Chapter 1

Rational Numbers

Introduction to Rational Numbers

What are Natural Numbers?

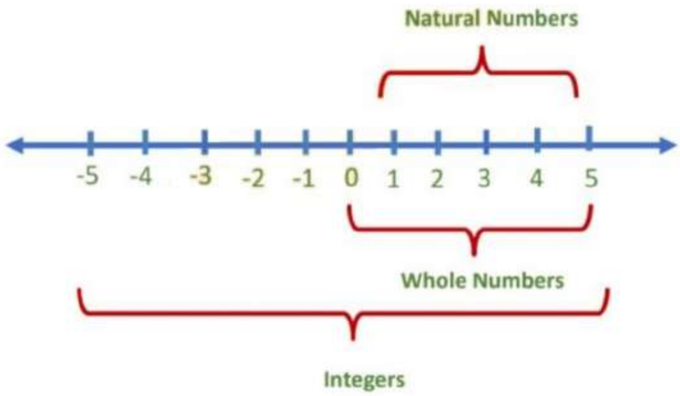
Counting numbers starting from 1 are known as natural numbers.
i.e., {1, 2, 3, 4....}

What are Whole numbers?

The natural numbers together with 0 are called whole numbers.
i.e., {0, 1, 2, 3, 4, 5......}

What are Integers?

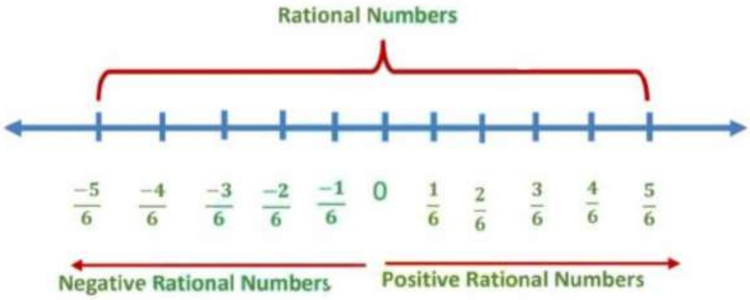
The whole numbers and negative of whole numbers together are called Integers.
i.e., {...-4, -3, -2, -1, 0, 1, 2, 3, 4...}



What are Rational Numbers?

A number which can be written in the form $\frac{p}{q}$, where p and q are integers and q $\neq 0$ is called a rational number.

For example: $-\frac{2}{3}, \frac{6}{7}$



So here if we multiply or divide numerator and denominator of a rational number by same non-zero integer, then we will get another equivalent rational number.

Thus, a rational number can be written in several equivalent forms. The general form for Equivalent rational number can be written as:

If $\frac{p}{q}$ is any rational number and $\frac{r}{s}$ be its equivalent rational number,

then $\frac{p}{q} = \frac{r}{s}$

such that ps = rq where p, q, r, and s are integers such that q and s are non-zero integers.

Closure Property of Rational Numbers

i) Closure Property
ii) Commutative Property
iii) Associative Property
iv) Additive Inverse
v) Multiplicative Inverse
vi) Distributive Property

i) Closure property

When we perform any operation on a specific set of numbers (natural number, whole number, integer, etc....) such that the resultant also belong to the same set then we say it follows closure property over that operation.

Closure Property of Whole Numbers

When we perform any operation on the whole number, such that the resultant also belong to the same set then we say it follows closure property of whole numbers over that operation.
So,

If any operation of two whole numbers satisfies the above- mentioned property, we say It is closed under that particular operation.

For example:
Let us take examples for various operations,
Let us take two whole numbers 2 and 3

1) For addition
$2+3=5$, which is also a whole number.
So, it is closed under addition.

2) For subtraction
$2-3=-1$, which is not a whole number.
So, it is not closed under subtraction.

3) For multiplication
$2*3=6$, which is also a whole number.
So, it is closed under multiplication.

4) For division
$2\div 3=\frac{2}{3}$, which is also a whole number.

So, it is closed under addition.

Let's summarize this in a table,

	Addition	Subtraction	Multiplication	Division
Operation	$a + b = c$	$a - b = c$	$a \times b = c$	$a \div b = c$
Example	$5 + 7 = 12$	$5 - 7 = -2$	$5 \times 4 = 20$	$5 \div 6 = \frac{5}{6}$
Remarks	Closed	Not Closed	Closed	Not Closed

Closure Property of Integers

When we perform any operation on integer, such that the resultant also belong to the same set then we say it follows closure property of integer over that operation.
So,

If any operation of two integers satisfies the above-mentioned property, we say It is closed under that particular operation.

For example:
Let us take examples for various operations,
Let us take two integers -2 and 3

1) For addition
-2+ 3 = 1, which is also an integer.
So, it is closed under addition.

2) For subtraction
-2 - 3 = -5, which is also an integer.
So, it is closed under subtraction.

3) For multiplication
-2*3 = -6, which is also an integer.
So, it is closed under multiplication.

4) For division
$-2 \div 3 = -\frac{2}{3}$, which is not an integer.
So, it is not closed under division.

Let's summarize this in a table,

	Addition	Subtraction	Multiplication	Division
Operation	$a + b = c$	$a - b = c$	$a \times b = c$	$a \div b = c$
Example	$5 + 2 = 7$	$-5 - 7 = -12$	$-5 \times 4 = -20$	$-5 \div 6 = -\frac{5}{6}$
Remarks	Closed	Closed	Closed	Not Closed

Closure Property of Rational Numbers

When we perform any operation on a rational number, such that the resultant also belong to the same set then we say it follows closure property of rational number over that operation.
So,

If any operation of two rational numbers satisfies the above- mentioned property, we say It is closed under that particular operation.

For example:
Let us take examples for various operations,

Let us take two rational numbers $\frac{1}{15}$ and - $\frac{1}{15}$

1) For addition

$\frac{1}{15}+(-\frac{1}{15})= 0$, which is also a rational number.

So, it is closed under addition.

2) For subtraction

$\frac{1}{15}- (-\frac{1}{15})= \frac{2}{15}$, which is also a rational number.

So, it is closed under subtraction.

3) For multiplication

$\frac{1}{15}*(-\frac{1}{15}) = -\frac{1}{225}$, which is also a rational number.

So, it is closed under multiplication.

4) For division

$\frac{1}{15} \div (-\frac{1}{15}) = 0$, which is also a rational number.

But if we take any rational number, 'a' and '0', then a÷ 0 is not defined and hence it is not closed.
But also, if we exclude '0', then it is closed under division.

Let's summarize this in a table,

	Addition	Subtraction	Multiplication	Division
Operation	$a + b = c$	$a - b = c$	$a \times b = c$	$a \div b = c$
Example	$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$	$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$	$\frac{-5}{6} \times \frac{1}{5} = \frac{-5}{30}$	$\frac{4}{5} \div 0$ = not defined
Remarks	Closed	Closed	Closed	Not Closed

Commutative Property of Rational Numbers

ii) Commutative property

An operation is said to be commutative when if we change the order of operands then the result remains the same, it does not change.

Commutative property of Whole Numbers

If any operation of two whole numbers satisfies the above-mentioned property, we say It is commutative under that particular operation.

For example:
Let us take examples for various operation

Let us take two whole numbers 2 and 3

1) For addition
2+ 3 = 5 and 3+2 = 5
Here, 2+3 = 3+2
So, it is commutative under addition.

2) For subtraction
2 - 3 = -1 and 3-2 = 51
Here, 2-3 ≠ 3-2
So, it is not commutative under subtraction.

3) For multiplication
2*3 = 6 and 3*2 = 6
Here, 2*3 = 3*2
So, it is commutative under multiplication.

4) For division
2÷3 = 2/3 and 3÷2 = 3/2
Here, 2÷3 ≠ 3÷2
So, it is not commutative under division.

	Addition	Subtraction	Multiplication	Division
Operation	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Example	$5 + 2 = 2 + 5$	$5 - 7 \neq 7 - 5$	$5 \times 4 = 4 \times 5$	$5 \div 6 \neq 6 \div 5$
Remarks	Commutative	Not Commutative	Commutative	Not Commutative

Commutative property of Integers

If any operation of two integers satisfies the above-mentioned property, we say It is commutative under that particular operation.

For example:

Let us take examples for various operations,

Let us take two integers -2 and 3

1) For addition
-2+ 3 = 1 and 3+(-2) = 1
Here, -2+3 = 3+(-2)
So, it is commutative under addition.

2) For subtraction

-2 -3 = -5 and 3-(-2) = 5
Here, -2-3 ≠ 3-(-2)
So, it is not commutative under subtraction.

3) For multiplication
-2*3 = -6 and 3*(-2) = -6
Here, -2*3 = 3*(-2)
So, it is commutative under multiplication.

4) For division
-2÷3 = -2/3 and 3÷ (-2) = 3/(-2)
Here, -2÷3 ≠ 3÷ (-2)
So, it is not commutative under division.

	Addition	Subtraction	Multiplication	Division
Operation	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Example	$5 + 2 = 2 + 5$	$-5 - 7 \neq 7 - (-5)$	$-5 \times 4 = 4 \times (5)$	$-5 \div 6 \neq 6 \div (-5)$
Remarks	Commutative	Not Commutative	Commutative	Not Commutative

Commutative Property of Rational Numbers

If any operation of two rational numbers satisfies the above- mentioned property, we say It is commutative under that particular operation.

For example:

Let us take examples for various operations,

Let us take two rational numbers $\frac{1}{5}$ and $\frac{2}{5}$
.

1) For addition

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ and $\frac{2}{5}+(\frac{1}{5}) = \frac{3}{5}$

Here, $\frac{1}{5} + \frac{2}{5} = \frac{2}{5} +(\frac{1}{5})$

So, it is commutative under addition.

2) For subtraction

$\frac{1}{5} - \frac{2}{5} = -\frac{1}{5}$ and $\frac{2}{5} - (\frac{1}{5}) = \frac{1}{5}$

Here, $\frac{1}{5} - \frac{2}{5} \neq \frac{2}{5} - (\frac{1}{5})$

So, it is not commutative under subtraction.

3) For multiplication

$\frac{1}{5} * \frac{2}{5} = \frac{2}{25}$ and $\frac{2}{5} * (\frac{1}{5}) = \frac{2}{25}$

Here, $\frac{1}{5} * \frac{2}{5} = \frac{2}{5} * (\frac{1}{5})$

So, it is commutative under multiplication.

4) For division

$\frac{1}{5} \div \frac{2}{5} = \frac{1}{2}$ and $\frac{2}{5} \div (\frac{1}{5}) = 2$

Here, $\frac{1}{5} \div \frac{2}{5} \neq \frac{2}{5} \div (\frac{1}{5})$

So, it is not commutative under division.

	Addition	Subtraction	Multiplication	Division
Operation	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Example	$\frac{1}{5} + \frac{2}{5} = \frac{2}{5} + \frac{1}{5}$	$\frac{4}{5} - \frac{1}{5} \neq \frac{1}{5} - \frac{4}{5}$	$\frac{-5}{7} \times \frac{4}{5} = \frac{4}{5} \times (\frac{-5}{7})$	$-5/7 \div 0 \neq 0 \div (-5/7)$
Remarks	Commutative	Not Commutative	Commutative	Not Commutative

Associative Property of Rational Numbers

iii) Associative property

A set of numbers is said to follow associative property over a particular operation if even after changing the grouping of numbers, we get the same result.

Associative Property of Whole Numbers

If any operation of whole numbers satisfies the above-mentioned property, we say It is associative under that particular operation.

For example:

Let us take examples for various operations,

Let us take three whole numbers 2, 4 and 3.

1) For addition
(2+4) +3 = 6+3 = 9 and 2+(4+3) = 2+7=9
Here, (2+4) +3 = 2+(4+3)
So, it is associative under addition.

2) For subtraction
(2-4) -3 = -2+(-3) = -5 and 2-(4-3) = 2-1 =1
Here, (2-4)-3 ≠ 2-(4-3)
So, it is not associative under subtraction.

3) For multiplication

(2*4)*3 = 8*3 = 24 and 2*(4*3) = 2*12=24
Here, (2*4) *3 = 2*(4*3)
So, it is associative under multiplication

4) For division

(2÷4) ÷3 = 0.5÷ (3) = $\frac{1}{6}$ and 2÷ (4÷3) = 2÷ $\frac{4}{3}$ = $\frac{3}{2}$

Here, (2÷4) ÷3 ≠ 2÷ (4÷3)
So, it is not associative under division.

	Addition	Subtraction	Multiplication	Division
Operation	a + (b + c) = (a + b) + c	a – (b – c) ≠ (a – b) – c	a × (b × c) = (a × b) × c	(a ÷ b) ÷ c ≠ a ÷ (b ÷ c)
Example	5 + (2 + 3) = (5 + 2) + 3	5 – (7 – 2) ≠ (5 – 7) - 2	5 × (4 × 2) = (5 × 4) × 2	(5 ÷ 6) ÷ 2 ≠ 5 ÷ (6 ÷ 2)
Remarks	Associative	Not Associative	Associative	Not Associative

Associative Property of integers

If any operation of integers satisfies the above-mentioned property, we say It is associative under that particular operation.

For example:

Let us take examples for various operations,

Let us take three whole numbers -2, 4 and 3.

1) For addition
(-2+4) +3 = 2+3 = 5 and -2+(4+3) = -2+7=5

Here, (-2+4) +3 = -2+(4+3)

So, it is associative under addition.

2) For subtraction
(-2-4) -3 = -6+(-3) = -9 and -2-(4-3) = -2-1 =-3
Here, (-2-4)-3 ≠ -2-(4-3)
So, it is not associative under subtraction.

3) For multiplication
(-2*4) *3 = -8*3 = -24 and -2*(4*3) = -2*12=-24
Here, (-2*4) *3 = -2*(4*3)
So, it is associative under multiplication

4) For division

(-2÷4) ÷3 = -0.5÷ (3) = - $\frac{1}{6}$ and -2÷ (4÷3) = -2÷ $\frac{4}{3}$ = - $\frac{3}{2}$

Here, (-2÷4) ÷3 ≠ -2÷ (4÷3)
So, it is not associative under division.

	Addition	Subtraction	Multiplication	Division
Operation	$a + (b + c) = (a + b) + c$	$a - (b - c) \neq (a - b) - c$	$a \times (b \times c) = (a \times b) \times c$	$(a \div b) \div c \neq a \div (b \div c)$
Example	$-5 + (2 + 3) = (-5 + 2) + 3$	$5 - (7 - 2) \neq (5 - 7) - 2$	$5 \times (4 \times 2) = (5 \times 4) \times 2$	$5 \div (6 \div 2) \neq (5 \div 6) \div 2$
Remarks	Associative	Not Associative	Associative	Not Associative

Associative Property of Rational Numbers

If any operation of a rational number satisfies the above-mentioned property, we say It is associative under that particular operation.

For example:

Let us take examples for various operations,

Let us take three whole numbers $-\frac{2}{5}, \frac{2}{5}$, and 1.

1) For addition

$(-\frac{2}{5}+\frac{2}{5}) +1 = 0+1 = 1$ and $-\frac{2}{5}+(\frac{2}{5}+1) = -\frac{2}{5}+\frac{7}{5}=1$

Here, $(-\frac{2}{5}+\frac{2}{5}) +3 = -\frac{2}{5}+(\frac{2}{5}+3)$

So, it is associative under addition.

2) For subtraction

$(-\frac{2}{5}-\frac{2}{5}) -1 = -\frac{4}{5}-1 = -\frac{9}{5}$ and $-\frac{2}{5}- (\frac{2}{5}-1) = -\frac{2}{5}- (-\frac{3}{5}) = \frac{1}{5}$

Here, $(-\frac{2}{5}-)-1 \neq -\frac{2}{5}- (\frac{2}{5}-1)$

So, it is not associative under subtraction.

3) For multiplication

$(-\frac{2}{5}*\frac{2}{5}) *1 = -\frac{4}{25}*1 = -\frac{4}{25}$ and $-\frac{2}{5}*(\frac{2}{5}*1) = -\frac{2}{5}*\frac{2}{5}= -\frac{4}{25}$

Here, $(-\frac{2}{5} * \frac{2}{5}) *1 = -\frac{2}{5}*(\frac{2}{5}*1)$

So, it is associative under multiplication

4) For division

$(-\frac{2}{5} \div \frac{2}{5}) \div 1 = -1 \div (1) = -1$ and $-\frac{2}{5} \div (\frac{2}{5} \div 1) = -\frac{2}{5} \div \frac{2}{5} = -1$

Here, in this case, it is holding equality but let's take 2 instead of 1.

$(-\frac{2}{5} \div \frac{2}{5}) \div 2 = -1 \div (2) = -\frac{1}{2}$ and $-\frac{2}{5} \div (\frac{2}{5} \div 2) = -\frac{2}{5} \div \frac{1}{5} = -2$

Here, $(-\frac{2}{5} \div \frac{2}{5}) \div 2 \neq -\frac{2}{5} \div (\frac{2}{5} \div 2)$

So, it is not associative under division.

	Addition	Subtraction	Multiplication	Division
Operation	a + (b + c) = (a + b) + c	a – (b – c) ≠ (a – b) – c	a × (b × c) = (a × b) × c	(a ÷ b) ÷ c ≠ a ÷ (b ÷ c)
Example	1/5 + (2/5 +1/5) = (1/5 + 2/5) + 1/5	1/5 - (2/5 -1/5) ≠ (1/5 - 2/5) - 1/5	1/5 × (2/5 ×1/5) = (1/5 × 2/5) × 1/5	1/5 ÷ (2/5 ÷1/5) ≠ (1/5 ÷ 2/5) ÷ 1/5
Remarks	Associative	Not Associative	Associative	Not Associative

Additive Identity of Rational Numbers

iv) Additive Inverse

When 0 is added to a number, the sum is the same number. Therefore, 0 is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers also.

For example:

$5 + 0 = 0 + 5 = 5$ (Addition of 0 to a whole number)

$-5 + 0 = 0 + (-5) = -5$ (Addition of 0 to an Integer)

$-\frac{5}{7} + 0 = 0 + -\frac{5}{7} = -\frac{5}{7}$ (Addition of 0 to a rational number)

For any rational number $\frac{a}{b}$,

$\frac{(-a)}{b}$ is the additive inverse of $\frac{a}{b}$

$\frac{a}{b}$ is the additive inverse of $\frac{-a}{b}$

$$\frac{a}{b} + \left(\frac{-a}{b}\right) = \left(\frac{-a}{b}\right) + \frac{a}{b}$$

$$\frac{3}{5} + \left(-\frac{3}{5}\right) = 0$$

Number ($\frac{3}{5}$) — Additive Inverse ($-\frac{3}{5}$)

0 is the only rational number which is equal to its own negative.

The sum of a number and its additive inverse is 0.

Write the additive inverse of the following:

a) $\frac{3}{8}$

b) $\frac{-7}{12}$

a) We know that additive inverse of $\frac{a}{b}$ is $\frac{(-a)}{b}$

$\therefore$ The additive inverse of $\frac{3}{8}$ is $\frac{(-3)}{8}$

b) The additive inverse of $\frac{(-a)}{b}$ is $\frac{a}{b}$

$\therefore$ additive inverse of $\frac{-7}{12}$ is $\frac{7}{12}$

What number should be added to $\frac{-11}{8}$ to get $\frac{5}{9}$.

Let the number be x.

$\frac{-11}{8} + x = \frac{5}{9}$

$x = \frac{5}{9} - \frac{(-11)}{8}$ (Transposing $\frac{-11}{8}$ to RHS)

$x = \frac{5}{9} + \frac{11}{8}$

$x = \frac{40+99}{72}$

$x = \frac{139}{72}$ Hence the required number is $\frac{139}{72}$

Multiplicative identity of Rational Numbers

v) Multiplicative Inverse
When 1 is multiplied by any number, the product is again the same number. Therefore 1 is the multiplicative identity for rational numbers, integers, and whole numbers.

For any non-zero rational number $\frac{a}{b}$ there exist a rational number $\frac{c}{d}$, such that

$\frac{a}{b} \times \frac{c}{d} = 1$, where $\frac{c}{d}$ is called the reciprocal or multiplicative inverse of $\frac{a}{b}$.

Find the multiplicative inverse of the following:

a) $\frac{5}{17}$

b) 13

a) Reciprocal of $\frac{5}{17}$ is $\frac{17}{5}$

b) Reciprocal of 13 is $\frac{1}{13}$

Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{6}$.

Reciprocal of $\frac{-7}{6}$ is $\frac{-6}{7}$

$\frac{6}{13} \times \frac{-6}{7} = \frac{-36}{91}$

Distributive Property of Rational Numbers

v) Distributive property of multiplication over addition and subtraction for Rational Numbers

To distribute means to divide.

According to the distributive property of multiplication over addition and subtraction, if we multiply the sum or subtraction of two numbers together then the resultant product will be same as the when we take the product of that numbers individually and then do addition or subtraction.

For any rational numbers a, b and c,

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Solve:

$\frac{(-2)}{5} \times \left\{\frac{3}{5} + \frac{2}{5}\right\}$

$= \frac{(-2)}{5} \times \frac{3}{5} + \frac{(-2)}{5} \times \frac{2}{5}$

(by Distributive property of multiplication over addition)

$= \frac{(-2)}{5} \times \left\{\frac{3}{5} + \frac{2}{5}\right\} = \frac{(-2)}{5} \times \left\{\frac{3+2}{5}\right\} = \frac{(-2)}{5} \times \frac{5}{5} = \frac{-2}{5}$

Solve:

$\frac{1}{3} \times \left\{\frac{2}{5} - \frac{(-3)}{5}\right\}$

$= \frac{1}{3} \times \frac{2}{5} - \frac{1}{3} \times \frac{(-3)}{5}$

(by Distributive property of multiplication over subtraction)

$= \frac{1}{3}\left\{\frac{2}{5} - \frac{(-3)}{5}\right\} = \frac{1}{3}\left\{\frac{2+3}{5}\right\} = \frac{1}{3} \times \frac{5}{5} = \frac{1}{3}$

Representations of Rational Numbers on a Number Line

Representation of Rational Numbers on Number Line

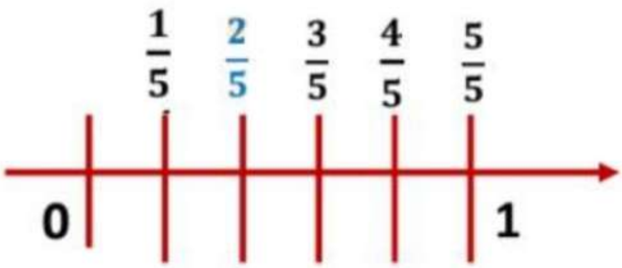
- In a rational number the denominator tells the number of equal parts into which the first unit is to be divided.
- The numerator tells 'how many' of these parts are considered.
- Therefore, the rational number $\frac{2}{5}$ means 2 parts out of 5 equal parts on the right of 0.
- The rational number $\frac{(-4)}{5}$ means 4 parts out of 5 equal parts on the left of 0.

Represent $\frac{2}{5}$ on the number line.

i) Draw a number line and mark O on it to represent zero.

ii) As the denominator of rational number is 5.the number line is divided into 5 equal parts.

iii) The numerator is 2 and so we will consider 2 parts out of 5 equal parts.



Rational Numbers between Two Rational Numbers

We know about different kinds of numbers. Let us understand the numbers that lie between two given numbers.

Let's take two natural numbers between 2 and 5, we can tell that 3 and 4 are natural numbers that lie between 2 and 5 and similarly if asked about integers

that lie between -2 and 2 we
can have -1, 0 and 1 as our answer.

But what about rational numbers,

If we are asked about rational numbers between $\frac{2}{5}$ and $\frac{4}{5}$. You might think that only $\frac{3}{5}$ will lie between them.

But $\frac{2}{5} = \frac{4}{10}$ and $\frac{4}{5} = \frac{8}{10}$

So $\frac{5}{10}$,$\frac{6}{10}$ and $\frac{7}{10}$ will also lie.

Moreover, $\frac{2}{5} = \frac{40}{100}$ and $\frac{4}{5} = \frac{80}{100}$

Now the numbers will be $\frac{41}{100}$,$\frac{42}{100}$, ……………,$\frac{78}{100}$,$\frac{79}{100}$

So in between two rational numbers, there can be infinitely many numbers possible.

Let us see how to find such numbers: -

Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$

Method 1

We will first find the mean of the two rational numbers.

$\frac{1}{4}$ and $\frac{1}{2}$

$\left(\frac{1}{4} + \frac{1}{2}\right) \div 2$

$\left(\frac{2+4}{8}\right) \div 2$

$\frac{6}{8} \times \frac{1}{2} = \frac{3}{8}$

Method 2

We will first convert the rational numbers $\frac{1}{4}$ and $\frac{1}{2}$ to the rational number with the same denominator.

$\frac{1}{4}$ and $\frac{1}{2}$

$\frac{1\times2}{4\times2}$ and $\frac{1\times4}{2\times4}$

$\frac{2}{8}$ and $\frac{4}{8}$

$\frac{2}{8}, \frac{3}{8}, \frac{4}{8}$

Hence, the required rational number is $\frac{3}{8}$.