13 Plane and Coordinate Geometry

Fauvism, Cubism, Symbolism, and other movements played an important role in the development of his own revolutionary approach to painting. Decrying literal representation, Kandinsky emphasised instead the importance of form, colour, rhythm, and the artist's inner need in expressing reality. Relying on his own unique terminology, he develops the idea of point as the "proto-element" of painting, the role of point in nature, music, and other art, and the combination of point and line that results in a unique visual language. He then turns to an absorbing discussion of line — the influence of force on line, lyric and dramatic qualities, and the translation of various phenomena into forms of linear expression. With profound artistic insight, Kandinsky points out the organic relationship of the elements of painting, touching on the role of texture, the element of time, and the relationship of all these elements to the basic material plane called upon to receive the content of a work of art.

A construction is a geometric drawing for which only a compass and a straightedge may be used. A 'point', a 'line' and a 'plane' are the basic concepts to be used in geometry.

Straight Line: A straight line may be drawn from any one point to any other point. A line is the collection of infinite number of points. A line is defined as a line of points that extends infinitely in two directions. It has one dimension, length; A line is breathless length. The ends of a line segment are points. The edges of surface are lines. A plane surface is that which lies evenly with the straight lines on itself. A terminated line or a line segment (AB) can be produced infinitely.

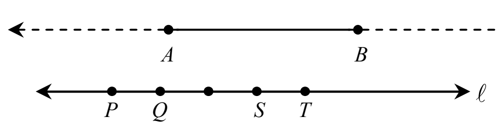


Figure: 13.1

Note

The distance between two points A and B is the length of line segment AB. Three or more points (P, Q, R, S and T) are said to be collinear if there is a line which contains all of them.

- **Concurrent Lines:** Three or more lines are said to be concurrent if there is a point which lies on all of them. Through a given point, infinite lines can be drawn.

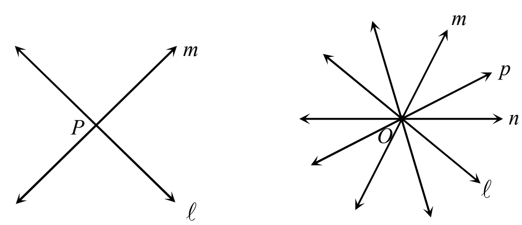


Figure: 13.2

- **Intersecting lines:** Two lines are intersecting if they have a common point. The common point is called the "point of intersection".
- **Parallel lines:** Two lines l and m in a plane are said to be parallel lines if they do not have a common point.

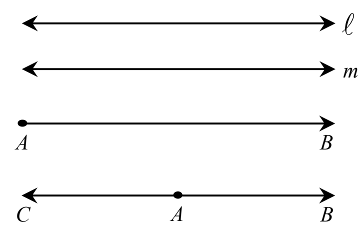


Figure: 13.3

- **Ray:** Directed line segment is called a ray. If AB is a ray then it is denoted by $\overline{AB}$. Point A is called initial point of ray.
- **Opposite rays:** Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of the two rays.

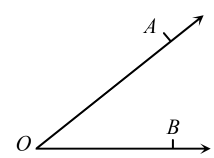


Figure: 13.4

- An angle is the union of two non-collinear rays with a common initial point. The common initial point is called the **'vertex'** of the angle and two rays are called the **'arms'** of the angles.

Note: Every angle has a measure and unit of measurement is degree.

- One right angle $= 90°$
- $1° = 60'$ (minutes)
- $1' = 60''$ (seconds)

Types of Angles:

- **Right angles:** An angle whose measure is $90°$ is called a right angle.

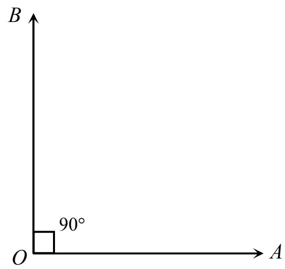


Figure: 13.5

- **Acute angle:** An angle whose measure is less than $90°$ is called an acute angle, $0° < \angle BOA < 90°$

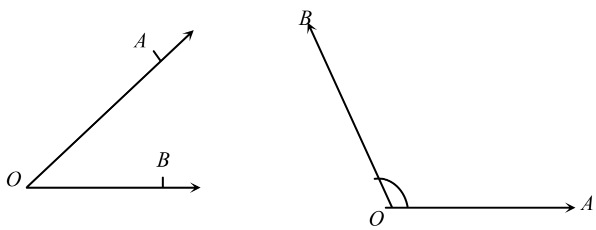


Figure: 13.6

- **Obtuse angle:** An angle whose measure is more than $90°$ but less than $180°$ is called an obtuse angle, $90° < \angle AOB < 180°$
- **Straight angle:** An angle whose measure is $180°$ is called a straight angle.

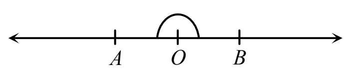


Figure: 13.7

- **Reflex angle :** An angle whose measure is more than $180°$ is called a reflex angle, $180° < \angle AOB < 360°$

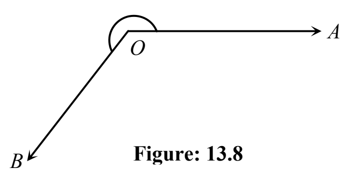


Figure: 13.8

Complementary angles: Two angles, the sum of whose measures is $90°$ are called complementary angles. $\angle AOC$ and $\angle BOC$ are complementary as $\angle AOC + \angle BOC = 90°$

- **Supplementary angles:** Two angles, the sum of whose measures is $180°$, are called the supplementary angles. $\angle AOC$ and $\angle BOC$ are supplementary as their sum is $180°$.

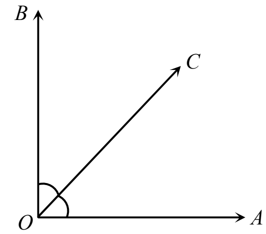


Figure: 13.9

- **Angle bisectors:** A ray *OX* is said to be the bisector of $\angle AOB$, if X is a point in the interior of $\angle AOB$, and $\angle AOX = \angle BOX$

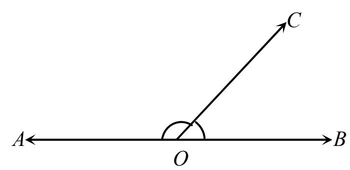


Figure: 13.10

- **Adjacent angles:** Two angles are called adjacent angles, it they have the same vertex, they have a common arm, non common arms are on either side of the common arm. $\angle AOX$ and $\angle BOX$ are adjacent angles, *OX* is common arm, *OA* and *OB* are non common arms and lies on either side of *OX*.

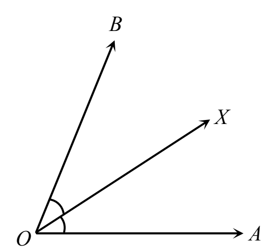


Figure: 13.11

- **Linear pair of angles:** Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays.

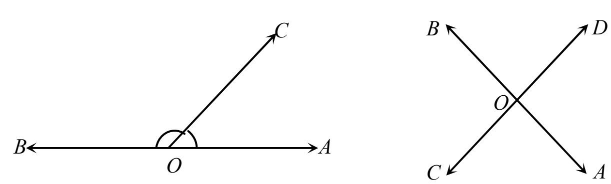
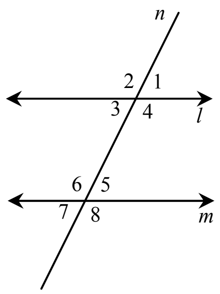


Figure: 13.12

- **Vertically opposite angles:** Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays. $\angle AOC$ and $\angle BOD$ from a pair of vertically opposite angles. Also $\angle OD$ and $\angle BOC$ form a pair of vertically opposite angles.

Angles Made by Transversal With Two Parallel Lines:

- **Corresponding angles:** Two angles on the same side of transversal are known as the corresponding angles if both lie either above the two lines or below the two lines, in figure $\angle 1$ & $\angle 5$, $\angle 4$ & $\angle 8$, $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$ are the pairs of corresponding angles.



$l \,||\, m \Rightarrow$ n is transversal

Figure: 13.13

- **Alternate interior angles:** $\angle 3$ & $\angle 5$, $\angle 4$ & $\angle 6$, are the pairs of alternate interior angles.

- **Consecutive interior angles :** The pair of interior angles on the same side of the transversal are called pairs of consecutive interior angles. In figure $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$, are the pair of consecutive interior angles.

Coordinate Geometry is an effective way to introduce points, lines, planes, origin, x-axis, y-axis, slope, intercept, geometric transformation and many other important mathematical concepts. Tap icons and use simple gestures to draw on a coordinate plane.

For many people, coordinate geometry is the familiar in many different ways. For example, in some cities streets are layed out as a coordinate grid. A Street, B Street C Street, intersects with First Avenue, Second Avenue and Third Avenue. The basic concept of coordinate geometry is used in spreadsheets, mapping, and many other systems for locating positions including GPS. In some way it seems that the human brain is intuitively adapt at using coordinate systems.

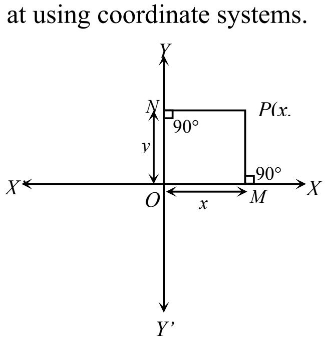


Figure: 13.14

In two dimensional coordinate geometry generally two types of co-ordinate systems are:

- **Cartesian or Rectangular coordinate system:** In cartesian coordinate system we represent any point by ordered pair (x,y) where x and y are called X and Y co-ordinate of that point respectively.

- **Polar coordinate system:** In polar coordinate system we represent any point by ordered pair (r, θ) where $'r'$ is called radius vector and $'\theta'$ is called vectorial angle of that point.

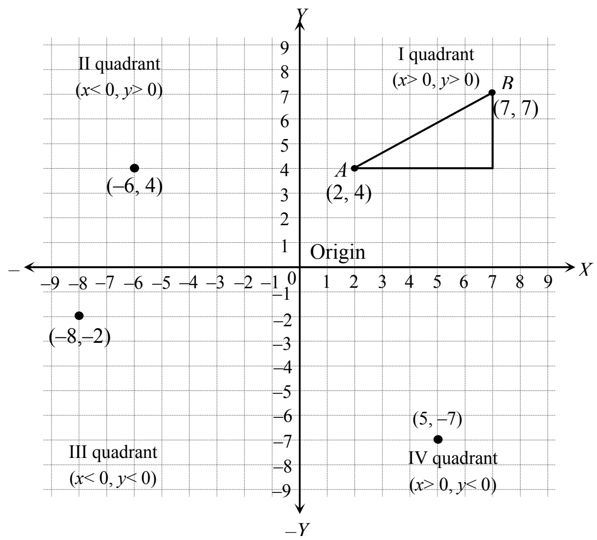


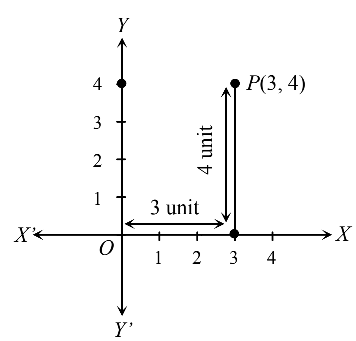
Figure: 13.15

Cartesian or rectangular coordinate system: Let X'OX and Y'OY are two lines such that X'OX is horizontal and Y'OY is vertical lines in the same plane and they intersect each other at O. This intersecting point is called origin.

- Let PM and PN be the perpendiculars on X-axis and Y-axis reservedly. The length of the line segment OM is called the x-coordinate be the or abscissa of point P. Let OM = x and ON = y. The position of the point P in the plane with respect to the coordinate axis is represented by the ordered pair (x,y). The ordered pair (x,y) is called the coordinates of point P.

Example 1. Plot the point (3, 4) on a graph paper.

Solution: Let $X'IX$ and $Y'OY$ be the coordinate axis. Here given point is P (3, 4), first we move 3 units along OX as 3 is positive then we arrive a point M. Now from M we move vertically upward as 4 is positive. Then we arrive at P (3,4).

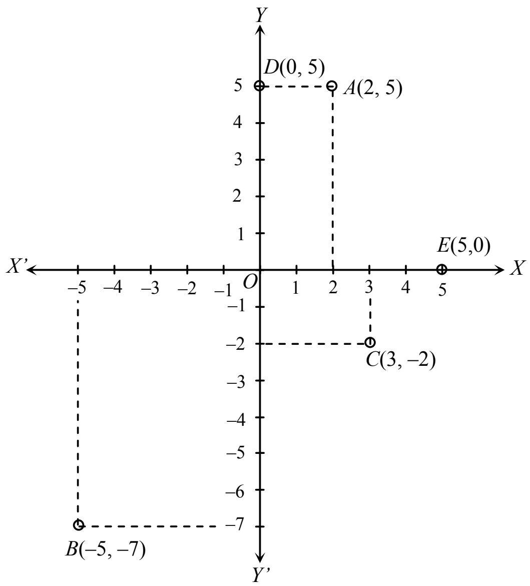


Example 2. Plot the following points on the graph paper.

(i) $A(2,5)$ (ii) $B(-5,-7)$ (iii) $C(3,-2)$

(iv) $D(0,5)$ (v) $E(5,0)$

Solution: Let XOX' and YOY' be the coordinate axis. Then the given points may be plotted as given below:



Distance between Two points

Let two points be $P(x_1, y_1)$ and $Q(x_2, y_2)$. Take two mutually perpendicular lines as the coordinate axis with O as origin. Mark the points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

We have $PR = AB = OB - OA = x_2 - x_1$

Similarly, $QR = QB - RB = QB - PA = y_2 - y_1$

Now, using Pythagoras Theorem, in right angled triangle PRQ,

We have, $PQ^2 = PR^2 + RQ^2$

Or $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since the distance or length of the line-segment PQ is always non-negative, on taking the positive square root, we get the distance as

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This result is known as **distance formula.**

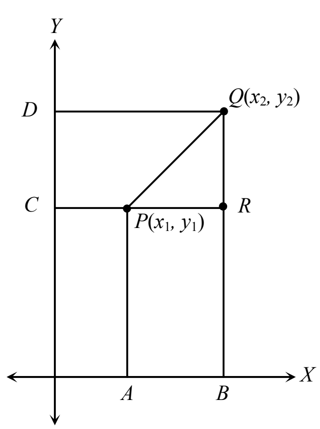


Figure: 13.16

Corollary: The distance of a point $P(x_1, y_1)$ from the origin (0, 0) is given by $OP = \sqrt{x_1^2 + y_1^2}$

Some useful points: In questions related to geometrical figures, take the given vertices in the given order and proceed as indicated.

- **For an isosceles triangle:** We have to prove that at least two sides are equal.
- **For an equilateral triangle:** We have to prove that three sides are equal.
- **For a right-angled triangle**: We have to prove that the sum of the squares of two sides is equal to the square of the third side.
- **For a square**: We have to prove that the four sides are equal, two diagonals are equal.
- **For a rhombus**: We have to prove that four sides are equal (and there is no need to establish that two diagonals are unequal as the square is also a rhombus).
- **For a rectangle**: We have to prove that the opposite sides are equal and two diagonals are equal.
- **For a Parallelogram**: We have to prove that the opposite sides are equal (and there is no need to establish that two diagonals are unequal sat the rectangle is also a parallelogram).
- **For three points to be collinear:** We have to prove that the sum of the distances between two pairs of points is equal to the third pair of points.

Example 3. Find the distance between

(i) $(5,3)$ and $(3,2)$; (ii) (a,b) and $(-b,a)$

Solution: Let d_1, d_2, d_3 be the required distances. By using the formula, we have

(i) $d_1 = \sqrt{(5-3)^2 + (3-2)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$

(ii) $d_3 = \sqrt{\{a+(-b)\}^2 + (b-a)^2} = \sqrt{(a+b)^2 + (a-b)^2} = \sqrt{2a^2 + 2b^2}$

Example 4. Prove that the points $(1,-1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) are the vertices of an isosceles triangle.

Solution: Let the point $(1,-1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) be denoted by P, Q and R, respectively. Now

$$PQ = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}+1\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$$QR = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$$PR = \sqrt{(1-1)^2 + (2+1)^2} = \sqrt{9} = 3$$

From the above, we see that $PQ = QR$ The triangle is isosceles.

Example 5. Using distance formula, show that the points $(-3, 2), (1, -2)$ and $(9, -10)$ are collinear.

Solution: Let the given points $(-3, 2), (1, -2)$ and $(9, -10)$ be denoted by *A, B* and *C*, respectively. Points *A, B* and *C* will be collinear, if the sum of the lengths of two line-segments is equal to the third.

Now, $AB = \sqrt{(1+3)^2 + (-2-2)^2} = \sqrt{16+16} = 4\sqrt{2}$

$$BC = \sqrt{(9-1)^2 + (-10+2)^2} = \sqrt{64+64} = 8\sqrt{2}$$

$$AC = \sqrt{(9+3)^2 + (-10-2)^2} = \sqrt{144+144} = 12\sqrt{2}$$

Since, $AB + BC = 4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} = AC$, the, points A, B and C are collinear.

Example 6. Find a point on the *X*-axis which is equidistant from the points (5, 4) and (– 2, 3).

Solution: Since the required point (say *P*) is on the X-axis, its ordinate will be zero. Let the abscissa of the point be x.

Therefore, coordinates of the point *P* are (x, 0).

Let *A* and *B* denote the points $(5, 4)$ and $(-2, 3)$ respectively.

Since we are given that *AP* = *BP*, we have $AP^2 = BP^2$

i.e., $(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$

or $x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$

or $-14x = -28$

or $x = 2$

Thus, the required point is (2, 0).

Example 7. The vertices of a triangle are (–2, 0), (2, 3) and (1, –3). Is the triangle equilateral, isosceles or scalene?

Solution: Let the points (–2, 0), (2, 3) and (1, –3) be denoted by *A, B* and *C* respectively. Then,

$$AB = \sqrt{(2+2)^2 + (3-0)^2} = 5$$

$$BC = \sqrt{(1-2)^2 + (-3-3)^2} = \sqrt{37}$$

and $AC = \sqrt{(1+2)^2 + (-0-0)^2} = 3\sqrt{2}$

Clearly, $AB \neq BC \neq AC$

Therefore, *ABC* is a scalene triangle.

Example 8. Show that the points (–2, 5), (3, –4) and (7, 10) are the vertices of a right triangle.

Solution: Let the three points be $A(-2, 5), B(3, -4)$ and $C(7, 10)$.

Then $AB^2 = (3+2)^2 + (-4-5)^2 = 106$

$$BC^2 = (7-3)^2 + (10+4)^2 = 212$$

$$AC^2 = (7+2)^2 + (10-5)^2 = 106$$

We see that $BC^2 = AB^2 + AC^2$

$212 = 106 + 106 \quad 212 = 212$

$\therefore \quad \angle A = 90°$

Thus, *ABC* is a right triangle, right angled at *A*.

Section Formulae

Formula for Internal Division: The coordinates of the pint which divided the line segment joining the pints (x_1, y_1) and (x_2, y_2) internally in the ratio ***m : n*** are given by:

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

Let *O* be the origin and let *OX* and *OY* be the X-axis and Y-axis respectively. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the given points; ΔAHP and ΔPKB are similar.

$$\frac{AP}{BP} = \frac{AH}{PK} = \frac{PH}{BK}$$

or $\frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$

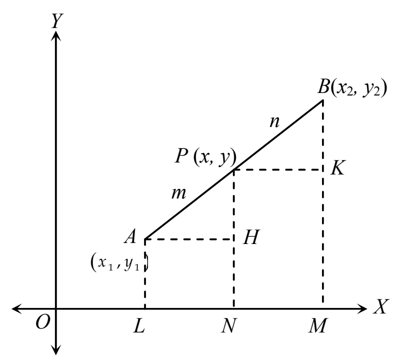


Figure: 13.17

Now, $\frac{m}{n} = \frac{x - x_1}{x_2 - x}$

$mx_2 - mx = nx - nx_1$

or $mx + nx = mx_2 + nx_1$

$$x = \frac{mx_2 + nx_1}{m+n}$$

And $\frac{m}{n} = \frac{y - y_1}{y_2 - y}$

$my_2 - my = ny - ny_1$ or $my + ny = my_2 + ny_1$

$$y = \frac{my_2 + ny_1}{m+n}$$

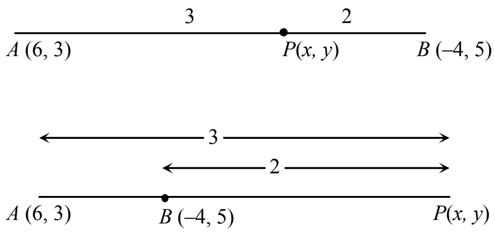
Thus, the coordinates of *P* are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Note: If *P* is the mid-point of *AB*, then it divides *AB* in the ratio 1 : 1, so its coordinates are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Formula for External Division: The coordinates of the points which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) externally in the ratio *m: n* are given by:

$$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$$

Example 9. Find the coordinates of the point which divides the line segment joining the points (6, 3) and (– 4, 5) in the ratio 3 : 2 (i) internally (ii) externally.



Solution: Let P(x, y) be the required point.

(i) For internal division, we have $x = \dfrac{3x - 4 + 2\times 6}{3+2}$

and $y = \dfrac{3\times 5 + 2\times 3}{3+2}$

$\Rightarrow$ $x = 0$ and $y = \dfrac{21}{5}$

So, the coordinates of P are $\left(0, \dfrac{21}{5}\right)$

(ii) For external division, we have $x = \dfrac{3x - 4 - 2\times 6}{3-2}$

and $y = \dfrac{3\times 5 - 2\times 3}{3-2}$

$\Rightarrow$ $x = -24$ and $y = 9$

So, the coordinates of P are (– 24, 9).

Centroid of a triangle: Prove that the coordinates of the triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$.

Also, deduce that the medians of a triangle are concurrent.

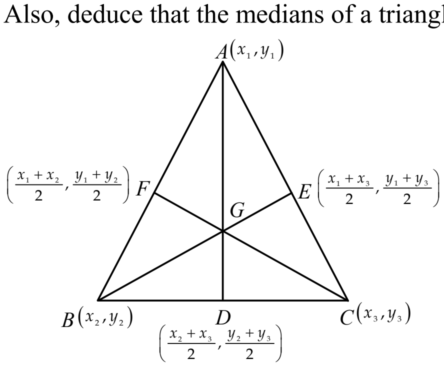


Figure: 13.18

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC whose medians are *AD, BE* and *CF* respectively. So. *D,E* and *F* are respectively the mid-points of *BC, CA* and *AB*.

Coordinates of *D* are $\left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right)$. Coordinates of a point dividing *AD* in the ratio 2 : 1 are:

$$\left(\frac{1\cdot x_1 + 2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{1\cdot y_1 + 2\left(\frac{y_2+y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of *E* are $\left(\dfrac{x_1+x_3}{2}, \dfrac{y_1+y_3}{1}\right)$. The coordinates of a point dividing *BE* in the ratio 2 : 1 are:

$$\left(\frac{1\cdot x_2 + \frac{2(x_1+x_3)}{2}}{1+2}, \frac{1\cdot y_2 + \frac{2(y_1+y_3)}{2}}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Similarly the coordinates of a point dividing *CF* in the ratio 2 : 1 are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

Thus, the point having coordinates $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$ is common to *AD*, *BE* and *CF* and divides them in the ratio 1 : 2.

Hence, medians of a triangle are concurrent and the coordinates of the centroid are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$.

Area of a triangle: Let *ABC* be any triangle whose vertices are $A(x_1, y_1), B(x_2, y_3)$.

Draw *BL*, *AM* and *CN* perpendicular from *B,A* and *C* respectively, to the *X*-axis. *ABLM, AMNC* and *BLNC* are all trapeziums.

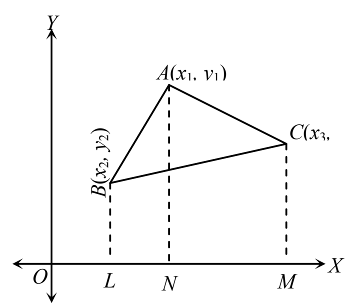


Figure: 13.19

Area of ΔABC = Area of trapezium *ABLM* + Area of trapezium *AMNC* – Area of trapezium *BLNC*

We know that,

Area of trapezium $= \dfrac{1}{2}$ (Sum of parallel sides) (distance b/w them)

Therefore,

$$\text{Area of } \Delta ABC = \frac{1}{2}(BL + AM)(LM) + \frac{1}{2}(AM + CN)MN - \frac{1}{2}(BL + CN)(LN)$$

$$\text{Area of } \Delta ABC = \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1)$$

$$-\frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y) + x_3(y_1 - y_2)\right]$$

Note

Condition for colinearity: Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if Area of $\Delta ABC = 0$

Area of Quadrilateral:

Let the vertices of Quadrilateral *ABCD* are $A(x_1, y_1)$, $B(x_2, y_2), C(x_3, y_3)$ and $D(x_4, y_4)$.

So, Area of quadrilateral *ABCD* = Area of ΔABC+ Area of ΔACD

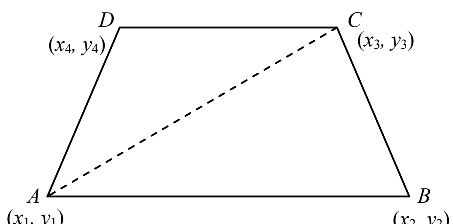


Figure: 13.20

Example 10. The vertices of ΔABC are $(-2,1),(5,4)$ and $(2,-3)$ respectively. Find the area of triangle.

Solution: $A(-2,1), B(-2,1)$ and $C(2,-3)$ be the vertices of triangle.

So, $x_1 = -2, y_1 = 1; x_2 = 5, y_2 = 4; x_3 = 2 y_3 = -3$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$$

$$= \frac{1}{2}\left|\left[(-2)(4+3) + (5)(-3-1) + 2(1-4)\right]\right|$$

$$= \frac{1}{2}\left|\left[-14 + (-20) + (-6) + 2(1-4)\right]\right|$$

$$= \frac{1}{2}|-40| = 20 \text{ sq. unit.}$$

Example 11. The area of a triangle is 5. Two of its vertices area $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. Find the third vertex.

Solution: Let the third vertex be (x_3, y_3) area of triangle

$$= \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$$

As $x_1 = 2\ y_1 = 1; x_2 = 3, y_2 = -2$

Area of $\Delta = 5$ sq. unit

$\Rightarrow \quad 5 = \frac{1}{2}\left|2(-2 - y_3) + 3(y_3 - 1) + x_3(1+2)\right|$

$\Rightarrow \quad 10 = |3x_3 + y_3 - 7|$

$\Rightarrow \quad 3x_3 + y_3 - 7 = \pm 10$

Taking positive sign $3x_3 + y_3 - 7 = 10$

$\Rightarrow \quad 3x + y_3 = 17 \qquad \ldots (i)$

Taking negative sign $3x_3 + y_3 - 7 = -10$

$\Rightarrow \quad 3x \therefore + y_3 = -3 \qquad \ldots (ii)$

Given that (x_3, y_3) lies on $y = x + 3$

So, $-x \therefore + y_3 = 3 \qquad \ldots (iii)$

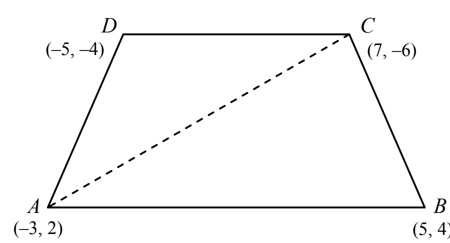
Solving eq. (*i*) and (*iii*)

$$x_3 = \frac{7}{2},\ y_3 = \frac{13}{2}$$

Solving eq. (*ii*) and (*iii*) $x_3 = \frac{-3}{2},\ y_3 = \frac{3}{2}$

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$.

Example 12. Find the area of quadrilateral whose vertices, taken in order, are $A(-3, 2)$, $B(5, 4)$, $C(7, -6)$ and $D(-5, -4)$.



Solution: Area of quadrilateral

= Area of ΔABC + Area of ΔACD

So, $\text{Area of } \Delta ABC = \frac{1}{2}\left|(-3)(4+6) + 5(-6-2) + 7(2-4)\right|$

$$= \frac{1}{2}|-30 - 40 - 14|$$

$$= \frac{1}{2}|-84| = 42 \text{ sq. units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left|-3(-6+4) + 7(-4-2) + (-5)(2+6)\right|$$

$$= \frac{1}{2}|+6 - 42 - 40| = \frac{1}{2}|-76| = 38 \text{ sq. units}$$

So, Area of quadrilateral

ABCD = 42 + 38 = 80 sq. units

Multiple Choice Questions

1. If two lines intersected by a transversal, then each pair of corresponding angles so formed is:
a. Equal **b.** Complementary
c. Supplementary **d.** None of these

2. Two parallel lines have:
a. a common point **b.** two common point
c. no any common point **d.** infinite common points

3. An angle is $14°$ more than its complementary angle then angle is:
a. $38°$ **b.** $52°$
c. $50°$ **d.** none of these

4. The angle between the bisectors of two adjacent supplementary angles is:
a. acute angle **b.** right angle
a. obtuse angle **d.** none of these

5. If one angle of triangle is equal to the sum of the other two then triangle is:
a. acute a triangle **b.** obtuse triangle
c. right triangle **d.** none

6. X lies in the interior of $\angle BAC$. If $\angle BAC = 70°$ and $\angle BAX = 42°$ then $\angle$ XAX =
a. $28°$ **b.** $29°$ **c.** $27°$ **d.** $30°$

7. If the supplement of an angle is three times its complement, then angle is:
a. $40°$ **b.** $35°$ **c.** $50°$ **d.** $45°$

8. Two angles whose measures are a and b are such that $2a - 3b = 60°$ then $\frac{4a}{5b} = ?$. If they form a linear pair:
a. 0 **b.** $\frac{8}{5}$ **c.** $\frac{1}{2}$ **d.** $\frac{2}{3}$

9. The abscissa of a point is distance of the point from:
a. x-axis **b.** y-axis
c. origin **d.** none of these

10. The y-coordinate of a point is distance of that point from:
a. x-axis **b.** y-axis
c. origin **d.** none of these

11. If both coordinates of any point are negative then that point will lie in:
a. first quadrant **b.** second quadrant
c. third quadrant **d.** fourth quadrant

12. If the abscissa of any point is zero then that point will lie :
a. on x-axis **b.** on y-axis
c. at origin **d.** none of these

13. The co-ordinates of one end point of a diameter of a circle are $(4,-1)$ and coordinates of the centre of the circle are $(1,-3)$ then coordinates of the other end of the diameter are:
a. $(2,5)$ **b.** $(-2,-5)$ **c.** $(3,2)$ **d.** $(-3,-2)$

14. The point $(-2,-1),(1,0),(4,3)$ and $(1,2)$ are the vertices of a:
a. Rectangle **b.** Parallelogram
c. Square **d.** Rhombus

15. The distance of the point (3, 5) from x-axis is:
a. $\sqrt{34}$ **b.** 3
c. 5 **d.** None of these

16. The points $(-a,-b),(0,0),(a,b)$ and (a^2,ab) are
a. collinear
b. vertices of a parallelogram
c. vertices of a rectangle
d. none of these

17. If the points $(5,1),(1,p)$ and $(4,2)$ are collinear then the value of p will be:
a. 1 **b.** 5 **c.** 2 **d.** -2

18. Length of the median from B on AC where $A(-1,3)$, $B(1,-1)$, $C(5,1)$ is:
a. $\sqrt{18}$ **b.** $\sqrt{10}$ **c.** $2\sqrt{3}$ **d.** 4

19. The points $(0,-1),(-2,3),(6,7)$ and $(8,3)$ are:
a. collinear
b. vertices of a parallelogram which is not a rectangle
c. vertices of a rectangle, which is not a square
d. none of these

20. If $(3,-4)$ and $(-6,5)$ are the extremities of the diagonal of a parallelogram and $(-2,1)$ is third vertex, then its fourth vertex is:
a. $(-1,0)$ **b.** $(0,-1)$
c. $(-1,1)$ **d.** none of these

21. The area of a triangle whose vertices are $(a, c+a),(a,c)$ and $(-a, c,-a)$ are:
a. a^2 **b.** b^2 **c.** c^2 **d.** $a^2 + c^2$

22. The area of the quadrilateral's the coordinates of whose verticals are $(1,-2)(6,2)$, $(5,3)$ and $(3,4)$ are:

a. $\frac{9}{2}$ **b.** 5 **c.** $\frac{1}{2}$ **d.** 11

23. The point (–2, –3) belongs to quadrant

a. Q_1 **b.** Q_2 **c.** Q_3 **d.** Q_4

24. The point (–2, 0) lies on

a. +ve x-axis **b.** +ve y-axis
c. –ve x-axis **d.** –ve y-axis

25. The point (0, –2) lies on

a. +ve x-axis **b.** +ve y-axis
c. –ve x-axis **d.** –ve y-axis

26. The points (a, a) $(-a, -a)$ and $\left(-\sqrt{3}a, \sqrt{3}a\right)$ form the vertices of an?

a. scalene triangle
b. right angled triangle
c. isosceles right angled triangle
d. equilateral triangle

27. In which ratio does the point (–1, –1) divides the line segment joining the pints (4, 4) and (7, 7)?

a. 5 : 8 **b.** 8 : 3 **c.** 3 : 8 **d.** 8 : 5

28. If (0, 0) and $\left(3, \sqrt{3}\right)$ are two vertices of an equilateral triangle then the third vertex is?

a. $\left(0, 2\sqrt{3}\right)$ or $\left(3, -\sqrt{3}\right)$ **b.** $\left(0, -2\sqrt{3}\right)$ or $\left(3, -\sqrt{3}\right)$
c. $\left(0, 2\sqrt{3}\right)$ or $\left(-3, \sqrt{3}\right)$ **d.** $\left(0, 2\sqrt{3}\right)$ or $\left(-3, -\sqrt{3}\right)$

29. The points (4, 4), (3, 5) and (–1, –1) form the vertices of

a. right angled triangle
b. right angled isosceles triangle
c. isosceles triangle
d. equilateral triangle

30. The area of ΔABC formed by vertices $A(1, 2), B(-3, -4)$ and $C(-2, 3)$ is?

a. $\sqrt{11}$ sq units **b.** 11 sq units
c. $\sqrt{13}$ sq units **d.** None

31. Find the value of k if the area of triangle formed by (1, k), (4, –3) and (–0, 7) is 15 sq units.

a. $\frac{21}{13}$ or –3 **b.** $-\frac{21}{13}$ or 3
c. $\frac{21}{13}$ or 3 **d.** $-\frac{21}{13}$ or –3

32. The value of k if (–3, 12), (7, 6) and (k, 9) are collinear is?

a. 3 **b.** 4
c. 2 **d.** 1

33. The coordinates of the point C dividing the join of points A (2, 6) and B (5, 1) in the ratio 2 : 3 is?

a. $\left(4, \frac{16}{5}\right)$ **b.** $\left(\frac{16}{5}, 4\right)$
c. $\left(\frac{16}{5}, \frac{4}{5}\right)$ **d.** None

34. The ratio by which P (4, 6) divides the join of A (–2, 3) and B (6, 7) is?

a. $\frac{3}{1}$ **b.** $\frac{3}{1}$
c. $\frac{2}{3}$ **d.** None

35. In what ratio is the segment joining the points (4, 6) and (–7, –1) divided by x-axis?

a. 1 : 6 **b.** 6 : 2
c. 2 : 6 **d.** 6 : 1

36. In what ratio is the segment joining the points (–3, 2) and (6, 1) divided by y-axis?

a. 1 : 3 **b.** 2 : 1
c. 1 : 2 **d.** 3 : 1

37. One end of diameter of a circle is (2, 3) and the centre is (–2, 5). The coordinates of the other end is?

a. (–6, 7) **b.** (6, –7)
c. (6, 7) **d.** None

38. The coordinates of the points of tri-section of a segment joining $A(-3,2)$ and B (9, 5) is?

a. (3, 1), (4, 5) **b.** (1, 3), (5, 4)
c. (1, 3), (4, 5) **d.** (3, 1), (5, 4)

39. If D (3, –1), E (2, 6) and F (–5, 7) are the midpoints of the sides of ΔABC, the area of the triangle is?

a. 96 sq units **b.** 24 sq units
c. 48 sq units **d.** 50 sq units

40. Area of quadrilateral formed by the vertices (–1, 6), (–3, –9), (5, –8) and (3, 9) is:

a. 96 sq units **b.** 18 sq units
c. 50 sq units **d.** 25 sq units

ANSWERS

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
d	c	b	b	c	a	d	b	b	a
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	b	b	b	c	a	b	b	c	a
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
a	c	c	c	d	d	a	a	a	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
a	c	b	a	d	c	a	b	a	a

SOLUTIONS

3. **(b)** $x-(90-x)=14°$

6. **(a)** $\angle CAX = \angle BAC - \angle BAX$

7. **(d)** $180-x=3(90-x)$

8. **(b)** $a+b=180°$

$2a-3b=60°$

13. **(b)** $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

14. **(b)** Calculate *AB, BC, CD, DA*
AC and *BD* to conclude.

17. **(d)** Prove value of triangle is zero.

27. **(a)** Suppose the point *C* (–1, –1) divides the line joining the points *A* (4, 4) and *B* (7, 7) in the ratio *k* : 1 Then, the coordinates of *C* are $\left(\frac{7k+4}{k+1},\frac{7k+4}{k+1}\right)$ But, we are given that the coordinates of the points *C* are (–1, –1).

$\therefore \quad \frac{7k+4}{k+1}=-1$

$\Rightarrow \quad k=-\frac{5}{8}$

Thus, *C* divides *AB* externally in the ratio 5 : 8.

34. **(a)** $(4.6)=\left[\frac{m(6)+n(-2)}{m+n},\frac{m(7)+n(3)}{m+n}\right]$

$=\frac{6m-3n}{m+n}=4$

$\Rightarrow \quad 6m-2n=4m+4n$

$\Rightarrow \quad 2m=6n$

$\Rightarrow \quad \frac{m}{n}=3:1=\frac{3}{1}$

35. **(d)** $(a,0)=\left[\frac{m(-7)+n(4)}{m+n},\frac{m(-1)+n(6)}{m+n}\right]$

$\Rightarrow \quad \frac{-m+6n}{m+n}=0$

$\Rightarrow \quad -m+6n=0$

$\Rightarrow \quad \frac{m}{n}=\frac{6}{1}=6:1$

36. **(c)** $(0,b)=\left[\frac{m(6)+n(-3)}{m+n},\frac{m(1)+n(2)}{m+n}\right]$

$\Rightarrow \quad \frac{6m-3n}{m+n}=0$

$\Rightarrow \quad 6m=3n=\frac{m}{n}=\frac{1}{2}$

37. **(a)** Centre = midpoint of diameter

$(-2,5)\left(\frac{x+2}{2},\frac{y+3}{2}\right)$

$\Rightarrow \quad \frac{x+2}{2}=-2$

and $\frac{y+3}{2}=5$

$\Rightarrow \quad x+2=-4,\ y+3=10$

$\Rightarrow \quad x=-6,\ y=7$

38. **(b)** **Case (i)** 1 : 2

$P=\left(\frac{1(9)+2(-3)}{1+2},\frac{1(5)+2(2)}{1+2}\right)=(1,3)$

Case (ii) 2 : 1

$P=\left(\frac{2(9)+1(-3)}{2+1},\frac{2(5)+1(2)}{2+1}\right)=(5,4)$.

39. **(a)** $Ar\ \Delta DEF=24$ sq units

$Ar\ \Delta ABC=4\times\Delta DEF=4\times4=96$ sq units

40. **(a)** Area

$=\frac{1}{2}\begin{vmatrix}x_1-x_3 & x_2-x_4\\ y_1-y_3 & y_2-y_4\end{vmatrix}$

$=\frac{1}{2}\begin{vmatrix}-6 & -6\\ 14 & -18\end{vmatrix}=96$ sq units

□□□