Sample Question Paper - 46
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time : 2 Hr. Max. Marks : 40

General Instructions :

1. The question paper consists of 14 questions divided into three Sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section - A

1. Solve : $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$.

2. From a point Q, 13 cm away from the centre of a circle, the length of tangent PQ to the circle is 12 cm. What will be it the radius of the circle (in cm) ?

3. If the n^{th} term of an A.P. is $(2n + 1)$, then find the sum of its first three terms.

4. The median of the following frequency distribution will be :

x	6	7	5	2	10	9	3
y	9	12	8	13	11	14	7

5. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface area of the two are equal then find the ratio of the radius and the slant height of the conical part.

6. The roots of the quadratic equation $x^2 - 0.04 = 0$.

OR

The difference in the roots of the equation $2x^2 - 11x + 5 = 0$.

Section - B

7. Consider the frequency distribution of the heights of 60 students of a class :

Height (in cm.)	No. of students	Cumulative frequency
150 – 155	16	16
155 – 160	12	28
160 – 165	9	37
165 – 170	7	44
170 – 175	10	54
175 – 180	6	60

Find the sum of the lower limit of the modal class and the upper limit of the median class.

8. The angle of elevation of the top of a tower at a point on the ground is 30°. If the height of the tower is tripled, find the angle of elevation of the top of the same point.

OR

The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

9. Draw a line-segment AB of length 8 cm. Taking A as centre draw a circle of radius 4 cm and taking B as centre, draw another circle B of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

10. Following table gives the ages in years of militants operating in a certain area of a country.

Age (in years)	40 – 43	43 – 46	46 – 49	49 – 52	52 – 54
Number of militants	31	58	60	k	27

If mean of the above distribution is 47.2, find how many militants in the age groups 49-52 are active in the area ?

Section - C

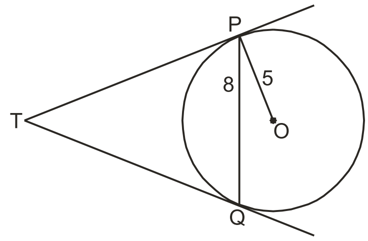
11. If S_n denotes the sum of the first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$.

OR

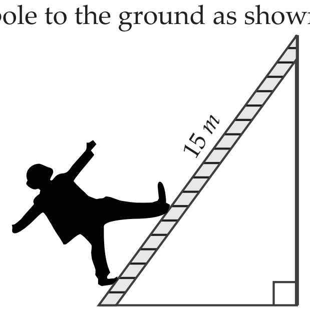
Find the sum of the following :

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right) + \ldots \text{ upto } n \text{ terms.}$$

12. In Fig. PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



13. A circus artist is showing stunts in a show climbing through a 15 m long rope which is highly stretched and tied from the top of a vertical pole to the ground as shown below :



Answer the following questions.

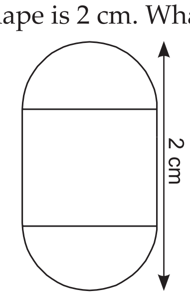
(i) What is the height of the pole, if angle made by rope to the ground level is 45° ?

(ii) If the angle made by the rope to the ground level remains same, then find the distance between artist and pole at ground level.

14. Mathematics teacher of a school took her 10th standard students to show Gol Gumbaz. It was a part of their Educational trip. The teacher had interest in history as well. She narrated the facts of Gol Gumbaz to students. Gol Gumbaz is the tomb of King Muhammad Adil Shah, Adil Shah Dynastry. Construction of the tomb, located in Vijayapura, Karanataka, India, was started in 1626 and completed in 1656. Then the teacher said in this monument one can find combination of solid figures. She pointed that there are cubical bases and hemispherical dome is at the top.

(i) Find the diagonal of the cubic portion of the Gol Gumbaz, if one side of cubical portion is 23 m.

(ii) A block of Gol Gumbaz is in the shape of a cylinder of diameter 0·5 cm with two hemispheres stuck to each of its ends. The length of shape is 2 cm. What is the volume of the block. (Use $\pi = 3{\cdot}14$)



Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section - A

1. Given, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\Rightarrow \quad x = \frac{2\sqrt{2} \pm \sqrt{(-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})}}{2(\sqrt{3})}$$

$$\left[\text{applying the formula : } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$\Rightarrow \quad x = \frac{2\sqrt{2} \pm \sqrt{8+24}}{2\sqrt{3}} = \frac{2\sqrt{2} \pm \sqrt{32}}{2\sqrt{3}}$$

$$= \frac{\sqrt{2} \pm 2\sqrt{2}}{\sqrt{3}} = \sqrt{6} \text{ and } -\sqrt{\frac{2}{3}}.$$

2. Given,

$$PQ = 13 \text{ cm and } OQ = 12 \text{ cm}$$

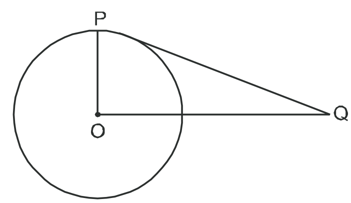
In ΔOPQ, by pythagoras theorem

Thus,

$$r = PO = \sqrt{PQ^2 - OQ^2}$$

$$= \sqrt{(13)^2 - (12)^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$$



As length cannot be negative.

3. Here,

$$a_n = 2n + 1$$

$$a_1 = 2(1) + 1 = 3$$

$$a_2 = 2(2) + 1 = 5$$

$$a_3 = 2(3) + 1 = 7$$

$$\therefore \quad a_1 + a_2 + a_3 = 3 + 5 + 7 = 15.$$

4.

x	f	$c.f.$	x	f	$c.f.$
2	13	13	7	12	49
3	7	20	9	14	63
5	8	28	10	11	74
6	9	37			

Here, $\frac{N}{2} = \frac{74}{2} = 37^{th}$ observation

$\therefore$ Median $= 37^{th}$ observation $= 6.$

5. Given, Radius of hemisphere = Radius of cone

Also, Curved surface area of hemisphere = Curved surface area of cone

$\Rightarrow \quad 2\pi r^2 = \pi r l$ [where l = slant height of the cone]

$\Rightarrow \quad 2r = l$

$\Rightarrow \quad r : l = 1 : 2.$

6. Given : $x^2 - 0.04 = 0$

$\Rightarrow \quad x^2 - (0.2)^2 = 0$

$\Rightarrow \quad (x + 0.2)(x - 0.2) = 0$

$\Rightarrow \quad x = -0.2, 0.2$

OR

Let α and β be the root of this quadratic equation

$$2x^2 - 11x + 5 = 0$$

$$\alpha + \beta = (11/2)$$

$$\alpha.\beta = (5/2)$$

We know that,

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\,\alpha.\beta$$

$$= \left(\frac{11}{2}\right)^2 - 4\left(\frac{5}{2}\right)$$

$$= \frac{121}{4} - \frac{20}{2} = \frac{121 - 40}{4} = \frac{81}{4} = \left(\frac{9}{2}\right)^2$$

Difference of roots $= (\alpha - \beta) = 4.5$

Section - B

7. Class having maximum frequency is the modal class.

Hence, modal class 150 – 155.

Lower limit of the modal class = 150

Now, $\frac{N}{2} = \frac{60}{2} = 30$

The cumulative frequency just greater than 30 is 37.

Hence, the median class is 1360 – 165.

Upper limit of the median class = 165

Required sum = 150 + 165 = 315

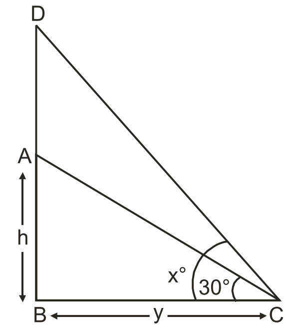
8. Let the height of the tower AB be h m and the distance between the tower and the point of observation C on the ground be y m.

So, $\tan 30° = \frac{h}{y}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{y}$

$\Rightarrow \qquad y = h\sqrt{3}$ ...(i)

Let the new height be DB and the new angle of elevation be $x°$.



So $\qquad \tan x° = \dfrac{3h}{y}$

$\Rightarrow \qquad \tan x° = \dfrac{3h}{h\sqrt{3}}$

$= \sqrt{3} = \tan 60°$

$\Rightarrow \qquad x° = 60°$

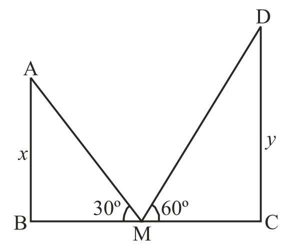
Thus, the angle of elevation is 60° from the same point when the height of the tower is tripled.

OR

Let AB and CD be two towers of height x and y respectively.

M is the mid-point of BC *i.e.,* BM = MC

In ΔABM, we have



$$\frac{AB}{BM} = \tan 30°$$

$\Rightarrow \qquad BM = \dfrac{x}{\tan 30°}$ ...(i)

In ΔCDM, we have

$$\frac{DC}{MC} = \tan 60°$$

$\Rightarrow \qquad \dfrac{y}{MC} = \tan 60°$

$\Rightarrow \qquad MC = \dfrac{y}{\tan 60°}$ ...(ii)

From equations (i) and (ii), we get

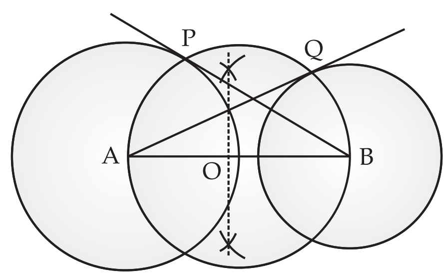
$$\frac{x}{\tan 30°} = \frac{y}{\tan 60°} \qquad (\because \ BM = MC)$$

$\Rightarrow \qquad \dfrac{x}{y} = \dfrac{\tan 30°}{\tan 60°}$

$\Rightarrow \qquad \dfrac{x}{y} = \dfrac{1/\sqrt{3}}{\sqrt{3}} = \dfrac{1}{3}$

$\therefore \qquad x : y = 1 : 3.$

9. Step I : Draw AB = 8 cm.



Step II : Construct a circle of radius 4 cm from point A.

Step III : Construct another circle of radius 3 cm from point B.

Step IV : Now draw perpendicular bisector of AB. Let O be the mid-point of AB.

Step V : Keeping O as the centre, draw circle with AO = OB as the radius.

Step VI : Mark point of intersection of this circle with the larger circle as P.

Step VII : Draw PB as tangent to the larger circle.

Step VIII : Similarly, mark the point of intersection of the circle with centre O with the smaller circle as Q.

Step IX : Draw AQ as the tangent to the smaller circle.

10.

Class Interval	Frequency (f_i)	x_i	f_ix_i
40 – 43	31	41.5	1286.5
43 – 46	58	44.5	2581
46 – 90	60	47.5	2850
49 – 52	k	50.5	$50.5k$
52 – 55	27	53.5	1444.5
	$\Sigma f_i = 176 + k$		$\Sigma f_ix_i = 8162 + 50.5k$

Mean $(\bar{x}) = 47.2$ [Given]

We know that, $\bar{x} = \dfrac{\Sigma f_ix_i}{\Sigma f_i}$

$$\Rightarrow \quad 47.2 = \frac{8162 + 50.5k}{176 + k}$$

$$\Rightarrow \quad 47.2\,(176 + k) = 8162 + 50.5k$$

$$\Rightarrow \quad 8307.2 + 47.2k = 8162 + 50.5k$$

$$\Rightarrow \quad 8307.2 - 8162 = 50.5k - 47.2k$$

$$\Rightarrow \quad 145.2 = 3.3k$$

$$\Rightarrow \quad k = \frac{145.2}{3.3} = 44$$

Thus, there are 44 militants operating in the age group 49 – 52.

Section - C

11. Let a be the first term of the series and d be the common difference.

$$\therefore \quad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

So, $$S_{30} = \frac{30}{2}\{2a + (30-1)d\}$$

$$\Rightarrow \quad S_{30} = 15\{2a + 29d] \quad \text{...(i)}$$

$$S_{20} = \frac{20}{2}\{2a + (20-1)d\}$$

$$\Rightarrow \quad S_{20} = 10\{2a + 19d\} \quad \text{...(ii)}$$

and $$S_{10} = \frac{10}{2}\{2a + (10-1)d\}$$

$$\Rightarrow \quad S_{10} = 5\{2a + 9d\} \quad \text{...(iii)}$$

Now,
$$S_{20} - S_{10} = 10\{2a + 19d\} - 5\{2a + 9d\}$$
$$= 20a + 190d - 10a - 45d$$
$$= 10a + 145d$$
$$= 5(2a + 29d) \quad \text{[From (iii)]}$$
$$3(S_{20} - S_{10}) = 3[5\,(2a + 29d]$$
$$S_{30} = 15\,(2a + 29d) \quad \text{[From (i)]}$$

Thus, $3(S_{20} - S_{10}) = S_{30}$

OR

Given,

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots\left(1-\frac{n}{n}\right) = (1+1+1+\ldots+1) - \left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\frac{4}{n}+\ldots+\frac{n}{n}\right)$$

$$= n - \left[\frac{n}{2}\left\{2\left(\frac{1}{n}\right)+(n-1)\frac{1}{n}\right\}\right]$$

$$= n - \left[\frac{n}{2}\left\{\frac{2}{n} - \frac{1}{n} + 1\right\}\right]$$

$$= n - \left[\frac{n}{2}\left\{\frac{1}{n} + 1\right\}\right]$$

$$= n - \frac{1+n}{2} = \frac{2n-n-1}{2}$$

$$= \frac{n-1}{2}$$

Thus, the sum of $\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots\left(1-\frac{n}{n}\right)$ is $\frac{n-1}{2}$

12. Join OT, let it intersect PQ at the point R.

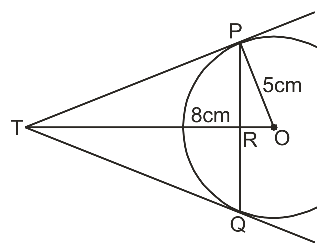
Now, ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle$PTQ. So, OT $\perp$ PQ and therefore, OT bisects PQ

$\therefore$ $PR = RQ = 4$ cm

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$

$= \sqrt{25-16}$

$= \sqrt{9} = 3$ cm



Now, $\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$ [$\because$ In ΔTRP, $\angle TRP = 90°$]

$\Rightarrow$ $\angle RPO = \angle PTR$

So, $\Delta TRP \sim \Delta PRO$ (ByAA rule)

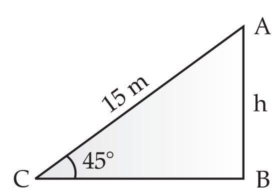
$\therefore$ $\frac{TP}{PO} = \frac{RP}{RO}$

or $\frac{TP}{5} = \frac{4}{3}$, or $TP = \frac{20}{3}$ cm

Hence, the length of TP = $\frac{20}{3}$ cm

13. (i) Consider h be the height of the pole.

In ΔABC,



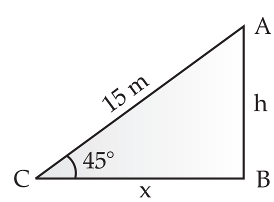
$\sin 45° = \frac{h}{15}$

$\Rightarrow$ $\frac{1}{\sqrt{2}} = \frac{h}{15}$

$\Rightarrow$ $h = \frac{15}{\sqrt{2}}$ m

(ii) Let x be the required distance.

In ΔABC,



$\Rightarrow$ $\frac{x}{15} = \cos 45°$

$\Rightarrow$ $\frac{x}{15} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ $x = \frac{15}{\sqrt{2}}$ m

14. (i) Diagonal of cubic portion $= a\sqrt{3}$

$= 23\sqrt{3}$ m

(ii) Volume of the block $= \pi r^2 h + 2 \times \frac{2}{3} \times \pi r^3$

$= \pi r^2 \left[h + \frac{4}{3} r \right]$

$= 3{\cdot}14 \times (0{\cdot}25)^2 \left[1{\cdot}5 + \frac{4}{3} \times 0{\cdot}25 \right]$ $[\because h = 2 - 2 \times 0.25 = 1.5]$

$= 3{\cdot}14 \times (0{\cdot}25)^2 \times \frac{5{\cdot}5}{3}$

$= 0{\cdot}36$ cm^3 (approx.)