NAME : .....

RACE
Regular Analysis through Continuous Exercise

GRP 1.0 MATHEMATICS

REVISION ASSINGMENT # 04 (CONIC SECTION) TIME : 90 MIN.

SECTION–I(i)

Straight Objective Type (3 Marks each, –1 for wrong answer)

1. Let L is distance between two parallel normals of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b, then maximum value of L is -

(A) 2a (B) 2b (C) a + b (D) 2(a – b)

2. Let P is any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. S_1 and S_2 its foci then maximum area of ΔPS_1S_2 is (in square units)

(A) b^2e (B) a^2e (C) ab (D) abe

3. Area of the quadrilaterals formed by drawing tangents at the ends of latus recta of $\frac{x^2}{4}+\frac{y^2}{1}=1$ is

(A) $\frac{16}{\sqrt{3}}$ (B) $\frac{8}{\sqrt{3}}$ (C) $\frac{4}{\sqrt{3}}$ (D) $4\sqrt{3}$

4. Locus of a point which divides a chord of slope 2 of parabola $y^2 = 4x$ internally in the ratio 1 : 2 is another parabola having vertex as-

(A) $\left(\frac{8}{3},\frac{2}{3}\right)$ (B) $\left(\frac{2}{3},\frac{8}{3}\right)$ (C) $\left(\frac{2}{9},\frac{8}{9}\right)$ (D) $\left(\frac{8}{9},\frac{2}{9}\right)$

5. Locus of a point which moves such that tangents drawn from it to $y^2= 4x + 4$ and $y^2 = 8x + 16$ are mutually perpendicular, is-

(A) y = x + 2 (B) y = x + 3 (C) x + 3 = 0 (D) x + 2 = 0

6. The tangent at the point A (12, 6) to a parabola intersect its directrix at the point $B(-1,2)$. If the focus of the parabola lies on positive x-axis then square of distance of focus from point A is

(A) 180 (B) 37 (C) 89 (D) 270

7. Let a second degree curve $x^2 + 4y^2 – 4 = 0$ is intersected by a variable pair of straight lines $x^2+4y^2+\lambda xy = 0$ (where 'λ' is a real parameter), at two points A and B respectively, then the locus of the point of intersection of tangents at A and B is

(A) $4x^2 + 2y^2 + 3xy = 0$ (B) $2x^2 + y^2 + 6xy = 0$

(C) $(2x – y)(2x + y) = 0$ (D) $x^2 – 4y^2 = 0$

8. Consider the hyperbola H : $x^2 – 4y^2 = 4$ and a point $P(x_1, y_1)$ on it with $x_1 > 2$ & $y_1 > 0$. The normal and tangent to H at P meet x-axis at Q and R respectively. If (ℓ, m) is the centroid of ΔPQR, then $\frac{d\ell}{dx_1}$ is

(A) $\frac{3x_1}{4}+\frac{4}{3x_1}$ (B) $\frac{3}{4}-\frac{4}{3x_1^2}$ (C) $\frac{5}{4}-\frac{4}{x_1^2}$ (D) negative

Space for Rough Work

9. Let PQ be the double ordinate of the ellipse and AA' be the major axis of this ellipse. If PA and QA' produced meets at a point R, then locus of R is a

(A) circle (B) parabola (C) ellipse (D) hyperbola

10. The equation of common tangent to the curves $y^2 = 8x$ and $xy = -1$ is-

(A) $3y = 9x + 2$ (B) $y = 2x + 1$ (C) $y = x + 2$ (D) $2y = x + 8$

SECTION–I(ii)

Multiple Correct Answer Type (4 Marks each, –1 for wrong answer)

11. If two distinct tangents to the parabola $y^2 = 4ax$ can become normals to the circle $x^2 + y^2 - 2ax - 2by + c = 0$, then

(A) $a^2 > 4b^2$ (B) $b^2 > 4a^2$ (C) $a \in \left(-\frac{|b|}{2}, \frac{|b|}{2}\right)$ (D) $b \in \left(-\frac{|a|}{2}, \frac{|a|}{2}\right)$

12. Consider parabola $y^2 = 4x$. Then locus of centroid of a triangle formed by vertex of parabola and a variable focal chord is a conic

(A) whose eccentricity is $\sqrt{2}$ (B) whose eccentricity is 1

(C) whose length of latus rectum is $\frac{3}{4}$ (D) whose length of latus rectum is $\frac{4}{3}$

13. Let 'P' be a point on $y^2 = 4x$ which is at shortest distance from centre of circle $x^2 + y^2 - 4x - 16y + 64 = 0$ and Q is point of intersection of circle and line segment joining point P and centre of circle then

(A) slope of tangent of circle at point Q is equal to $\left(\frac{1}{2}\right)$.

(B) distance between centre of circle and point P is $2\sqrt{5}$.

(C) co-ordinates of point P is (4, 4).

(D) x-intercept made by normal at point P is 4.

14. A hyperbola intersects an ellipse $x^2 + 9y^2 = 9$ orthogonally. The eccentricity of the hyperbola is reciprocal of that of ellipse. If the axes of the hyperbola are along coordinate axes, then-

(A) vertices of hyperbola are $\left(\pm\frac{8}{3}, 0\right)$

(B) y coordinate of point of intersection of ellipse and hyperbola is either $\frac{1}{3}$ or $-\frac{1}{3}$

(C) latus rectum of hyperbola is $\frac{2}{3}$

(D) latus rectum of hyperbola is $\frac{4}{3}$

Space for Rough Work

15. Let from the point with abscissa 25, two tangents are drawn to the ellipse $24x^2 + 25y^2 = 600$ with foci at S_1 and S_2. The points of contact of tangents are A and B. If the distance of A from S_1 is $\frac{60}{13}$ units, then

(A) The distance of A from S_2 is $\frac{80}{13}$ units

(B) The distance of B from S_1 is 5 units

(C) The distance of B from S_2 is 5 units

(D) The distance of A from the directrix corresponding to S_2 is $\frac{350}{13}$ units.

16. A line through origin O meets the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 25$ at P and Q respectively on same side of origin. PR is drawn parallel to y-axis and QR is drawn parallel to x-axis. Locus of R will be a conic S = 0 such that

(A) S = 0 will touch both the given circles (B) eccentiricity of S = 0 will be 3 / 5

(C) eccentinicity of S = 0 will be 5/3 (D) Foci of S = 0 lies on $x^2 + y^2 = 16$

17. Let distance between parallel tangents of ellipse $x^2 + 2y^2 = 2$ is $\sqrt{5}$. If quadrilateral ABCD is formed by these tangents, then

(A) ABCD is a parallelogram. (B) area of quadrilateral ABCD is $\frac{10}{\sqrt{3}}$

(C) slope of tangents are $\pm\frac{1}{\sqrt{3}}$ (D) slope of tangents are ±3

18. If (2, 0) is one end of transverse axis, $x^2 + y^2 - 4y = 0$ is the director circle and (α, β) be the one focus of a hyperbola then $\alpha + \beta$ is less than

(A) – 2 (B) 1 (C) 4 (D) 6

SECTION–I(iii)

Linked Comprehension Type (Single Correct Answer Type) (3 Marks each, –1 for wrong answer)

Paragraph for Question 19 to 20

If the locus of the circumcentre of a variable triangle having sides y-axis, y = 2 and $\ell x + my = 1$, where (ℓ, m) lies on the parabola $y^2 = 4x$, is curve C, then

On the basis of above information, answer the following questions :

19. Vertex of curve C is -

(A) $\left(-2, \frac{3}{2}\right)$ (B) $\left(-2, -\frac{3}{2}\right)$ (C) $\left(2, \frac{3}{2}\right)$ (D) $\left(2, -\frac{3}{2}\right)$

20. Length of smallest focal chord of the curve C is -

(A) $\frac{1}{4}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

Space for Rough Work

Paragraph for Question 21 & 22

Let slope of normal at point P in first quadrant on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $-\frac{1}{2}$. Tangent and normal at point P meet x-axis at points T & N respectively such that $\frac{SN}{ST}=2$ (S is focus nearest to point P)

21. Eccentricity of hyperbola, is -

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{\frac{7}{2}}$ (D) $\sqrt{\frac{5}{2}}$

22. $\left(\frac{SP}{a}\right)$ is equal to -

(A) 1 (B) 2 (C) $\sqrt{7}$ (D) $\sqrt{5}$

Paragraph for Question 23 & 24

Let $S(x_1,0)$ for $x_1 > 0$ is foci of the ellipse $E:\frac{x^2}{9}+\frac{y^2}{4}=1$. Suppose a parabola whose vertex is $V\left(x_1\sqrt{5},0\right)$ touches the ellipse at points A and B in I and IV quadrant respectively. Axis of parabola is x-axis.

On the basis of above information, answer the following questions :

23. Equation of normal to the ellipse E at A is-

(A) $9x\sqrt{2}+3y+5\sqrt{2}=0$ (B) $9x\sqrt{2}-3y+5\sqrt{2}=0$

(C) $9x\sqrt{2}-3y=5\sqrt{2}$ (D) $9x\sqrt{2}-3y=5$

24. Tangent to the ellipse E at A and B meet at C, then area of ΔABC is-

(A) $32\sqrt{2}$ (B) $\frac{32\sqrt{2}}{3}$ (C) $12\sqrt{2}$ (D) $\frac{16\sqrt{2}}{3}$

Space for Rough Work

SECTION–I(v)

Matching list type (4 × 4) (Single option correct) (3 Marks each, –1 for wrong answer)

25. Let tangent of parabola $y^2 = 4x$ at point $P(x_1, y_1)$ and $Q(x_2, y_2)$ intersect each other at $R(-15, -2)$. If S is area of ΔPQR and (x_0, y_0) is circumcentre of ΔPQR, (where $y_2 > y_1$).

Match List-I with List-II and select the correct answer using the code given below the list.

	List-I		**List-II**
(P)	$x_1x_2x_0 - 2S$	(1)	208
(Q)	$y_1y_0 + \sqrt{S}x_0$	(2)	147
(R)	$\frac{x_1 + x_2 + 2S}{x_0}$	(3)	163
(S)	$\sqrt{S}x_2 + x_0$	(4)	182

Codes :

	P	Q	R	S
(A)	3	1	4	2
(B)	1	2	3	4
(C)	1	3	4	2
(D)	4	2	3	1

26. Match List-I with List-II and select the correct answer using the code given below the list.

	List-I		**List-II**
(P)	End points of a stick 'AB' of length 10m slides on the coordinate axes (Point A on x-axis) then locus of the point M dividing this stick such that $\frac{BM}{AM} = \frac{3}{2}$ is a curve whose eccentricity is e, then 3e is	(1)	$\sqrt{6}$
(Q)	AA' is the major axis of the ellipse $3x^2 + 2y^2 + 6x - 4y - 1 = 0$ and P is a variable point on ellipse, then greatest area of $\Delta APA'$ is	(2)	$2\sqrt{7}$
(R)	The distance between the foci of the curve represented by the equation $x = 1 + 4\cos\theta$, $y = 2 + 3\sin\theta$ is	(3)	$\frac{128}{3}$
(S)	Tangents are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ at the end points of the latusrectum, the area of the quadrilateral so formed is	(4)	$\sqrt{5}$

Codes :

	P	Q	R	S
(A)	1	2	3	4
(B)	4	1	3	2
(C)	4	2	1	3
(D)	4	1	2	3

SECTION–II (iii)

Numerical Grid Type (Upto Second Decimal place) (4 Marks each, –1 for wrong answer)

1. PQ and PR are two tangents to a parabola $y^2 = 4ax$ and tangent to parabola at a third point S cuts these tangents at A and B respectively then value of $\frac{PA}{PQ} + \frac{PB}{PR}$ is

2. If the hyperbola $x^2 - \frac{y^2}{4} = 1$ passes through the foci of the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1, (b < 2)$ then length of semi latus rectum of ellipse is

3. If circles C_1 & C_2 are drawn by taking foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ as centres, and a common tangent L is drawn to touch all three curves, then product of the radii of C_1 & C_2 is given by

4. If set of all values of k for which curves $y^2 = 4x$ and $y^2 = (x - k)$ have a common normal other than x-axis, is (α,∞), then α is

Space for Rough Work

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	C	C	C	C	D	A	A	B	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A,C	A,C	A,B,C	A,C,D	A,B	A	C	B	A	B
	Q.	21	22								
	A.	A	B								
SECTION-II	Q.	1	2	3	4	5					
	A.	3.00	7.20	2.50	3.00	2.00					