural numbers increased or reduced by certain number. For example, in the series 3, 8, 15, 24, …

$$\begin{array}{cccc} 3, & 8, & 15, & 24 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2^2-1 & 3^2-1 & 4^2-1 & 5^2-1 \end{array}$$

We have {Squares of natural numbers – 1} as the terms. The first term is $2^2 - 1$; the second term is $3^2 - 1$; the third term is $4^2 - 1$, and so on. Hence, the next term will be $6^2 - 1$, i.e. 35. [Please note that the above series can also be looked at as a series with increasing differences. The differences between the 1st and 2nd terms, the 2nd and 3rd terms, and so on are 5, 7, 9, and so on. Hence, the next difference should be 11 giving us the next term as 35.] There could also be a series with {squares of natural numbers + some constant}.

Like we have seen series with squares of numbers, we can have similar series with cubes of numbers. For example, take the series 1, 8, 27, 64, …

$$\begin{array}{cccc} 1, & 8, & 27, & 64 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1^3 & 2^3 & 3^3 & 4^3 \end{array}$$

Here, all the terms are cubes of natural numbers. So, the next term will be 5^3, i.e. 125.

Consider the series 2, 9, 28, 65, …

$$\begin{array}{cccc} 2, & 9, & 28, & 65 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1^3+1 & 2^3+1 & 3^3+1 & 4^3+1 \end{array}$$

Here, the terms are {Cubes of natural numbers + 1}. The first term is $1^3 + 1$; the second term is $2^3 + 1$; the third term is $3^3 + 1$, and so on. Hence, the next term will be $5^3 + 1$, i.e. 125.

Miscellaneous Series

There are series that do not come under the other patterns and are of general nature but are important and are fairly common. Even here, some times, there can be a specific pattern in some cases.

Take the series 3, 5, 7, 11, 13, … . This is a series of consecutive PRIME NUMBERS. It is an important series and the student should look out for this as one of the patterns. The next term in this series is 17.

There can also be variations using prime numbers. Take the series 9, 25, 49, 121, … . In this series, the terms are squares of prime numbers. Hence, the next term is 13^2, i.e. 169.

Take the series 15, 35, 77, … . The first term is 3×5; the second term is 5×7; the third term is 7×11; here, the terms are product of two consecutive prime numbers. So, the next term will be the product of 11 and 13, i.e. 143.

Take the series 8, 24, 48, 120, 168, … Here, the 2nd term is 3 times the first term and the 3rd term is 2 times the 2nd term, but after that, it does not follow this pattern any more. If you look at the terms carefully, you will find that the terms are {one less than squares of prime numbers}. Hence, the next term will be $17^2 - 1$, i.e. 288.

Consider the series 1, 4, 9, 1, 6, 2, 5, 3, …

At first sight, there is nothing we can say about the series. This is actually a series formed by squares of natural numbers. However, if any of the squares is in two or more digits, each of the digits is written as a separate term of the series. Thus, the first terms are 1, 4, and 9, the squares of 1, 2, and 3, respectively. After this, we should get 16 (which is the square of 4). Since this has two digits 1 and 6, these two digits are written as two different terms 1 and 6 in the series. Similarly, the next square 25 is written as two different terms 2 and 5 in the series. So, the next square 36 should be written as two terms 3 and 6. Of these, 3 is already given. So, the next term of the series is 6.

Consider the series 1, 1, 2, 3, 5, 8, …

$$\begin{array}{cccccc} 1, & 1, & 2, & 3, & 5, & 8 \\ & & \downarrow & \downarrow & \downarrow & \downarrow \\ & & 1+1 & 1+2 & 2+3 & 3+5 \end{array}$$

Here, each term, starting with the third number, is the sum of the two preceding terms. After taking the first two terms as given (1 and 1), then onwards, to get any term, we need to add the two terms that come immediately before that position. Hence, to get the next term of the series, we should take the two preceding terms 5 and 8 and add them up to get 13. So, the next term of the series is 13. The term after this will be 21 (= 8 + 13).

Combination Series

A number series which has more than one type of (arithmetic) operation performed or more than one series combined together is a combination series. The series that are combined can be two series of the same type or could be different types of series as described earlier. Let us look at some examples.

First, let us look at those series that are formed by more than one arithmetic operation performed on the terms to get the subsequent terms.

Consider the series: 2, 6, 10, 3, 9, 13, 4, 12, … Here, the first term 2 is multiplied by 3 to get the second term, and 4 is added to get the third term. The next term is 3 (one more than the first term 2), and it is multiplied by 3 to get 9 (which is the next term) and then 4 is added to get the next term 13. The next term 4 (which is one more than 3), which is multiplied with 3 to get 12. Then, 4 is added to this to get the next number 16.

Consider the series: 1, 2, 6, 21, 88, …. Here, we can observe that 88 is close to 4 times 21. It is in fact $21 \times 4 + 4$. So, if we now look at the previous term 21, it is related to the previous term 6 as $6 \times 3 + 3$. Now we get the general pattern: to get any term, multiply the previous term with k and then add k where k is a natural number with values in increasing order from 1. So, to get the second term, the first term has to be multiplied with 1 and then 1 is added. To get the third term, the second term is multiplied with 2 and then 2 is added and so on. Hence, after 88, the next term is $88 \times 5 + 5$, i.e. 445.

Now, let us look at a series that is formed by combining two (or more) different series. The two (or more) series can be of the same type or of different types described earlier.

Consider the series: 8, 12, 9, 13, 10, 14, …. Here the 1st, 3rd, 5th, … terms, which are 8, 9, 10, …, form one series whereas the 2nd, 4th, 6th, etc. terms, which are 12, 13, 14, form another series. Here, both series that are being combined are two simple constant difference series. Therefore, the missing number will be the next term of the first series 8, 9, 10, …, which is equal to 11.

Consider the series: 0, 7, 2, 17, 6, 31, 12, 49, 20, …. Here, the series consisting of 1st, 3rd, 5th, … terms (i.e. the series consisting of the odd terms), which is 0, 2, 6, 12, 20, … is combined with another series consisting of 2nd, 4th, 6th, … terms (i.e. the series consisting of the even terms) which is 7, 17, 31, 49, … . The first series has the differences in increasing order 2, 4, 6, 8, 10, and so on. The second series also has the difference in increasing order 10, 14, 18, …. Since, the last term 20 belongs to the first series, a number from the second series should follow next. The next term of the second series will be obtained by adding 22 to 49, that is 71.

Consider the series: 1, 1, 2, 4, 3, 9, 4, 16, …. Here, one series consisting of odd terms, which is 1, 2, 3, 4, …, is combined with the series of even terms which is 1, 4, 9, 16, …. The first series is a series of natural numbers. The second series is the squares of natural numbers. Hence, the next term is 5.

Consider the series: 1, 1, 4, 8, 9, 27, …. Here, the series of squares of natural numbers is combined with the series of cubes of natural numbers. The next term in the series will be 4.

Consider the series: 2, 4, 5, 9, 9, 16, 14, ? , 20, …. Here, we have to find out the term that should come in place of the question mark. The odd terms form one series 2, 5, 9, 14, 20, … where the difference is increasing. The differences are 3, 4, 5, 6, … This series is combined with the series of even terms 4, 9, 16, … where the terms are squares of numbers 2, 3, 4, … . Hence, the term that should come in place of the question mark is the next term of the second series which is 5^2, i.e. 25.

A general approach to the number Series: The best way of approaching the number series questions is to first observe the difference between terms. If the difference is constant, it is a constant difference series. If the difference is increasing or decreasing by a constant number, then it is a series with a constant increasing or decreasing difference. If there is no constant increasing or decreasing difference, then try out the product series approach. For this, first divide the second term with the first term, third with the second, and so on. If the numbers obtained are the same, then it is a product series. Alternatively, try writing each term of the series as a product of two factors and see if there is any pattern that can be observed. If still there is no inference, but the difference is increasing or decreasing in a rapid manner, then check out the square series. If the increase is very high, and it is not a square series, then try out the cube series.

If the difference is alternately decreasing and increasing (or increasing for some time and alternately decreasing), then it should most probably be a mixed series. Therefore, test out the series with alternate numbers. If still the series is not solved, try out the general series.

Letter Series

The questions here are similar to the questions in Number Series Type I. Instead of numbers, we have letters of the alphabet given here. We have to first identify the pattern that the series of letters follow. Then, we have to find the missing letter based on the pattern already identified. Ins number series, we saw different patterns that the numbers in the series can follow—like squares, cubes. In letter series, obviously, patterns like squares, cubes will not be possible. In letter series, in general, we have a series with constant or increasing or decreasing differences. The position of the letters in the English alphabet is considered to be the value of the alphabet in questions on letter series. Also, when we are counting, after we count from A to Z, we again start with A, i.e. we treat the letters as being cyclic in nature. Like in number series, in this type of letter series also, we can have a 'combination' of series, i.e. two series are combined and given. We need to identify the pattern in the two series to find out the missing letter. Sometimes, there will be some special types of series also. Let us look at a few examples to understand questions on letter series.

Solved Examples

Example 1

Find the next letter in the series

D, G, J, M, P, ______.

(A) Q (B) R (C) S (D) T

Solution

Three letters are added to each letter to get the next letter in the series.

i.e. D^{+3}, G^{+3}, J^{+3}, M^{+3}, P^{+3}, $\underline{S}$

P + 3 and P = 16 and 16 + 3 = 19 and the 19th letter in the alphabet is S.

Example 2

Find the next letter in the series

A, B, D, H, ______.

(A) L (B) N (C) R (D) P

Solution

Each letter in the given series is multiplied with 2 to get the next letter in the series.

$A \times 2 \Rightarrow 1 \times 2 = 2$ and the 2^{nd} letter is B, $B \times 2 \Rightarrow 2 \times 2 = 4$ and the 4^{th} letter is D.

Similarly, $H \times 2 \Rightarrow 8 \times 2 = 16$ and the 16^{th} letter is P.

Example 3

What is the next letter in the series?

B, D, G, K, P, ______

(A) S (B) V (C) W (D) X

Solution

$B^{+2}, D^{+3}, G^{+4}, K^{+5}, P^{+6}$, ______

$P + 6 = 16 + 6 = 22$ and the 22^{nd} letter is V.

Example 4

I, X, J, W, K, V, L, ______.

(A) M (B) U (C) S (D) T

Solution

The given series is an alternate series.

I^{+1}, J^{+1}, K^{+1}, L is one series and X^{-1}, W^{-1}, V^{-1}, ______ is the other series.

$X - 1 = W$, $W - 1 = V$ and $V - 1 = 22 - 1 = 21$ and the 21^{st} letter is U.

Example 5

97, 83, 73, 67, 59, ______

(A) 53 (B) 49 (C) 47 (D) 51

Solution

The given numbers are alternate prime numbers in decreasing order, starting with 97. Hence, the next number in the series is 47.

Example 6

75, 291, 416, 480, 507, ______

(A) 515 (B) 532 (C) 511 D) 521

Solution

$75^{+216}, 291^{+125}, 416^{+64}, 480^{+27}$, 507, ______

The differences are cubes of consecutive natural numbers in decreasing order. Hence, the next number in the series in $507 + (B)^3 = 515$.

Exercises

Direction for questions 1 to 25: Complete the following series.

1. 17, 19, 23, 29, 31, 37, ______
(A) 41 (B) 43 (C) 40 (D) 42

2. 225, 196, 169, ______, 121, 100, 81
(A) 156 (B) 144 (C) 136 (D) 125

3. 64, 125, 216, 343, ______
(A) 64 (B) 424 (C) 317 (D) 512

4. 54, 66, 82, 102, 126, ______
(A) 146 (B) 130 (C) 154 (D) 144

5. 7, 11, 20, 36, 61, ______, 146
(A) 25 (B) 91 (C) 97 (D) 92

6. 8, 16, 48, 96, 288, 576, ______
(A) 1152 (B) 1728
(C) 1052 (D) 1428

7. 125, 375, 377, 1131, 1133, ______
(A) 3399 (B) 1136
(C) 1135 (D) 1234

8. 12, 35, 106, 317, 952, ______
(A) 2851 (B) 2855
(C) 1851 (D) 1849

9. 2, 4, 7, 35, 42, 462, ______
(A) 5016 (B) 470 (C) 4712 (D) 475

10. $13\frac{1}{3}$, 15, $\frac{120}{7}$, 20, 24, ______
(A) 30 (B) 36 (C) 40 (D) $37^1/_3$

11. 6, 15, 35, 77, 143, 221, ______
(A) 357 (B) 437 (C) 323 (D) 383

12. 29, 29, 27, 23, 25, 19, 23, 17, ______, ______
(A) 19, 13 (B) 19, 15 (C) 21, 13 (D) 19, 13

13. 24, 625, 26, 729, 28, 841, ______
(A) 30 (B) 29 (C) 900 (D) 961

14. 3731, 2923, 1917, 1311, ______
(A) 117 (B) 119 (C) 917 (D) 75

15. 11, 28, 327, 464, ______
(A) 525 (B) 5625 (C) 5125 (D) 5250

16. 6, 24, 60, 120, 210, ______
(A) 336 (B) 343 (C) 368 (D) 322

17. 132, 182, 306, 380, 552, 870, ______
(A) 930 (B) 1010 (C) 992 (D) 1142

18. KPD, LOE, MNF, NMG, ______
(A) ONF (B) OLH (C) MLH (D) MNH

19. BEP, CIQ, DOR, FUS, GAT, ______
(A) HEV (B) HIT (C) IET (D) IEU

20. GKF, IPC, LTY, PWT, UYN, ______
(A) ABZ (B) XBX
(C) XAH (D) AZG

21. QLR, JPD, RNU, GNC, SPX, DLB, ______
(A) TRA (B) AJA
(C) BTU (D) KJE

22. GTB, CYV, YDP, ______, QND
(A) DIV (B) UIJ
(C) DDV (D) UVV

23. ABDH, BDHP, CFLX, DHPF, ______
(A) EKNT (B) TNEK
(C) EJTN (D) JNTE

24. TCFK, RADI, OXAF, JSVA, ______
(A) DMPU (B) DMOT
(C) CMOT (D) CLOT

25. KJAM, GGWJ, ______, YAOD, UXKA
(A) CDUI (B) DFTC
(C) DCTF (D) CDSG

Answer Keys

1. A	**2.** B	**3.** D	**4.** C	**5.** C	**6.** B	**7.** A	**8.** B	**9.** D	**10.** A
11. C	**12.** C	**13.** A	**14.** D	**15.** C	**16.** A	**17.** C	**18.** B	**19.** A	**20.** D
21. A	**22.** B	**23.** C	**24.** D	**25.** D					