01

Units and Measurements

TOPIC 1
Units

01 If E and H represent the intensity of electric field and magnetising field respectively, then the unit of E / H will be **[2021, 27 Aug Shift-I]**

(a) ohm (b) mho
(c) joule (d) newton

Ans. *(a)*

Unit of intensity of electric field E is Vm^{-1}.

Unit of intensity of magnetising field H is Am^{-1}.

Unit of E/H can be calculated as

$$\frac{\text{Unit of } E}{\text{Unit of } H}=\frac{Vm^{-1}}{Am^{-1}}=\frac{V}{A}=\text{ohm}.$$

Thus, the unit of E/H will be ohm.

02 Match List-I with List-II.

List-I	List-II
A. R_H(Rydberg constant)	1. $kg\,m^{-1}s^{-1}$
B. h(Planck's constant)	2. $kg\,m^2s^{-1}$
C. μ_B(Magnetic field energy density)	3. m^{-1}
D. η(Coefficient of viscosity)	4. $kg\,m^{-1}s^{-2}$

Choose the most appropriate answer from the options given below. **[2021, 27 Aug Shift-II]**

	A	B	C	D
(a)	2	3	4	1
(b)	3	2	4	1
(c)	4	2	1	3
(d)	3	2	1	4

Ans. *(b)*

As we know that, SI unit of following terms are

(a) R_H (Rydberg constant) $= m^{-1}$

(b) h(Planck's constant)

$$= J\text{-}s = kg\text{-}m^2.s^{-2}.s = kg\text{-}m^2\,s^{-1}$$

(c) Magnetic field energy density

$$(\mu_B)=\frac{\text{Energy}}{\text{Volume}}=\frac{kg\text{-}m^2s^{-2}}{m^3}=kgm^{-1}s^{-2}$$

(d) η (coefficient of viscosity)

$$\because \quad F=\eta A\frac{dv}{dx}$$

$$\therefore \quad \eta=\frac{Fdx}{Adv}=\frac{kg\text{-}ms^{-2}\cdot m}{m^2.ms^{-1}}=kgm^{-1}s^{-1}$$

So, the correct match is A-3, B-2, C-4 and D-1.

03 The density of a material in SI units is $128\ kg\ m^{-3}$. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is **[2019, 10 Jan Shift-I]**

(a) 40 (b) 16
(c) 640 (d) 410

Ans. *(a)*

To convert a measured value from one system to another system, we use

$$N_1u_1=N_2u_2$$

where, N is numeric value and u is unit.

We get

$$128\cdot\frac{kg}{m^3}=N_2\frac{50\,g}{(25\,cm)^3}\quad\left[\because \text{density}=\frac{\text{mass}}{\text{volume}}\right]$$

$$\Rightarrow \frac{128\times 1000\,g}{100\times 100\times 100\,cm^3}=\frac{N_2\times 50\,g}{25\times 25\times 25\,cm^3}$$

$$\Rightarrow N_2=\frac{128\times 1000\times 25\times 25\times 25}{50\times 100\times 100\times 100}=40$$

04 The 'rad' is the correct unit used to report the measurement of **[AIEEE 2006]**

(a) the ability of a beam of gamma ray photons to produce ions in a target
(b) the energy delivered by radiation to a target
(c) the biological effect of radiation
(d) the rate of decay of a radioactive source

Ans. *(c)*

'Rad' is used to measure biological effect of radiation.

TOPIC 2
Errors in Measurement and Significant Figures

05 Two resistors $R_1=(4\pm 0.8)\,\Omega$ and $R_2=(4\pm 0.4)\ \Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be **[2021, 1 Sep Shift-II]**

(a) $(4\pm 0.4)\Omega$ (b) $(2\pm 0.4)\Omega$
(c) $(2\pm 0.3)\Omega$ (d) $(4\pm 0.3)\Omega$

Ans. *(c)*

Given, $R_1=(4\pm 0.8)\,\Omega \Rightarrow R_2=(4\pm 0.4)\,\Omega$

Equivalent resistance when the resistors are connected in parallel is given by

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}\Rightarrow \frac{1}{R_{eq}}=\frac{1}{4}+\frac{1}{4}$$

$$R_{eq}=2\,\Omega$$

Now, $\dfrac{\Delta R_{eq}}{R_{eq}^2} = \dfrac{\Delta R_1}{R_1^2} + \dfrac{\Delta R_2}{R_2^2}$

Substituting the values in the above equation, we get

$$\frac{\Delta R_{eq}}{4} = \frac{0.8}{16} + \frac{0.4}{16} \Rightarrow \Delta R_{eq} = 0.3\,\Omega$$

$\therefore$ The equivalent resistance in parallel combination is $R_{eq} = (2 \pm 0.3)\,\Omega$.

06 A student determined Young's modulus of elasticity using the formula $Y = \dfrac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2, without any significant error, his observations are as following.

Physical quantity	Least count of the equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then, the fractional error in the measurement of Y is

[2021, 1 Sep Shift-II]

(a) 0.0083 (b) 0.0155
(c) 0.155 (d) 0.083

Ans. *(b)*

The given formula of Young's modulus of elasticity,

$$Y = \frac{mgL^3}{4bd^3\delta}$$

where, Y = Young's modulus of elasticity,
m = mass of the bar,
L = length of the bar,
b = breadth of the bar,
d = thickness of the bar
and δ = depression of the bar.

There is no error in the value of the g.

The fractional error in the measurement of Y,

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L} + \frac{\Delta b}{b} + 3\frac{\Delta d}{d} + \frac{\Delta \delta}{\delta}$$

Substituting the values in the above expression, we get

$$\frac{\Delta Y}{Y} = \frac{10^{-3}}{2} + 3\frac{(1 \times 10^{-3})}{1} + \frac{0.1 \times 10^{-3}}{4 \times 10^{-2}} + 3\frac{(0.01 \times 10^{-3})}{0.4 \times 10^{-2}} + \frac{(0.01 \times 10^{-3})}{5 \times 10^{-3}}$$

$$\frac{\Delta Y}{Y} = 0.0155$$

The fractional error in the measurement of the Young's modulus is 0.0155.

07 The diameter of a spherical bob is measured using a Vernier callipers. 9 divisions of the main scale, in the vernier calipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8th division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is .... $\times 10^{-2}$ cm.

[2021, 31 Aug Shift-II]

Ans. *(52)*

Given, 9 divisions of main scale are equal to 10 divisions of Vernier scale.

i.e. 9 MSD = 10 VSD

$\Rightarrow$ VSD $= \dfrac{9}{10}$ MSD ...(i)

Size of 1 main scale division,

1 MSD = 1 mm

Now, least count, LC = 1 MSD − 1VSD ....(ii)

Using Eqs. (i) and (ii), we get

$$\text{LC} = 1\,\text{MSD} - \frac{9}{10}\,\text{MSD} = \frac{1}{10}\,\text{MSD} = \frac{1}{10}\,\text{mm}$$

While measuring the diameter of bob.

Main Scale Reading, MSR = 10 mm

Vernier Scale Reading, VSR = 8

Zero error, e = 0.04 cm

Now, diameter, $d = [\text{MSR} + \text{LC} \times \text{VSR}] - e$

$$= \left(10\text{ mm} + \frac{1}{10} \times 8\text{ mm}\right) - 0.04\text{ cm}$$

$= (10.8)$ mm − 0.04 cm

$= 1.08$ cm − 0.04 cm = 1.04 cm

Radius, $r = \dfrac{d}{2} = \dfrac{1.04}{2}$ cm = 0.52 cm

$= 52 \times 10^{-2}$ cm

$\therefore$ Correct answer is 52.

08 If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is

[2021, 26 Aug Shift-II]

(a) 86.4 s (b) 4.32 s
(c) 43.2 s (d) 8.64 s

Ans. *(c)*

Increase in length of pendulum is 0.1%.

i.e. $\dfrac{\Delta L}{L} \times 100 = 0.1$

Time period of pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Here, 2π and g are constant.

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta L}{L} \times 100$$

$$\Rightarrow \quad \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times 0.1$$

$$\Rightarrow \quad \Delta T = 0.05 \times \frac{T}{100}$$

In one single day, the time in seconds is $T = 24 \times 60 \times 60 = 86400$ s

$$\therefore \Delta T = 0.05 \times \frac{86400}{100} = 43.2\text{ s}$$

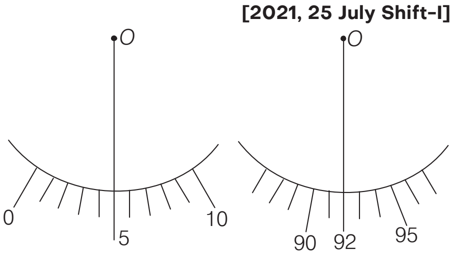
Thus, the error in time per day is 43.2 s.

09 Student A and student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is ....

[Figure shows position of reference O when jaws of screw gauge are closed]

Given, pitch = 0.1 cm.

[2021, 25 July Shift-I]



Screw gauge (A) Screw gauge (B)

Ans. *(13)*

Given,

Number of circular scale division = 100

True value of radius, R = 0.322 cm

Least count (LC) = 0.1 cm/100 = 0.001 cm

Now,

As we know that,

True value (TV) = Main scale reading (MSR) + Circular scale reading (CSR) + Error

where, Error = nth division × LC

Now,

For student A,

$0.322 = 0.300 + \text{CSR}_A + 5 \times 0.001$

$CSR_A = 0.322 - 0.300 - 0.005 = 0.017$

and for student B,

$0.322 = 0.2 + CSR_B + 92 \times 0.001$

$CSR_B = 0.322 - 0.2 - 0.092 = 0.030$

$\therefore$ Difference $= CSR_B - CSR_A$

$= 0.030 - 0.017 = 0.013$ cm

Now, division on circular scale

$$= \frac{0.013}{0.001} = 13$$

10 Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1.	64.0	8	128.0	16.0
2.	64.0	4	64.0	16.0
3.	20.0	4	36.0	9.0

(Least count of length = 0.1m, least count for time = 0.1s)

If E_1, E_2 and E_3 are the percentage errors in g for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student number ....

[2021, 22 July Shift-II]

Ans. *(1)*

Given, observation of three students named S_1, S_2 and S_3.

Let $l_1, l_2, l_3, T_1, T_2, T_3$ be the measured length and time period by student S_1, S_2 and S_3 respectively,

and E_1, E_2 and E_3 be the errors in g

and $\Delta T = 0.1$ s, $\Delta l = 0.1$ m

As we know that,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

On squaring both sides,

$$T^2 = 4\pi^2 \frac{l}{g}$$

By using concept of relative error,

$$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta l}{l}$$

$$E_1 = 2\times\frac{0.1}{16} + \frac{0.1}{64}$$

$$= 0.0125 + 0.0016$$

$$= 0.0141$$

$$E_2 = 2\times\frac{0.1}{16} + \frac{0.1}{64} = 0.0141$$

$$E_3 = 2\times\frac{0.1}{9} + \frac{0.1}{20} = 0.027$$

From above error calculations $E_1 = E_2$

But, since number of oscillations in ε_1 is more, so more precise observation, less error.

$\therefore$ Student number 1 will have least error.

11 The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement, it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is .... cm.

[2021, 17 March Shift-I]

(a) 8.36 (b) 8.54
(c) 8.58 (d) 8.56

Ans. *(b)*

Given, positive zero error

$= 0.2$ mm $= 0.02$ cm

[$\therefore$ Least count LC = 0.01 cm]

Main scale reading = 8.5 cm

Vernier scale reading

= Vernier scale coincidence × Least count

$= 6 \times 0.01 = 0.06$ cm

Final reading = Main scale reading + Vernier scale reading − Zero error

$= 8.5 + 0.06 - 0.02 = 8.54$ cm

12 In order to determine the Young's modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's modulus determined by this experiment ?

[2021, 16 March Shift-II]

(a) 0.14% (b) 0.9% (c) 9% (d) 1.4%

Ans. *(d)*

Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{Al}$

$$\Rightarrow \quad Y = \frac{mgL}{\pi R^2 l} \quad \text{... (i)}$$

[$\because F = mg$ and $A = \pi R^2$]

To determine the fractional errors, we can write

Eq. (i) as follows

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + \frac{2\cdot\Delta R}{R} + \frac{\Delta l}{l}$$

$$\Rightarrow \frac{\Delta Y}{Y}\times 100 = \frac{\Delta m}{m}\times 100 + \frac{\Delta L}{L}\times 100 + \frac{2\Delta R}{R}\times 100 + \frac{\Delta l}{l}\times 100$$

$$\Rightarrow \frac{\Delta Y}{Y}\times 100 = 100\left[\frac{\Delta m}{m} + \frac{\Delta L}{L} + \frac{2\Delta R}{R} + \frac{\Delta l}{l}\right]$$

$$= 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$$

$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$$

13 One main scale division of a vernier callipers is a cm and nth division of the vernier scale coincide with $(n-1)$th division of the main scale. The least count of the callipers (in mm) is **[2021, 16 March Shift-I]**

(a) $\frac{10na}{(n-1)}$ (b) $\frac{10a}{(n-1)}$

(c) $\left(\frac{n-1}{10n}\right)a$ (d) $\frac{10a}{n}$

Ans. *(d)*

According to the question,

One division of main scale reading

$= a$ cm

nth vernier scale division

$= (n-1)$th main scale division

$\therefore$ One division of vernier scale reading

$$= \frac{(n-1)\times a}{n} \quad \text{...(i)}$$

We know that,

Least count (LC) =

[1 main scale division − 1 vernier scale division] cm

$$= a - \frac{(n-1)a}{n} \quad \text{[using Eq. (i)]}$$

$$= \frac{a(n-n+1)}{n} = \frac{a}{n}\text{ cm} = \frac{a}{n}\times 10 \text{ mm}$$

$$\Rightarrow \quad LC = \frac{10a}{n}\text{ mm}$$

14 The resistance $R = \frac{V}{I}$, where $V = (50 \pm 2)$ V and $I = (20 \pm 0.2)$ A. The percentage error in R is x %. The value of x to the nearest integer is......... . **[2021, 16 March Shift-I]**

Ans. *(5)*

Given, $R = \frac{V}{I}$ ...(i)

where, $V = (50 \pm 2)\,\text{V}$

$I = (20 \pm 0.2)\,\text{A}$

From Eq. (i)

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$\% \text{ error in } R = \left[\frac{2}{50} \times 100 + \frac{0.2}{20} \times 100\right]\%$$

$$= [2 \times 2 + 0.2 \times 5]\%$$

$$= 5\%$$

Comparing with the given value in the question i.e., $x\%$, the value of $x = 5$.

15 A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be **[2021, 26 Feb Shift-I]**

(a) $\frac{2T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ (b) $\frac{2T}{rJ}$

(c) $\frac{3T}{rJ}$ (d) $\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

Ans. *(d)*

Given, radius of small drop $= r$

Radius of big drop $= R$

Surface tension $= T$

and mechanical equivalent of heat $= J$

As, small drops combine to form big drop.

$\therefore$ Volume of big drop $(V_B) = n \times$ Volume of small drop (V_S)

$$\Rightarrow \frac{4}{3}\pi R^3 = n.\frac{4}{3}\pi r^3$$

$$\Rightarrow nr^3 = R^3$$

$$\Rightarrow r = \frac{R}{n^{1/3}} \quad \text{...(i)}$$

Surface energy of small drop

$(E_S) =$ Surface tension $(T) \times$ Area (A)

$\Rightarrow \quad E_S = n \times 4\pi r^2 T$

and $\quad E_B = 4\pi R^2 T$

Now, change in energy will be

$$\Delta E = E_B - E_S = 4\pi T(nr^2 - R^2)$$

$\therefore$ Heat energy per unit volume

$$= \frac{\Delta E}{V} = \frac{4\pi T(nr^2 - R^2)}{J \times \frac{4}{3}\pi R^3}$$

$$= \frac{3T}{J}\left(\frac{nr^2}{R^3} - \frac{1}{R}\right)$$

$$= \frac{3T}{J}\left(n\frac{R^2}{n^{2/3}R^3} - \frac{1}{R}\right)$$

$$= \frac{3T}{J}\left[\frac{n^{1/3}}{R} - \frac{1}{R}\right] \quad \text{[from Eq. (i)]}$$

$$= \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

16 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 1.0 m from metre scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of g will be **[2021, 24 Feb Shift-II]**

(a) 1.13% (b) 1.03%

(c) 1.33% (d) 1.30%

Ans. *(a)*

Given,

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{... (i)}$$

where, time period, $T = 1.95$ s

Length of string, $l = 1$ m

Acceleration due to gravity $= g$

Error in time period, $\Delta T = 0.01\,\text{s} = 10^{-2}$ s

Error in length, $\Delta L = 1\,\text{mm} = 1 \times 10^{-3}$ m

Squaring Eq. (i) on both sides, we get

$$T^2 = 4\pi^2\frac{L}{g}$$

$$\Rightarrow \quad g = 4\pi^2\frac{L}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = \frac{10^{-3}}{1} + \frac{2 \times 10^{-2}}{1.95}$$

$$= 10^{-3} + 1.025 \times 10^{-2}$$

$$= 10^{-3} + 10.25 \times 10^{-3}$$

$$= 11.25 \times 10^{-3}$$

$$\because \quad \Delta g / g \times 100 = 11.25 \times 10^{-3} \times 10^2$$

$$= 1.125\% \simeq 1.13\%$$

17 A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{2/3}}{\sqrt{c}d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is **[2020, 5 Sep Shift-I]**

(a) 13.5% (b) 16.5%

(c) 14.5% (d) 12.25%

Ans. *(c)*

Given, $\quad z = \frac{a^2 b^{2/3}}{\sqrt{c}\,d^3}$

According to question,

% error in $z = (2)\%$ error in $a + \left(\frac{2}{3}\right)\%$ error in $b + \left(\frac{1}{2}\right)\%$ error in $c + (3)\%$ error in d

$$\frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$$

$$= 2 \times 2\% + \frac{2}{3} \times 1.5\% + \frac{1}{2} \times 4\% + 3 \times 2.5\%$$

$$= 14.5\%$$

18 The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is .... **[2020, 6 Sep Shift-I]**

Ans. *(1050)*

Given, relative error in mass,

$$\frac{\Delta m}{m} \times 100 = 6\%$$

Relative error in diameter,

$$\frac{\Delta d}{d} \times 100 = 1.5\%$$

Density of sphere,

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{M}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

$$\rho = \frac{6}{\pi}Md^{-3} \text{ or } \rho \propto Md^{-3}$$

For maximum error in density,

$$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3 \times \frac{\Delta d}{d}$$

$$= 6\% + 3 \times 1.5\%$$

$$\frac{\Delta \rho}{\rho} \times 100 = 10.5\%$$

$$\Rightarrow \quad \frac{1050}{100}\% = \frac{x}{100}\% \quad \text{(given)}$$

$$\therefore \quad x = 1050$$

19 For the four sets of three measured physical quantities as given below. Which of the following options is correct ? **[2020, 9 Jan Shift-II]**

(i) $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$

(ii) $A_2 = 24.44, B_2 = 16.082, C_2 = 240.2$

(iii) $A_3 = 25.2, B_3 = 19.2812,$
$C_3 = 236.183$

(iv) $A_4 = 25, B_4 = 236.191, C_4 = 19.5$

(a) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3$
$< A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

(b) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1$
$= A_2 + B_2 + C_2$
$= A_3 + B_3 + C_3$

(c) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1$
$= A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

(d) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2$
$= A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

Ans. *(*)*

Given , $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$

$\therefore A_1 + B_1 + C_1 = 280.6324$

As sum contains same number of digits after decimal as present in the number having the least number of decimal places.

So, $A_1 + B_1 + C_1 = 280.6$

Similarly,

$A_2 + B_2 + C_2 = 280.7$

$A_3 + B_3 + C_3 = 280.7$ (rounded off)

$A_4 + B_4 + C_4 = 280.7$ (rounded off)

So, $A_1 + B_1 + C_1 < A_2 + B_2 + C_2$
$= A_3 + B_3 + C_3$
$= A_4 + B_4 + C_4$

None of the options is matching with result.

20 In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained 55.0 cm. The percentage error in the determination of g is close to **[2019, 8 April Shift-II]**

(a) 0.7% (b) 6.8%
(c) 3.5% (d) 0.2%

Ans. *(b)*

Relation used for finding acceleration due to gravity by using a pendulum is

$$g = \frac{4\pi^2 l}{T^2}$$

So, fractional error in value of g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \quad \text{...(i)}$$

Given, $\Delta l = 0.1$ cm, $l = 55$ cm, $\Delta T = 1$ s and T for 20 oscillations $= 30$ s

Substituting above values in Eq. (i), we get

$$\frac{\Delta g}{g} = \frac{0.1}{55} + 2 \times \frac{1}{30}$$

Hence, percentage error in g is

$$= \frac{\Delta g}{g} \times 100 = \frac{10}{55} + \frac{20}{3} = 6.8\%$$

21 In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is **[2019, 9 April Shift-I]**

(a) 0.01 kg/m^3 (b) 0.10 kg/m^3
(c) 0.07 kg/m^3 (d) 0.31 kg/m^3

Ans. *(*)*

Given, mass $= (10.00 \pm 0.10)$ kg

Edge length $= (0.10 \pm 0.01)$ m

Error in mass, $\frac{\Delta M}{M} = \frac{0.1}{10}$ ...(i)

and error in length, $\frac{\Delta l}{l} = \frac{0.01}{0.1}$ ...(ii)

Density of the cube is given by

$$r = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{l^3}$$

$\therefore$ Permissible error in density is

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} \pm 3\frac{\Delta l}{l} \quad \text{...(iii)}$$

Substituting the value from Eqs. (i) and (ii) in Eq. (iii), we get

$$\frac{\Delta\rho}{\rho} = \frac{0.1}{10} + 3 \times \frac{0.01}{0.1} = \frac{1}{100} + \frac{3}{10} = \frac{31}{100}$$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{31}{100} = 0.31$$

Since, $\frac{\Delta\rho}{\rho}$ should be unitless quantity.

But there is no option with unitless error. Hence, no option is correct.

22 The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is **[2019, 9 April Shift-II]**

(a) 37.030 cm^2 (b) 37.0 cm^2
(c) 37.03 cm^2 (d) 37 cm^2

Ans. *(c)*

Area of 1 square = 5.29 cm^2

Area of seven such squares

= 7 times addition of area of 1 square

= 5.29 + 5.29 + 5.29 ... 7 times

= 37.03 cm^2

As we know that, if in the measured values to be added/subtracted the least number of significant digits after the decimal is n.

Then, in the sum or difference also, the number of significant digits after the decimal should be n.

Here, number of digits after decimal in 5.29 is 2, so our answer also contains only two digits after decimal point.

$\therefore$ Area required = 37.03 cm^2

23 The pitch and the number of divisions, on the circular scale for a given screw gauge are 0.5 mm and 100, respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale for a thin sheet are 5.5 mm and 48 respectively, the thickness of this sheet is **[2019, 9 Jan Shift-II]**

(a) 5.950 mm (b) 5.725 mm
(c) 5.755 mm (d) 5.740 mm

Ans. *(c)*

For a measuring device, the least count is the smallest value that can be measured by measuring instrument.

Least count

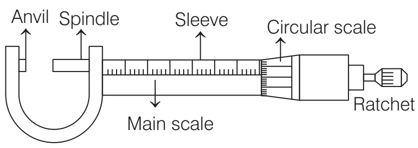
$$= \frac{\text{Minimum reading on main - scale}}{\text{Total divisions on the scale}}$$

Here, screw gauge is used for measurement therefore,

$$\text{LC} = \frac{\text{Pitch}}{\text{number of division}}$$

$$\text{LC} = \frac{0.5}{100} \text{ mm}$$

$$\text{LC} = 5 \times 10^{-3} \text{ mm} \quad \text{...(i)}$$



According to question, the zero line of its circular scale lies 3 division below the mean line and the readings of main scale = 5.5 mm

The reading of circular scale = 48

then the actual value is given by

actual value of thickness (t) = (main scale reading) + (circular scale reading + number of division below mean line) × LC

$\Rightarrow \quad t = 5.5 \text{ mm} + (48 + 3) \times 5 \times 10^{-3}$ mm

$\Rightarrow \quad t = 5.755$ mm

24 The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ? **[2019, 10 Jan Shift-II]**

(a) $4300 \pm 80\ cm^3$
(b) $4260 \pm 80\ cm^3$
(c) $4264.4 \pm 81.0\ cm^3$
(d) $4264 \pm 81\ cm^3$

Ans. *(b)*

Volume of a cylinder of radius 'r' and height h is given by

$$V = \pi r^2 h$$

or $V = \frac{1}{4}\pi D^2 h$, where D is the diameter of circular surface. Here, $D = 12.6$ cm and $h = 34.2$ cm

$$\Rightarrow V = \frac{\pi}{4} \times (12.6)^2 \times (34.2)$$

$$V = 4262.22\ cm^3$$

$V = 4260$ (in three significant numbers)

Now, error calculation can be done as

$$\frac{\Delta V}{V} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta h}{h}$$

$$= \frac{2 \times 0.1}{12.6} + \frac{0.1}{34.2}$$

$$\Rightarrow \frac{\Delta V}{V} = 0.0158 + 0.0029$$

$$\Rightarrow \Delta V = (0.01879) \times (4262.22)$$

$$\Rightarrow \Delta V = 79.7 \simeq 80\ cm^3$$

$\therefore$ For proper significant numbers, volume reading will be

$$V = 4260 \pm 80\ cm^3$$

25 The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is **[2019, 12 Jan Shift-I]**

(a) 50
(b) 200
(c) 500
(d) 100

Ans. *(b)*

In a screw gauge,

Least count

$$= \frac{\text{Measure of 1 main scale division (MSD)}}{\text{Number of division on circular scale}}$$

Here, minimum value to be measured/least count is 5 μm.

$$= 5 \times 10^{-6}\ \text{m}$$

$\therefore$ According to the given values,

$$5 \times 10^{-6} = \frac{1 \times 10^{-3}}{N}$$

or $$N = \frac{10^{-3}}{5 \times 10^{-6}} = \frac{1000}{5}$$

$$= 200 \text{ divisions}$$

26 The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1% , the maximum error in determining the density is **[JEE Main 2018]**

(a) 2.5%
(b) 3.5%
(c) 4.5%
(d) 6%

Ans. *(c)*

$\because$ Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3}$ or $\rho = \frac{M}{L^3}$

$\Rightarrow$ Error in density $\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$

So, maximum % error in measurement of ρ is

$$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{3\Delta L}{L} \times 100$$

or % error in density $= 1.5 + 3 \times 1$

% error $= 4.5\%$

27 The following observations were taken for determining surface tension T of water by capillary metho(d) Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80\ \text{m/s}^2$ and the simplified relation $T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to **[JEE Main 2017]**

(a) 1.5%
(b) 2.4%
(c) 10%
(d) 0.15%

Ans. *(a)*

By ascent formula, we have surface tension,

$$T = \frac{rhg}{2} \times 10^3 \frac{\text{N}}{\text{m}} = \frac{dhg}{4} \times 10^3 \frac{\text{N}}{\text{m}} \quad \left(\because r = \frac{d}{2}\right)$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} \text{ [given, } g \text{ is constant]}$$

So, percentage

$$= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}}\right) \times 100$$

$$= 1.5\%$$

$$\therefore \frac{\Delta T}{T} \times 100 = 1.5\%$$

28 A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91s, 92s and 95s. If the minimum division in the measuring clock is 1s, then the reported mean time should be **[JEE Main 2016]**

(a) (92 ± 2) s
(b) (92 ± 5) s
(c) (92 ± 1.8) s
(d) (92 ± 3) s

Ans. *(b)*

Arithmetic mean time of a oscillating simple pendulum $= \frac{\Sigma x_i}{N}$

$$= \frac{90 + 91 + 92 + 95}{4}$$

$$= 92\ \text{s}$$

Mean deviation of a simple pendulum

$$= \frac{\Sigma |\bar{x} - x_i|}{N} = \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Given, minimum division in the measuring clock, i.e. simple pendulum = 1 s. Thus, the reported mean time of a oscillating simple pendulum = (92 ± 2) s

29 A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[JEE Main 2014]**

(a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Ans. *(b)*

If student measure 3.50 cm, it means that there in an uncertainly of order 0.01 cm.

For vernier scale with 1 MSD

= 1mm and 9 MSD
= 10 VSD

$\therefore$ LC of VC = 1 MSD − 1 VSD

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right) = \frac{1}{100}\ \text{cm}$$

30 The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01V while measuring the current of 5 mA at 300K, what will be the error in the value of current in mA? **[JEE Main 2013]**

(a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

Ans. *(a)*

Given, $I = (e^{1000V/T} - 1)$ mA

$dV = \pm 0.01$ V, $T = 300$ K

$I = 5$ mA $\Rightarrow I = e^{1000V/T} - 1$

$I + 1 = e^{1000V/T}$

Taking log on both sides, we get

$$\log(I+1) = \frac{1000V}{T}$$

$$\Rightarrow \frac{d(I+1)}{I+1} = \frac{1000}{T}dV \Rightarrow \frac{dI}{I+1} = \frac{1000}{T}dV$$

$$\Rightarrow dI = \frac{1000}{T} \times (I+1)\,dV$$

$$dI = \frac{1000}{300} \times (5+1) \times 0.01 = 0.2 \text{ mA}$$

So, error in the value of current is 0.2 mA.

31 A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is **[AIEEE 2012]**

(a) 58.59 degree (b) 58.77 degree
(c) 58.65 degree (d) 59 degree

Ans. *(c)*

1 Vernier scale division = 29/30 main scale division

$$1\text{ VSD} = \frac{29}{30} \times 0.5° = \left(\frac{29}{60}\right)°$$

Thus, least count = 1 MSD − 1 VSD

$$= \left(\frac{1}{2}\right)° - \left(\frac{29}{60}\right)° = \left(\frac{1}{60}\right)°$$

∴ Reading = Main scale reading + Vernier scale reading

$= \text{MSR} + n \times \text{LC}$

$= 58.5° + 9 \times (1/60)°$

$= 58.65°$

32 Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is **[AIEEE 2012]**

(a) 6% (b) zero (c) 1% (d) 3%

Ans. *(a)*

From Ohm's law, $R = V/I$

By error method, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

$= 3\% + 3\% = 6\%$

33 The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are **[AIEEE 2010]**

(a) 5, 1, 2 (b) 5, 1, 5
(c) 5, 5, 2 (d) 4, 4, 2

Ans. *(a)*

The reliable digit plus the first uncertain digit is known as significant figures.

For the number 23.023, all the non-zero digits are significant, hence 5.

For the number 0.0003, number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant, hence 1.

For the number 2.1×10^{-3}, significant figures are 2.

34 In an experiment, the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half a degree (= 0.5°), then the least count of the instrument is **[AIEEE 2009]**

(a) one minute
(b) half minute
(c) one degree
(d) half degree

Ans. *(a)*

Least count

$$= \frac{\text{Value of main scale division}}{\text{Number of divisions on vernier scale}}$$

$$= \frac{1}{30}\text{ MSD} = \frac{1}{30} \times \frac{1°}{2}$$

$$= \frac{1°}{60} = 1 \text{ min}$$

35 A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as **[AIEEE 2008]**

(a) 17.6 kg ms^{-1} (b) 17.565 kg ms^{-1}
(c) 17.56 kg ms^{-1} (d) 17.57 kg ms^{-1}

Ans. *(a)*

So, momentum, $p = mv = 17.565$ kg ms^{-1}
where $m = 3.513$ kg and $v = 5.00$ ms^{-1}

As the number of significant digits in m is 4 and in v is 3, so, p must have 3 (minimum) significant digits.

Hence, $p = 17.6$ kg ms^{-1}

TOPIC 3
Dimensions

36 Which of the following equations is dimensionally incorrect ?
Where, t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, V = volume , p = pressure, W = work done, τ = torque, ε = permittivity, E = electric field, J = current density, L = length. **[2021, 31 Aug Shift-I]**

(a) $V = \frac{\pi p a^4}{8\eta L}$ (b) $h = \frac{2s\cos\theta}{\rho r g}$
(c) $J = \varepsilon\frac{\partial E}{\partial t}$ (d) $W = \tau\theta$

Ans. *(a)*

As we know that,

Dimensional formula of volume

$$= [M^0 L^3 T^0] \quad \text{...(i)}$$

Since, $F = 6\pi\eta r v$

∴ $\eta \propto \frac{F}{rv}$

where, η is viscosity and F is force.

$$\therefore [\eta] = \frac{[M L T^{-2}]}{[L \cdot L T^{-1}]} = [M L^{-1} T^{-1}]$$

$$\text{So, } \left[\frac{\pi p a^4}{8\eta L}\right] = \frac{[M L^{-1} T^{-2}][L^4]}{[M L^{-1} T^{-1}][L^1]}$$

$$= [M^0 L^3 T^{-1}] \quad \text{...(ii)}$$

Since, Eq. (i) is not equal to Eq (ii), so option (a) is wrong .

Now, since formula of capillary rise in tube, $h = \frac{2s\cos\theta}{\rho g r}$

Dimensional formula of LHS part,

∴ $[h] = [L]$

Dimensional formula of RHS part

$$= \frac{[s]}{[\rho][g][r]} = \frac{[MT^{-2}]}{[ML^{-3}][LT^{-2}][L]} = [L]$$

Hence, $h = \frac{2s\cos\theta}{\rho rg}$ is dimensionally correct.

So, option (b) will also be dimensionally correct.

In option (c),

$$J = \varepsilon \frac{dE}{\partial t} \quad \text{...(iii)}$$

$$\Rightarrow \quad J = \varepsilon \frac{E}{t}$$

Dimension of current density J is calculated as

Since, $\quad J = \frac{I}{A}$

$$\therefore \quad [J] = \frac{[I]}{[A]} = \frac{[A]}{[L^2]}$$

$$\Rightarrow \quad [J] = [AL^{-2}] \quad \text{...(iv)}$$

Again, we know that

$$E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{r^2}$$

$$\Rightarrow \quad \varepsilon E = \frac{1}{4\pi} \cdot \frac{q}{r^2} \Rightarrow \frac{\varepsilon E}{t} = \frac{1}{4\pi} \cdot \frac{q}{tr^2}$$

$$\Rightarrow \quad \left[\frac{\varepsilon E}{t}\right] = \frac{[q]}{[t][r^2]} = \frac{[AT]}{[T][L^2]}$$

$$\Rightarrow \quad \left[\frac{\varepsilon E}{t}\right] = [AL^{-2}] \quad \text{...(v)}$$

Form Eqs. (iv) and (v), we see that Eq. (iii) is dimensionally correct.

In option (d) $W = \tau\theta$

and $\quad \tau = r \times F$

So, dimensional formula of

$$[\tau][\theta] = [r][F] = [L][MLT^{-2}]$$
$$= [ML^2T^{-2}] = [W]$$

37 If velocity $[v]$, time $[T]$ and force $[F]$ are choosen as the base quantities, the dimensions of the mass will be **[2021, 31 Aug Shift-II]**

(a) $[FT^{-1}v^{-1}]$ (b) $[FTv^{-1}]$
(c) $[FT^2v]$ (d) $[FvT^{-1}]$

Ans. ***(b)***

When the velocity (v), time (T) and force (F) are chosen as base quantities. Then, mass is given by

$$m \propto v^x T^y F^z \quad \text{...(i)}$$

Using dimensional formula of all quantities,

$$[ML^0T^0] = [LT^{-1}]^x [T]^y [MLT^{-2}]^z$$
$$[M^1L^0T^0] = [M^zL^{x+z}T^{-x+y-2z}]$$

Comparing the powers of dimensions on both sides, we get

$$z = 1$$
$$x + z = 0 \text{ and } -x + y - 2z = 0$$
$$\Rightarrow \quad x + 1 = 0 \Rightarrow x = -1$$
and $-(-1) + y - 2(1) = 0$
$$\Rightarrow \quad 1 + y - 2 = 0 \Rightarrow y = 1$$

Substituting these values in Eq. (i), we get

$$m \propto v^{-1}T^1F^1$$
$$\Rightarrow \quad m = [FTv^{-1}]$$

38. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density? **[2021, 27 Aug Shift-II]**

(a) $[FL^{-4}T^2]$ (b) $[FL^{-3}T^2]$
(c) $[FL^{-5}T^2]$ (d) $[FL^{-3}T^3]$

Ans. ***(a)***

As we know that, the dimensional formula of

density $[D] = [ML^{-3}T^0]$

Since, dimensional formula of force
$$[F] = [MLT^{-2}]$$

Dimensional formula of length
$$[L] = [M^0L^1T^0]$$

Dimensional formula of time
$$[T] = [M^0L^0T^1]$$

$$\therefore \quad [D] = [F]^a [L]^b [T]^c$$
$$\Rightarrow \quad [ML^{-3}T^0] = [MLT^{-2}]^a [M^0L^1T^0]^b [M^0L^0T^1]^c$$
$$= [M^aL^{a+b}T^{-2a+c}]$$

Comparing powers of dimensions on both sides, we get

$$a = 1$$
$$a + b = -3$$
$$\Rightarrow \quad b = -3 - 1 = -4$$
and $\quad -2a + c = 0$
$$c = 2a$$
$$\Rightarrow \quad c = 2 \times 1 = 2$$

$\therefore$ Dimensional formula of density will be $[F^1L^{-4}T^2]$.

39 Which of the following is not a dimensionless quantity ? **[2021, 27 Aug Shift-I]**

(a) Relative magnetic permeability (μ_r)
(b) Power factor
(c) Permeability of free space (μ_0)
(d) Quality factor

Ans. ***(c)***

Relative magnetic permeability (μ_r) It is the ratio of permeability of medium to the permeability of free space i.e.,

$$\mu_r = \frac{\mu_m}{\mu_0}$$

As, it is a ratio of permeabilities. So, it is unitless and dimensionless quantity.

Power factor $(\cos\phi)$ It is cosine of phase difference between alternating current and alternating voltage. Hence, it has only a numerical value, so it is also a unitless and dimensionless quantity.

Permeability of free space (μ_0) It is the ratio of magnetising field induction (B) to magnetising field intensity (H)

i.e. $\quad \mu_0 = \frac{B}{H}$

$$\Rightarrow \quad [\mu_0] = \frac{[B]}{[H]} \quad \text{...(i)}$$

Since, $\quad H = nI$

where, n = number of turns per unit length

and $\quad I$ = current

$$[H] = [n][I] = [L^{-1}][A] = [AL^{-1}]$$

We know that, force on a current carrying conductor in magnetic field B is given as

$$F = IBl$$

where, $\quad l$ = length of the conductor,
B = magnetic field
and $\quad I$ = current.

$$\therefore \quad B = \frac{F}{Il}$$

$$\Rightarrow [B] = \frac{[F]}{[I][l]} = \frac{[MLT^{-2}]}{[A][L]} = [MT^{-2}A^{-1}]$$

$\therefore$ From Eq. (i), we have

$$[\mu_0] = \frac{[MT^{-2}A^{-1}]}{[AL^{-1}]} = [MLT^{-2}A^{-2}]$$

Hence, permeability of free space is not a dimensionless quantity.

Quality factor It is the ratio of energy stored to the energy dissipated per cycle.

Quality factor

$$= \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$$

As, it is ratio of energies so, it will be unitless and dimensionless quantity.

40 Match List-I with List-II.

	List-I		List-II
A.	Magnetic induction	1.	$[ML^2T^{-2}A^{-1}]$
B.	Magnetic flux	2.	$[ML^{-1}A]$
C.	Magnetic permeability	3.	$[MT^{-2}A^{-1}]$
D.	Magnetisation	4.	$[MLT^{-2}A^{-2}]$

Choose the most appropriate answer from the options given below. **[2021, 26 Aug Shift-II]**

	A	B	C	D
(a)	2	4	1	3
(b)	2	1	4	3
(c)	3	2	4	1
(d)	3	1	4	2

Ans. ***(*)***

Magnetic induction

Magnetic force of induction can be given as

$$F = qvB \Rightarrow B = \frac{F}{qv}$$

$$[B] = \frac{[F]}{[q][v]} = \frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [M^1T^{-2}A^{-1}]$$

Magnetic Flux Magnetic flux is the product of magnetic induction and area.

$$\phi = B \cdot A$$

$$[\phi] = [B][A] = [M^1T^{-2}A^{-1}][L^2]$$

$$= [ML^2T^{-2}A^{-1}]$$

Magnetic permeability

We know that, magnetic field inside the solenoid.

$$B = \mu_0 nI$$

$$\Rightarrow \quad \mu_0 = \frac{B}{nI}$$

(where, n is number of turns per unit length)

$\Rightarrow$

$$[\mu_0] = \frac{[B[}{[n][I]} = \frac{[MT^{-2}A^{-1}]}{[L^{-1}][A]} = [MLT^{-2}A^{-2}]$$

Magnetisation The magnetic dipole moment acquired per unit volume is known as magnetisation.

$$I = \frac{M}{V}$$

$$[I] = \frac{[L^2A]}{[L^3]} = [L^{-1}A]$$

Hence, no option is correct.

41 If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimension of P in the formula $P = EL^2M^{-5}G^{-2}$ is **[2021, 26 Aug Shift-I]**

(a) $[M^0L^1T^0]$ (b) $[M^{-1}L^{-1}T^2]$

(c) $[M^1L^1T^{-2}]$ (d) $[M^0L^0T^0]$

Ans. ***(d)***

Dimension of energy, $E = [ML^2T^{-2}]$

Angular momentum, $L = [ML^2T^{-1}]$

Mass, $M = [M]$

and gravitational constant, $G = [M^{-1}L^3T^{-2}]$

The dimension of P in formula, $P = EL^2M^{-5}G^{-2}$ is given as follows

$$[P] = \frac{[E][L^2]}{[M^5][G^2]}$$

$$= \frac{[ML^2T^{-2}] \times [M^2L^4T^{-2}]}{[M^5][M^{-2}L^6T^{-4}]}$$

$$= [M^0L^0T^0]$$

42 The force is given in terms of time t and displacement x by the equation

$$F = A\cos Bx + C\sin Dt$$

The dimensional formula of $\frac{AD}{B}$ is **[2021, 25 July Shift-II]**

(a) $[M^0LT^{-1}]$ (b) $[ML^2T^{-3}]$

(c) $[M^1L^1T^{-2}]$ (d) $[M^2L^2T^{-3}]$

Ans. ***(b)***

Given,

Equation of force, $F = A\cos Bx + C\sin Dt$

where, x is displacement and t is time.

Now, Bx should be dimensionless.

$$\therefore \quad [Bx] = [M^0L^0T^0]$$

$$\Rightarrow \quad [B][L^1] = [M^0L^0T^0]$$

$$\Rightarrow \quad [B] = [M^0L^{-1}T^0] \quad \text{...(i)}$$

Similarly, $[Dt] = [M^0L^0T^0]$

$$\Rightarrow [D] = \frac{[M^0L^0T^0]}{[t]} = [M^0L^0T^{-1}] \quad \text{...(ii)}$$

and $[A] = [F] = [MLT^{-2}]$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\frac{[AD]}{[B]} = \frac{[MLT^{-2}][M^0L^0T^{-1}]}{[M^0L^{-1}T^0]}$$

$$= \frac{[MLT^{-3}]}{[L^{-1}]} = [ML^2T^{-3}]$$

43 If time (t), velocity (v) and angular momentum (l) are taken as the fundamental units, then the dimension of mass (m) in terms of t, v and l is **[2021, 20 July Shift-II]**

(a) $[t^{-1}v^1l^{-2}]$ (b) $[t^1v^2l^{-1}]$

(c) $[t^{-2}v^{-1}l^1]$ (d) $[t^{-1}v^{-2}l^1]$

Ans. ***(d)***

Let us suppose,

$$m \propto t^a$$

$$m \propto v^b$$

and $m \propto l^c$

$$\Rightarrow \quad m \propto t^a v^b l^c$$

or $m = kt^a v^b l^c$ ...(i)

where, k is any constant and $k = 1$

$\because$ We know that dimensions of

Mass, $m = [M^1L^0T^0]$

Time, $t = [M^0L^0T^1]$

Velocity, $v = [M^0L^1T^{-1}]$

Angular momentum, $l = [M^1L^2T^{-1}]$

Substituting all these dimensions in Eq. (i), we get

$$[M^1L^0T^0] = [M^0L^0T^1]^a [M^0L^1T^{-1}]^b [M^1L^2T^{-1}]^c$$

$$[M^1L^0T^0] = [M^cL^{b+2c}T^{a-b-c}]$$

On comparing powers, we get

$$c = 1,$$

$$b + 2c = 0$$

$$b + 2 = 0$$

$$\Rightarrow \quad b = -2$$

$$a - b - c = 0$$

$$\Rightarrow \quad a - (-2) - 1 = 0$$

$$\Rightarrow \quad a = -1$$

$\therefore$ We can write $m \propto t^{-1}v^{-2}l^1$

44 The entropy of any system is given by

$$S = \alpha^2\beta \ln\left[\frac{\mu kR}{J\beta^2} + 3\right]$$

where, α and β are the constants; μ, J, k and R are number of moles, mechanical equivalent of heat, Boltzmann constant and gas constant, respectively.

$\left[\text{Take, } S = \frac{dQ}{T}\right]$ **[2021, 20 July Shift-I]**

Choose the incorrect option.

(a) α and J have the same dimensions.

(b) S, β, k and μR have the same dimensions.

(c) S and α have different dimensions.

(d) α and k have the same dimensions.

Ans. ***(d)***

Since, entropy of the system is given by

$$S = \alpha^2\beta\ln\left[\frac{\mu kR}{J\beta^2} + 3\right] \quad \text{...(i)}$$

As, $S = \frac{Q}{\Delta T}$ [given]

$$\Rightarrow \quad [S] = \frac{[ML^2T^{-2}]}{[K]} \quad \text{...(ii)}$$

$\because$ Dimensions of $Q = [ML^2T^{-2}]$

Dimension of $T = [K]$

Boltzmann constant, $k = \frac{\text{energy}}{T}$

[$\because$ Dimensions of energy $= [ML^2T^{-2}]$]

$$\Rightarrow \quad [k] = \frac{[ML^2T^{-2}]}{[K]} \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we can write

$$[S] = [k] = \frac{[ML^2T^{-2}]}{[K]} \quad \text{...(iv)}$$

$\because$ Gas constant,

$$[R]=\frac{[\text{Energy}]}{[nT]}=\frac{[ML^2T^{-2}]}{[\text{mol K}]} \quad \text{...(v)}$$

and mechanical equivalent of heat

$$[J]=[M^0L^0T^0] \quad \text{...(vi)}$$

As, $[\mu kR]=[J\beta]^2$

Using Eqs. (iii), (v) and (vi), we get

$$\Rightarrow [\text{mol}]\times\frac{[ML^2T^{-2}]}{[K]}\times\frac{[ML^2T^{-2}]}{[\text{mol K}]}=[\beta^2]$$

$$\Rightarrow \quad [\beta]=[ML^2T^{-2}K^{-1}] \quad \text{...(vii)}$$

Using Eq. (i), we can write,

$$[\alpha^2]=\frac{[S]}{[\beta]}=\frac{[ML^2T^{-2}K^{-1}]}{[ML^2T^{-2}K^{-1}]}$$

$$\Rightarrow \quad \alpha=[M^0L^0T^0] \quad \text{...(viii)}$$

So, from Eqs. (iii) and (viii), we can say that α and k have different dimensions.

45 If C and V represent capacity and voltage respectively, then what are the dimensions of λ, where $\frac{C}{V}=\lambda$?

[2021, 26 Feb Shift-II]

(a) $[M^{-2}L^{-3}I^2T^6]$ (b) $[M^{-3}L^{-4}I^3T^7]$
(c) $[M^{-1}L^{-3}I^{-2}T^{-7}]$ (d) $[M^{-2}L^{-4}I^3T^7]$

Ans. *(d)*

Given, C and V represent capacity and voltage, respectively.

Dimensional formula of $[C]=[M^{-1}L^{-2}T^4I^2]$

Dimensional formula of $[V]=[ML^2T^{-3}I^{-1}]$

Therefore, dimensional formula of $[C/V]$

$$=\frac{[M^{-1}L^{-2}T^4I^2]}{[ML^2T^{-3}I^{-1}]}$$

$$=[M^{-2}L^{-4}T^7I^3]$$

46 In a typical combustion engine, the work done by a gas molecule is given $W=\alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be

[2021, 26 Feb Shift-I]

(a) $[MLT^{-2}]$ (b) $[M^0LT^0]$
(c) $[M^2LT^{-2}]$ (d) $[MLT^{-1}]$

Ans. *(b)*

Given, work done by gas molecule,

$$W=\alpha^2\beta e^{-\beta x^2/kT}$$

Here, x is displacement, k is Boltzmann constant, α and β are constants and T is temperature.

Dimensional formula of $[W]=[ML^2T^{-2}]$

$\therefore$ Dimensions of $[\alpha^2\beta]=[ML^2T^{-2}]$

$$\Rightarrow \quad \alpha=\left[\frac{ML^2T^{-2}}{\beta}\right]^{1/2} \quad \text{...(i)}$$

The term $[e^{-\beta x^2/kT}]$ should be dimensionless, i.e. $[M^0L^0T^0]$.

$$\Rightarrow \quad \left[\frac{\beta x^2}{kT}\right]=[M^0L^0T^0]$$

$$\Rightarrow \quad [\beta]=\frac{[k][T]}{[x^2]} \quad \text{...(ii)}$$

Energy of gaseous molecule $(E)=\frac{7}{2}kT$

$$\Rightarrow \quad [k]=[E]/[T]=[ML^2T^{-2}K^{-1}]$$

Substituting the value of k in Eq. (ii), we get

$$[\beta]=\frac{[ML^2T^{-2}K^{-1}][K]}{[L^2]}=[MT^{-2}]$$

Substituting the value of β in Eq. (i), we get

$$[\alpha]=\left\{\frac{[ML^2T^{-2}]}{[MT^{-2}]}\right\}^{1/2}=[M^0L^1T^0]$$

47 If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of

[2021, 25 Feb Shift-I]

(a) $[MLT^0]$ (b) $[MLT^{-1}]$
(c) $[M^0L^0T^0]$ (d) $[LC^{-1}]$

Ans. *(c)*

Dimensional formula of $[e]=[IT]$

$$[h]=[M^1L^2T^{-1}]$$

$$[c]=[M^0L^1T^{-1}]$$

$$\left[\frac{1}{4\pi\varepsilon_0}\right]=[M^1L^3T^{-4}I^{-2}]$$

where, $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant.

Therefore, dimensional formula of $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$

$$=[M^1L^3T^{-4}I^{-2}]\cdot\frac{[IT]^2}{[M^1L^2T^{-1}][M^0LT^{-1}]}$$

$$=[M^0L^0T^0I^0]$$

or $[M^0L^0T^0]$

48 Match List-I with List-II

	List-I		List-II
A.	h (Planck's constant)	1.	$[MLT^{-1}]$
B.	E (kinetic energy)	2.	$[ML^2T^{-1}]$
C.	V (electric potential)	3.	$[ML^2T^{-2}]$
D.	P (linear momentum)	4.	$[ML^2I^{-1}T^{-3}]$

Choose the correct answer from the options given below.

[2021, 25 Feb Shift-I]

	A	B	C	D
(a)	3	4	2	1
(b)	2	3	4	1
(c)	1	2	4	3
(d)	3	2	4	1

Ans. *(b)*

The dimensional formulae of given terms are

Planck's constant $(h)=[ML^2T^{-1}]$

Kinetic energy $(E)=[ML^2T^{-2}]$

Electric potential $(V)=[ML^2I^{-1}T^{-3}]$

Linear momentum $(p)=[MLT^{-1}]$

So, the correct match is

$A\rightarrow 2$, $B\rightarrow 3$, $C\rightarrow 4$, $D\rightarrow 1$.

49 The work done by a gas molecule in an isolated system is given by, $W=\alpha\beta^2e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants.

Then, the dimensions of β will be

[2021, 24 Feb Shift-I]

(a) $[M^2LT^2]$
(b) $[M^0LT^0]$
(c) $[MLT^{-2}]$
(d) $[ML^2T^{-2}]$

Ans. *(c)*

Given, work done, $W=\alpha\cdot\beta^2e^{-\frac{x^2}{\alpha kT}}$

where, k is Boltzmann constant, T is temperature and x is displacement.

We know that, $\frac{x^2}{\alpha kT}$ is a dimensionless quantity.

$$\therefore \quad \left[\frac{x^2}{\alpha kT}\right]=[M^0L^0T^0]\Rightarrow[\alpha]=\frac{[x^2]}{[k][T]}$$

$$\Rightarrow \quad [\alpha]=\frac{[L^2]}{[k][T]} \quad \text{...(i)}$$

Since, dimensions of k are

$$[k]=[M^1L^2T^{-2}K^{-1}] \quad \text{...(ii)}$$

Dimensions of temperature are

$$[T]=[K] \quad \text{...(iii)}$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$[\alpha]=\frac{[L^2]}{[M^1L^2T^{-2}K^{-1}][K]}$$

$$[\alpha]=[M^{-1}T^2]$$

According to dimensional analysis,

$$[W]=[\alpha\beta^2] \Rightarrow [\beta^2]=\frac{[W]}{[\alpha]}$$

$$\Rightarrow \quad [\beta^2]=\frac{[M^1L^2T^{-2}]}{[M^{-1}T^2]}=[M^2L^2T^{-4}]$$

$$\Rightarrow \quad [\beta]=[MLT^{-2}]$$

50 If speed V, area A and force F are chosen as fundamental units, then the dimensional formula of Young's modulus will be **[2020, 2 Sep Shift-I]**

(a) $[FA^2V^{-3}]$ (b) $[FA^{-1}V^0]$
(c) $[FA^2V^{-2}]$ (d) $[FA^2V^{-1}]$

Ans. ***(b)***

Let Young's modulus is related to speed, area and force, as

$$Y=F^xA^yV^z$$

Substituting dimensions, we have

$$[ML^{-1}T^{-2}]=[MLT^{-2}]^x[L^2]^y[LT^{-1}]^z$$

Comparing power of similar quantities, we have

$x=1, x+2y+z=-1$ and $-2x-z=-2$

Solving these, we get

$$x=1, y=-1, z=0$$

So, $\quad Y=FA^{-1}V^0$

Hence, correct option is (b).

51 If momentum P, area A and time T are taken to be the fundamental quantities, then the dimensional formula for energy is **[2020, 2 Sep Shift-II]**

(a) $[P^2AT^{-2}]$ (b) $[PA^{-1}T^{-2}]$
(c) $[PA^{1/2}T^{-1}]$ (d) $[P^{1/2}AT^{-1}]$

Ans. ***(c)***

Let dimensions of energy E in terms of momentum P, area A and time T are

$$[E]=[P]^x[A]^y[T]^z$$

Substituting dimensions of fundamental quantities for E, P, A and T, we have

$$[ML^2T^{-2}]=[MLT^{-1}]^x[L^2]^y[T]^z$$

$$[ML^2T^{-2}]=[M^xL^{x+2y}T^{-x+z}]$$

Equating powers of same physical quantities on both sides, we have

$x=1, x+2y=2$ and $-x+z=-2$

so, $\quad x=1, y=1/2, z=-1$

$\therefore$ Dimensional formula of $[E]=[PA^{1/2}T^{-1}]$

Hence, correct option is (c).

52 Amount of solar energy received on the earth's surface per unit area per unit time is defined as solar constant. Dimensional formula of solar constant is **[2020, 3 Sep Shift-II]**

(a) $[MLT^{-2}]$ (b) $[ML^0T^{-3}]$
(c) $[M^2L^0T^{-1}]$ (d) $[ML^2T^{-2}]$

Ans. ***(b)***

$$\text{Solar constant}=\frac{\text{Solar energy}}{\text{Area}\times\text{Time}}=\frac{[ML^2T^{-2}]}{[L^2][T]}$$

$$=[ML^0T^{-3}]$$

Hence, correct option is (b).

53 Dimensional formula for thermal conductivity is (Here, K denotes the temperature) **[2020, 4 Sep Shift-I]**

(a) $[MLT^{-2}K]$ (b) $[MLT^{-2}K^{-2}]$
(c) $[MLT^{-3}K^{-1}]$ (d) $[MLT^{-3}K]$

Ans. ***(c)***

For conduction of heat,

$$\frac{dQ}{dt}=KA\frac{dT}{dx}$$

$$K=\frac{\left(\frac{dQ}{dt}\right)}{A\left(\frac{dT}{dx}\right)}=\frac{dQ\times dx}{A\times dt\times dT}$$

$$=\frac{\text{joule}\times\text{metre}}{(\text{metre})^2\times\text{second}\times\text{kelvin}}$$

$$=\frac{\frac{\text{kilogram}\times(\text{metre})^2}{(\text{second})^2}\times\text{metre}}{(\text{metre})^2\times\text{second}\times\text{kelvin}}$$

$$=\frac{\text{kg}\cdot\text{m}^2\cdot\text{m}}{\text{s}^2\cdot\text{m}^2\cdot\text{s}\cdot\text{K}}=\frac{\text{kg}\cdot\text{m}}{\text{s}^3\cdot\text{K}}$$

$$[K]=\frac{[M^1][L^1]}{[T^3][K^1]}$$

$$\Rightarrow \quad [K]=[M^1L^1T^{-3}K^{-1}]$$

Hence, correct option is (c).

55 A quantity x is given by $x=\frac{IFv^2}{WL^4}$ where, I is moment of inertia, F is force, v is work and L is length. The dimensional formula for x is same as that of **[2020, 4 Sep Shift-II]**

(a) Planck's constant
(b) force constant
(c) coefficient of viscosity
(d) energy density

Ans. ***(d)***

Given that, $x=\frac{IFv^2}{WL^4}$

Dimensionally,

$$[x]=\frac{[I][F][v]^2}{[W][L]^4}$$

$$=\frac{[M^1L^2][M^1L^1T^{-2}][L^1T^{-1}]^2}{[M^1L^2T^{-2}][L^1]^4}$$

$$=[M^1L^2]\frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[M^1L^2T^{-2}][L^4]}$$

$$=[M^1L^{-1}T^{-2}] \quad \text{...(i)}$$

On checking the alternatives:

(a) Planck's constant $\Rightarrow [h]=[M^1L^2T^{-1}]$ doesn't match with dimensional formula of x.

(b) Force constant $\Rightarrow K]=[M^1T^{-2}]$ doesn't match with dimensional formula of x.

(c) Coefficient of viscosity $\Rightarrow$ $[\eta]=[M^1L^{-1}T^{-1}]$ doesn't match with dimensional formula of x.

(d) Energy density $\Rightarrow [E_d]=[M^1L^{-1}T^{-2}]$ matches with dimensional formula of x.

Hence, option (d) is correct.

55 The quantities $x=\frac{1}{\sqrt{\mu_0\varepsilon_0}}, y=\frac{E}{B}$ and $z=\frac{l}{CR}$ are defined, where C is capacitance, R is resistance, l is length, E is electric field, B is magnetic field, ε_0 is free space permittivity and μ_0 is permeability, respectively. Then, **[2020, 5 Sep Shift-II]**

(a) x, y and z have the same dimension
(b) Only x and z have the same dimension
(c) Only x and y have the same dimension
(d) Only y and z have the same dimension

Ans. ***(a)***

$$x=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=\text{speed of light in vacuum}$$

$\therefore$ Dimension of x, $[x]=[M^0L^1T^{-1}]$

$$y=\frac{E}{B}=\text{speed of EM wave}$$

$\therefore$ Dimension of y, $[y]=[M^0L^1T^{-1}]$

$$z=\frac{l}{RC}=\frac{l}{\tau}=\frac{\text{length}}{\text{time}}$$

$\therefore$ Dimension of z, $[z]=[M^0L^1T^{-1}]$

Thus, all quantities have same dimensions i.e., of velocity.

Hence, correct option is (a).

56 The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is **[2020, 7 Jan Shift-II]**

(a) $[ML^{-1}T^{-2}]$
(b) $[MLT^{-2}]$
(c) $[ML^2T^{-1}]$
(d) $[ML^2T^{-2}]$

Ans. *(a)*

As, $\frac{B^2}{2\mu_0}$ = energy density of magnetic field

$= \frac{\text{Energy}}{\text{Volume}}$

So, $\left[\frac{B^2}{2\mu_0}\right] = [\text{Energy /Volume}]$

$= \frac{[ML^2T^{-2}]}{[L^3]}$

$= [ML^{-1}T^{-2}]$

57 The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant h, speed of light c and gravitational constant G and ampere A is **[2020, 8 Jan Shift-I]**

(a) $h^{-2/3}c^{-1/3}G^{4/3}A^{-1}$
(b) $h^{1/3}G^{2/3}c^{1/3}A^{-1}$
(c) $h^2G^{3/2}c^{1/3}A^{-1}$
(d) $h^{2/3}c^{5/3}G^{1/3}A^{-1}$

Ans. *(*)*

Let $V_0 = (h)^a \cdot (c)^b \cdot (G)^c \cdot (A)^d$ ...(i)

Then, $[V_0] = [\text{potential}]$

$= \left[\frac{\text{potential energy}}{\text{charge}}\right]$

$= \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$

$[h] = \left[\frac{\text{Energy}}{\text{Frequency}}\right] = \frac{[ML^2T^{-2}]}{[T^{-1}]}$

$= [ML^2T^{-1}]$

$[c] = [\text{Speed}] = [LT^{-1}]$

$[G] = \left[\frac{\text{Force} \times (\text{Distance})^2}{(\text{Mass})^2}\right]$

$= \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

Substituting the dimensions of V_0, h, C, G and A in Eq. (i) and equating dimension on both sides, we get

$[ML^2T^{-3}A^{-1}] = [ML^2T^{-1}]^a \times [LT^{-1}]^b \times [M^{-1}L^3T^{-2}]^c \times [A]^d$

$\Rightarrow \quad a - c = 1$ ...(ii)

$2a + b + 3c = 2$ ...(iii)

$-a - b - 2c = -3$ ...(iv)

$d = -1$ ...(v)

On solving above equations, we get

$a = 0, b = 5, c = -1, d = -1$

Substituting these values in Eq. (i), we get

$V_0 = h^0 \cdot c^5 \cdot G^{-1} \cdot A^{-1}$

None of the given options matches with the result.

58 A quantity f is given by $f = \sqrt{hc^5/G}$, where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of **[2020, 9 Jan Shift-I]**

(a) area
(b) volume
(c) momentum
(d) energy

Ans. *(d)*

Dimensions of quantity f are

$$[f] = \frac{[h]^{1/2}[c]^{5/2}}{[G]^{1/2}} \quad \text{...(i)}$$

[**Note** To produce dimensions of different constants, just remember/recall nearest formulae containing there constants.]

As, $h = \frac{E}{\nu}$; $[h] = [ML^2T^{-2}][T] = [ML^2T^{-1}]$

$c = [LT^{-1}]$ and $G = \frac{F \cdot r^2}{m^2}$

$\Rightarrow \quad [G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

So, dimensions of f using Eq. (i),

$$[f] = \frac{[ML^2T^{-1}]^{\frac{1}{2}}[LT^{-1}]^{\frac{5}{2}}}{[M^{-1}L^3T^{-2}]^{\frac{1}{2}}}$$

$$= \left[M^{\frac{1}{2}+\frac{1}{2}}, L^{\frac{5}{2}-\frac{3}{2}+1}, T^{-\frac{1}{2}-\frac{5}{2}+\frac{2}{2}}\right]$$

$= [ML^2T^{-2}]$

Thus, it is the dimensions of energy.

59 In SI units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is **[2019, 8 April Shift-I]**

(a) $[A^{-1}TML^3]$ (b) $[AT^2M^{-1}L^{-1}]$
(c) $[AT^{-3}ML^{3/2}]$ (d) $[A^2T^3M^{-1}L^{-2}]$

Ans. *(d)*

Dimensions of ε_0 (permittivity of free space) are

$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$

As, c = speed of light.

$\therefore$ Dimension of $[c] = [LT^{-1}]$

So, dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ are

$$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \left[\sqrt{\frac{\varepsilon_0^2}{\varepsilon_0\mu_0}}\right] = [\varepsilon_0 c] \quad \left[\because c^2 = \frac{1}{\mu_0\varepsilon_0}\right]$$

$= [M^{-1}L^{-3}T^4A^2][LT^{-1}]$

$= [M^{-1}L^{-2}T^3A^2]$

60 If surface tension (S), moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be **[2019, 8 April Shift-II]**

(a) $S^{1/2}I^{1/2}h^{-1}$ (b) $S^{3/2}I^{1/2}h^0$
(c) $S^{1/2}I^{1/2}h^0$ (d) $S^{1/2}I^{3/2}h^{-1}$

Ans. *(c)*

Suppose, linear momentum (p) depends upon the Planck's constant (h) raised to the power (a), surface tension (S) raised to the power (b) and moment of inertia (I) raised to the power (c).

Then, $p \propto (h)^a (S)^b (I)^c$ or $p = kh^a S^b I^c$

where, k is a dimensionless proportionality constant.

Thus, $[p] = [h]^a [S]^b [I]^c$ ...(i)

Then, the respective dimensions of the given physical quantities, i.e.

$[p] = [\text{mass} \times \text{velocity}] = [MLT^{-1}]$

$[I] = [\text{mass} \times \text{distance}^2]$

$= [ML^2T^0]$

$[S] = [\text{force} \times \text{length}] = [ML^0T^{-2}]$

$[h] = [ML^2T^{-1}]$

Then, substituting these dimensions in Eq. (i), we get

$[MLT^{-1}] = [ML^2T^{-1}]^a [MT^{-2}]^b [ML^{+2}]^c$

For dimensional balance, the dimensions on both sides should be same.

Thus, equating dimensions, we have

$a + b + c = 1$

$2(a + c) = 1$ or $a + c = \frac{1}{2}$

$-a - 2b = -1$ or $a + 2b = 1$

Solving these three equations, we get

$a = 0, b = \frac{1}{2}, c = \frac{1}{2}$

$\therefore \quad p = h^0 S^{\frac{1}{2}} I^{\frac{1}{2}}$ or $p = S^{\frac{1}{2}} I^{\frac{1}{2}} h^0$

61 In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **[2019, 10 April Shift-II]**

(a) $[M^{-1}L^{-2}T^4A^2]$ (b) $[M^{-2}L^0T^{-4}A^{-2}]$
(c) $[M^{-3}L^{-2}T^8A^4]$ (d) $[M^{-2}L^{-2}T^6A^3]$

Ans. *(c)*

To find dimensions of capacitance in the given relation, we can use formula for energy.

Capacitors energy is $U = \frac{1}{2}CV^2$

So, dimensionally,

$$\Rightarrow [C] = \left[\frac{U}{V^2}\right]$$

As, $V = \text{potential} = \frac{\text{potential energy}}{\text{charge}}$

We have,

$$[C] = \frac{[U]}{\left[\frac{U^2}{q^2}\right]} = \frac{[q^2]}{[U]} = \left[\frac{A^2T^2}{ML^2T^{-2}}\right]$$

$$\therefore \quad X = [M^{-1}L^{-2}A^2T^4]$$

To get dimensions of magnetic field, we use force on a current carrying conductor in magnetic field,

$$F = BIl \Rightarrow [B] = \frac{[F]}{[I][l]} = \left[\frac{MLT^{-2}}{AL}\right]$$

$$\therefore \quad Z = [M^1L^0T^{-2}A^{-1}]$$

Now, using given relation,

$$X = 5YZ^2$$

$$[Y] = \frac{[X]}{[Z^2]} = \left[\frac{M^{-1}L^{-2}A^2T^4}{(M^1L^0T^{-2}A^{-1})^2}\right]$$

$$= \frac{M^{-1}L^{-2}A^2T^4}{M^2T^{-4}A^{-2}}$$

$$\therefore [Y] = [M^{-3}L^{-2}T^8A^4]$$

62 Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)? **[2019, 12 April Shift-I]**

(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$ (c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

Ans. *(a)*

Key Idea A formula is valid only, if the dimensions of LHS and RHS are same. So, we need to balance dimensions of given options with the dimension of electrical resistance.

Let dimensions of resistance R, permittivity ε_0 and permeability μ_0 are $[R]$, $[\varepsilon_0]$ and $[\mu_0]$, respectively.

So, $[R] = [\varepsilon_0]^\alpha [\mu_0]^\beta$ ...(i)

$$[R] = [M^1L^2T^{-3}A^{-2}],$$

$$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2],$$

$$[\mu_0] = [M^1L^1T^{-2}A^{-2}]$$

Now, from Eq. (i), we get

$$[M^1L^2T^{-3}A^{-2}] = [M^{-1}L^{-3}T^4A^2]^\alpha [M^1L^1T^{-2}A^{-2}]^\beta$$

$$[M^1L^2T^{-3}A^{-2}] = M^{-\alpha+\beta}L^{-3\alpha+\beta}T^{4\alpha-2\beta}A^{2\alpha-2\beta}$$

On comparing both sides, we get

$$-\alpha + \beta = 1 \quad \text{...(ii)}$$

$$-3\alpha + \beta = 2 \quad \text{...(iii)}$$

$$4\alpha - 2\beta = -3 \quad \text{...(iv)}$$

$$2\alpha - 2\beta = -2 \quad \text{...(v)}$$

Value of α and β can be found using any two Eqs. from (ii) to (v),

On subtracting Eq. (iii) from Eq. (ii), we get

$$(-\alpha + \beta) - (-3\alpha + \beta) = 1 - 2$$

$$\Rightarrow \quad 2\alpha = -1 \text{ or } \alpha = \frac{-1}{2}$$

Put the value of α in Eq. (ii), we get

$$\beta = +\frac{1}{2}$$

$$\therefore \quad [R] = [\varepsilon_0]^{-1/2}[\mu_0]^{1/2} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

63 In form of G (universal gravitational constant), h (Planck constant) and c (speed of light), the time period will be proportional to **[2019, 9 Jan Shift-II]**

(a) $\sqrt{\frac{Gh}{c^5}}$ (b) $\sqrt{\frac{hc^5}{G}}$

(c) $\sqrt{\frac{c^3}{Gh}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

Ans. *(a)*

Key Idea According to dimensional analysis, if a physical quantity (let y) depends on another physical quantities (let A, B, C), then the dimension formula is given by

$$y = A^aB^bC^c$$

(where, a, b, c are the power of physical quantity and k = constant)

For the given question, the time (t) depends on the G (gravitational constant), h (Planck's constant) and c (velocity of light), then according to dimensional analysis

$$t = k(G)^a(h)^b(c)^c \quad \text{...(i)}$$

For calculating the values of a, b and c, compare the dimensional formula for both side.

LHS $t = \text{time} = [M^0L^0T^1]$

RHS $G = \frac{F\cdot r^2}{m_1m_2}$

$$= \frac{\text{kg ms}^{-2} \times \text{m}^2}{(\text{kg})^2} = [M^{-1}L^3T^{-2}]^a$$

$$h = [M^1L^2T^{-1}]^b$$

$$c = \frac{d}{t} = [M^0L^1T^{-1}]^c$$

Compare both side for powers of M, L and T,

$$M \Rightarrow 0 = -a + b \quad \text{...(i)}$$

$$L \Rightarrow 0 = 3a + 2b + c \quad \text{...(ii)}$$

$$T \Rightarrow 1 = -2a - b - c \quad \text{...(iii)}$$

Solving Eqs. (i), (ii), (iii), we get

$$a = \frac{1}{2},\ b = \frac{1}{2} \text{ and } c = \frac{-5}{2}$$

So, put these values in Eq. (i), we get

$$t = kG^{1/2}h^{1/2}c^{-5/2}$$

$$t = k\sqrt{\frac{Gh}{c^5}}$$

64 The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is **[2019, 11 Jan Shift-I]**

(a) $[MLT^{-2}]$ (b) $[M^0L^2T^{-4}]$

(c) $[M^2LT^{-4}]$ (d) $[M^2L^2T^{-2}]$

Ans. *(c)*

Force of interaction between two atoms is given as

$$F = \alpha\beta\exp(-x^2/\alpha kT)$$

As we know, exponential terms are always dimensionless, so

$$\text{dimensions of}\left(\frac{-x^2}{\alpha kT}\right) = [M^0L^0T^0]$$

$\Rightarrow$ Dimensions of α = Dimension of (x^2/kT)

Now, substituting the dimensions of individual term in the given equation, we get $= \frac{[M^0L^2T^0]}{[M^1L^2T^{-2}]}$ {$\because$ Dimensions of kT} equivalent to the dimensions of energy

$$= [M^1L^2T^{-2}]\} = [M^{-1}L^0T^2] \quad \text{...(i)}$$

Now from given equation, we have dimensions of

F = dimensions of $\alpha \times$ dimensions of β

$\Rightarrow$ Dimensions of β = Dimensions of $\left(\frac{F}{\alpha}\right)$

$$= \frac{[M^1L^1T^{-2}]}{[M^{-1}L^0T^2]} \quad [\because \text{using Eq. (i)}]$$

$$= [M^2L^1T^{-4}]$$

65 If speed (v), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be **[2019, 11 Jan Shift-II]**

(a) $[v^{-4}A^{-2}F]$
(b) $[v^{-2}A^{2}F^{2}]$
(c) $[v^{-2}A^{2}F^{-2}]$
(d) $[v^{-4}A^{2}F]$

Ans. ***(d)***

Dimensions of speed are, $[v]=[LT^{-1}]$
Dimensions of acceleration are, $[A]=[LT^{-2}]$
Dimensions of force are, $[F]=[MLT^{-2}]$
Dimension of Young modulus is,
$$[Y]=[ML^{-1}T^{-2}]$$
Let dimensions of Young's modulus is expressed in terms of speed, acceleration and force as;
$$[Y]=[v]^{\alpha}[A]^{\beta}[F]^{\gamma} \quad ...(i)$$
Then substituting dimensions in terms of M, L and T we get,
$$[ML^{-1}T^{-2}]=[LT^{-1}]^{\alpha}[LT^{-2}]^{\beta}[MLT^{-2}]^{\gamma}$$
$$=[M^{\gamma}L^{\alpha+\beta+\gamma}T^{-\alpha-2\beta-2\gamma}]$$
Now comparing powers of basic quantities on both sides we get, $\gamma=1$
$$\alpha+\beta+\gamma=-1$$
and $-\alpha-2\beta-2\gamma=-2$
Solving these, we get
$$\alpha=-4,\ \beta=2,\ \gamma=1$$
Substituting α, β, and γ in Eq. (i) we get;
$$[Y]=[v^{-4}A^{2}F^{1}]$$

66 Let l, r, c, and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{l}{rcv}$ in SI units will be **[2019, 12 Jan Shift-II]**

(a) $[LT^{2}]$ (b) $[LTA]$
(c) $[A^{-1}]$ (d) $[LA^{-2}]$

Ans. ***(c)***

Dimensions of given quantities are
l = inductance $=[M^{1}L^{2}T^{-2}A^{-2}]$
r = resistance $=[M^{1}L^{2}T^{-3}A^{-2}]$
c = capacitance $=[M^{-1}L^{-2}T^{4}A^{2}]$
v = voltage $=[M^{1}L^{2}T^{-3}A^{-1}]$
So, dimensions of $\frac{l}{rcv}$ are
$$\left[\frac{l}{rcv}\right]=\frac{[ML^{2}T^{-2}A^{-2}]}{[M^{1}L^{2}T^{-2}A^{-1}]}=[A^{-1}]$$

67 Let $[\varepsilon_0]$ denotes the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then **[JEE Main 2013]**

(a) $[\varepsilon_0]=[M^{-1}L^{-3}T^{2}A]$
(b) $[\varepsilon_0]=[M^{-1}L^{-3}T^{4}A^{2}]$
(c) $[\varepsilon_0]=[M^{-2}L^{2}T^{-1}A^{-2}]$
(d) $[\varepsilon_0]=[M^{-1}L^{2}T^{-1}A^{2}]$

Ans. ***(b)***

Electrostatic force between two charges,
$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2}\Rightarrow\varepsilon_0=\frac{q_1q_2}{4\pi FR^2}$$
Substituting the units.
$$\text{Hence, }\varepsilon_0=\frac{C^2}{N\text{-}m^2}=\frac{[AT]^2}{[MLT^{-2}][L^2]}$$
$$=[M^{-1}L^{-3}T^{4}A^{2}]$$

68 The dimensions of magnetic field in M, L, T and C (coulomb) is given as **[AIEEE 2008]**

(a) $[MLT^{-1}C^{-1}]$ (b) $[MT^{2}C^{-2}]$
(c) $[MT^{-1}C^{-1}]$ (d) $[MT^{2}C^{-1}]$

Ans. ***(c)***

From the relation $F=qvB$
$$\Rightarrow [MLT^{-2}]=[C][LT^{-1}][B]$$
$$\Rightarrow [B]=[MC^{-1}T^{-1}]$$

69 Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Q denotes the electric charge? **[AIEEE 2006]**

(a) Wb/m^2 (b) henry (H)
(c) H/m^2 (d) weber (Wb)

Ans. ***(b)***

$$\text{Magnetic energy}=\frac{1}{2}LI^2=\frac{Lq^2}{2t^2}\left[\text{as } I=\frac{q}{t}\right]$$
where L = inductance, I = current
Energy has the dimensions $=[ML^{2}T^{-2}]$
Equate the dimensions, we have
$$[ML^{2}T^{-2}]=[\text{henry}]\times\frac{[Q^2]}{[T^2]}$$
$$\Rightarrow [\text{henry}]=\frac{[ML^2]}{[Q^2]}$$

70 Out of the following pairs, which one does not have identical dimensions? **[AIEEE 2005]**

(a) Angular momentum and Planck's constant
(b) Impulse and momentum
(c) Moment of inertia and moment of a force
(d) Work and torque

Ans. ***(c)***

Moment of inertia, $I=mr^2 \quad \therefore [I]=[ML^2]$
and τ = Moment of force $= r\times F$
$\therefore \quad [\tau]=[L][MLT^{-2}]=[ML^{2}T^{-2}]$

71 Which one of the following represents the correct dimensions of the coefficient of viscosity? **[AIEEE 2004]**

(a) $[ML^{-1}T^{-2}]$
(b) $[MLT^{-1}]$
(c) $[ML^{-1}T^{-1}]$
(d) $[ML^{-2}T^{-2}]$

Ans. ***(c)***

By Newton's formula, $\eta=\frac{F}{A(\Delta v_x/\Delta z)}$
$\therefore$ Dimensions of η
$$=\frac{\text{Dimensions of force}}{\text{Dimensions of area}\times\text{Dimensions of velocity gradient}}$$
$$=\frac{[MLT^{-2}]}{[L^2][T^{-1}]}=[ML^{-1}T^{-1}]$$

72 Dimensions of $1/\mu_0\varepsilon_0$, where symbols have their usual meaning, are **[AIEEE 2003]**

(a) $[L^{-1}T]$ (b) $[L^{2}T^{2}]$
(c) $[L^{2}T^{-2}]$ (d) $[LT^{-1}]$

Ans. ***(c)***

As we know that formula of velocity is
$$v=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$$
$$\Rightarrow \quad v^2=\frac{1}{\mu_0\varepsilon_0}=[LT^{-1}]^2$$
$$\therefore \quad \frac{1}{\mu_0\varepsilon_0}=[L^{2}T^{-2}]$$

73 The physical quantities not having same dimensions are **[AIEEE 2003]**

(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

Ans. ***(b)***

Planck's constant (in terms of unit)
$$h=\text{J-s}=[ML^{2}T^{-2}][T]=[ML^{2}T^{-1}]$$
Momentum (p) = kg-ms^{-1}
$$=[M][L][T^{-1}]$$
$$=[MLT^{-1}]$$