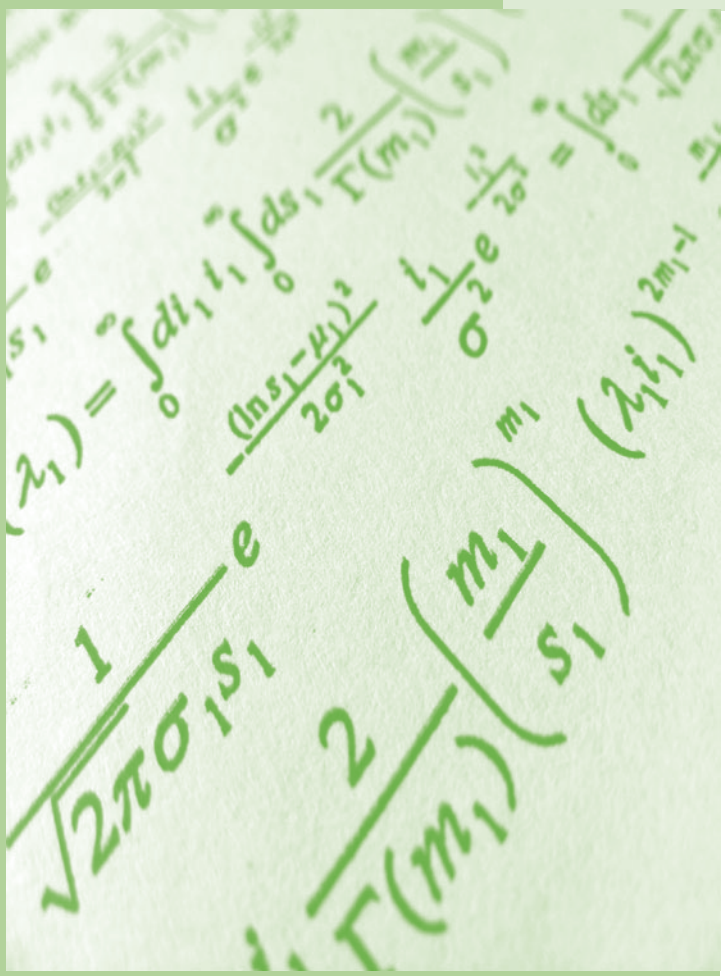
Chapter

2

Polynomials and Rational Expressions



REMEMBER

Before beginning this chapter, you should be able to:

- Explain about polynomials, their types, and basic operations on polynomials
- Work on factorization of polynomials

KEY IDEAS

After completing this chapter, you would be able to:

- Define terms such as HCF and LCM
- Understand the methods to find LCM and HCF of polynomials
- Study relation among the LCM, the HCF and the product of polynomials
- Learn fundamental operations on rational expressions

INTRODUCTION

In this chapter we will learn about HCF and LCM of polynomials and the method of finding them. Further we will learn about rational expressions and also the method of expressing them in their lowest terms.

POLYNOMIAL OF *n*TH DEGREE

The expression of the form $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$ is a polynomial of nth degree ($a_0 \neq 0$) in one variable. Here $a_0, a_1, \ldots, a_n$ are real numbers.

HCF of Given Polynomial

For the given two polynomials, $f(x)$ and $g(x)$, $r(x)$ can be taken as the highest common factor, if

1. $r(x)$ is a common factor of $f(x)$ and $g(x)$ and
2. every common factor of $f(x)$ and $g(x)$ is also a factor of $r(x)$.

Highest common factor is generally referred to as HCF.

Method for Finding HCF of the Given Polynomials

Step 1: Express each polynomial as a product of powers of irreducible factors which also requires the numerical factors to be expressed as the product of the powers of primes.

Step 2: If there is no common factor then HCF is 1 and if there are common irreducible factors, we find the least exponent of these irreducible factors in the factorised form of the given polynomials.

Step 3: Raise the common irreducible factors to the smallest or the least exponents found in step 2 and take their product to get the HCF.

Examples:

1. Find the HCF of $42a^2b^2$ and $48ab^3$.
   Let $f(x) = 42a^2b^2$ and $g(x) = 48ab^3$
   Writing $f(x)$ and $g(x)$ as a product of powers of irreducible factors,
   $f(x) = 2 \times 3 \times 7 \times a^2 \times b^2$
   $g(x) = 2 \times 2 \times 2 \times 2 \times 3 \times a \times b^3 = 2^4 \times 3 \times a \times b^3$
   The common factors with the least exponents are 2, 3 and ab^2.
   $\therefore$ The HCF of the given polynomials $= 2 \times 3 \times ab^2 = 6ab^2$.
2. Find the HCF of $96(x-1)(x+1)^2(x+3)^3$ and $64(x^2-1)(x+3)(x+2)^2$
   Let $f(x) = 96(x-1)(x+1)^2(x+3)^3$ and
   $g(x) = 64(x^2-1)(x+3)(x+2)^2$.
   Writing $f(x)$ and $g(x)$ as the product of the powers of irreducible factors,
   $f(x) = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times (x-1) \times (x+1)^2 \times (x+3)^3$
   $g(x) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times (x-1)(x+1)(x+3)(x+2)^2$.
   The common factors with the least exponents are 2^5, $(x-1)$, $(x+1)$, $(x+3)$.
   $\therefore$ The HCF of the given polynomials $= 32 \times (x-1)(x+1)(x+3)$
   $= 32(x^2-1)(x+3)$.

LCM of the Given Polynomials

Least Common Multiple or the Lowest Common Multiple is the product of all the factors (taken once) of the polynomials given with their highest exponents respectively.

Method to Calculate LCM of the Given Polynomials

Step 1: First express each polynomial as a product of powers of irreducible factors.

Step 2: Consider all the irreducible factors (only once) occurring in the given polynomials. For each of these factors, consider the greatest exponent in the factorised form of the given polynomials.

Step 3: Now raise each irreducible factor to the greatest exponent and multiply them to get the LCM.

Examples:

1. Find the LCM of $42a^2b^2$ and $48ab^3$.

Let $p(x) = 42a^2b^2$ and $q(x) = 48ab^3$

Writing $p(x)$ and $q(x)$ as the product of the powers of irreducible factors,

$p(x) = 2 \times 3 \times 7 \times a^2 \times b^2$

$q(x) = 2 \times 2 \times 2 \times 2 \times 3 \times a \times b^3$

Now all the factors (taking only once) with the highest exponents are 2^4, 3, 7, a^2, b^3

$\Rightarrow$ The LCM of the given polynomials $= 2^4 \times 3 \times 7 \times a^2 \times b^3 = 336a^2b^3$.

2. Find the LCM of $96(x - 1)(x + 1)^2(x + 3)^3$ and $64(x^2 - 1)(x + 3)(x + 2)^2$

Let $f(x) = 96(x - 1)(x + 1)^2(x + 3)^3$ and $g(x) = 64(x^2 - 1)(x + 3)(x + 2)^2$

$f(x) = 2 \times 2 \times 2 \times 2 \times 2 \times 3\ (x - 1)(x + 1)^2\ (x + 3)^3$

$g(x) = 2 \times 2 \times 2 \times 2 \times 2 \times 2\ (x - 1)(x + 1)(x + 3)(x + 2)^2$

Now all the factors (taking only once) with the highest exponents are 2^6, 3, $(x - 1)$, $(x + 1)^2$, $(x + 3)^3$, $(x + 2)^2$.

$\Rightarrow$ The LCM of the given polynomials $= 192(x - 1)(x + 1)^2(x + 2)^2(x + 3)^3$.

Relation among the HCF, the LCM and the Product of the Polynomials

If $f(x)$ and $g(x)$ are two polynomials then we have the following relation, (HCF of $f(x)$ and $g(x)$) $\times$ (LCM of $f(x)$ and $g(x)$) $= \pm$ ($f(x) \times g(x)$).

Example:

Let $f(x) = (x + 3)^2(x - 1)(x + 2)^3$ and $g(x) = (x + 3)(x - 1)^2(x + 2)^2$ be two polynomials.

The common factors with the least exponents are $(x + 3)$, $(x - 1)$, $(x + 2)^2$.

$\Rightarrow$ HCF $= (x - 1)(x + 3)(x + 2)^2$

All the factors (taken only once) with the highest exponents are $(x + 3)^2$, $(x - 1)^2$, $(x + 2)^3$.

$\Rightarrow$ LCM $= (x - 1)^2(x + 2)^3(x + 3)^2$

Now $f(x) \times g(x) = \{(x + 3)^2(x - 1)(x + 2)^3\} \times \{(x + 3)(x - 1)^2(x + 2)^2\} = (x - 1)^3(x + 2)^5\ (x + 3)^3$

LCM $\times$ HCF $= \{(x - 1)(x + 3)(x + 2)^2\} \times \{(x - 1)^2(x + 2)^3(x + 3)^2\} = (x - 1)^3(x + 3)^3\ (x + 2)^5$

Thus, we say, LCM $\times$ HCF = Product of two polynomials.

EXAMPLE 2.1

If $f(x) = (x - 2)(x^2 - x - a)$, $g(x) = (x + 2)(x^2 + x - b)$ and their HCF is $x^2 - 4$, then find the value of $a - b$. (a and b are constants) from the following options.

(a) 2 **(b)** 3 **(c)** -4 **(d)** 0

SOLUTION

$f(x) = (x - 2)(x^2 - x - a)$

$g(x) = (x + 2)(x^2 + x - b)$

HCF $= x^2 - 4 = (x + 2)(x - 2)$

$\therefore$ $x + 2$ is a factor of $(x^2 - x - a)$

$\therefore$ $(-2)^2 - (-2) - a = 0 \quad \Rightarrow \quad a = 6$

$x - 2$ is a factor of $x^2 + x - b$

$\therefore$ $2^2 + 2 - 3 = 0$

$b = 6$ $\therefore$ $a - b = 0$.

EXAMPLE 2.2

If the LCM of the polynomials $f(x) = (x + 1)^5 (x + 2)^a$ and $g(x) = (x + 1)^b (x + 2)^a$ is $(x + 1)^a (x + 2)^b$, then find the minimum value of $a + b$ from the following options.

(a) is 10 **(b)** is 14 **(c)** is 15 **(d)** Cannot say

SOLUTION

$f(x) = (x + 1)^5 (x + 2)^a$

$g(x) = (x + 1)^b (x + 2)^5$

LCM $= (x + 1)^a (x + 2)^b$

As $(x + 1)^a$ is a multiple of both $(x + 1)^5$ and $(x + 1)^b$, $a \geq 5$ and $a \geq b$ (1)

As $(x + 2)^b$ is a multiple of both $(x + 2)^a$ and $(x + 2)^5$, $b \geq 5$ and $b \geq a$ (2)

From Eqs. (1) and (2),

$a = b$, $a \geq 5$ and $b \geq 5$

$\therefore$ Minimum value of $a + b$ is $5 + 5$, i.e., 10.

Rational Expressions

We know that any number of the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$ is called a rational number. As integers and polynomials behave in the same manner, we observe most of the properties satisfied by rational numbers are also satisfied by the algebraic expressions of the form $\frac{f(x)}{g(x)}$ which are called rational expressions. Rational expression is 'an algebraic expression which is of the form $\frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are polynomials and $g(x)$ is not a zero polynomial.

For any rational number of the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$. p and q are called numerator and denominator respectively. Even though p and q are integers $\frac{p}{q}$ need not be an integer. Similarly for any rational expression $\frac{f(x)}{g(x)}$, $f(x)$ is called numerator and $g(x)$ is called denominator.

Eventhough $f(x)$ and $g(x)$ are polynomials $\frac{f(x)}{g(x)}$ need not be a polynomial.

Examples:

1. $\frac{2x-1}{x^2-3x+1}$ is a rational expression.
2. $\frac{x^3+5x^2-\sqrt{3x}+\sqrt{5}}{2x^2-\sqrt{5x}+8}$ is a rational expression.
3. $\frac{x^2-5\sqrt{x}-1}{3x-5}$ is not a rational expression as the numerator is not a polynomial.

Notes

1. Every polynomial is a rational expression as $f(x)$ can be written as $\frac{f(x)}{1}$.
2. $\frac{f(x)}{g(x)}$ is not a rational expression if either numerator $f(x)$ or denominator $g(x)$ or both $f(x)$ and $g(x)$ are not polynomials.

WORKOUT

Which of the following algebraic expressions is/are not polynomials?

(A) $x^3+2x^3+\sqrt{7}x+4$ (B) $5x^2+4\sqrt{x}-11$

(C) $\frac{x^3+3x^2-8x+11}{4x\sqrt{x}-3x+3}$ (D) $\frac{x^3+3x^2-6x+13}{x^2+1}$

(a) A, B and C **(b)** A and C **(c)** B and C **(d)** A and D

SOLUTION

Recall definition of polynomials.

Rational Expressions in Lowest Terms

Let $f(x)$ and $g(x)$ have integer coefficients and HCF of $f(x)$ and $g(x)$ is 1, then the rational expression $\frac{f(x)}{g(x)}$ is said to be in its lowest terms.

If HCF of $f(x)$ and $g(x)$ is not equal to 1, then by cancelling HCF of $f(x)$ and $g(x)$ from both numerator and denominator, we can reduce the given rational expression in its lowest terms.

EXAMPLE 2.3

Verify whether the rational expression $\frac{x^2-1}{(2x+1)(x+2)}$ is in its lowest terms.

SOLUTION

The given rational expression can be written as $\frac{(x+1)(x-1)}{(2x+1)(x+2)}$ and clearly HCF of numerator and denominator of the given expression is 1.

$\therefore$ It is in its lowest terms.

EXAMPLE 2.4

Express the rational expression $\frac{x^2-2x-3}{2x^2-3x-5}$ in its lowest terms.

SOLUTION

Factorizing both the numerator and the denominator of the given expression we have

$$\frac{x^2-2x-3}{2x^2-3x-5}=\frac{(x+1)(x-3)}{(2x-5)(x+1)}$$

HCF of numerator and denominator is $x+1$.

Dividing both numerator and denominator of the given expression by $(x+1)$, we have $\frac{x-3}{2x-5}$ which is in its lowest terms.

Addition/Subtraction of Rational Expressions

The sum of any two rational expressions $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{p(x)}$ is written as

$$\frac{f(x)}{g(x)}+\frac{h(x)}{p(x)}=\frac{f(x)\,p(x)+h(x)\,g(x)}{g(x)\,p(x)}$$

If the denominators $g(x)$ and $p(x)$ are equal then $\frac{f(x)}{g(x)}+\frac{h(x)}{p(x)}=\frac{f(x)+h(x)}{g(x)}$.

The difference of the above rational expressions can be written as

$$\frac{f(x)}{g(x)}-\frac{h(x)}{p(x)}=\frac{f(x)\,p(x)-h(x)\,g(x)}{g(x)\,p(x)}.$$

Notes

1. Sum or difference of two rational expressions is also a rational expression.
2. For any rational expression $\frac{f(x)}{g(x)}+\frac{-f(x)}{g(x)}$ is called the additive inverse of $\frac{f(x)}{g(x)}$.

   That is, $\frac{f(x)}{g(x)}+\left(\frac{-f(x)}{g(x)}\right)=0.$

EXAMPLE 2.5

Simplify $\frac{x+1}{2x-1}+\frac{3x-2}{x-1}$.

SOLUTION

$$\frac{x+1}{2x-1}+\frac{3x-2}{x-1}=\frac{(x+1)(x-1)+(3x-2)(2x-1)}{(2x-1)(x-1)}=\frac{x^2-1+6x^2-7x+2}{2x^2-3x+1}=\frac{7x^2-7x+1}{2x^2-3x+1}.$$

EXAMPLE 2.6

Find the sum of the rational expressions $\frac{2x-3}{x^2+x-2}$ and $\frac{3x-1}{2x^2+5x+2}$.

SOLUTION

$$\frac{2x-3}{x^2+x-2}+\frac{3x-1}{2x^2+5x+2}=\frac{2x-3}{(x+2)(x-1)}+\frac{3x-1}{(2x+1)(x+2)}$$

$$=\frac{(2x-3)(2x+1)+(3x-1)(x-1)}{(x-1)(x+2)(2x+1)}$$

$$=\frac{4x^2-4x-3+3x^2-4x+1}{(x-1)(x+2)(2x+1)}=\frac{7x^2-8x-2}{2x^2+3x^2-4x-2}.$$

EXAMPLE 2.7

If $A=\frac{x+1}{2x-1}$, $B=\frac{2x+1}{3x+2}$ and $C=\frac{4x-5}{2x^2+5x-3}$, then find $4A-3B+C$.

SOLUTION

$$4A-3B+C=\frac{4(x+1)}{2x-1}-\frac{3(2x-1)}{3x+2}+\frac{4x-5}{(2x-1)(x+3)}\ (\because 2x^2+5x-3=(2x-1)(x+3))$$

$$=\frac{4(x+1)(3x+2)(x+3)-3(2x-1)^2(x+3)+(4x-5)(3x+2)}{(2x-1)(3x+2)(x+3)}$$

$$=\frac{4(3x^3+14x^2+17x+6)-3(4x^3+8x^2-11x+3)+12x^2-7x-10}{(2x-1)(3x+2)(x+3)}$$

$$=\frac{44x^2+94x+5}{6x^2+19x^2+x-6}.$$

EXAMPLE 2.8

Which one should be added among the following options to $\frac{1}{x-2}+\frac{1}{x+2}$ to get $\frac{4x^3}{x^4-16}$?

(a) $\frac{2x^2}{x^2+4}$

(b) $\frac{2x}{x^2+4}$

(c) $\frac{2x^2}{x^2-4}$

(d) $\frac{2}{x^2+4}$

SOLUTION

Let A be the required expression

$$\frac{1}{x-2}+\frac{1}{x+2}+A=\frac{4x^3}{x^4-16}$$

$$\therefore\ A=\frac{4x^3}{x^4-16}-\left(\frac{1}{x-2}+\frac{1}{x+2}\right)$$

$$=\frac{4x^3}{x^4-16}-\frac{2x}{x^2-4}$$

$$=\frac{2x}{x^2-4}\left[\frac{2x^2}{x^2+4}-1\right]$$

$$=\frac{2x}{x^2-4}\left[\frac{2x^2-x^2-4}{x^2+4}\right]=\frac{2x}{x^2+4}.$$

Multiplication of Rational Expressions

The product of two rational expressions $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{p(x)}$ is given by

$$\frac{f(x)}{g(x)}\times\frac{h(x)}{p(x)}=\frac{f(x)\cdot h(x)}{g(x)\cdot p(x)}.$$

Notes

1. The process of finding the
   (i) product of two rational expressions is similar to the process of finding the product of two rational numbers.
   (ii) product of two rational expressions is also a rational expression.
2. After finding the product of two rational expressions the resultant rational expression must be put in its lowest terms.

EXAMPLE 2.9

Find the product of the rational expressions $\frac{3x^2+8x-3}{2x^2-x-6}$ and $\frac{x^2-4}{x+3}$.

SOLUTION

Product of the given expressions is

$$\frac{3x^2+8x-3}{2x^2-x-6}\times\frac{x^2-4}{x+3}$$

$$=\frac{(x+3)(3x-1)}{(2x+3)(x-2)}\times\frac{(x-2)(x+2)}{(x+3)}$$

$$=\frac{(3x-1)(x+2)}{2x+3}=\frac{3x^2+5x-2}{2x+3}.$$

EXAMPLE 2.10

Simplify $\left[\frac{2x-1}{x+3}-\frac{x^2-4}{2x+1}\right]\times\frac{2x^2+7x+3}{x+2}$.

SOLUTION

$$\left[\frac{2x-1}{x+3}-\frac{x^2-4}{2x+1}\right]\times\frac{2x^2+7x+3}{x+2}$$

$$=\frac{(2x-1)(2x+1)-(x^2-4)(x+3)}{(x+3)(2x+1)}\times\frac{(x+3)(2x+1)}{x+2}$$

$$=\frac{4x^2-1-x^2-3x^2+4x+12}{x+2}=\frac{-(x^3-x^2-4x-11)}{x+2}.$$

Note For every rational expression of the form $\frac{f(x)}{g(x)}, (g(x)\neq 0)$ there exists a rational expression of the form $\frac{g(x)}{f(x)}$ such that $\frac{f(x)}{g(x)}\times\frac{g(x)}{f(x)}=1$, then $\frac{g(x)}{f(x)}$ is called the multiplicative inverse of $\frac{f(x)}{g(x)}$ and vice versa.

Division of Rational Expressions

Let $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{p(x)}$ be two non-zero rational expressions, then $\frac{f(x)}{g(x)}\div\frac{h(x)}{p(x)}=\frac{f(x)}{g(x)}\times\frac{p(x)}{h(x)}$. That is, $\frac{f(x)p(x)}{g(x)h(x)}$ which is also a rational expression.

Note The process of dividing two rational expressions is similar to the process of dividing two rational numbers.

EXAMPLE 2.11

Express $\frac{2x^2+6x}{3x^2+7x+2}\div\frac{x^2+x-6}{3x^2+7x+2}$ as a rational expression in its lowest terms.

SOLUTION

Given, $\frac{2x^2+6x}{3x^2+7x+2}\div\frac{x^2+x-6}{3x^2+7x+2}$

$$\frac{2x(x+3)}{(x+2)(3x+1)}\div\frac{(x+3)(x-2)}{(3x+1)(x+2)}$$

$$\frac{2x(x+3)}{(x+2)(3x+1)}\times\frac{(3x+1)(x+2)}{(x+3)(x-2)}=\frac{2x}{x-2}.$$

EXAMPLE 2.12

If $A = 2x^3 + 5x^2 + 4x + 1$ and $B = 2x^2 + 3x + 1$, then find the quotient from the following four option, when A is divided by B.

(a) $x - 1$ **(b)** $x + 1$ **(c)** $2x + 1$ **(d)** $2x - 1$

SOLUTION

$$\frac{A}{B} = \frac{2x^3 + 5x^2 + 4x + 1}{2x^2 + 3x + 1}$$

$$= \frac{2x^3 + (3x^2 + 2x^2) + (x + 3x) + 1}{2x^2 + 3x + 1}$$

$$= \frac{(2x^3 + 3x^2 + x) + (2x^2 + 3x + 1)}{2x^2 + 3x + 1}$$

$$= \frac{(2x^2 + 3x + 1)(x + 1)}{2x^2 + 3x + 1} = x + 1.$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. The LCM of $18x^2y3$ and $8x3y^2$ is _____.
2. The HCF of $24a^2b$ and $32ab^2$ is _____.
3. The HCF of $9a^2 - 16b^2$ and $12a^2 - 16ab$ is _____.
4. The LCM of $2(x - 3)^2$ and $3(x - 2)^2$ is _____.
5. The HCF of $24x^5$ and $36x^6y^k$ is $12x^5$, then the value of k is _____.
6. The HCF of $a^m + b^m$ and $a^n - b^n$, where m is odd positive integer and n is even positive integer is _____.
7. The LCM of $12x^3y^2$ and $18x^py^3$ is $36x^4y^3$. Then the number of integer values of p is _____.
8. The HCF of $(x + 2)(x^2 - 7x + k)$ and $(x - 3)(x^2 + 3x + l)$ is $(x + 2)(x - 3)$. Then the values of k and l are _____ respectively.
9. The LCM of $x^2 + 2x - 8$ and $x^2 + 3x - 4$ is _____.
10. The rational expression $\frac{x^2+1}{x^2+4x+3}$ is not in its lowest terms. (True/False)
11. The additive inverse of $\frac{x^2+1}{x^2-1}$ is _____.
12. $\frac{x^2}{x^2-1} \div \frac{x^3}{x^2+1} =$ _____.
13. The product of two rational expressions is not always a rational expression. (True/False)
14. The rational expression $\frac{(x+1)^2}{x^2-1}$ in its lowest terms is _____.
15. The reciprocal of $x - \frac{1}{x}$ is $\frac{1}{x} - x$. (True/False)
16. If the rational expression $\frac{x-a}{x^2-b}$ is in its lowest terms, then _____. $[a \neq b/a^2 \neq b]$
17. Every polynomial is a rational expression. (True/False)
18. $\frac{x^3+\sqrt{6}x^2-7}{2x^3+\sqrt{x}+1}$ is a rational expression. (True/False)
19. The HCF of the polynomials $8a^3b^2c$, $16a^2bc^3$ and $20ab^3c^2$ is _____.
20. The LCM of the polynomials $8(x^3 + 8)$ and $12(x^2 - 4)$ is _____.
21. The HCF of the polynomials $12xy^2z^3$, $18x^2y^2z$ and $28x^3yz^2$ is _____.
22. The LCM of the polynomials $15a^2b(a^2 - b^2)$ and $40ab^2(a - b)$ is _____.
23. The HCF of the polynomials $15(a + 1)^2 (a - 2)$, $65(a - 1)^2 (a + 1)$ and $90 (a - 2)^2 (a - 1)$ is _____.
24. The HCF of the polynomials $(x + 3)^2 (x - 2)$ $(x + 1)^2$ and $(x + 1)^3 (x + 3)(x + 4)$ is _____.
25. The LCM of the polynomials $x^2 - 1$, $x^2 + 1$ and $x^4 - 1$ is _____.
26. The multiplicative inverse of $x - \frac{x-1}{1-x}$ is _____.
27. Which of the following algebraic expressions are polynomials?

    (a) $x^3 - \sqrt{7}x + 13$

    (b) $3x^4 + 2x^3 - \sqrt{5x} + 8$

    (c) $11 - x^2 + x\sqrt{x} + 7x$

    (d) $-\sqrt{3}x^3 + 8x^2 - 9$
28. The additive inverse of $\frac{x+1}{x^2-1}$ is _____.
29. If $A = \frac{p(x)}{q(x)}$ and $B = \frac{f(x)}{g(x)}$ are two rational expressions where $q(x)$, $g(x) \neq 0$, then $A + B$ and $A - B$ are also rational expressions. (True/False)
30. The rational expression whose numerator is a linear polynomial with -3 as zero and whose denominator is a quadratic polynomial with zeroes $\frac{1}{2}$ and -1 is _____.

PRACTICE QUESTIONS

Short Answer Type Questions

31. Find the HCF and the LCM of the following monomials.
(i) x^3y^6 and x^2y^8
(ii) $3a^2b^3c^4$ and $9a^4b^3c^2$
(iii) $p^4q^2r^3$ and $q^3p^6r^5$

32. Find the LCM and HCF of the polynomials $(x^2 - 4)(x^2 - x - 2)$ and $(x^2 + 4x + 4)(x^2 - 3x + 2)$. Verify that the product of the LCM and HCF is equal to the product of the polynomials.

33. If HCF of the two polynomials $(x - 1)(x - 6)(x - 2)$ and $(x - 6)^3(x - 4)(x - 2)^2$ is $(x - 6)(x - 2)$, then find their LCM.

34. If LCM of the two polynomials $(x^2 + 3x)(x^2 + 3x + 2)$ and $(x^2 + 6x + 8)(x^2 + kx + 6)$ is $x(x + 1)(x + 2)^2(x + 3)(x + 4)$, then find k.

35. If $(x + 2)(x + 5)$ is the HCF of the polynomials $(x + 2)(x^2 + 6x + a)$ and $(x + 5)(x^2 + 8x + b)$, then find the values of a and b.

36. If the LCM and HCF of the two polynomials are $(x - 1)^2(x - 2)^2(x - 3)^3$ and $(x - 1)(x - 2)(x - 3)$ respectively and one of the polynomials is $(x - 1)(x - 2)^2(x - 3)$, then find the other polynomial.

37. Find the LCM and HCF of the polynomials $(8 - x^3)$ and $(x^2 - 4)(x + 3)$. Verify that the product of the LCM and HCF is equal to the product of the polynomials.

38. The HCF and LCM of the two polynomials are $(x - 3)(x + 1)$ and $(x^2 - 9)(x^2 - 1)$ respectively. If one of the polynomial is $(x - 3)(x^2 - 1)$, then find the second polynomial.

39. Find the HCF and LCM of $(x - 1)(x + 2)(x - 3)$, $(x + 2)^2(x + 1)$ and $(x + 3)(x^2 - 4)$.

40. Find the sum of the following rational expressions.

$$\frac{x+1}{(x-1)^2} \text{ and } \frac{x-2}{x^2-1}.$$

41. If $P = \dfrac{3x+1}{3x-1}$ and $Q = \dfrac{3x-1}{3x+1}$, then find $P - Q$ and $P + Q$.

42. Reduce the following rational expressions to their lowest terms.

(i) $\dfrac{16x^2 - (x^2 - 9)^2}{4x + 9 - x^2}$

(ii) $\dfrac{8x^5 - 8x}{(3x^2 + 3)(4x + 4)}$

43. If $X = \dfrac{1+a}{1-a}$ and $Y = \dfrac{1-a}{1+a}$, then find $X^2 + Y^2 + XY$.

44. What should be added to $\dfrac{a}{a-b} + \dfrac{b}{a+b}$ to get 1?

45. Simplify:

$$\frac{x+1}{x-1} - \frac{x-1}{x+1} + \frac{8x}{1+x^2} - \frac{12x^3}{x^4-1}$$

Essay Type Questions

46. Simplify: $\left[\dfrac{x+2}{x-2} - \dfrac{x-2}{x+2} - \dfrac{8x}{x^2-16}\right] \div \dfrac{8x}{x^4-16}$

47. Simplify the rational expression

$$\frac{2x}{1+x^2+x^4} + \frac{1}{1+x+x^2} - \frac{1}{1-x+x^2}.$$

48. Find the product of the following rational expressions and express the result in lowest terms:

$$\frac{8x^2+10x-3}{12x^2+x-3} \text{ and } \frac{6x^2-7x-3}{8x^2-10x-3}$$

49. Simplify the rational expression

$$\frac{1}{x+p} + \frac{1}{x+q} + \frac{1}{x+r} + \frac{px}{x^3+px^2} + \frac{qx}{x^3+qx^2} + \frac{rx}{x^3+rx^2}.$$

50. Simplify:

$$\frac{8x^3 - y^3 + z^3 + 6xyz}{a^3 - 8b^3 + 27c^3 + 18abc} \div \frac{4x^2 + y^2 + z^2 + 2xy + yz - 2xz}{a^2 + 4b^2 + 9c^2 + 2ab + 6bc - 3ac}.$$

PRACTICE QUESTIONS

CONCEPT APPLICATION

Level 1

1. The LCM of the polynomials $18(x^4 - x^3 + x^2)$ and $24(x^6 + x^3)$ is _____.
   (a) $72x^2(x + 1)(x^2 - x + 1)^2$
   (b) $72x^3(x^2 - 1)(x^3 - 1)$
   (c) $72x^3(x^3 - 1)$
   (d) $72x^3(x^3 + 1)$

2. The LCM of the polynomials $f(x) = 9(x^3 + x^2 + x)$ and $g(x) = 3(x^3 + 1)$ is _____.
   (a) $27x(x + 1)(x^2 + x + 1)$
   (b) $9x(x + 1)(x^2 + x + 1)(x^2 - x + 1)$
   (c) $9(x + 1)(x^2 - x + 1)(x^2 + x + 1)$
   (d) $9x(x + 1)(x^2 + x + 1)$

3. The LCM of polynomials $14(x^2 - 1)(x^2 + 1)$ and $18(x^4 - 1)(x + 1)$ is _____.
   (a) $126(x + 1)(x^2 + 1)(x - 1)$
   (b) $126(x + 1)(x^2 + 1)(x^2 - 1)$
   (c) $126(x + 1)^2 (x^2 + 1)(x - 1)^2$
   (d) $126(x + 1)(x^2 + 1)(x - 1)^2$

4. If HCF and LCM of two polynomials $P(x)$ and $Q(x)$ are $x(x + p)$ and $12x^2 (x - p)(x^2 - p^2)$ respectively. If $P(x) = 4x^2 (x + p)$, then $Q(x)$ = _____.
   (a) $3x(x - p)^2 (x + p)$
   (b) $3x(x - p)(x + p)$
   (c) $3x(x + p)(x^2 - p^2)$
   (d) $3x(x + p)(x^2 + p^2)$

5. The product of HCF and LCM of two polynomials is $(x^2 - 1)(x^4 - 1)$, then the product of the polynomials is _____.
   (a) $(x^2 - 1)(x^2 + 1)$
   (b) $(x^2 - 1)(x^2 + 1)^2$
   (c) $(x^2 - 1)^2 (x^2 + 1)$
   (d) None of these

6. If $(x + 4)(x - 2)(x + 1)$ is the HCF of the polynomials $f(x) = (x^2 + 2x - 8)(x^2 + 4x + a)$ and $g(x) = (x^2 - x - 2)(x^2 + 3x - b)$, then (a, b) = _____.
   (a) $(3, -4)$ (b) $(-3, -4)$
   (c) $(-3, 4)$ (d) $(3, 4)$

7. Find the LCM of $x^3 - x^2 + x - 1$ and $x^3 - 2x^2 + x - 2$.
   (a) $(x + 1)(x - 1)$
   (b) $x - 1$
   (c) $(x^2 + 1)(x - 1)(x - 2)$
   (d) None of these

8. The HCF of the polynomials $p(x)$ and $q(x)$ is $6x - 9$, then $p(x)$ and $q(x)$ could be _____.
   (a) $3, 2x - 3$
   (b) $12x - 18, 2$
   (c) $3(2x - 3)^2, 6(2x - 3)$
   (d) $3(2x - 3), 6(2x + 3)$

9. If the HCF of the polynomials $f(x)$ and $g(x)$ is $4x - 6$, then $f(x)$ and $g(x)$ could be _____.
   (a) $2, 2x - 3$
   (b) $8x - 12, 2$
   (c) $2(2x - 3)^2, 4(2x - 3)$
   (d) $2(2x + 3), 4 (2x + 3)$

10. If the HCF of the polynomials $f(x) = (x + 3)(3x^2 - 7x - a)$ and $g(x) = (x - 3)(2x^2 + 3x + b)$ is $(x + 3)(x - 3)$, then $a + b$ = _____.
    (a) 3 (b) -15
    (c) -3 (d) 15

11. Find the HCF of the polynomials $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 - x^2 + x - 1$.
    (a) $x(x + 1)$ (b) $x - 1$
    (c) $x^2 + 1$ (d) $x + 1$

12. If the HCF of $x^3 + 2x^2 - ax$ and $2x^3 + 5x^2 - 3x$ is $x(x + 3)$, then a = _____.
    (a) 3 (b) -3
    (c) 6 (d) -4

13. The HCF of the polynomials $70(x^3 - 1)$ and $105(x^2 - 1)$ is _____.
    (a) $15(x - 1)$
    (b) $35(x - 1)$
    (c) $35(x^2 - 1)(x^2 + x + 1)$
    (d) $15(x^2 - 1)$

PRACTICE QUESTIONS

PRACTICE QUESTIONS

14. What should be subtracted from $\frac{7x}{(x^2 - x - 12)}$ to get $\frac{3}{x+3}$?

(a) $\frac{5}{x+4}$ (b) $\frac{4}{x-4}$

(c) $\frac{2}{x-4}$ (d) $\frac{1}{x-4}$

15. Which of the following is/are true?

(A) The sum of two rational expressions is always a rational expression.

(B) The difference of two rational expressions is always a rational expression.

(C) $\frac{p(x)}{q(x)}$ is in its lowest terms if LCM $[p(x), q(x)] = 1$.

(D) Reciprocal of $\frac{-2x}{x^2-1}$ is $\frac{x^2-1}{2x}$.

(a) A, B (b) A, B, D

(c) A, C (d) A, B, C

16. The product of additive inverses of $\frac{x^2-1}{2x}$ and $\frac{x^2-4}{3-x}$ is _____.

(a) $x^2 + 5x + 6$ (b) $x^2 + x - 6$

(c) $x^2 - x - 6$ (d) $x^2 - 5x + 6$

17. What should be added to $\frac{1}{x^2-7x+12}$ to get $\frac{2}{x^2-6x+18}$?

(a) $\frac{2}{(x+3)(x-2)}$ (b) $\frac{4}{(x+3)(x+2)}$

(c) $\frac{1}{x^2-5x+6}$ (d) $\frac{-1}{x^2+5x-6}$

18. The rational expression $\frac{x^3-3x^2+2x}{x^2y-2xy}$ in lowest terms is _____.

(a) $\frac{x-2}{y}$ (b) $\frac{x+1}{xy}$

(c) $\frac{x-2}{y}$ (d) $\frac{x+1}{y}$

19. If $(x - 4)$ is the HCF of $p(x) = x^2 - nx - 12$ and $q(x) = x^2 - mx - 8$, then the simplest form of $\frac{p(x)}{q(x)}$ is _____.

(a) $\frac{x-3}{x+2}$ (b) $\frac{x+3}{x-2}$

(c) $\frac{x+2}{x+3}$ (d) $\frac{x+3}{x+2}$

20. What should be added to $\frac{2}{(x^2+x-6)}$ to get $\frac{-4x}{(x^2-4)(x^2-9)}$?

(a) $\frac{4}{x^2-x-6}$ (b) $\frac{-2}{x^2-x-6}$

(c) $\frac{4x}{x^2+x+6}$ (d) $\frac{-3}{x^2+x+6}$

21. The HCF of the polynomials $4(x+8)^2(x^2-5x+6)$ and $6(x^2+12x+32)(x^2-7x+12)$ is _____.

(a) $2(x+8)(x-3)$ (b) $(x-3)(x+8)$

(c) $(x-8)(x+3)$ (d) $2(x-8)(x+3)$

22. The LCM of the polynomials $8(x^2-2x)(x-6)^2$ and $2x(x^2-4)(x-6)^2$ is _____.

(a) $8(x^2-4)(x^2-6x)^2$

(b) $(x^3-4x)(x-6)^2$

(c) $8(x^3-4x)(x-6)^2$

(d) $8x(x^2+4)(x-6)^2$

23. The HCF of polynomials $(x^2-2x+1)(x+4)$ and $(x^2+3x-4)(x+1)$ is _____.

(a) $(x+4)(x-1)$

(b) $(x+1)(x+4)$

(c) $(x+1)(x-4)$

(d) $(x^2-1)(x+4)$

24. If the zeroes of the rational expression $(3x + 2a)(2x + 1)$ are $\frac{-1}{2}$ and $\frac{b}{3}$, then the value of a is _____.

(a) $-2b$ (b) $\frac{-b}{2}$

(c) $\frac{-b}{3}$ (d) None of these

25. The LCM of the polynomials $(x^2 - 8x + 16)(x^2 - 25)$ and $(x^2 - 10x + 25)(x^2 - 2x - 24)$ is _____.

(a) $(x^4 - 41x + 400)(x - 6)$

(b) $(x^4 + 41x + 400)(x^2 - 9x + 20)$

(c) $(x^4 - 41x + 400)(x^2 - 9x + 20)(x - 6)$

(d) $(x^4 - 41x + 400)(x^2 - 9x + 20)(x + 6)$

26. If the HCF of the polynomials $(x^2 + 8x + 16)(x^2 - 9)$ and $(x^2 + 7x + 12)$ is $x^2 + 7x + 12$, then their LCM is _____.

(a) $(x + 4)(x^2 - 9)$

(b) $(x + 4)^2 (x^2 - 9)$

(c) $(x^2 - 4)(x^2 + 9)$

(d) $(x + 4)^2 (x + 3)$

27. If $h(x) = x^2 + x$ and $g(y) = y^3 - y$, then the HCF of $h(b) - h(a)$ and $g(b) - g(a)$ is _____.

(a) $a + b$ (b) $b - a$

(c) $b^2 + a^2$ (d) $b^2 + ab + a^2$

28. If $h(y) = y^3$ and $g(z) = z^4$, then HCF of $h(b) - h(a)$ and $g(b) - g(a)$ is _____.

(a) $b - a$

(b) $b^2 - a^2$

(c) $b^3 - a^3$

(d) $b^2 + ab + a^2$

29. If the HCF of the polynomials $(x + 4)(2x^2 + 5x + a)$ and $(x + 3)(x^2 + 7x + b)$ is $(x^2 + 7x + 12)$ then $6a + b$ is _____.

(a) -6 (b) 5

(c) 6 (d) -5

Level 2

30. The HCF and LCM of the polynomials $p(x)$ and $q(x)$ are $5(x - 2)(x + 9)$ and $10(x^2 + 16x + 63)(x - 2)^2$. If $p(x)$ is $10(x + 9)(x^2 + 5x - 14)$, then $q(x)$ is _____.

(a) $5(x + 9)(x - 2)$

(b) $10(x - 2)^2 (x + 7)$

(c) $10(x + 9)(x - 2)$

(d) $5(x - 2)^2 (x + 9)$

31. If the zeroes of the rational expression $(ax + b)(3x + 2)$ are $\frac{-2}{3}$ and $\frac{1}{2}$, then $a + b =$ _____.

(a) -1 (b) 0

(c) $-b$ (d) None of these

32. Simplify:

$$\frac{x^2 - (y - 2z)^2}{x - y + 2z} + \frac{y^2 - (2x - z)^2}{y + 2x - z} + \frac{z^2 - (x - 2y)^2}{z - x + 2y}.$$

(a) 0 (b) 1

(c) $x + y + z$ (d) None of these

33. If the HCF of the polynomials $x^2 + px + q$ and $x^2 + ax + b$ is $x + l$, then their LCM is _____.

(a) $(x + a - l)(x + l - p)$

(b) $(x - (l + a))(x + l - p)(x + l)$

(c) $(x + a - l)(x + p - l)(x + l)$

(d) $(x - l + a)(x - p + l)(x + l)$

34. The expression $\frac{1}{1-x} - \frac{1}{1+x} - \frac{x^3}{1-x} + \frac{x^2}{1+x}$ in lowest terms is _____.

(a) $2x^3 + 1$ (b) $x^2 + 2$

(c) $x^2 + 2x$ (d) $x^2 - 2x$

35. Simplify:

$$\frac{a^2 - (b - c)^2}{(a + c)^2 - b^2} + \frac{b^2 - (a - c)^2}{(a + b)^2 - c^2} + \frac{c^2 - (a - b)^2}{(b + c)^2 - a^2}.$$

(a) 0 (b) 1

(c) $a + b + c$ (d) $\frac{1}{a + b + c}$

36. Simplify: $\frac{x+1}{x-1} + \frac{x-1}{x+1} - \frac{2x^2 - 2}{x^2 + 1}$.

(a) $\frac{4x^4 + 2}{x^4 - 1}$ (b) $\frac{4x^2}{x^4 - 1}$

(c) $\frac{8x^2}{x^4 - 1}$ (d) 1

37. If the LCM of the polynomials $x^9 + x^6 + x^3 + 1$ and $x^6 - 1$ is $x^{12} - 1$, then their HCF is _____.

(a) $x^3 + 1$ (b) $x^6 + 1$

(c) $x^3 - 1$ (d) $x^6 - 1$

38. If $x^2 + x - 1$ is a factor of $x^4 + px^3 + qx^2 - 1$, then the values of p and q can be

(a) 2, 1 (b) 1, -2

(c) -1, -2 (d) -2, -1

39. The HCF of two polynomials $p(x)$ and $q(x)$ using long division method was found to be $x + 5$, If their first three quotients obtained are x, $2x + 5$, and $x + 3$ respectively. Find $p(x)$ and $q(x)$. (The degree of $p(x)$ > the degree of $q(x)$)

(a) $p(x) = 2x^4 + 21x^3 + 72x^2 + 88x + 15$
$q(x) = 2x^3 + 21x^2 + 71x + 80$

(b) $p(x) = 2x^4 - 21x^3 - 72x^2 - 88x + 15$
$q(x) = 2x^3 + 21x^2 - 71x + 80$

(c) $p(x) = 2x^4 + 21x^3 + 88x + 15$
$q(x) = 2x^3 + 71x + 80$

(d) $p(x) = 2x^4 - 21x^2 - 72x^2 + 80x + 15$
$q(x) = 2x^3 - 21x^2 + 71x + 80$

40. If the HCF of the polynomials $x^3 + px + q$ and $x^3 + rx^2 + lx + x$ is $x^2 + ax + b$, then their LCM is _____. $(r \neq 0)$

(a) $(x^2 + ax + b)(x + a)(x + a - r)$

(b) $(x^2 + ax + b)(x - a)(x - a + r)$

(c) $(x^2 + ax + b)(x - a)(x - a - r)$

(d) $(x^2 - ax + b)(x - a)(x - a + r)$

41. If the HCF of the polynomials $(x - 3)(3x^2 + 10x + b)$ and $(3x - 2)(x^2 - 2x + a)$ is $(x - 3)(3x - 2)$, then the relation between a and b is _____.

(a) $3a + 8b = 0$ (b) $8a - 3b = 0$

(c) $8a + 3b = 0$ (d) $a - 2b = 0$

42. The HCF of the polynomials $12(x + 2)^3 (x^2 - 7x + 10)$ and $18(x^2 - 4)(x^2 - 6x + 5)$ is _____.

(a) $(x^2 + 3x + 10)(x - 2)$

(b) $6(x^2 + 3x + 10)(x + 2)$

(c) $(x^2 - 3x - 10)(x - 2)$

(d) $6(x^2 - 3x - 10)(x - 2)$

43. The HCF of the polynomials $(x^2 - 4x + 4)(x + 3)$ and $(x^2 + 2x - 3)(x - 2)$ is

(a) $x + 3$ (b) $x - 2$

(c) $(x + 3)(x - 2)$ (d) $(x + 3)(x - 2)^2$

44. The HCF of the polynomials $5(x^2 - 16)(x + 8)$ and $10(x^2 - 64)(x + 4)$ is _____.

(a) $x^2 + 12x + 32$ (b) $5(x^2 + 12x + 32)$

(c) $x^2 - 12x + 32$ (d) $5(x^2 - 12x + 32)$

45. Find the HCF of $6x^4y$ and $12xy$.

(a) $6x^2y$ (b) $6x$

(c) $6y$ (d) $6xy$

46. Find the LCM of $p^4q^2r^3$ and $q^3p^6r^5$.

(a) $p^4q^3r^3$ (b) $p^4q^2r^5$

(c) $p^6q^3r^5$ (d) $p^6q^2r^5$

47. The LCM of the polynomials $12(x^3 + 27)$ and $18(x^2 - 9)$ is _____.

(a) $6(x + 3)$

(b) $36(x^2 - 9)(x^2 + 3x + 9)$

(c) $36(x + 3)^2 (x^2 + 3x + 9)$

(d) $36(x^2 - 9)(x^2 - 3x + 9)$

48. The rational expression $\dfrac{x^2 + 2x + 3}{x^4 + 4x^3 + 4x^2 - 9}$ in its lowest terms is _____.

(a) $\dfrac{1}{x^2 + 3x + 3}$ (b) $\dfrac{1}{x^2 + 2x - 3}$

(c) $\dfrac{1}{x^2 + 4x - 3}$ (d) None of these

49. The LCM of the polynomials $(x + 3)^2 (x - 2)(x + 1)^2$ and $(x + 1)^3 (x + 3)(x^2 - 4)$ is _____.

(a) $(x + 1)^3 (x + 3)(x^2 - 4)$

(b) $(x + 3)^2 (x + 1)^3 (x^2 - 4)$

(c) $(x + 3)^2 (x + 1)^3 (x + 2)$

(d) $(x + 3)^2 (x + 1)^2 (x - 2)$

Level 3

50. The HCF of two polynomials $p(x)$ and $q(x)$ using long division method was found in two steps to be $3x - 2$, and the first two quotients obtained are $x + 2$ and $2x + 1$. Find $p(x)$ and $q(x)$. (The degree of $p(x)$ > the degree of $q(x)$).

(a) $p(x) = 6x^3 + 11x^2 + x + 6$, $q(x) = 6x^2 + x + 2$

(b) $p(x) = 6x^3 + 11x^2 - x + 6$, $q(x) = 6x^2 - x + 2$

(c) $p(x) = 6x^3 - 11x^2 + x - 6$, $q(x) = 6x^2 - x - 2$

(d) $p(x) = 6x^3 + 11x^2 - x - 6$, $q(x) = 6x^2 - x - 2$

51. Simplify: $\dfrac{x+2}{x-2} + \dfrac{x-2}{x+2} - \dfrac{3x^2-3}{x^2+4}$.

(a) $\dfrac{-x^4 + 31x^2 + 20}{x^4 - 16}$ (b) $\dfrac{x^4 + 31x^2 + 20}{x^4 - 16}$

(c) $-\dfrac{x^4 + 31x^2 - 20}{x^4 - 16}$ (d) $-\dfrac{x^4 - 21x^2 + 20}{x^4 - 16}$

PRACTICE QUESTIONS

52. If $P = \frac{x+1}{x-1}$ and $Q = \frac{x-1}{x+1}$, then $P^2 + Q^2 - 2PQ$ = _____.

(a) $\frac{16x^2}{x^4 - 2x + 1}$ (b) $\frac{4x^4 + 8x^2 + 4}{x^4 - 2x + 1}$

(c) $\frac{4x^2}{x^4 + 2x^2 + 1}$ (d) $\frac{8x^2}{x^4 - 2x^2 + 1}$

53. Simplify: $\frac{81x^4 - 16x^2 + 32x - 16}{9x^2 - 4x + 4}$

(a) $9x^2 + 4x - 4$ (b) $9x^2 - 4x - 4$

(c) $9x^2 - 2x - 8$ (d) $9x^2 + 2x - 8$

54. The rational expression

$A = \left(\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1}\right)$ is multiplied with the additive inverse of $B = \frac{1-x^4}{4x}$ to get C. Then, C = _____.

(a) $\frac{32x^2}{x^4 - 1}$ (b) $\frac{2x}{x^4 - 1}$

(c) 2 (d) 1

55. If the HCF of $x^3 + qx^2 + px$ and $qx^3 + 11x^2 - 4x$ is $x(x + 4)$, then p = _____.

(a) -4 (b) 3

(c) -2 (d) 5

56. If degree of both $f(x)$ and $[f(x) + g(x)]$ is 18, then degree of $g(x)$ can be

(a) 18 (b) 9

(c) 6 (d) Any one of these

57. The product of additive inverse and multiplicative inverse of $\frac{x-2}{x^2-4}$ is _____.

(a) $x^2 + 4x + 4$ (b) $x^2 - 4x + 4$

(c) $x^2 - 6x + 9$ (d) None of these

58. What should be multiplied to $(2x^2 + 3x - 4)$ to get $4x^4 - 9x^2 + 24x - 16$?

(a) $2x^2 - 3x - 4$ (b) $2x^2 + 24x - 16$

(c) $2x^2 + 3x + 4$ (d) $2x^2 - 3x + 4$

59. If $f(x) = (x + 2)^7 (x + 4)^p$, $g(x) = (x + 2)^q (x + 4)^9$ and the LCM of $f(x)$ and $g(x)$ is $(x + 2)^q (x + 4)^9$, then find the maximum value of $p - q$.

(a) 4

(b) 3

(c) 2

(d) Cannot be determined

60. If $f(x) = x^2 - 7x + 12$ and $g(x) = x^2 - 8x + 15$, then find the HCF of $f(x)$ and $g(x)$.

(a) $x - 4$ (b) $x - 3$

(c) $x - 5$ (d) $x - 6$

61. If $f(x) = (x + 2)(x^2 + 8x + 15)$ and $g(x) = (x + 3)(x^2 + 9x + 20)$, then find the LCM of $f(x)$ and $g(x)$.

(a) $(x + 2)(x + 3)(x + 4)(x + 5)$

(b) $(x + 2)^2 (x + 3)(x + 5)$

(c) $(x + 2)(x + 3)(x + 5)(x + 1)$

(d) None of these

62. If $f(x) = x^2 + 6x + a$, $g(x) = x^2 + 4x + b$, $h(x) = x^2 + 14x + c$ and the LCM of $f(x)$, $g(x)$ and $h(x)$ is $(x + 8)(x - 2)(x + 6)$, then find $a + b + c$. (a, b and c are constants).

(a) 20 (b) 16

(c) 32 (d) 10

63. Simplify: $\frac{(x^2 + 11x + 28)}{(x^2 + 13x + 40)} \div \frac{(x^2 + 6x + 8)}{(x^2 + 11x + 24)}$.

(a) $\frac{(x + 2)(x + 7)}{(x + 3)(x + 3)}$ (b) $\frac{x + 4}{x + 8}$

(c) $\frac{x + 8}{x + 4}$ (d) $\frac{(x + 3)(x + 7)}{(x + 2)(x + 5)}$

64. If LCM of $f(x)$ and $g(x)$ is $6x^2 + 13x + 6$, then which of the following cannot be the HCF of $f(x)$ and $g(x)$?

(a) $2x + 3$ (b) $3x + 1$

(c) $(2x + 3)(3x + 2)$ (d) $3x + 2$

65. If the LCM of $f(x)$ and $g(x)$ is $a^6 - b^6$, then their HCF can be

(a) $a - b$ (b) $a^2 + ab + b^2$

(c) $a^2 - ab + b^2$ (d) All of these

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $72x^3y^3$
2. $8ab$
3. $3a - 4b$
4. $6(x-2)^2\,(x-3)^2$
5. any whole number
6. $a + b$
7. 4
8. 12, 2
9. $(x-1)(x-2)(x+4)$
10. False
11. $\dfrac{x^2+1}{1-x^2}$
12. $\dfrac{1}{x(x-1)}$
13. False
14. $\dfrac{x+1}{x-1}$
15. False
16. $a^2 \neq b$
17. True
18. False
19. $4abc$
20. $24(x+2)^2\,(x-2)(x^2-2x+4)$
21. $2xyz$
22. $120\,a^2b^2\,(a^2-b^2)$
23. 5
24. $(x+3)(x+1)^2$
25. $(x^2+1)(x^2-1)$
26. $\dfrac{1}{1+x}$
27. a, d
28. $\dfrac{1}{1-x}$
29. True
30. $\dfrac{x+3}{\left(x-\frac{1}{2}\right)(x+1)}$

Short Answer Type Questions

31. (i) HCF = x^2y^6
    LCM = x^3y^8
    (ii) HCF = $3a^2b^3c^2$, LCM = $9a^4b^3c^4$
    (iii) HCF = $p^4q^2r^3$ and LCM = $p^6q^3r^5$
33. LCM = $(x-1)(x-2)^2\,(x-4)(x-6)^3$
34. $k = 5$
35. $a = 5,\ b = 12$
36. $(x-1)^2\,(x-2)(x-3)^3$
38. $(x+1)(x^2-9)$
39. HCF = $(x+2)$
    LCM = $(x^2-9)(x^2-4)(x^2-1)(x+2)$
40. $\dfrac{2x^2-x+3}{(x-1)^2(x+1)}$
41. $P - Q = \dfrac{12x}{(3x-1)(3x+1)}$

    $P + Q = \dfrac{2(9x^2+1)}{(9x^2-1)}$
42. (i) $x^2 + 4x - 9$
    (ii) $\dfrac{2x(x-1)}{3}$
43. $\dfrac{(a^2+3)(3a^2+1)}{(1-a^2)^2}$
44. $\dfrac{2ab}{b^2-a^2}$
45. $\dfrac{4x}{1-x^4}$

Essay Type Questions

46. 8

47. 0

48. $\frac{2x+3}{4x+1}$

49. $\frac{3}{x}$

50. $\frac{(2x-y+z)}{a-2b+3c}$

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (c)
11. (c)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (b)
21. (a)	**22.** (c)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (a)	

Level 2

30. (d)	**31.** (c)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (a)
40. (b)	**41.** (b)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (d)	**48.** (b)	**49.** (b)

Level 3

50. (d)	**51.** (a)	**52.** (a)	**53.** (a)	**54.** (c)	**55.** (a)	**56.** (d)	**57.** (d)	**58.** (d)	**59.** (c)
60. (b)	**61.** (a)	**62.** (a)	**63.** (d)	**64.** (b)	**65.** (d)				

CONCEPT APPLICATION

Level 1

1. Factorize the given polynomials. The product of all the factors with highest exponents is LCM.

2. Factorize the given polynomials. The product of all the factors with highest exponents is LCM.

3. Factorize the given polynomials. The product of all the factors with highest exponents is LCM.

4. We gave, $P(x)\ Q(x) = \text{LCM} \times \text{HCF}$.

5. We have, $P(x)\ Q(x) = \text{LCM} \times \text{HCF}$.

6. If $(x - k)$ is a factor if $f(x)$, then $f(k) = 0$.

7. Factorize the given polynomials. The product of all the factors with highest exponents is LCM.

10. If $(x - k)$ is a factor of $f(x)$, then $f(k) = 0$.

11. Factorize the given polynomials.

12. If $(x - k)$ is a factor of $f(x)$, then $f(k) = 0$.

14. Factorize the given polynomials. The product of all the factors with highest exponents is LCM.

15. Sum or difference of two rational expressions is always a rational expression.

17. Subtract the first expression from the second expression.

18. Factorise numerator and denominator and eliminate the common factors.

20. Factorize the given polynomials. The product of all the factors with highest exponents is LCM.

21. (i) Factorize the given polynomials.
(ii) $x^2 - 5x + 6 = (x - 2)(x - 3)$
$x^2 + 12x + 32 = (x + 8)(x + 4)$
$x^2 - 7x + 12 = (x - 4)(x - 3)$.

22. (i) Factorize the given polynomials.
(ii) $x^2 - 2x = x(x - 2)$
$x^2 - 4 = (x + 2)(x - 2)$.

23. (i) Factorize the given polynomials.
(ii) $x^2 - 2x + 1 = (x - 1)^2$
$x^2 + 3x - 4 = (x + 4)(x - 1)$.

24. (i) Equate the given expression to zero and compare with the given values.

(ii) As zero of $2x + 1$ is $-\frac{1}{2}$, zero of the expression $3x + 2a$ is $\frac{b}{3}$.

25. (i) Factorize the given polynomials.
(ii) $(x^2 - 8x + 16)(x^2 - 25) = (x - 4)^2\ (x + 5)(x - 5)$
$(x^2 - 10x + 25)(x^2 - 2x - 24)$
$= (x - 5)^2\ (x - 6)(x + 4)$.

26. (i) $\text{LCM} \times \text{HCF} = \pm f(x) \times g(x)$
(ii) $x^2 + 8x + 16 = (x + 4)^2$.

27. (i) $h(b) - h(a) = (b^2 + b) - (a^2 + a)$.
(ii) Now factorize the above expression.
(iii) Similarly factorize $g(b) - g(a)$.

28. (i) Divide the given polynomials with $(x + l)$.
(ii) $h(b) - h(a) = b^3 - a^3$, $g(b) - g(a) = b^4 - a^4$.
(iii) Now, factorize the above expressions.

29. (i) Find the values of a and b using the concept of HCF then obtain the required relation.

(ii) $x^2 + 7x + 12 = (x + 4)(x + 3)$.

(iii) $x + 3$ is a factor of $2x^2 + 5x + a$ and $x + 4$ is a factor of $x^2 + 7x + b$.

Level 2

30. (i) $p(x)\ q(x) = \text{LCM} \times \text{HCF}$.
(ii) $\text{LCM} \times \text{HCF} = \pm f(x) \times g(x)$
(iii) $x^2 + 16x + 63 = (x + 7)(x + 9)$
$x^2 + 5x - 14 = (x + 7)(x - 2)$.

31. (i) Equate the given expression to zero to get the values of a and b then find $a + b$.

(ii) As the zero of $3x + 2$ is $\frac{-2}{3}$, zero of $ax + b$ is $\frac{1}{2}$.

32. (i) Factorize and then simplify.
(ii) $x^2 - (y - 2z)^2 = (x + y - 2z)(x - y + 2z)$ and so on.

33. (i) If $(x - k)$ is a of factor $f(x)$, then $f(k) = 0$.
(ii) Find the quotients by dividing each of the expressions by $x + l$.

34. (i) Add the terms which have the same denominators.

HINTS AND EXPLANATION

(ii) LCM of denominators is $(1 - x)(1 + x)$.
(iii) $a^2 - 1 = (a + 1)(a - 1)$;
$a^3 - 1 = (a - 1)(a^2 + a + 1)$.

35. (i) Factorize numerator and denominator then simplify.
(ii) $a^2 - (b - c)^2 = (a + b - c)(a - b + c)$; $(a + c)^2 - b^2 = (a + c + b)(a + c - b)$.

36. (i) Simplify the first two terms.
(ii) LCM of denominators is $(x^2 - 1)(x^2 + 1)$.

37. (i) We have, $P(x)\ Q(x) = \text{LCM} \times \text{HCF}$.
(i) $P^2 + Q^2 - 2PQ = (P - Q)^2$.
(ii) Find $P - Q$ and then find $(P - Q)^2$.

39. (i) Use division rule.
(ii) Apply the concept of finding HCF by long division method.

40. Find the quotients by dividing each of the given expressions by $x^2 + ax + b$.

41. (i) Find the values of a, b using the concept of HCF then obtain the relation between a and b.
(ii) $3x - 2$ is a factor of $3x^2 + 10x + b$.
(iii) $x - 3$ is a factor of $x^2 - 2x + a$.
(iv) Apply remainder theorem to find the values of a and b.

42. (i) Factorize the given polynomials.
(ii) $x^2 - 7x + 10 = (x - 5)(x - 2)$
$(x^2 - 6x + 5) = (x - 5)(x - 1)$.

44. (i) Factorize the given polynomials.
(ii) $x^2 - 16 = (x + 4)(x - 4)$
$x^2 - 64 = (x + 8)(x - 8)$.

45. The factors of $6x^4y$ and $12xy$ are $(6xy)x^3$ and $(6xy)\cdot 2$ respectively.
$\therefore$ HCF is $6xy$.

46. The factors of $p^4q^2r^3$ and $q^3p^6r^5$ are $(p^4q^2r^3)$ and $(p^4q^2r^3)\ p^2qr^2$ respectively.
$\therefore$ LCM is $p^6q^3r^5$.

47. Let $f(x) = 12(x^3 + 27)$ and $g(x)$
$= 18(x^2 - 9)$
$f(x) = 12(x^3 + 3^3) = 12(x + 3)(x^2 - 3x + 9)$
$g(x) = 18(x^2 - 3^2) = 18(x + 3)(x - 3)$.
LCM of 12 and 18 is 36.
$\therefore$ LCM of $f(x)$ and $g(x)$ is $36(x + 3)(x - 3)$
$(x^2 - 3x + 9) = 36(x^2 - 9)(x^2 - 3x + 9)$.

48. $\dfrac{x^2 + 2x + 3}{x^4 + 4x^3 + 4x^2 - 9}$

$= \dfrac{x^2 + 2x + 3}{(x^2 + 2x)^2 - 3^2}$

$= \dfrac{x^2 + 2x + 3}{(x^2 + 2x + 3)(x^2 + 2x - 3)} = \dfrac{1}{x^2 + 2x - 3}$.

49. Let $f(x) = (x + 3)^2\ (x - 2)(x + 1)^2$ and
$g(x) = (x + 1)^3\ (x + 3)(x^2 - 4)$
$g(x) = (x + 1)^3\ (x + 3)(x + 2)(x - 2)$
$\therefore$ LCM of $f(x)$ and $g(x)$ is $(x + 3)^2\ (x + 1)^3\ (x + 2)$ $(x - 2)$
$= (x + 3)^2\ (x + 1)^3\ (x^2 - 4)$.

Level 3

50. (i) Use division rule.
(ii) Apply the concept of finding HCF by division method.

52. (i) $P^2 + Q^2 - 2PQ = (P - Q)^2$.
(ii) Find $P - Q$ and then find $(P - Q)^2$.

54. (i) Simplify the expression A
(ii) Find additive inverses of the expression B and then multiply with A.

55. Let $f(x) = x^3 + qx^2 + px = x(x^2 + qx + p)$ and $g(x) = qx^3 + 11x^2 - 4x = x(qx^2 + 11x - 4)$.
Given, HCF $= x(x + 4)$
$\Rightarrow$ $x + 4$ is the factor of $g(x)$
$\Rightarrow$ $g(-4) = 0$
$\Rightarrow$ $q(-4)^2 + 11(-4) - 4 = 0$
$\Rightarrow$ $q = 3$
$x + 4$ is also factor of $f(x)$
$\Rightarrow$ $f(-4) = 0$
$\Rightarrow$ $(-4)^2 + 3\ (-4) + p = 0$ $\Rightarrow$ $p = -4$.

56. Degree of $f(x) = 18$
Degree of $[f(x) + g(x)] = 18$
$\therefore$ The degree of $g(x)$ can be less than or equal to 18.

57. $\dfrac{x - 2}{x^2 - 4} = \dfrac{1}{x + 2}$

Additive inverse of $\dfrac{1}{x + 2}$ is $\dfrac{-1}{x + 2}$

Multiplicative inverse of $\dfrac{1}{x + 2}$ is $x + 2$

$\therefore$ The required product is $\left(\frac{-1}{x+2}\right)(x+2)$, i.e., -1.

58. Let A be the required expression

$\therefore A(2x^2 + 3x - 4) = 4x^4 - 9x^2 + 24x - 16$

$$\Rightarrow \quad A = \frac{4x^4 - (9x^2 - 24x + 16)}{2x^2 + 3x - 4}$$

$$= \frac{(2x^2)^2 - (3x - 4)}{2x^2 + 3x - 4}$$

$$= \frac{(2x^2 + 3x - 4)(2x^2 - 3x + 4)}{(2x^2 + 3x - 4)}$$

$$= 2x^2 - 3x + 4.$$

59. Given

$f(x) = (x + 2)^7 (x + 4)^p$

$g(x) = (x + 2)^q (x + 4)^9$

LCM $= (x + 2)^q (x + 4)^9$

$\therefore q \geq 7$ and $p \leq 9$

For maximum value of $p - q$, p is maximum and q is minimum.

$\therefore$ maximum value of $p - q$ is $9 - 7 = 2$.

60. Given

$f(x) = x^2 - 7x + 12 = (x - 3)(x - 4)$

$g(x) = x^2 - 8x + 15 = (x - 3)(x - 5)$

$\therefore$ HCF $= x - 3$.

61. Given $f(x) = (x + 2)(x^2 + 8x + 15)$

$= (x + 2)(x + 3)(x + 5)$

$g(x) = (x + 3)(x^2 + 9x + 20)$

$= (x + 3)(x + 4)(x + 5)$

LCM $= (x + 2)(x + 3)(x + 4)(x + 5)$.

62. $f(x) = x^2 + 6x + a$

$g(x) = x^2 + 4x + b$

$h(x) = x^2 + 14 + c$

LCM $= (x + 8)(x - 2)(x + 6)$

As three given polynomials are quadratic and their LCM is cubic, each polynomial is the product of two of the factors of LCM. By inspection method,

$f(x) = x^2 + 6x + a = (x + 8)(x - 2)$

$\Rightarrow \quad a = -16$

$g(x) = x^2 + 4x + b = (x + 2)(x + 6)$

$\Rightarrow \quad b = -12$

$h(x) = x^2 + 14x + c = (x + 8)(x + 6)$

$\Rightarrow \quad c = 48$

$a + b + c = -16 - 12 + 48 = 20.$

63. $\frac{(x^2 + 11x + 28)}{(x^2 + 13x + 40)} \div \frac{(x^2 + 6x + 8)}{(x^2 + 11x + 24)}$

$$= \frac{(x + 4)(x + 7)}{(x + 5)(x + 8)} \div \frac{(x + 2)(x + 4)}{(x + 3)(x + 8)}$$

$$= \frac{(x + 4)(x + 7)(x + 3)(x + 8)}{(x + 5)(x + 8)(x + 2)(x + 4)}$$

$$= \frac{(x + 7)(x + 3)}{(x + 5)(x + 2)}.$$

64. LCM of $f(x)$ and $g(x) = 6x^2 + 13x + 6$

$= (2x + 3)(3x + 2)$.

We know that, HCF is a factor of LCM.

$\therefore$ $3x + 1$ cannot be the HCF.

65. LCM of $f(x)$ and $g(x)$ is $a^6 - b^6$

HCF of $f(x)$ and $g(x)$ is a factors $a^6 - b^6$

All $(a - b)$, $(a^2 + ab + b^2)$

and $(a^2 - ab + b^2)$ are factors of $a^6 - b^6$.