

Ex 26.1

Scalar Triple Product Ex 26.1 Q1(i)

We have

$$\begin{aligned} [\hat{i} \hat{j} \hat{k}] + [\hat{j} \hat{k} \hat{i}] + [\hat{k} \hat{i} \hat{j}] &= (\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{k} \times \hat{i}) \cdot \hat{j} \\ &= \hat{k} \cdot \hat{k} + \hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j} \\ &= 1 + 1 + 1 \\ &= 3 \end{aligned}$$

$$\text{Therefore, } [\hat{i} \hat{j} \hat{k}] + [\hat{j} \hat{k} \hat{i}] + [\hat{k} \hat{i} \hat{j}] = 3$$

Scalar Triple Product Ex 26.1 Q1(ii)

We have

$$\begin{aligned} [2\hat{i} \hat{j} \hat{k}] + [\hat{i} \hat{k} \hat{j}] + [\hat{k} \hat{j} 2\hat{i}] &= (2\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{i} \times \hat{k}) \cdot \hat{j} + (\hat{k} \times \hat{j}) \cdot 2\hat{i} \\ &= 2\hat{k} \cdot \hat{k} + (-\hat{j}) \cdot \hat{j} + (-\hat{i}) \cdot 2\hat{i} \\ &= 2 - 1 - 2 \\ &= -1 \end{aligned}$$

$$\text{Therefore, } [2\hat{i} \hat{j} \hat{k}] + [\hat{i} \hat{k} \hat{j}] + [\hat{k} \hat{j} 2\hat{i}] = -1$$

Scalar Triple Product Ex 26.1 Q2(i)

We have

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} \\ &= 2(-1-0) + 3(-1+3) \\ &= -2 + 6 \\ &= 4 \end{aligned}$$

$$\text{Therefore, } [\vec{a} \vec{b} \vec{c}] = 4$$

Scalar Triple Product Ex 26.1 Q2(ii)

We have

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} \\ &= 1(1+1) + 2(2+0) + 3(2-0) \\ &= 2 + 4 + 6 \\ &= 12 \end{aligned}$$

$$\text{Therefore, } [\vec{a} \vec{b} \vec{c}] = 12$$

Scalar Triple Product Ex 26.1 Q3(i)

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $|\vec{a} \vec{b} \vec{c}|$.

We have

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} \\ &= 2(4-1) - 3(2+3) + 4(-1-6) \\ &= 6 - 15 - 28 \\ &= -9 - 28 \\ &= -37 \end{aligned}$$

$$\text{Therefore, the volume of the parallelepiped is } |\vec{a} \vec{b} \vec{c}| = |-37| = 37 \text{ cubic unit.}$$

Scalar Triple Product Ex 26.1 Q3(ii)

$$\text{Let } \vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $|\vec{a} \vec{b} \vec{c}|$.

We have

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix} \\ &= 2(-4-1) + 3(-2+3) + 4(-1-6) \\ &= -10 + 3 - 28 \\ &= -10 - 25 \\ &= -35 \end{aligned}$$

$$\text{Therefore, the volume of the parallelepiped is } |\vec{a} \vec{b} \vec{c}| = |-35| = 35 \text{ cubic unit.}$$

Scalar Triple Product Ex 26.1 Q3(iii)

$$\text{Let } \vec{a} = 11\hat{i}, \vec{b} = 2\hat{j}, \vec{c} = 13\hat{k}$$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $|\vec{a} \cdot \vec{b} \times \vec{c}|$.

We have

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix} \\ &= 11(26 - 0) + 0 + 0 \\ &= 286 \end{aligned}$$

Therefore, the volume of the parallelepiped is $|\vec{a} \cdot \vec{b} \times \vec{c}| = |286| = 286$ cubic unit.

Scalar Triple Product Ex 26.1 Q1(iv)

$$\text{Let } \vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $|\vec{a} \cdot \vec{b} \times \vec{c}|$.

We have

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} \\ &= 1(1 - 2) - 1(-1 - 1) + 1(2 + 1) \\ &= -1 + 2 + 3 \\ &= 4 \end{aligned}$$

Therefore, the volume of the parallelepiped is $|\vec{a} \cdot \vec{b} \times \vec{c}| = |4| = 4$ cubic unit.

Scalar Triple Product Ex 26.1 Q4(i)

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff their scalar triple product is zero i.e. $[\vec{a} \vec{b} \vec{c}] = 0$.

Here,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & 2 & -1 \\ 3 & 2 & 7 \\ 5 & 6 & 5 \end{vmatrix} \\ &= 1(10 - 42) - 2(15 - 35) - 1(18 - 10) \\ &= -32 + 40 - 8 \\ &= 0 \end{aligned}$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q4(ii)

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff their scalar triple product is zero i.e. $[\vec{a} \vec{b} \vec{c}] = 0$.

Here,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \\ &= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32) \\ &= -60 + 126 - 66 \\ &= 0 \end{aligned}$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q5(i)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a} \vec{b} \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow [\vec{a} \vec{b} \vec{c}] = 0$$

$$\begin{aligned} \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} &= 0 \\ &= 1(\lambda - 1) + 1(2\lambda + \lambda) + 1(-2 - \lambda) \\ &= \lambda - 1 + 3\lambda - 2 - \lambda \\ 3 &= 3\lambda \\ 1 &= \lambda \end{aligned}$$

Scalar Triple Product Ex 26.1 Q5(ii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a} \vec{b} \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow [\vec{a} \vec{b} \vec{c}] = 0$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix} = 0$$

$$= 2(10 + 3\lambda) + 1(5 + 3\lambda) + 1(\lambda - 2\lambda)$$

$$= 20 + 6\lambda + 5 + 3\lambda - \lambda$$

$$-25 = 8\lambda$$

$$\lambda = -\frac{25}{8}$$

Scalar Triple Product Ex 26.1 Q5(iii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a} \vec{b} \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow [\vec{a} \vec{b} \vec{c}] = 0$$

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

$$= 1(2\lambda - 2) - 2(6 - 1) - 3(6 - \lambda)$$

$$= 2\lambda - 2 - 12 + 2 - 18 + 3\lambda$$

$$= 5\lambda - 30$$

$$30 = 5\lambda$$

$$\lambda = 6$$

Scalar Triple Product Ex 26.1 Q5(iv)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $[\vec{a} \vec{b} \vec{c}] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$$\Rightarrow [\vec{a} \vec{b} \vec{c}] = 0$$

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix} = 0$$

$$= 1(0 + 5) - 3(0 - 5\lambda) + 0$$

$$= 5 + 15\lambda$$

$$-5 = 15\lambda$$

$$\lambda = -\frac{1}{3}$$

Scalar Triple Product Ex 26.1 Q6

Let

$$OA = 6\hat{i} - 7\hat{j}, OB = 16\hat{i} - 19\hat{j} - 4\hat{k}, OC = 3\hat{j} - 6\hat{k}, OD = 2\hat{i} + 5\hat{j} + 10\hat{k}$$

$$AB = OB - OA = 10\hat{i} - 12\hat{j} - 4\hat{k}$$

$$AC = OC - OA = -16\hat{i} - 16\hat{j} + 2\hat{k}$$

$$CD = OD - OC = 2\hat{i} + 2\hat{j} + 16\hat{k}$$

$$AD = OD - OA = -4\hat{i} + 12\hat{j} + 10\hat{k}$$

The four points are co-planar if vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are co-planar.

$$\begin{vmatrix} 10 & -12 & -4 \\ -16 & -16 & 2 \\ -4 & 12 & 10 \end{vmatrix} = 16(-160 - 24) + 25(-160 + 8) - 4(-144 + 64) \neq 0$$

Hence the points are not coplanar.

Scalar Triple Product Ex 26.1 Q7

$AB = \text{position vector of } B - \text{position vector of } A$

$$= 4\hat{i} - 2\hat{j} - 2\hat{k}$$

$AC = \text{position vector of } C - \text{position vector of } A$

$$= -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$AD = \text{position vector of } D - \text{position vector of } A$

$$= -2\hat{i} - 2\hat{j} + 4\hat{k}$$

The four points are co-planar if the vectors are co-planar.

$$\text{Thus, } \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix} = 4[16 - 4] + 2[-8 - 4] - 2[4 + 8] = 48 - 24 - 24 = 0$$

Hence proved.

Scalar Triple Product Ex 26.1 Q8

$$\text{Let } \vec{OA} = 6\hat{i} - 7\hat{j}, \vec{OB} = 16\hat{i} - 19\hat{j} - 4\hat{k}, \vec{OC} = 3\hat{i} - 6\hat{k}, \vec{OD} = 2\hat{i} - 5\hat{j} + 10\hat{k}$$

Thus,

$$\vec{AB} = \vec{OB} - \vec{OA} = 10\hat{i} - 12\hat{j} - 4\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = -3\hat{i} + 7\hat{j} - 6\hat{k}$$

$$\vec{AD} = \vec{OD} - \vec{OA} = -4\hat{i} + 2\hat{j} + 10\hat{k}$$

The four points are co-planar if vectors $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are co-planar.

Thus, we have

$$\begin{vmatrix} 10 & -12 & -4 \\ -3 & 7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 10(70 + 12) + 12(-30 - 24) - 4(-6 + 28) = 820 - 648 - 88$$

Scalar Triple Product Ex 26.1 Q9

Let

$$\text{Position vector of } A = -\hat{j} - \hat{k}$$

$$\text{Position vector of } B = 4\hat{i} + 5\hat{j} + \lambda\hat{k}$$

$$\text{Position vector of } C = 3\hat{i} + 9\hat{j} + 4\hat{k}$$

$$\text{Position vector of } D = -4\hat{i} + 4\hat{j} + 4\hat{k}$$

The four points are coplanar if the vectors $\vec{AB}$, $\vec{AC}$, $\vec{AD}$ are coplanar.

$$\vec{AB} = 4\hat{i} + 6\hat{j} + (\lambda + 1)\hat{k}$$

$$\vec{AC} = 3\hat{i} + 10\hat{j} + 5\hat{k}$$

$$\vec{AD} = -4\hat{i} + 5\hat{j} + 5\hat{k}$$

$$\begin{vmatrix} 4 & 6 & (\lambda + 1) \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$$

$$4(50 - 25) - 6(15 + 20) + (\lambda + 1)(15 + 40) = 0$$

$$100 - 210 + 55 + 55\lambda = 0$$

$$55\lambda = 55$$

$$\lambda = 1$$

Scalar Triple Product Ex 26.1 Q10

$$(\vec{a} - \vec{b}) \cdot \{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$$

$$= [(\vec{a} - \vec{b}) \quad (\vec{b} - \vec{c}) \quad (\vec{c} - \vec{a})]$$

$$= [a \quad (\vec{b} - \vec{c}) \quad (\vec{c} - \vec{a})] + [-b \quad (\vec{b} - \vec{c}) \quad (\vec{c} - \vec{a})]$$

$$= 6[a \quad b \quad c] - 6[a \quad b \quad c]$$

$$= 0$$

Scalar Triple Product Ex 26.1 Q11

If $\vec{a}$ represents the sides AB, If $\vec{b}$ represents the sides BC, If $\vec{c}$ represents the sides AC of the triangle ABC.

$\vec{a} \times \vec{b}$ is perpendicular to the plane of the triangle ABC.

$\vec{b} \times \vec{c}$ is perpendicular to the plane of the triangle ABC.

$\vec{c} \times \vec{a}$ is perpendicular to the plane of the triangle ABC.

Hence $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a vector perpendicular to the plane of the triangle ABC.

Scalar Triple Product Ex 26.1 Q12(i)

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if

$$[a \quad b \quad c] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & c_3 \end{vmatrix} = 0$$

$$0 - 1(c_3) + 1(2) = 0$$

$$c_3 = 2$$

Scalar Triple Product Ex 26.1 Q12(ii)

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if

$$[a \quad b \quad c] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & c_2 & 1 \end{vmatrix} = 0$$

$$0 - 1 + 1(c_2) = 0$$

$$c_2 = 1$$

Scalar Triple Product Ex 26.1 Q13

Let

Position vector of $OA = 3\hat{i} + 2\hat{j} + \hat{k}$

Position vector of $OB = 4\hat{i} + \lambda\hat{j} + 5\hat{k}$

Position vector of $OC = 4\hat{i} + 2\hat{j} - 2\hat{k}$

Position vector of $OD = 6\hat{i} + 5\hat{j} - \hat{k}$

The four points are coplanar if the vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar.

$$\overrightarrow{AB} = \hat{i} + (\lambda - 2)\hat{j} + 4\hat{k}$$

$$\overrightarrow{AC} = \hat{i} + 0\hat{j} - 3\hat{k}$$

$$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\begin{vmatrix} 1 & (\lambda - 2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$1(9) - (\lambda - 2)(-2 + 9) + 4(3 - 0) = 0$$

$$9 - 7\lambda + 14 + 12 = 0$$

$$7\lambda = 35$$

$$\lambda = 5$$