Ray Optics and Optical Instruments

- **Optics :** It is the branch of physics which deals with the study of light and the phenomena associated with it. It is divided into two branches :
  - Geometrical optics or ray optics
  - Physical optics or wave optics
- **Refraction of light :** When a ray of light passes from one medium to another, in which it has a different velocity, there occurs a change in the direction of propagation of light except when it strikes the surface of separation of two media normally. This bending of a ray of light is known as refraction.
  - ▶ The angles made by the incident ray and the refracted ray with the normal to the separating surface at the point of incidence are known as the angle of incidence and of refraction respectively.
  - ▶ **Laws of refraction :** The two laws of refraction are as follows :
    - The incident ray, the normal and the refracted ray all lie in the same plane.
    - The ratio of the sine of angle of incidence to the sine of angle of refraction for any two media is constant for a light of definite colour. This constant is denoted by $^1\mu_2$ or μ_{21} called the refractive index of the second medium with respect to the first, the subscripts 1 and 2 indicating that the light passes from medium 1 to medium 2.

    $$\frac{\sin i}{\sin r} = {}^1\mu_2$$

    This is also known as Snell's law.
    where i = angle of incidence,
    r = angle of refraction.

- ▶ **Absolute refractive index :** Refractive index of a medium with respect to vacuum (or in practice, air) is known as absolute refractive index of the medium.

  $$\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$
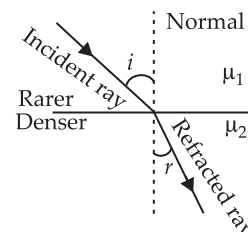
  General expression for Snell's law

  $$^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\left(\dfrac{c}{v_2}\right)}{\left(\dfrac{c}{v_1}\right)} = \frac{v_1}{v_2}$$

  where c is the speed of light in air, v_1 and v_2 be the speeds of light in medium 1 and medium 2 respectively. According to Snell's law,
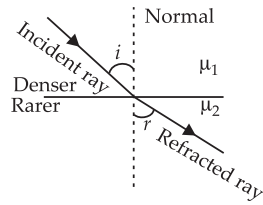
  $$^1\mu_2 = \frac{\sin i}{\sin r}; \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

 or $\mu_1 \sin i = \mu_2 \sin r$

- ▶ When a light ray passes from a rarer to denser medium ($\mu_2 > \mu_1$), it will bend towards the normal as shown in the figure.

- ▶ When a light ray passes from a denser medium to rarer medium ($\mu_1 > \mu_2$) it will bend away from the normal as shown in the figure.

▶ If a light ray passes through a number of parallel media and if the first and the last media are same. The emergent ray is parallel to the incident ray as shown in figure.
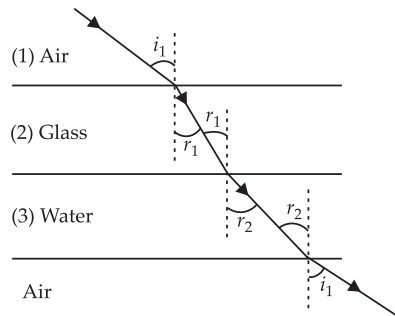
$$^1\mu_2 = \frac{\sin i_1}{\sin r_1}, \; ^2\mu_3 = \frac{\sin r_1}{\sin r_2} \; \text{and} \; ^3\mu_1 = \frac{\sin r_2}{\sin i_1}$$
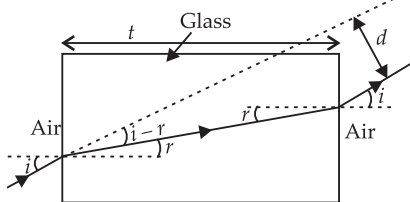
Hence,

$$^1\mu_2 \times ^2\mu_3 \times ^3\mu_1 = \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i_1} = 1$$

▶ **Lateral shift :** When the medium is same on both sides of a glass slab, then the deviation of the emergent ray is zero. That is the emergent ray is parallel to the incident ray but it does suffer lateral displacement/shift with respect to the incident ray and is given by

$$\text{Lateral shift}, d = t\,\frac{\sin(i - r)}{\cos r}$$

where t is the thickness of the slab.

▶ **Real depth and apparent depth :** When one looks into a pool of water, it does not appear to be as deep as it really is. Also when one looks into a slab of glass, the material does not appear to be as thick as it really is. This all happens due to refraction of light.

– If a beaker is filled with water and a point lying at its bottom is observed by someone located in air, then the bottom point appears raised. The apparent depth is less than the real depth. It can be shown that

$$\text{apparent depth} = \frac{\text{real depth}}{\text{refractive index } (\mu)}$$

If there is an ink spot at the bottom of a glass slab, it appears to be raised by a distance

$$d = t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$$

where t is the thickness of the glass slab and μ is its refractive index.

– If a beaker is filled with immiscible transparent liquids of refractive indices μ_1, μ_2, μ_3 and individual depths d_1, d_2, d_3 respectively, then the apparent depth of the beaker is $= \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + \dfrac{d_3}{\mu_3}$

● **Total internal reflection**
It is a phenomenon of reflection of light into denser medium from the boundary of denser medium and rarer medium. Two essential conditions for the phenomenon of total internal reflection are :
▶ Light should travel from a denser to a rarer medium.
▶ Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.
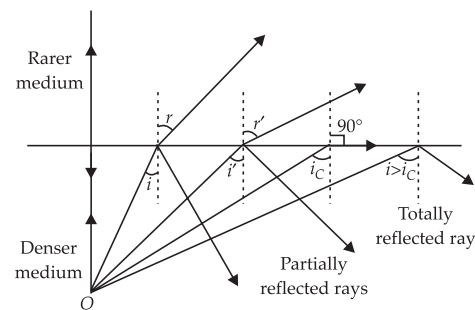

▶ **Critical angle :** It is that angle of incidence for which the angle of refraction becomes 90. It is given by $\sin i_C = \dfrac{1}{^R\mu_D}$

If the rarer medium is air or vacuum, then $\sin i_C = \dfrac{1}{\mu}$. Critical angle for red light is more than that for blue light.

- ▶ Critical angle depends on
  - – nature of medium
  - – wavelength of light
- ▶ A diver in water at a depth d sees the world outside through a horizontal circle of radius $r = d \tan i_C = \dfrac{d}{\sqrt{\mu^2 - 1}}$
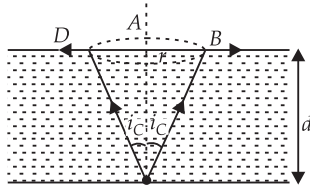
- ▶ **Applications of total internal reflection**
  - – The brilliance of diamond is due to the phenomenon of total internal reflection.
  - – Mirages in deserts are also due to total internal reflection.
  - – The working of optical fibre is based on the phenomenon of total internal reflection.

- ● **Refraction from a spherical surface**
  The portion of a refracting medium, whose curved surface forms the part of a sphere, is known as spherical refracting surface. Sign conventions for spherical refracting surface are the same as those for spherical mirrors.
  - ▶ Spherical refracting surfaces are of two types :
    - – Convex refracting spherical surface
    - – Concave refracting spherical surface
  - ▶ When the object is situated in rarer medium, the relation between μ_1 (refractive index of rarer medium), μ_2 (refractive index of the spherical refracting surface) and R (radius of curvature) with the object and image distances is given by
    $$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$
  - ▶ When the object is situated in denser medium, the relation between μ_1, μ_2, R, u and v can be obtained by interchanging μ_1 and μ_2. In that case, the relation becomes
    $$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R} \text{ or } -\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$
 - ▶ These formulae are valid for both convex and concave spherical surfaces.

- ● **Lens :** A lens is a portion of a transparent refracting medium bound by two spherical surfaces or one spherical surface and the other plane surface. Lenses are divided into two classes :
 - ▶ **Convex lens or converging lens :** When a lens is thicker in the middle than at the edges, it is known as convex lens or converging lens. These are of three types :
    - – Double convex lens or biconvex lens
    - – Plano convex lens
    - – Concavo convex lens

Biconvex lens	Plano convex lens	Concavo convex lens
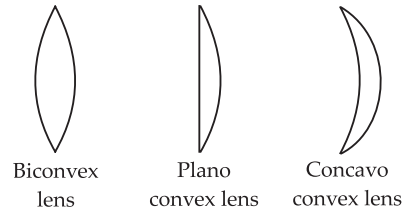
  - ▶ **Concave lens or diverging lens :** When the lens is thicker at the edges than in the middle it is known as concave lens or diverging lens. These are of three types :
    - – Double concave lens or biconcave lens
    - – Plano concave lens
    - – Convexo concave lens

|		
Biconcave lens	Plano concave lens	Convexo concave lens
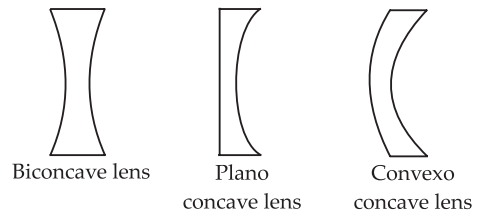
  - ▶ **Sign Conventions :** The sign conventions for thin lenses are the same as those of spherical mirrors except that instead of the pole of the mirror, we now use optical centre of a lens.
 - ▶ **Lens maker's formula**
    $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$
    where R_1 and R_2 are radii of curvature of the two surfaces of the lens and μ is refractive index of material of lens w.r.t. medium in which lens is placed. This formula is valid for thin lenses. It is valid for both convex and concave lenses. As per sign convention, for a convex lens, R_1 is positive and R_2 is negative and for a concave lens, R_1 is negative and R_2 is positive.

▶ When the refractive index of the material of the lens is greater than that of the surroundings, then biconvex lens acts as a converging lens and a biconcave lens acts as a diverging lens as shown in the figure :

$$\mu_2 > \mu_1$$
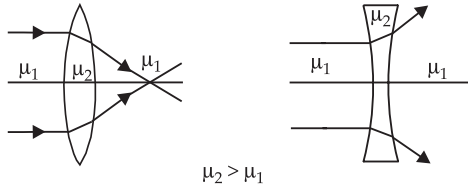
▶ When the refractive index of the material of the lens is smaller than that of the surrounding medium, then biconvex lens acts as a diverging lens and a biconcave lens acts as a converging lens as shown in the figure.

$$\mu_2 < \mu_1$$
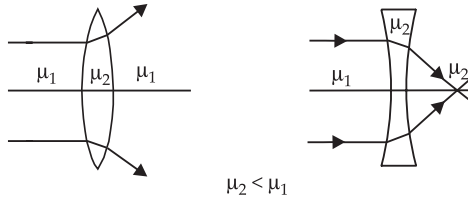
▶ **Thin lens formula**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where

u = distance of the object from the optical centre of the lens

v = distance of the image from the optical centre of the lens

f = focal length of a lens

f is positive for converging or convex lens and f is negative for diverging or concave lens.

▶ **Linear magnification**

$$m = \frac{\text{size of image } (I)}{\text{size of object } (O)} = \frac{v}{u}$$

where m is positive for erect image and m is negative for inverted image.

▶ **Power of a lens**

$$P = \frac{1}{\text{focal length in metres}}$$

– The SI unit of power of lens is dioptre (D).
– $1 \text{ D} = 1 \text{ m}^{-1}$

– For a convex lens, P is positive.
– For a concave lens, P is negative.
When focal length (f) of lens is in cm, then

$$P = \frac{100}{f \text{ (in cm)}} \text{ dioptre}$$

▶ **Combination of thin lenses in contact:**
When a number of thin lenses of focal length f_1, f_2, ...etc., are placed in contact coaxially, the equivalent focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$$

The total power of the combination is given by

$P = P_1 + P_2 + P_3 + ...$

▶ When two thin lenses of focal lengths f_1 and f_2 are placed coaxially and separated by a distance d, the focal length of a combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In terms of power $P = P_1 + P_2 - dP_1P_2$.

● **Refraction through a prism**

▶ **Prism :** It is a homogeneous, transparent medium enclosed by two plane surfaces inclined at an angle. These surfaces are called the refracting surfaces and angle between them is known as the refracting angle or the angle of prism. The angle between the incident ray and the emergent ray is known as the angle of deviation.

▶ For refraction through a prism it is found that
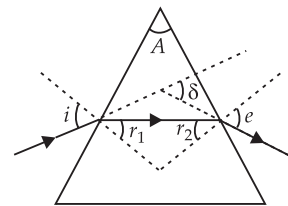
$\delta = i + e - A$ where $A = r_1 + r_2$

When A and i are small

$\delta = (\mu - 1)A$

In a position of minimum deviation

$\delta = \delta_m$, $i = e$ and $r_1 = r_2 = r$

$$\therefore \quad i = \left(\frac{A + \delta_m}{2}\right) \text{ and } r = \frac{A}{2}$$

▶ The refractive index of the material of the prism is

$$\mu = \dfrac{\sin\left[\dfrac{(A+\delta_m)}{2}\right]}{\sin\left(\dfrac{A}{2}\right)}$$

This is known as prism formula where A is the angle of prism and δ_m is the angle of minimum deviation.

● **Optical Instruments**

▶ **Simple microscope :** It is also known as magnifying glass or simple magnifier. It consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual, enlarged and on same side of lens between object and infinity.
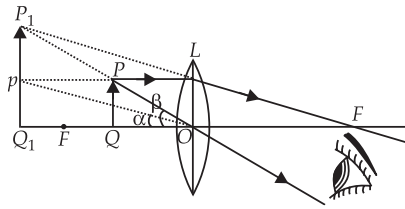
– When the image is formed at infinity (far point),

$$M = \dfrac{D}{f}$$

– When the image is formed at the least distance of distinct vision D (near point),

$$M = 1 + \dfrac{D}{f}$$

– Magnifying power

$$M = \dfrac{\text{angle subtended by image at the eye}}{\text{angle subtended by the object at the eye}}$$

$$= \dfrac{\tan\beta}{\tan\alpha} = \dfrac{\beta}{\alpha}$$

where both the object and image are situated at the least distance of distinct vision.

▶ **Compound microscope :** It consists of two convergent lenses of short focal lengths and apertures arranged co-axially. Lens (of focal length f_o) facing the object is known as objective or field lens while the lens (of focal length f_e) facing the eye, is known as eye-piece or ocular. The objective has a smaller aperture and smaller focal length than eye-piece. Magnifying power of a compound microscope

$$M = m_o \times m_e$$
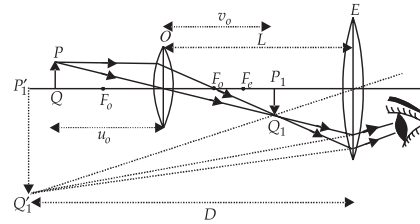
– When the final image is formed at infinity (normal adjustment),

$$M = \dfrac{v_o}{u_o}\left(\dfrac{D}{f_e}\right)$$

Length of tube, $L = v_o + f_e$

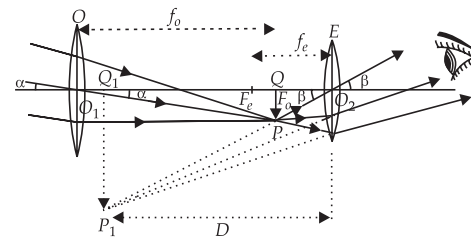– When the final image is formed at least distance of distinct vision,

$$M = \dfrac{v_o}{u_o}\left(1+\dfrac{D}{f_e}\right)$$

where u_o and v_o represent the distance of object and image from the objective lens, f_e is the focal length of an eye lens.

Length of the tube, $L = v_o + \left(\dfrac{f_e D}{f_e + D}\right)$

▶ **Astronomical telescope (refracting type)**

It consists of two converging lenses. The one facing the object is known as objective or field lens and has large focal length and aperture while the other facing the eye is known as eye-piece or ocular and has small focal length and aperture.

– When the final image is formed at infinity (normal adjustment),

$$M = \dfrac{f_o}{f_e}$$

Length of tube, $L = f_o + f_e$

– When the final image is formed at least distance of distinct vision,

$$M = \dfrac{f_o}{f_e}\left(1+\dfrac{f_e}{D}\right)$$

Length of tube, $L = f_o + \dfrac{f_e D}{f_e + D}$

Practice Time

## ➲ Multiple Choice Questions (MCQs)

1. From a point source, a light falls on a spherical glass surface (μ = 1.5 and radius of curvature = 10 cm). The distance between point source and glass surface is 50 cm. The position of image is
(a) 25 cm
(b) 50 cm
(c) 100 cm
(d) 150 cm

2. A screen is placed 90 cm away from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Find the focal length of lens.
(a) 42.8 cm
(b) 21.4 cm
(c) 10.7 cm
(d) 5.5 cm

3. A convergent beam of light passes through a diverging lens of focal length 0.2 m and comes to focus 0.3 m behind the lens. The position of the point at which the beam would converge in the absence of the lens is
(a) 0.12 m
(b) 0.6 m
(c) 0.3 m
(d) 0.15 m

4. The focal length of the lenses of an astronomical telescope are 50 cm and 5 cm. The length of the telescope when the image is formed at the least distance of distinct vision is
(a) 45 cm
(b) 55 cm
(c) $\dfrac{275}{6}$ cm
(d) $\dfrac{325}{6}$ cm

5. Two lenses of power +10 D and –5 D are placed in contact. Where should an object be held from the lens, so as to obtain a virtual image of magnification 2?
(a) 5 cm
(b) –5 m
(c) 10 cm
(d) –10 cm

6. Mirage is a phenomenon due to
(a) refraction of light
(b) total internal reflection of light
(c) diffraction of light
(d) none of these.

7. A biconvex lens has a focal length 2/3 times the radius of curvature of either surface. The refractive index of the lens material is
(a) 1.75
(b) 1.33
(c) 1.5
(d) 1.0

8. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m s^{-1} and stops at the focus. The image
(a) moves away from the lens with an uniform speed 5 m s$^{-1.}$
(b) moves away from the lens with an uniform acceleration.
(c) moves away from the lens with a non-uniform acceleration.
(d) moves towards the lens with a non-uniform acceleration.

9. A convex lens of focal length 0.2 m and made of glass ($^a\mu_g$ = 1.5) is immersed in water ($^a\mu_w$ = 1.33). Find the change in the focal length of the lens.
(a) 5.8 m
(b) 0.58 cm
(c) 0.58 m
(d) 5.8 cm

10. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the separation between the objective and the eyepiece?
(a) 0.75 m
(b) 1.38 m
(c) 1.0 m
(d) 1.5 m

11. The far point of a near sighted person is 6.0 m from her eyes, and she wears contacts that enable her to see distant objects clearly. A tree is 18.0 m away and 2.0 m high. How high is the image formed by the contacts?
(a) 1.0 m
(b) 1.5 m
(c) 0.75 m
(d) 0.50 m

12. A ray of light is incident on a thick slab of glass of thickness t as shown in the figure. The emergent ray is parallel to the incident ray but

displaced sideways by a distance d. If the angles are small then d is,
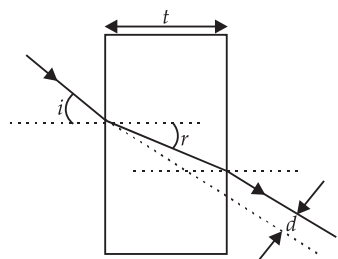
(a) $t\left(1-\dfrac{i}{r}\right)$ (b) $rt\left(1-\dfrac{i}{r}\right)$

(c) $it\left(1-\dfrac{r}{i}\right)$ (d) $t\left(1-\dfrac{r}{i}\right)$

13. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen,
(a) half the image will disappear
(b) complete image will disappear
(c) intensity of image will decreases
(d) intensity of image will increases.

14. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. If water is replaced by a liquid of refractive index 1.63 upto the same height, by what distance would the microscope have to be moved to focus on the needle again?
(a) 1.00 cm (b) 2.37 cm
(c) 1.73 cm (d) 3.93 cm

15. A mark placed on the surface of a sphere is viewed through glass from a position directly opposite. If the diameter of the sphere is 10 cm and refractive index of glass is 1.5. The position of the image will be
(a) –20 cm (b) 30 cm
(c) 40 cm (d) – 10 cm

16. Two lenses of focal lengths 20 cm and –40 cm are held in contact. The image of an object at infinity will be formed by the combination at
(a) 10 cm (b) 20 cm
(c) 40 cm (d) infinity

17. The final image in an astronomical telescope with respect to an object is
(a) virtual and erect (b) real and erect
(c) real and inverted (d) virtual and inverted

18. An object is placed at a distance of 1.5 m from a screen and a convex lens is interposed between them. The magnification produced is 4. The focal length of the lens is
(a) 1 m (b) 0.5 m (c) 0.24 m (d) 2 m

19. The image of the needle placed 45 cm from a lens is formed on a screen placed 90 cm on the other side of the lens. The displacement of the image, if the needle is moved by 5.0 cm away from the lens is
(a) 10 cm, towards the lens
(b) 15 cm, away from the lens
(c) 15 cm, towards the lens
(d) 10 cm, away from the lens

20. A real image of a distant object is formed by a planoconvex lens on its principal axis. Spherical aberration is
(a) absent.
(b) smaller, if the curved surfaces of the lens face the object.
(c) smaller, if the plane surface of the lens faces the object.
(d) same, whichever side of the lens faces the object.

21. A convex lens is dipped in a liquid whose refractive index is equal to the refractive index of the lens. Then its focal length will
(a) become zero
(b) become infinite
(c) become small, but non-zero
(d) remain unchanged

22. A concave mirror of focal length f_1 is placed at a distance d from a convex lens of focal length f_2. A beam of light coming from infinity and falling on this convex lens – concave mirror combination returns to infinity. The distance d must equal
(a) $f_1 + f_2$ (b) $-f_1 + f_2$
(c) $2f_1 + f_2$ (d) $-2f_1 + f_2$

23. A ray incident at a point at an angle of incidence of 60° enters a glass sphere of refractive index $\sqrt{3}$ and is reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is
(a) 50° (b) 60°
(c) 90° (d) 40°

24. For a glass prism $(\mu = \sqrt{3})$ the angle of minimum deviation is equal to the angle of the prism. The angle of the prism is
(a) 45° (b) 30° (c) 90° (d) 60°

25. Light travels in two media A and B with speeds 1.8×10^8 m s^{-1} and 2.4×10^8 m s^{-1} respectively. Then the critical angle between them is

(a) $\sin^{-1}\left(\dfrac{2}{3}\right)$

(b) $\tan^{-1}\left(\dfrac{3}{4}\right)$

(c) $\tan^{-1}\left(\dfrac{2}{3}\right)$

(d) $\sin^{-1}\left(\dfrac{3}{4}\right)$

26. Two identical glass $\left(\mu_g = \dfrac{3}{2}\right)$ equiconvex lenses of focal length f are kept in contact. The space between the two lenses is filled with water $\left(\mu_w = \dfrac{4}{3}\right)$. The focal length of the combination is

(a) f

(b) $\dfrac{f}{2}$

(c) $\dfrac{4f}{3}$

(d) $\dfrac{3f}{4}$

27. A point source of light is placed at a depth of h below the surface of water of refractive index μ. A floating opaque disc is placed on the surface of water so that light from the source is not visible from the surface. The minimum diameter of the disc is

(a) $\dfrac{2h}{(\mu^2 - 1)^{1/2}}$

(b) $2h(\mu^2 - 1)^{1/2}$

(c) $\dfrac{h}{2(\mu^2 - 1)^{1/2}}$

(d) $h(\mu^2 - 1)^{1/2}$

28. Critical angle for light going from medium (i) to (ii) is θ. The speed of light in medium (i) is v, then the speed of light in medium (ii) is

(a) $v(1 - \cos\theta)$

(b) $\dfrac{v}{\sin\theta}$

(c) $\dfrac{v}{\cos\theta}$

(d) $\dfrac{v}{(1 - \sin\theta)}$

29. A ray of light passes through an equilateral prism (refractive index 1.5) such that angle of incidence is equal to angle of emergence and the latter is equal to 3/4th of the angle of prism. The angle of deviation is

(a) 60°
(b) 30°
(c) 45°
(d) 120°

30. A concave lens is placed in contact with a convex lens of focal length 25 cm. The combination produces a real image at a distance of 80 cm, If an object is at a distance of 40 cm, the focal length of concave lens is

(a) − 400 cm
(b) − 200 cm
(c) + 400 cm
(d) + 200 cm

31. Double convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

(a) 11 cm
(b) 22 cm
(c) 7 cm
(d) 6 cm

32. A small angle prism (μ = 1.62) gives a deviation of 4.8°. The angle of prism is

(a) 5°
(b) 6.36°
(c) 3°
(d) 7.74°

33. The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?
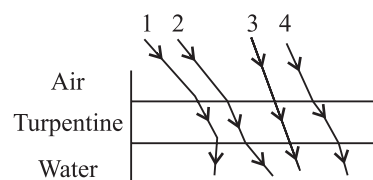
(a) 1
(b) 2
(c) 3
(d) 4

34. The power of a biconvex lens is 10 dioptre and the radius of curvature of each surface is 10 cm. Then the refractive index of the material of the lens is

(a) $\dfrac{3}{2}$

(b) $\dfrac{4}{3}$

(c) $\dfrac{9}{8}$

(d) $\dfrac{5}{3}$

35. A vessel of depth x is half filled with oil of refractive index μ_1 and the other half is filled with water of refractive index μ_2. The apparent depth of the vessel when viewed from above is

(a) $\dfrac{x(\mu_1 + \mu_2)}{2\mu_1\mu_2}$

(b) $\dfrac{x\,\mu_1\mu_2}{2(\mu_1 + \mu_2)}$

(c) $\dfrac{x\,\mu_1\mu_2}{(\mu_1 + \mu_2)}$

(d) $\dfrac{2x(\mu_1 + \mu_2)}{\mu_1\mu_2}$

36. There are certain material developed in laboratories which have a negative refractive index. A ray incident from air (medium 1) into such a medium (medium 2) shall follow a path given by
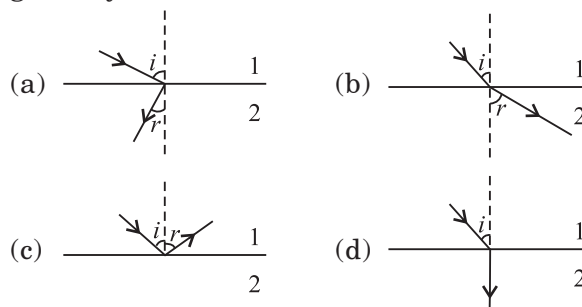
37. A point luminous object (O) is at a distance h from front face of a glass slab of width d and of refractive index μ. On the back face of the slab is a reflecting plane mirror. An observer sees the image of object in mirror [figure]. Distance of image from front face as seen by observer will be
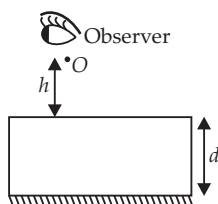
(a) $h + \dfrac{2d}{\mu}$

(b) $2h + 2d$

(c) $h + d$

(d) $h + \dfrac{d}{\mu}$

38. You are given four sources of light each one providing a light of a single colour – red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statements is correct, if the source of yellow light is replaced with that of other lights without changing the angle of incidence?

(a) The beam of red light would undergo total internal reflection.

(b) The beam of red light would bend towards normal while it gets refracted through the second medium.

(c) The beam of blue light would undergo total internal reflection.

(d) The beam of green light would bend away from the normal as it gets refracted through the second medium.

39. An astronomical refractive telescope has an objective of focal length 20 m and an eyepiece of focal length 2 cm. Then

(a) the magnification is 1000.

(b) the length of the telescope tube is 20.02 m

(c) the image formed is inverted

(d) all of these.

40. A compound microscope consists of an objective lens with focal length 1.0 cm and eye piece of focal length 2.0 cm and a tube length 20 cm the magnification will be

(a) 100   (b) 200   (c) 250   (d) 300

⊃ Case Based MCQs

Case I : Read the passage given below and answer the following questions from 41 to 45.

Refraction Through Lens

A convex or converging lens is thicker at the centre than at the edges. It converges a parallel beam of light on refraction through it. It has a real focus. Convex lens is of three types : (i) Double convex lens (ii) Plano-convex lens (iii) Concavo-convex lens. Concave lens is thinner at the centre than at the edges. It diverges a parallel beam of light on refraction through it. It has a virtual focus.

41. A point object O is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in figure.
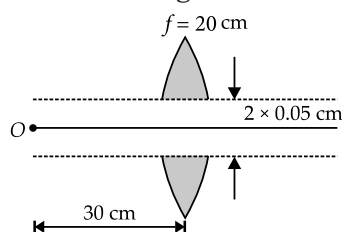
What will be the location of the image?

(a) 30 cm right of lens (b) 60 cm right of lens

(c) 70 cm left of lens   (d) 40 cm left of lens

42. Two thin lenses are in contact and the focal length of the combination is 80 cm. If the focal length of one lens is 20 cm, the focal length of the other would be.

(a) –26.7 cm   (b) 60 cm

(c) 80 cm   (d) 20 cm

43. A spherical air bubble is embedded in a piece of glass. For a ray of light passing through the bubble, it behaves like a

(a) converging lens

(b) diverging lens

(c) plano-converging lens

(d) plano-diverging lens

44. Lens used in magnifying glass is

(a) Concave lens

(b) Convex lens

(c) Both (a) and (b)

(d) Neither concave lens nor convex lens

45. The magnification of an image by a convex lens is positive only when the object is placed
(a) at its focus F
(b) between F and $2F$
(c) at $2F$
(d) between F and optical centre

Case II : Read the passage given below and answer the following questions from 46 to 49.

Compound Microscope

A compound microscope is an optical instrument used for observing highly magnified images of tiny objects. Magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye by the object, when both the final image and the object are situated at the least distance of distinct vision from the eye. It can be given that : $m = m_e \times m_o$, where m_e is magnification produced by eye lens and m_o is magnification produced by objective lens.

Consider a compound microscope that consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm.

46. The object distance for eye-piece, so that final image is formed at the least distance of distinct vision, will be
(a) 3.45 cm
(b) –5 cm
(c) –1.29 cm
(d) 2.59 cm

47. How far from the objective should an object be placed in order to obtain the condition described in Q.No. 46?
(a) 4.5 cm
(b) 2.5 cm
(c) 1.5 cm
(d) 3.0 cm

48. The intermediate image formed by the objective of a compound microscope is
(a) real, inverted and magnified
(b) real, erect, and magnified
(c) virtual, erect and magnified
(d) virtual, inverted and magnified.

49. The magnifying power of a compound microscope increases when
(a) the focal length of objective lens is increased and that of eye lens is decreased.
(b) the focal length of eye lens is increased and that of objective lens is decreased.
(c) focal lengths of both objective and eye-piece are increased.
(d) focal lengths of both objective and eye-piece are decreased.

➲ Assertion & Reasoning Based MCQs

For question numbers 50-55, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

50. Assertion (A) : Higher is the refractive index of a medium or denser the medium, lesser is the velocity of light in that medium.
Reason (R) : Refractive index is inversely proportional to velocity.

51. Assertion (A) : Convergent lens property of converging remains same in all media.
Reason (R) : Property of lens whether the ray is diverging or converging is independent of the surrounding medium.

52. Assertion (A) : Endoscopy involves use of optical fibres to study internal organs.
Reason (R) : Optical fibres are based on phenomena of total internal reflection.

53. Assertion(A): If optical density of a substance is more than that of water, then the mass density of substance can be less than water.
Reason (R) : Optical density and mass density are not related.

54. Assertion (A) : Microscope magnifies the image.
Reason (R) : Angular magnification for image is more than object in microscope.

55. Assertion (A) : A double convex lens ($\mu = 1.5$) has focal length 10 cm. When the lens is immersed in water ($\mu = 4/3$) its focal length becomes 40 cm.
Reason (R) : $\dfrac{1}{f} = \dfrac{\mu_g - \mu_m}{\mu_m}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

## ➲ Very Short Answer Type Questions (VSA)

1. When a monochromatic light travels from one medium to another its wavelength changes but frequency remains the same. Explain.

2. When red light passing through a convex lens is replaced by light of blue colour, how will the focal length of the lens change?

3. Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid?

4. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light?

5. A compound microscope is used because a realistic simple microscope does not have _____ magnification.

6. Two thin lenses of power –4 D and 2 D are placed in contact coaxially. Find the focal length of the combination.

7. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15 , 25 and 35 respectively. In which media would the velocity of light be minimum?

8. A biconvex lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens? Give reason.

9. An optical instrument uses an objective lens of power 100 D and an eyepiece of power 40 D. The final image is formed at infinity when the tube length of the instrument is kept at 20 cm. Identify the optical instrument.

10. What is the angle of incidence for maximum deviation through a prism?

## ➲ Short Answer Type Questions (SA-I)

11. A ray of light falls on a transparent sphere with centre C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the angle of refraction at A if refractive index of the material of the sphere is $\sqrt{3}$.
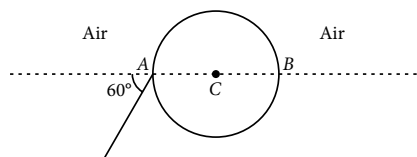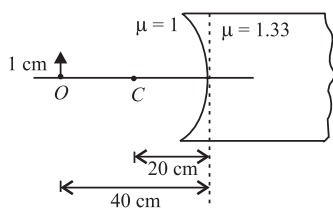

12. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens?

13. When a ray passes a glass slab of thickness t at an angle i with an angle of refraction r, what is the lateral shift of the emergent ray?
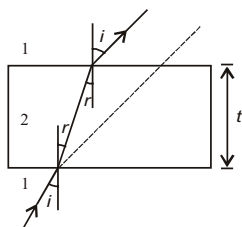
14. Calculate the focal length of a biconvex lens if the radii of its surfaces are 60 cm and 15 cm, and index of refraction of the lens glass = 1.5.

15. Draw the image, when an object is kept beyond the focal point of a convex lens.

16. Draw the image, when an object is placed between the focus and pole of a convex lens.

17. A thin prism of angle $A = 6°$ produces a deviation $\delta = 3°$. Find the refractive index of the material of prism.

18. A parallel beam of light is incident on a thin lens as shown. The radius of curvature of both surfaces is R. Determine the focal length of this system.
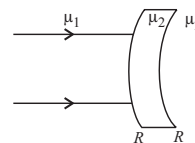
19. A point object O is kept in air as shown. The radius of the curved part is 20 cm and medium on the other side is of refractive index 1.33. Find the image distance.
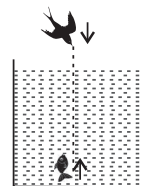
A fish in the tank is rising upward along the same line with speed 8 cm/s.

[Take : μ_{water} = 4/3] Find :

(i) speed of the image of fish as seen by the bird directly

20. A bird in air is diving vertically over a tank with speed 9 cm/s. Base of the tank is silvered.

(ii) speed of image of bird relative to the fish looking upwards.

➲ Short Answer Type Questions (SA-II)

21. A coin 2 cm in diameter is embedded in a solid glass ball of radius 30 cm. The index of refraction of the ball is 1.5, and the coin is 20 cm from the surface. Find the position and height of the image of the coin.

22. What is the difference in the construction of an astronomical telescope and a compound microscope? The focal lengths of the objective and eyepiece of a compound microscope are 1.25 cm and 5.0 cm, respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 when the final image is formed at the near point.

23. Define power of a lens. Write its units. Deduce the relation $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$ for two thin lenses kept in contact coaxially.

24. (a) State the necessary conditions for producing total internal reflection of light.

(b) Draw ray diagrams to show how specially designed prisms make use of total internal reflection to obtain inverted image of the object by deviating rays (i) through 90° and (ii) through 180°.

25. Which two of the following lenses L_1, L_2, and L_3 will you select as objective and eyepiece for constructing best possible (i) telescope (ii) microscope? Give reason to support your answer.

Lens	Power (P)	Aperture (A)
L_1	6 D	1 cm
L_2	3 D	8 cm
L_3	10 D	1 cm

26. A small bulb (assumed to be a point source) is placed at the bottom of a tank containing water to a depth of 80 cm. Find out the area of the surface of water through which light from the bulb can emerge. Take the value of the refractive index of water to be 4/3.

27. (a) Obtain relation between the critical angle of incidence and the refractive index of the medium.

**(b)** Explain briefly how the phenomenon of total internal reflection is used in fibre optics.

28. Three rays of light red (R), green (G) and blue (B) fall normally on one of the sides of an isosceles right angled prism as shown. The refractive index of prism for these rays is 1.39, 1.44 and 1.47 respectively. Find which of these rays get internally reflected and which get only refracted from AC. Trace the paths of rays. Justify your answer with the help of necessary calculations.
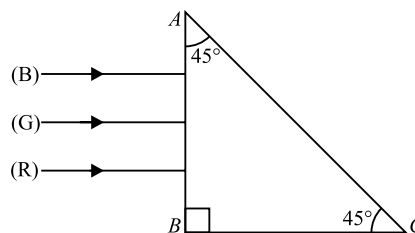
29. (a) A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. Find the magnifying power of the telescope for viewing distant objects when

(i) the telescope is in normal adjustment,

(ii) the final image is formed at the least distance of distinct vision.

(b) Also find the separation between the objective lens and the eyepiece in normal adjustment.

30. Find the equivalent focal length of the combination of lens.
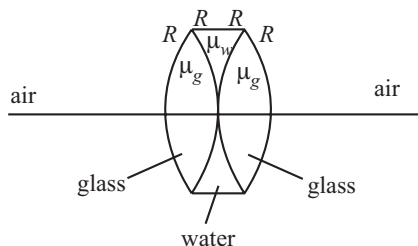
**31.** An isosceles prism has one of the refracting surface silvered. A ray of light is incident normally on the refracting face AB. After two reflections the ray emerges from the base of the prism perpendicular to it. Find the angle of the prism.

32. A sphere of radius R is made of material of refractive index μ_2. Where would an object be placed so that a real image is formed at equidistant from the sphere?
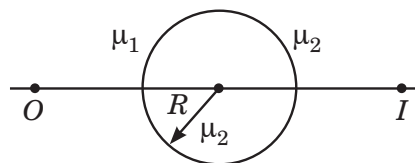
33. Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination.

➲ Long Answer Type Questions (LA)

34. (a) A point object is placed in front of a double convex lens (of refractive index $n = n_2/n_1$ with respect to air) with its spherical faces of radii of curvature R_1 and R_2. Show the path of rays due to refraction at first and subsequently at the second surface to obtain the formation of the real image of the object. Hence obtain the lens-maker's formula for a thin lens.

(b) A double convex lens having both faces of the same radius of curvature has refractive index 1.55. Find out the radius of curvature of the lens required to get the focal length of 30 cm.

35. (a) Plot a graph for angle of deviation as a function of angle of incidence for a triangular prism.

(b) Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation.

36. (a) Define magnifying power of a telescope. Write its expression.

(b) A small telescope has an objective lens of focal length 150 cm and an eye piece of focal length 5 cm. If this telescope is used to view a 100 m high tower 3 km away, find the height of the final image when it is formed 25 cm away from the eye piece.

ANSWERS

1. (b) : Using, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

Here, $\mu_1 = 1$, $\mu_2 = 1.5$, $u = -50$ cm, $R = 10$ cm

∴ $\dfrac{1.5}{v} - \dfrac{1}{(-50)} = \dfrac{(1.5-1)}{10}$

or $\dfrac{1.5}{v} = 0.05 - 0.02 = 0.03 \implies v = 50$ cm

2. (b) : The image of the object can be located on the screen for two positions of convex lens such that u and v are exchanged.

The separation between two positions of the lens is $d = 20$ cm

From the figure,
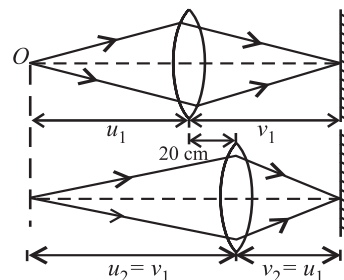
$u_1 + v_1 = 90$ cm

$v_1 - u_1 = 20$ cm

On solving,

$v_1 = 55$ cm, $u_1 = 35$ cm

From lens formula,

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{55} - \dfrac{1}{-35} = \dfrac{1}{f}$ or $\dfrac{1}{55} + \dfrac{1}{35} = \dfrac{1}{f}$

$\implies f = \dfrac{55 \times 35}{90} = 21.4$ cm

3. (a) : Here, $f = -0.2$ m, $v = +0.3$ m

The lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \therefore \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{0.3} + \frac{1}{0.2} = \frac{0.5}{0.06}$$

$$\therefore \quad u = \frac{0.06}{0.5} = 0.12 \text{ m}$$

4. (d) : Here, $f_o = 50$ cm, $f_e = 5$ cm, $D = 25$ cm

The length of the telescope when the image is formed at the least distance of distinct vision is

$$L = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5 \times 25}{5 + 25} = 50 + \frac{25}{6} = \frac{325}{6} \text{ cm}$$

5. (d) : Here, $P_1 = 10$ D and $P_2 = -5$ D

Therefore, power of the combined lens is

$P = P_1 + P_2 = +10 + (-5) = +5$ D

Now, magnification, $m = \dfrac{f}{u+f}$

Here, $m = 2$ and $f = \dfrac{1}{P} = \dfrac{1}{5} = 0.2$ m $= 20$ cm

$$\therefore \quad 2 = \frac{20}{u+20} \Rightarrow u + 20 = 10$$

$$\Rightarrow \quad u = -10 \text{ cm}$$

6. (b) : Mirage is phenomenon due to total internal reflection of light.

7. (a) : $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

Here, $f = \dfrac{2}{3}R$, $R_1 = +R$, $R_2 = -R$

$$\therefore \quad \frac{1}{(2/3)R} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{(\mu-1) \times 2}{R}$$

$$\mu - 1 = \frac{3}{4} = 0.75 \Rightarrow \mu = 1.75$$

8. (c) : When an object moving towards a convergent lens from the left of the lens with a uniform speed of 5 m s^{-1}, the image moves away from the lens with a non uniform acceleration.

9. (c) : Using $\dfrac{1}{f_a} = (^a\mu_g)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

Here, $f_a = 0.2$ m, $^a\mu_g = 1.50$

$$\therefore \quad \frac{1}{0.2} = (1.50 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0.50\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{R_1} - \frac{1}{R_2} = 10$$

Consider f_w be the focal length of the lens, when immersed in water. If $^w\mu_g$ be refractive index of glass w.r.t. water, then

$$^w\mu_g = \frac{^a\mu_g}{^a\mu_w} = \frac{1.50}{1.33} = 1.128$$

Now, $\dfrac{1}{f_w} = (^w\mu_g - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (1.128 - 1) \times 10 = 1.28$

or $f_w = \dfrac{1}{1.28} = 0.78$

Hence, change in focal length of the lens,

$f_w - f_a$

$= 0.78 - 0.2 = 0.58$ m

10. (d) : The separation between the objective and the eye piece = Length of the telescope tube *i.e.* $f = f_o + f_e$

Here, $f_o = 144$ cm $= 1.44$ m

$f_e = 6.0$ cm $= 0.06$ m

$\therefore \quad f = 1.44 + 0.06 = 1.5$ m

11. (d) : The far point of 6.0 m tell us that the focal length of the lens is $f = -6.0$ m, $u = -18$ m and $h = 2$ m

Using, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} \Rightarrow \dfrac{1}{v} = \dfrac{1}{f} + \dfrac{1}{u} = \dfrac{1}{-6.0} - \dfrac{1}{18.0}$

$\Rightarrow \quad v = -4.5$ m

$\therefore$ The image size, $h' = h\left(\dfrac{-v}{u}\right) = 2 \times \left(-\dfrac{4.5}{18.0}\right) = 0.50$ m

12. (c) : From figure, in right angled $\triangle CDB$, $\angle CBD = (i - r)$

$$\therefore \quad \sin(i - r) = \frac{CD}{BC} = \frac{d}{BC}$$

or $d = BC \sin(i - r)$ ...(i)

Also, in right angled $\triangle CNB$

$$\cos r = \frac{BN}{BC} = \frac{t}{BC}$$

or $BC = \dfrac{t}{\cos r}$ ...(ii)

Substitute equation (ii) in equation (i), we get

$$d = \frac{t}{\cos r} \sin(i - r)$$

For small angles $\sin(i - r) \approx i - r$, $\cos r \approx 1$

$$\therefore \quad d = t(i - r), \quad d = it\left[1 - \frac{r}{i}\right]$$

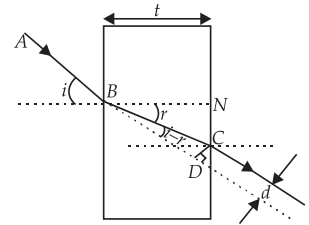**13. (c) :** Image formed is complete but has decreased intensity.

14. (c) : Actual depth of the needle in water, $h_1 = 12.5$ cm

Apparent depth of needle in water, $h_2 = 9.4$ cm

$$\therefore \quad \mu_{\text{water}} = \frac{h_1}{h_2} = \frac{12.5}{9.4} = 1.33$$

Hence, $\mu_{\text{water}} = 1.33$

When water is replaced by a liquid of refractive index $\mu' = 1.63$, the actual depth remains the same, but its apparent depth changes.

Let H be the new apparent depth of the needle.

$\therefore \quad \mu' = \dfrac{h_1}{H}$ or $H = \dfrac{h_1}{\mu'} = \dfrac{12.5}{1.63} = 7.67$ cm

Here, H is less than h_2. Thus to focus the needle again, the microscope should be moved up. Distance by which the microscope should be moved up
$= 9.4 - 7.67 = 1.73$ cm

15. (a) : As refraction occurs from denser to rarer medium
$\therefore \quad \dfrac{\mu_2}{-u} + \dfrac{\mu_1}{v} = \dfrac{\mu_1 - \mu_2}{R}$

Here, $\mu_1 = 1$, $\mu_2 = 1.5$, $R = 5$ cm, $u = -10$ cm
$\dfrac{1.5}{10} + \dfrac{1}{v} = \dfrac{1-1.5}{-5} = \dfrac{1}{10}$ or $\dfrac{1}{v} = \dfrac{1}{10} - \dfrac{3}{20} = \dfrac{-1}{20}$ or $v = -20$ cm

16. (c) : Equivalent focal length, $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

Here, $f_1 = 20$ cm and $f_2 = -40$ cm
$\dfrac{1}{f} = \dfrac{1}{20} - \dfrac{1}{40} = \dfrac{2-1}{40}$, $f = 40$ cm
From lens formuls, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$
Here, $f = 40$ cm, $u = \infty$
$\therefore \quad \dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{\infty} \Rightarrow v = f = 40$ cm

17. (d) : The final image in an astronomical telescope with respect an object is virtual and inverted.

18. (c) : Here, $m = \dfrac{v}{u} = -4$ or $u = \dfrac{-v}{4}$
Also, $|u| + |v| = 1.5$
$\dfrac{v}{4} + v = 1.5$ or $v = 1.2$ m and $u = \dfrac{-1.2}{4} = -0.3$ m
$\therefore \quad f = \dfrac{uv}{u-v} = \dfrac{-0.3 \times 1.2}{-0.3 - 1.2} = 0.24$ m

19. (c) : Here, $u = -45$ cm, $v = 90$ cm
$\therefore \quad \dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{90} + \dfrac{1}{45} = \dfrac{1}{30} \Rightarrow f = 30$ cm
When the needle is moved 5 cm away from the lens,
$u = -(45 + 5) = -50$ cm
$\therefore \quad \dfrac{1}{v'} = \dfrac{1}{f} + \dfrac{1}{u'} = \dfrac{1}{30} + \dfrac{1}{-50} = \dfrac{2}{150}$
or $v' = 75$ cm
$\therefore$ Displacement of image $= v - v' = 90 - 75 = 15$ cm, towards the lens.

20. (b) : When the curved surface of the lens faces the object, the spherical aberration is smaller. The total deviation is shared between the curved and the plane surfaces.

21. (b) : When refractive index of lens is equal to the refractive index of liquid, the lens behave like a plane surface with focal length infinity.

22. (c) :
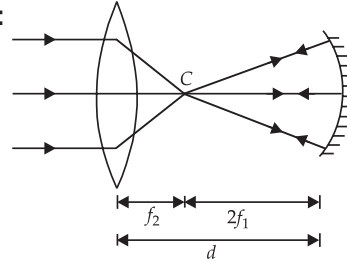
$\therefore \quad d = 2f_1 + f_2$

23. (c) : Refraction at P,
$\dfrac{\sin 60°}{\sin r_1} = \sqrt{3}$

$\sin r_1 = \dfrac{1}{2}$
or $r_1 = 30°$
Since, $r_2 = r_1$
$\therefore \quad r_2 = 30°$

Refraction at Q, $\dfrac{\sin r_2}{\sin i_2} = \dfrac{1}{\sqrt{3}}$ or $\dfrac{\sin 30°}{\sin i_2} = \dfrac{1}{\sqrt{3}}$ or $i_2 = 60°$

At point Q, $r_2' = r_2 = 30°$
$\therefore \quad \alpha = 180° - (r_2' + i_2) = 180° - (30° + 60°) = 90°$

24. (d) : Using, $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin \dfrac{A}{2}}$

According to question,
$\delta_m = A$

$\therefore \quad \sqrt{3} = \dfrac{\sin\left(\dfrac{A+A}{2}\right)}{\sin \dfrac{A}{2}}$ (Given, $\mu = \sqrt{3}$)

$\sqrt{3} = \dfrac{\sin A}{\sin \dfrac{A}{2}} = \dfrac{2\sin \dfrac{A}{2}\cos \dfrac{A}{2}}{\sin \dfrac{A}{2}}$

or $\cos \dfrac{A}{2} = \dfrac{\sqrt{3}}{2} = \cos 30° \Rightarrow \dfrac{A}{2} = 30°$ or $A = 60°$

25. (d) : Here, $v_A = 1.8 \times 10^8$ m s^{-1},
$v_B = 2.4 \times 10^8$ m s^{-1}
Light travels slower in denser medium. Hence medium A is a denser medium and medium B is a rarer medium.
Here, light travels from medium A to medium B. Let C be the critical angle between them.
$\therefore \quad \sin C = {}^A\mu_B = \dfrac{1}{{}^B\mu_A}$

Refractive index of medium B w.r.t. to medium A is
${}^A\mu_B = \dfrac{\text{Velocity of light in medium } A}{\text{Velocity of light in medium } B} = \dfrac{v_A}{v_B}$

$\therefore \ \sin C = \dfrac{v_A}{v_B} = \dfrac{1.8 \times 10^8}{2.4 \times 10^8} = \dfrac{3}{4}$ or $C = \sin^{-1}\left(\dfrac{3}{4}\right)$

26. (d) : Let R be the radius of curvature of each surface.

$\dfrac{1}{f} = (1.5 - 1)\left(\dfrac{1}{R} + \dfrac{1}{R}\right)$ $\therefore \ R = f$

For the water-lens, $\dfrac{1}{f'} = \left(\dfrac{4}{3} - 1\right)\left(-\dfrac{1}{R} - \dfrac{1}{R}\right)$

$= \dfrac{1}{3}\left(-\dfrac{2}{f}\right)$

or $\dfrac{1}{f'} = -\dfrac{2}{3f}$

Using, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$,

$\dfrac{1}{F} = \dfrac{1}{f} + \dfrac{1}{f} + \dfrac{1}{f'} = \dfrac{2}{f} - \dfrac{2}{3f} = \dfrac{4}{3f}$

$\therefore \ F = \dfrac{3f}{4}$

27. (a) : The figure shows incidence from water at critical angle θ_c for the limiting case.

Now, $\sin\theta_c = \dfrac{1}{\mu}$

so that $\tan\theta_c = \dfrac{1}{(\mu^2 - 1)^{1/2}}$

From figure, $\tan\theta_c = \dfrac{r}{h}$

where r is the radius of the disc.
Therefore, diameter of the disc is

$2r = 2h\tan\theta_c = \dfrac{2h}{(\mu^2 - 1)^{1/2}}$

28. (b) : $\sin\theta = \dfrac{\mu_2}{\mu_1} = \dfrac{v}{v'}$

where v and v' are the speeds of light in medium (i) and medium (ii) respectively.

$\therefore \ v' = \dfrac{v}{\sin\theta}$

29. (b) : Using, $A + \delta = i + e$
Here, $A = 60°$, $\mu = 1.5$,

$i = e = \dfrac{3}{4}A = \dfrac{3}{4} \times 60° = 45°$

$\therefore \ \ 60° + \delta = 45° + 45° \Rightarrow \delta = 90° - 60° = 30°$

30. (a) : $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

Here, $u = -40$ cm, $v = +80$ cm

$\therefore \ \ \dfrac{1}{f} = \dfrac{1}{80} - \dfrac{1}{-40} = \dfrac{1+2}{80} = \dfrac{3}{80}$

For the combination of focal length

$\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2} \Rightarrow \dfrac{1}{f_2} = \dfrac{1}{f} - \dfrac{1}{f_1}$

$\dfrac{1}{f_2} = \dfrac{3}{80} - \dfrac{1}{25} = \dfrac{15 - 16}{400}$

$\dfrac{1}{f_2} = \dfrac{-1}{400} \Rightarrow f_2 = -400$ cm

31. (b) : Lens maker formula,

$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

Here, $f = 20$ cm, $\mu = 1.55$, $R_1 = R$ and $R_2 = -R$

$\therefore \ \ \dfrac{1}{20} = (1.55 - 1)\left(\dfrac{1}{R} - \dfrac{1}{(-R)}\right) = 0.55 \times \left(\dfrac{1}{R} + \dfrac{1}{R}\right)$

or $\dfrac{1}{20} = 0.55 \times \dfrac{2}{R}$

$\therefore \ \ R = 1.1 \times 20 = 22$ cm
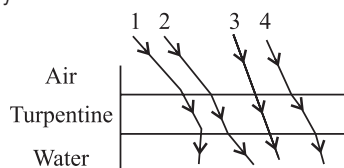
32. (d) : Here, $\mu = 1.62$, $\delta = 4.8°$
Using, $\delta = (\mu - 1)A$

$A = \dfrac{\delta}{(\mu - 1)} = \dfrac{4.8}{(1.62 - 1)}$

$A = 7.74°$

33. (b) : In the figure, the path shown for the ray 2 is correct. The ray suffers two refractions : At A, ray goes from air to turpentine, bending towards normal. At B, ray goes from turpentine to water (*i.e.*, From denser to rarer medium), bending away from normal.

34. (a) : Power of lens, P (in dioptre)

$= \dfrac{100}{\text{focal length } f \text{ (in cm)}}$

$\therefore \ f = \dfrac{100}{10} = 10$ cm
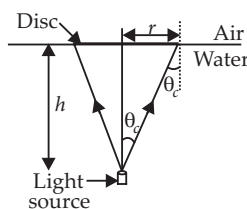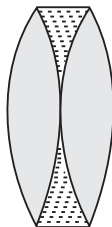
According to lens maker's formula

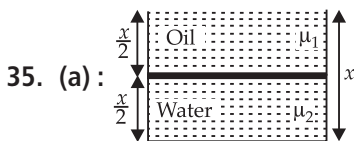$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

For biconvex lens, $R_1 = +R$, $R_2 = -R$

$\therefore \ \dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R} + \dfrac{1}{R}\right)$

$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{2}{R}\right) \Rightarrow \dfrac{1}{10} = (\mu - 1)\left(\dfrac{2}{10}\right)$

$(\mu - 1) = \dfrac{1}{2} \Rightarrow \mu = \dfrac{1}{2} + 1 = \dfrac{3}{2}$

35. (a) :

As refractive index, $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

∴ Apparent depth of the vessel when viewed from above is

$d_{apparent} = \dfrac{x}{2\,\mu_1} + \dfrac{x}{2\,\mu_2} = \dfrac{x}{2}\left(\dfrac{1}{\mu_1} + \dfrac{1}{\mu_2}\right)$

$= \dfrac{x}{2}\left(\dfrac{\mu_2 + \mu_1}{\mu_1\mu_2}\right) = \dfrac{x(\mu_1 + \mu_2)}{2\,\mu_1\mu_2}$

36. (a) : Using Snell's law

$\mu = \dfrac{\sin i}{\sin r} \Rightarrow \sin r = \dfrac{\sin i}{\mu}$

According to question, μ is negative

∴ $\sin r$ is negative.

Hence, r is negative, therefore the ray follows path as shown in option (a).

37. (a) : As shown in figure glass slab will form the image of bottom

i.e., mirror MM' at a depth $\left(\dfrac{d}{\mu}\right)$

from its front face. So the distance of object O from virtual mirror mm'

will be $h + \left(\dfrac{d}{\mu}\right)$.
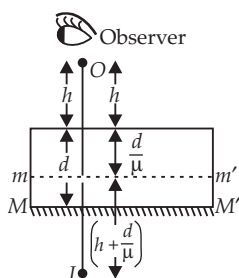
Now as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of

image I from mm' will be $h + \left(\dfrac{d}{\mu}\right)$ and as the distance of

virtual mirror from the front face of slab is $\left(\dfrac{d}{\mu}\right)$, the distance

of image I from front face as seen by observer will be

$= \left[h + \dfrac{d}{\mu}\right] + \dfrac{d}{\mu} = h + \dfrac{2d}{\mu}$

38. (c) : Here, for yellow light, $r = 90°$ when $i = C$. As i is kept same, C must be smaller for total internal reflection.

From $\mu = \dfrac{1}{\sin C}$, C will be smallest for blue colour.

Therefore, blue light would undergo total internal reflection.

39. (d) : In normal adjustment,

Length of telescope tube, $L = f_o + f_e$

Here, $f_o = 20$ m and $f_e = 2$ cm $= 0.02$ m

∴ $L = 20 + 0.02 = 20.02$ m

and magnification, $m = \dfrac{f_o}{f_e} = \dfrac{20}{0.02} = 1000$

The image formed is inverted with respect to the object.

40. (c) : Magnification, of compound microscope

$m = m_o \times m_e = \left(\dfrac{L}{f_o}\right) \times \left(\dfrac{D}{f_e}\right)$

Here, $L = 20$ cm, $D = 25$ cm (near point), $f_o = 1$ cm and $f_e = 2$ cm

∴ $m = \dfrac{20}{1} \times \dfrac{25}{2} = 250$

41. (b) : Each half of the lens will form an image in the same plane. The optic axes of the lenses are displaced,

$\dfrac{1}{v} - \dfrac{1}{(-30)} = \dfrac{1}{20}$; $v = 60$ cm

42. (a) : Here $f_1 = 20$ cm ; $f_2 = ?$

$F = 80$ cm

As $\dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{1}{F} \Rightarrow \dfrac{1}{f_2} = \dfrac{1}{F} - \dfrac{1}{f_1}$

$\dfrac{1}{f_2} = \dfrac{1}{80} - \dfrac{1}{20} = \dfrac{-3}{80}$

$f_2 = \dfrac{-80}{3} = -26.7\,\text{cm}$

43. (b) : The bubble behaves like a diverging lens.

44. (b) : Convex lens is used in magnifying glass.

45. (d) : Magnification of an image by a convex lens is positive when object is placed between F and optical centre.

46. (b) : Here $f_0 = 2.0$ cm, $f_e = 6.25$ cm, $u_0 = ?$

When the final image is obtained at the least distance of distinct vision, $v_e = -25$ cm

As $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

∴ $\dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} = \dfrac{1}{-25} - \dfrac{1}{6.25} = \dfrac{-1-4}{25} = \dfrac{-5}{25} = -\dfrac{1}{5}$

or $u_e = -5$ cm

47. (b) : Distance between objective and eye-piece $= 15$ cm

∴ Distance of the image from objective is

$v_0 = 15 - 5 = 10$ cm

∴ $\dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{10} - \dfrac{1}{2} = \dfrac{1-5}{10} = -\dfrac{2}{5}$

or $u_0 = -\dfrac{5}{2} = -2.5$ cm

∴ Distance of object from objective $= 2.5$ cm

48. (a) : The intermediate image formed by the objective of a compound microscope is real, inverted and magnified.

49. (d) : Magnifying power of a compound microscope increases when fixed length of both objective and eye-piece are decreased.

50. (a) : Both assertion and reason are true and reason is the correct explanation of assertion.
According to Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$

or, $\mu_1 v_1 = \mu_2 v_2$

This shows that higher is the refractive index of a medium or denser the medium, lesser is the velocity of light in that medium.

51. (d) : A convex lens made of glass behaves as a convergent lens when placed in air or water. However when the same lens is immersed in carbon disulphide ($\mu = 1.63$), it behaves as a divergent lens. Therefore when a convergent lens is placed inside a transparent medium of refractive index greater than that of material of the lens, it behaves as a divergent lens. Behaviour of a lens depends on the refractive index of a surrounding medium.

52. (a) : An endoscope is made of optical fibres. Its core is made of optically denser material. Its outer cladding is made of optically rarer material. It is based on total internal reflection.

53. (a) : Optical density and mass density are not related to each other. Mass density is mass per unit volume. It is possible that mass density of an optically denser medium be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media). *e.g.*, turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher.

54. (a) : Microscope is an optical instrument which forms a magnified image of a small nearby object and thus, increases the visual angle subtended by the image at the eye so that the object is seen to be bigger and distinct. Therefore, angular magnification for image is more than object.

55. (a) : $\dfrac{1}{f'} = (^{w}\mu_g - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right] = \left(\dfrac{\mu_g}{\mu_w} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ ...(i)

$\dfrac{1}{f} = \left(\dfrac{\mu_g}{\mu_\alpha} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$ ...(ii)

From (i) and (ii)

$\dfrac{f'}{f} = \dfrac{\left(\dfrac{\mu_g}{\mu_\alpha} - 1\right)}{\left(\dfrac{\mu_g}{\mu_w} - 1\right)} \Rightarrow f' = \dfrac{10 \times 8}{2} = 40 \text{ cm}$

1. Frequency being a characteristic of source of light, does not change with change of medium.
Refractive index μ of medium is defined as,

$$\mu = \frac{c}{v} = \frac{\text{(speed of light in vacuum)}}{\text{(speed of light in medium)}}$$

As, $v = \upsilon\lambda$

$\therefore \quad \mu \propto \dfrac{1}{\lambda}$ ($\because \upsilon$ is same in different media)

Hence, wavelength of light is different in different media.

2. Focal length of the lens decreases when red light is replaced by blue light.

3. When the refractive index of the biconvex lens is equal to the refractive index of the liquid in which lens is immersed then the biconvex lens behaves as a plane glass sheet. In this case,

$\dfrac{1}{f} = 0$ or $f \to \infty$.

4. $\because \ \lambda_{red} > \lambda_{violet}$ and $\lambda \propto \dfrac{1}{\mu}$

$\therefore \ \mu_{red} < \mu_{violet}$

$\mu = \dfrac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin\dfrac{A}{2}} \qquad \therefore \ (\delta_m)_{violet} > (\delta_m)_{red}$

5. A compound microscope is used because a realistic simple microscope does not have large magnification.

6. Net power $P = P_1 + P_2 = -4 + 2 = -2$ D

Focal length $f = \dfrac{1}{P} = \dfrac{1}{-2}$ m $= -0.5$ m $= -50$ cm

7. Refractive index, $\mu = \dfrac{c}{v} = \dfrac{\sin i}{\sin r}$

As $\sin 15° < \sin 25° < \sin 35°$
So, $v_A < v_B < v_C$
Hence in medium A, velocity of light is minimum.

8. The lens will act as a diverging lens as the refractive index of water is greater than that of lens.

9. (a) $P_o = 100$ D, $\therefore f_o = 1$ cm, $P_e = 40$ D,
$\therefore \quad f_e = 2.5$ cm.
Since $f_o < f_e$, the instrument is a compound microscope.

10. The deviation is maximum when angle is 90°.

11. From Snell's law, we have : $\dfrac{\sin(i)}{\sin(r)} = \mu$

At A, $i = 60°$; $\mu = \sqrt{3}$

Now, $\sin(r) = \dfrac{\sin(i)}{\mu}$

$\Rightarrow \quad \sin(r) = \dfrac{\sin(60°)}{\sqrt{3}} = \dfrac{1}{2} \Rightarrow r = \sin^{-1}\left(\dfrac{1}{2}\right)$

$\therefore \quad r = 30°$

12. Focal length of a concave lens is negative.
Using lens maker's formula,

$$\frac{1}{f} = \left(\frac{\mu_l}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu_l = 1.5$, $\mu_m = 1.65$

Also, $\dfrac{\mu_l}{\mu_m} < 1$, so $\left(\dfrac{\mu_l}{\mu_m} - 1\right)$ is negative and focal length of the given lens becomes positive. Hence, it behaves as a converging lens.

13. Using Snell's law, for ray 1 and its refraction,
$\sin i = \mu \sin r$ and $\sin e = \mu \sin r$
$\Rightarrow \angle i = \angle e$
The emergent ray is parallel to the incident ray.
Further,

$$AB = \frac{AN}{\cos r} = \frac{t}{\cos r}$$

And $BC = AB \sin(i - r)$

$\Rightarrow$ lateral shift $(d) = \dfrac{t}{\cos r} \sin(i - r)$

14. $R_1 = 60$ cm $R_2 = 15$ cm

μ_1 μ_2 μ_1
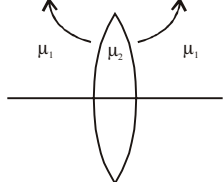
Using the Lens maker's Formula,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{60} - \frac{1}{-15}\right) = (0.5)\left[\frac{1}{60} + \frac{1}{15}\right]$$

$$\frac{1}{f} = (0.5)\left[\frac{1+4}{60}\right] \Rightarrow f = +24 \text{ cm.}$$

15. When the object is outside the focal point of a converging lens, the image is real and inverted.
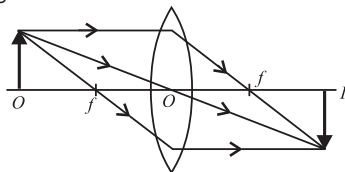
16. The object is between the pole and focal point of a converging lens. The image is virtual, enlarged and erect.
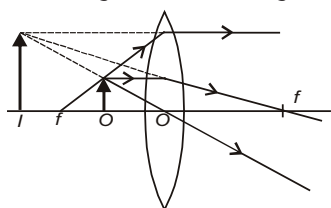
17. We know that $\delta = A(\mu - 1)$

or $\mu = 1 + \dfrac{\delta}{A}$

Here $A = 6°$, $\delta = 3°$ therefore $\mu = 1 + \dfrac{3}{6} = 1.5$

18. for Ist surface, $u_1 = \infty$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{\mu_2}{v_1} - \frac{\mu_1}{\infty} = \frac{\mu_2 - \mu_1}{+R} \quad \text{... (i)}$$

for 2nd surface, $u_2 = v_1$, $v_2 = f$

$$\frac{\mu_3}{f} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{R} \quad \text{... (ii)}$$

Adding eqn. (i) and (ii), we get

$$\frac{\mu_3}{f} = \frac{\mu_3 - \mu_1}{R} \Rightarrow f = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

19. $u = -40$ cm, $R = -20$ cm, $\mu = 1$, $\mu_2 = 1.33$
Applying equation for refraction through spherical surface, we get

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{or} \quad \frac{1.33}{v} - \frac{1}{-40} = \frac{1.33 - 1}{-20}$$

After solving, $v = -32$ cm

20. (i) Velocity of fish in air

$$= \frac{8}{4/3} = \frac{3}{4}(8) = 6 \text{ cm s}^{-1} \quad \text{(upwards)}$$

Velocity of fish w.r.t bird $= 6 + 9 = 15$ cm s^{-1} (upwards)

(ii) Velocity of bird in water $= 9 \times \dfrac{4}{3} = 12$ cm s^{-1} (downwards)

Velocity of bird w.r.t fish $= 12 + 8 = 20$ cm s^{-1} (downwards)

21. $u = -20$ cm,
$v = ?$,
$\mu_1 = 1.5$, $\mu_2 = 1$,
$R = -30$ cm

$$\Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1}{v} - \frac{1.5}{-20} = \frac{1 - 1.5}{-30} \Rightarrow v = -17.1 \text{ cm.}$$
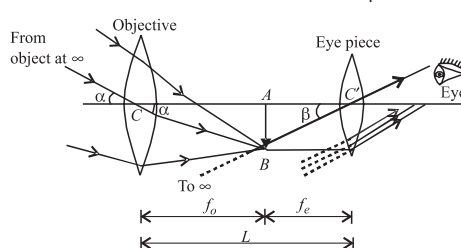
The positive sign indicates that the image is formed inside the glass or the image is virtual.

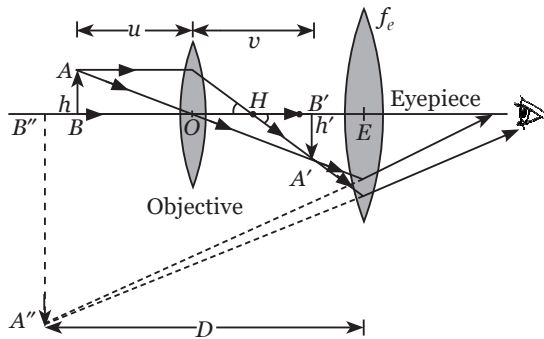$$m = \frac{v/\mu_2}{u/\mu_1} = \frac{-17.1/1}{-20/1.5} = \frac{-17.1 \times 1.5}{-20} = +1.28.$$

The positive value of m indicates it is an erect image.
(dia)$_{image}$ = $m \times$ (dia)$_{coin}$ = $1.28 \times 2 = 2.56$ cm.

22. Construction for astronomical telescope.

Construction for compound microscope :

Angular magnification, $m = \dfrac{v_0}{u_0}\left(1 + \dfrac{D}{fe}\right)$

$\Rightarrow\quad 30 = \dfrac{v_0}{u_0}\left(1 + \dfrac{25}{5}\right)$

$\Rightarrow\quad v_0 = 5u_0$ ...(i)
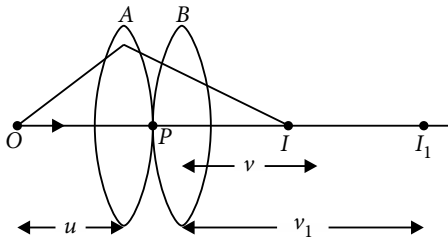
From lens formula,

$\dfrac{1}{f_0} = \dfrac{1}{v_0} - \dfrac{-1}{(-u_0)} \Rightarrow \dfrac{1}{1.25} = \dfrac{u_0 + v_0}{v_0 u_0}$ ...(ii)

Substituting (i) in (ii), $u_0 = 1.5$ cm

23. Power of lens : It is the reciprocal of focal length of a lens.

$P = \dfrac{1}{f}$ (f is in metre)

Unit of power of lens : Dioptre

An object is placed at point O. The lens A produces an image at I_1 which serves as a virtual object for lens B which produces final image at I.

Given, the lenses are thin. The optical centres (P) of the lenses A and B coincide with each other.

For lens A, we have

$\dfrac{1}{v_1} - \dfrac{1}{u} = \dfrac{1}{f_1}$ ...(i)

For lens B, we have $\dfrac{1}{v} - \dfrac{1}{v_1} = \dfrac{1}{f_2}$ ...(ii)

Adding equations (i) and (ii),

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$ ...(iii)

If two lenses are considered as equivalent to a single lens of focal length f, then

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ ...(iv)

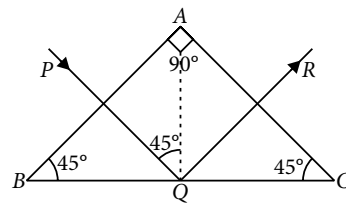From equation (iii) and equation (iv), we can write

$\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

24. (a) Essential conditions for total internal reflection:

(i) Light should travel from a denser medium to a rarer medium.

(ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.

(b) (i) To deviate a ray of light through 90° :

A totally reflecting prism is used to deviate the path of the ray of light through 90°, when it is inconvenient to view the direct light. In Michelson's method to find velocity of light, the direct light from the octagonal mirror is avoided from direct viewing by making use of totally reflecting prism.

(ii) To deviate a ray of light through 180° : When the ray of light comes to meet the hypotenuse face BC at right angles to it, it is refracted out of prism as such along the path RS. The path of the ray of light has been turned through 180° due to two total internal reflections.
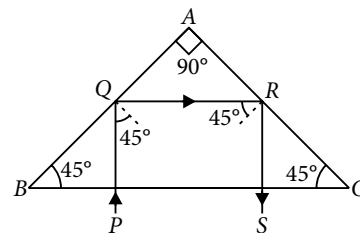
25. An astronomical telescope should have an objective of larger aperture and longer focal length while an eyepiece of small aperture and small focal length. Therefore, we will use L_2 as an objective and L_3 as an eyepiece.

For constructing microscope, L_3 should be used as objective and L_1 as eyepiece because both the lenses of microscope should have short focal lengths and the focal length of objective should be smaller than the eyepiece.

26. The light rays starting from bulb can pass through the surface if angle of incidence at surface is less than or equal to critical angle (C) for water air interface. If h is the depth of bulb from the surface, the light will emerge only through a
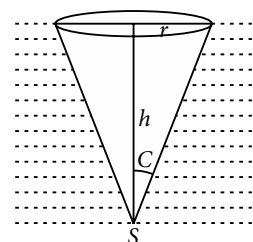
circle of radius r given by $r = \dfrac{h}{\sqrt{\mu^2 - 1}}$

Area of water surface $= \dfrac{\pi h^2}{\mu^2 - 1} = \dfrac{22}{7} \times \dfrac{(0.80)^2}{(1.33)^2 - 1} = 2.6 \text{ m}^2$

**27.** (a) Relation between refractive index and critical angle
When $i = C$, $r = 90°$
From Snell's law, $\mu_b \sin C = \mu_a \sin 90° = \mu_a \times 1$

$\dfrac{\mu_b}{\mu_a} = \dfrac{1}{\sin C}$; $^a\mu_b = \dfrac{1}{\sin C}$

(b) Optical fibre is made up of very fine quality glass or quartz of refractive index about 1.7.

A light beam incident on one end of an optical fibre at appropriate angle refracts into the fibre and undergoes repeated total internal reflection.

This is because the angle of incidence is greater than critical angle. The beam of light is received at other end of fibre with nearly no loss in intensity. To send a complete image, the image of different portion is send through separate fibres and thus a complete image can be transmitted through an optical fibre.

cladding $\mu = 1.5$

glass core $\mu = 1.7$
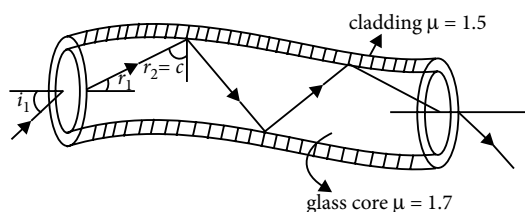
28. Critical angle for

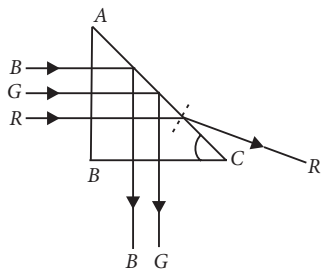(i) Red light is $\sin c_1 = \dfrac{1}{1.39} = 0.7194$

or $c_1 = 46°$

(ii) Green light is $\sin c_2 = \dfrac{1}{1.44} = 0.6944$

or $c_2 = 44°$

(iii) Blue light is $\sin c_3 = \dfrac{1}{1.47} = 0.6802$

or $c_3 = 43°$

As angle of incidence $i = 45°$ of red light ray on face AC is less that its critical angle of 46°, so red light ray will emerge out of face AC.

29. (a) (i) Given $f_0 = 140$ cm, $f_e = 5$ cm
When final image is at infinity, magnifying power,

$m = \dfrac{-f_0}{f_e} = -\dfrac{140}{5.0}$

$m = -28$

Negative sign shows that the image is inverted.

(ii) When final image is at the least distance of distinct vision,

magnifying power, $m = \dfrac{-f_0}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

$= \dfrac{-140}{5.0}\left(1 + \dfrac{5.0}{25}\right) = -33.6$

(b) Separation between objective and eye piece when final image is formed at infinity,

$L = f_0 + f_e$

$L = 140$ cm $+ 5.0$ cm

$L = 145$ cm

30. $\dfrac{1}{f_{eq}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$

First lens is glass lens with surrounding air.

$\dfrac{1}{f_1} = \left(\dfrac{\mu_g}{\mu_a} - 1\right)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right) = \dfrac{2}{R}(\mu_g - 1)$

Second lens is water lens with surrounding air.

$\dfrac{1}{f_2} = \left(\dfrac{\mu_w}{\mu_a} - 1\right)\left(\dfrac{1}{-R} - \dfrac{1}{+R}\right) = -\dfrac{2}{R}(\mu_w - 1)$

Third lens is glass lens with surrounding air.

$\dfrac{1}{f_3} = \left(\dfrac{\mu_g}{\mu_a} - 1\right)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right) = \dfrac{2}{R}(\mu_g - 1)$

$\dfrac{1}{f_{eq}} = \dfrac{2}{R}(\mu_g - 1) - \dfrac{2}{R}(\mu_w - 1) + \dfrac{2}{R}(\mu_g - 1)$

$\Rightarrow \quad f_{eq} = \dfrac{R}{4\mu_g - 2\mu_w - 2}$

Equivalent lens will have a focal length f_{eq} with air surrounding it.
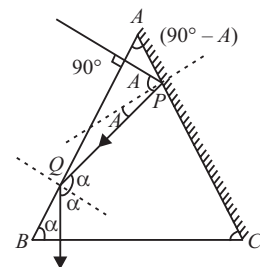
31. The incident ray passes without deviation from face AB. It suffers reflections at P and Q. From figure, incident ray and normal at Q are parallel; therefore
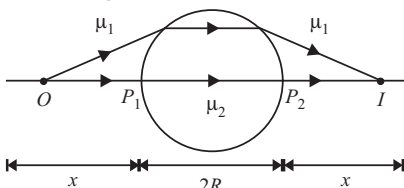
$\alpha = 2A$ ...(i)

Also, $2\alpha + A = 180°$ (Sum of angles of isosceles prism)   ...(ii)

On solving eqns. (i) and (ii), we get

$A = 36°$, $\alpha = 72°$

32. Let the object be placed at a distance x from the pole P_1 of the sphere. If a real image is to be formed at equidistant from the sphere, then the ray must pass symmetrically through the sphere, as shown in the figure. Applying the equation at the first surface, we get

$$\frac{\mu_2}{+\infty} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{-R} \text{ or } x = \left(\frac{\mu_1}{\mu_2 - \mu_1}\right)R$$

Note that the real image is formed only when the refractive index of the sphere is more than that of the surrounding, i.e., $\mu_2 > \mu_1$.

33.
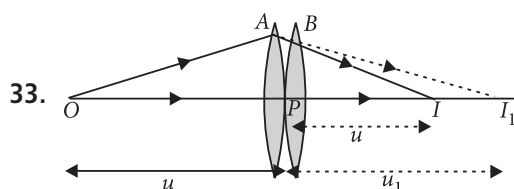
Consider two lenses placed close to each other. The focal lengths of lens A and B is f_1 and f_2 respectively.

For lens A,

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \qquad \text{... (i)}$$

For lens B,

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \qquad \text{...(ii)}$$

Adding (i) and (ii),

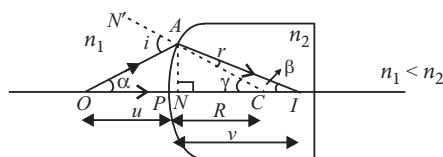$$\frac{1}{v'} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v'} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ then, $\frac{1}{f_T} = \frac{1}{f_1} + \frac{1}{f_2}\left(\because P = \frac{1}{f}\right)$

$P_T = P_1 + P_2$

34. (a) Refraction at convex spherical surface :
When object is in rarer medium and image formed is real.

In $\triangle OAC$, $i = \alpha + \gamma$
and in $\triangle AIC$, $\gamma = r + \beta$ or $r = \gamma - \beta$

∴   By Snell's law $^1n_2 = \frac{\sin i}{\sin r} \approx \frac{i}{r} = \frac{\alpha + \gamma}{\gamma - \beta}$

or $\frac{n_2}{n_1} = \frac{\alpha + \gamma}{\gamma - \beta}$ or $n_2\gamma - n_2\beta = n_1\alpha + n_1\gamma$

or $(n_2 - n_1)\gamma = n_1\alpha + n_2\beta$ ...(i)

As α, β and γ are small and P and N lie close to each other,

So, $\alpha \approx \tan\alpha = \frac{AN}{NO} \approx \frac{AN}{PO}$

$\beta \approx \tan\beta = \frac{AN}{NI} \approx \frac{AN}{PI}$

$\gamma \approx \tan\gamma = \frac{AN}{NC} \approx \frac{AN}{PC}$

On using them in equation (i), we get

$(n_2 - n_1)\frac{AN}{PC} = n_1\frac{AN}{PO} + n_2\frac{AN}{PI}$

or $\frac{n_2 - n_1}{PC} = \frac{n_1}{PO} + \frac{n_2}{PI}$ ...(ii)

where, $PC = + R$, radius of curvature

$PO = -u$, object distance

$PI = +v$, image distance

So $\frac{n_2 - n_1}{R} = \frac{n_1}{-u} + \frac{n_2}{v}$ or $\frac{n_2 - n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}$

This gives formula for refraction at spherical surface when object is in rarer medium.

(b) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here $f = 30$ cm, $\mu = 1.55$, $R_1 = -R_2 = R$

∴ $\frac{1}{30} = (1.55 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$ or $\frac{1}{30} = 0.55 \times \frac{2}{R}$

or $R = 0.55 \times 2 \times 30 = 33$ cm.

35. (a) If graph is plotted between angle of incidence i and angle of deviation δ, it is found that the angle of deviation δ first decreases with increase in angle of incidence i and then becomes minimum 'δ_m' when $i = e$ and then increases with increase in angle of incidence i. Figure shows the path of a ray of light suffering refraction through a prism of refracting angle 'A'.
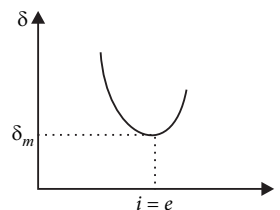
(b) At minimum deviation, the inside beam travels parallel to base of the prism.

$i = e$

$r = r'$
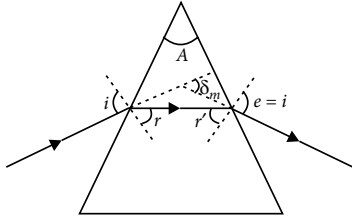
$\delta_m = (i + e) - (r + r')$

$\delta_m = 2i - 2r$ ...(i)

Also $r + r' = A = 2r$ ...(ii)

So, angle of incidence using equation (i)

$i = \dfrac{A + \delta_m}{2}$, angle of refraction $r = \dfrac{A}{2}$

Now refractive index of the material of prism

$^a\mu_g = \dfrac{\sin i}{\sin r} = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin \dfrac{A}{2}}$

where A is the "refracting angle" of the prism and $A = 60°$ for an equiangular prism.

36. (a) Magnifying power of refracting telescope (M) is defined as the ratio of the angle subtended by the image (β) at the eye to the angle subtended by the distant object at the unaided eye (α).

$$M = \dfrac{\beta}{\alpha}$$

(b)   Here, $f_o = 150$ cm, $f_e = 5$ cm

Angle subtended by 100 m tall tower at 3 km is

$$\alpha = \dfrac{100}{3 \times 1000} = \dfrac{1}{30}\ \text{rad}$$

If h is the height of image formed by the objective, then

$$\alpha = \dfrac{h}{f_o} = \dfrac{h}{150}$$

$\therefore$ $\dfrac{h}{150} = \dfrac{1}{30}$ or $h = \dfrac{150}{30}\ \text{cm} = 5\ \text{cm}$

Magnification produced by eyepiece

$$m_e = \left(1 + \dfrac{D}{f_e}\right) = \left(1 + \dfrac{25}{5}\right) = 6$$

$\therefore$ Height of final image $= h \times m_e = 5 \times 6 = 30$ cm

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