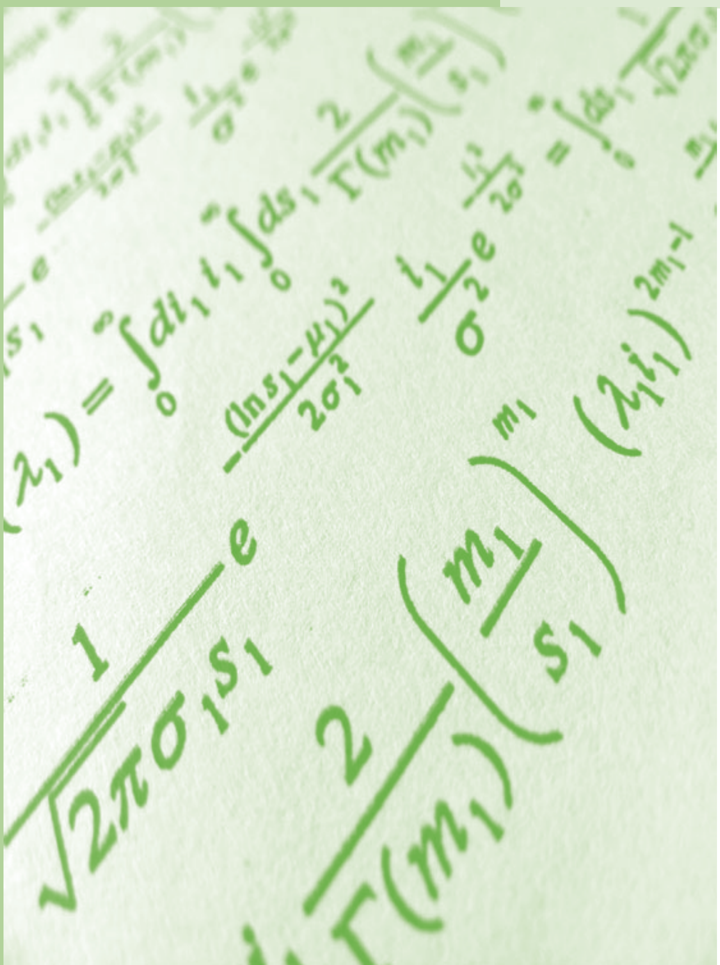
Chapter

5

Statements



REMEMBER

Before beginning this chapter, you should be able to:

- Have knowledge of different types of numbers
- Apply fundamental operations on numbers

KEY IDEAS

After completing this chapter, you would be able to:

- Understand statements, truth tables of different compound statements
- Study laws of algebra of statements
- Learn methods of proof and disproof to explain validity of a statement
- Understand application to switching networks etc

INTRODUCTION

In this chapter, we will learn about statements, truth tables of different compound statements, laws of algebra of statements, application of truth tables to switching networks, etc.

STATEMENT

A sentence which can be judged either true or false but not both is called a **statement**. Statements are denoted by lower case letters like p, q, r, etc.

Examples:

1. p: 2 is a prime number. This statement is true.
2. q: $2 + 3 = 6$. This statement is false.

Truth Value

The truthness or falsity of a statement is called its truth value.

Truthness of a statement is denoted by T, while its falsity is denoted by F.

Examples:

1. The truth value of the statement, p: The sun rises in the east, is True.
2. The truth value of the statement q: All odd numbers are prime, is False.

Negation of a Statement

The denial of a statement is called its negation. Negation of a statement p is denoted by $\sim p$ and read as *not p* or *negation p*.

Truth Table

p	~p
T	F
F	T

Examples:

1. p: $2 + 4 = 6$
   $\sim p$: $2 + 4 \neq 6$
2. p: 3 is a factor of 10
   $\sim p$: 3 is not a factor of 10
3. p: Charminar is in Delhi
   $\sim p$: Charminar is not in Delhi

Compound Statement

A statement obtained by combining two or more simple statements using connectives is called a compound statement.

Examples:

Consider the two statements.
p: 2 is a prime number and
q: 2 is an even number.

Some compound statements that can be formed by using the statements p and q are:

1. 2 is a prime number and 2 is an even number.
2. 2 is a prime number or 2 is an even number.
3. 2 is neither a prime number nor an even number.

Let us look at some basic compound statements.

Conjunction

If p and q are two simple statements, the compound statement p and q is called the conjunction of p and q. It is denoted by $p \wedge q$.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

We observe that $p \wedge q$ is true only when both p and q are true.

Examples:

1. Let p: 4 is a perfect square and q: 2 is an odd number.

   $p \wedge q$: 4 is a perfect square and 2 is an odd number.

   As p is T and q is F, the truth value of $p \wedge q$ is F.

2. Let p: $3 > 2$ and q: $\sqrt{2}$ is an irrational number.

   Then, $p \wedge q$: $3 > 2$ and $\sqrt{2}$ is an irrational number.

   The truth value of $p \wedge q$ is true as both p and q are true.

Disjunction

If p and q are two simple statements, then the compound statement p or q is called the disjunction of p and q. It is denoted by $p \vee q$.

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

We observe that, $p \vee q$ is false only when both p and q are false.

Examples:

1. Let p: The set of even primes is an empty set.

   q: 1 is a factor of every natural number.

   $p \vee q$: The set of even primes is an empty set or 1 is a factor of every natural number.

   As p is false and q is true, truth value of $p \vee q$ is T.

2. Let p: 5 is a factor of 18.

q: 12 divides 6.

Then, $p \vee q$: 5 is factor of 18 or 12 divides 6.

The truth value of $p \vee q$ is false as both p and q are false.

Implication or Conditional

If p and q are two statements, the compound statement *if p then q*, is called a conditional statement. It is denoted by $p \Rightarrow q$.

The statement p is called the hypothesis (or given) and the statement q is called the conclusion (or result).

Truth Table

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

We observe that, a true statement cannot imply a false statement.

Examples:

1. Let p: Every set is a subset of itself.

q: $3 + 5 = 8$

$p \Rightarrow q$: If every set is a subset of itself, then $3 + 5 = 8$.

As p is true and q is true, the truth value of $p \Rightarrow q$ is true.

2. Let p: ABC is a right triangle if $\angle A = 100°$.

q: $\angle A + \angle B + \angle C = 180°$

$p \Rightarrow q$: ABC is a right triangle if $\angle A = 100°$, then $\angle A + \angle B + \angle C = 180°$.

As p is false and q is true, the truth value of $p \Rightarrow q$ is true.

Bi-conditional or Bi-implication

If p and q are two statements, then the compound statement *p if and only if q* is called the bi-conditional of p and q. It is denoted by $p \Leftrightarrow q$.

Truth Table

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

We observe that, $p \Leftrightarrow q$ is true if both p and q have the same truth values.

Examples:

1. Let p: $2 \times 3 = 6$
   q: $2 + 8 = 10$
   $p \Leftrightarrow q$: $2 \times 3 = 6$ if and only if $2 + 8 = 10$.
   Since both p and q are true, the truth value of $p \Leftrightarrow q$ is T.
2. Let p: Every triangle is equilateral.
   q: Charminar is in Hyderabad.
   $p \Leftrightarrow q$: Every triangle is equilateral if and only if Charminar is in Hyderabad.
   As p is false and q is true, the truth value of $p \Leftrightarrow q$ is F.

Converse, Inverse and Contrapositive of a Conditional Let $p \Rightarrow q$ or *if p then q* be a conditional,

1. If q then p, i.e., $q \Rightarrow p$, is called the converse of $p \Rightarrow q$.
2. If not p then not q, i.e., $\sim p \Rightarrow \sim q$, is called the inverse of $p \Rightarrow q$.
3. If not q then not p, i.e., $\sim q \Rightarrow \sim p$ is called the contrapositive of $p \Rightarrow q$.

Truth Table

p	q	$\sim p$	$\sim q$	Conditional $p \Rightarrow q$	Converse $q \Rightarrow p$	Inverse $\sim p \Rightarrow \sim q$	Contrapositive $\sim q \Rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

EXAMPLE 5.1

Write the converse, inverse and contrapositive of the conditional 'If x is odd then x^2 is odd'.

SOLUTION

Conditional: If x is odd, then x^2 is odd.

Converse: If x^2 is odd, then x is odd.

Inverse: If x is not odd, then x^2 is not odd.

Contrapositive: If x^2 is not odd, then x is not odd.

EXAMPLE 5.2

Write the converse, and the contrapositive of the conditional, 'If ABC is a triangle, then $\angle A + \angle B + \angle C = 180°$'.

SOLUTION

Conditional: If ABC is a triangle, then $\angle A + \angle B + \angle C = 180°$.

Converse: If $\angle A + \angle B + \angle C = 180°$, then ABC is a triangle.

Contrapositive: If $\angle A + \angle B + \angle C \neq 180°$, then ABC is not a triangle.

Let us now look at the truth tables of some compound statements:

EXAMPLE 5.3

Write the truth table of $p \vee \sim q$.

SOLUTION

Truth Table

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

EXAMPLE 5.4

Write the truth table of $\sim p \vee (p \wedge q)$.

SOLUTION

Truth Table

p	q	$\sim p$	$p \wedge q$	$\sim p \vee (p \wedge q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

EXAMPLE 5.5

Write the truth table of $\sim p \Rightarrow p \vee q$.

SOLUTION

Truth Table

p	q	$\sim p$	$p \vee q$	$\sim p \Rightarrow p \vee q$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

Tautology

A compound statement which always takes **True** as its truth value is called a tautology.

Example: The truth table of $p \vee \sim p$ is

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

We observe that $p \vee \sim p$ takes T as its truth value always. So, $p \vee \sim p$ is a tautology.

EXAMPLE 5.6

Show that the compound statement $p \Rightarrow p \vee q$ is a tautology.

SOLUTION

Truth Table

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

We observe that $p \Rightarrow p \vee q$ is always True.

Hence, $p \Rightarrow p \vee q$ is a tautology.

Contradiction

A compound statement which always takes **False** as its truth value is called a contradiction.

Example:

The truth table of $p \wedge \sim p$ is

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

We observe that $p \wedge \sim p$ takes F as its truth value always. So $p \wedge \sim p$ is a contradiction.

EXAMPLE 5.7

Show that the compound statement $(p \vee \sim p) \Rightarrow (q \wedge \sim q)$ is a contradiction.

SOLUTION

Truth Table

p	q	$\sim p$	$\sim q$	$p \vee \sim p$	$q \wedge \sim q$	$p \vee \sim p \Rightarrow q \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

We observe that $p \vee \sim p \Rightarrow q \wedge \sim q$ is always False.

Hence, $p \vee \sim p \Rightarrow q \wedge \sim q$ is a contradiction.

Contingency

A compound statement which is neither a tautology nor a contradiction is called a contingency.

Example: $p \vee q \Rightarrow \sim p$

Truth Table

p	q	$\sim p$	$p \vee q$	$p \vee q \Rightarrow \sim p$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

Logically Equivalent Statements

Two statements r and s are said to be logically equivalent, if the last columns of their truth tables are identical. (OR)

Two statements r and s are said to be logically equivalent if $r \Leftrightarrow s$ is a tautology. Generally, r and s will be compound statements.

If the statements r and s are logically equivalent, then we denote this as $r \equiv s$.

Note $r \Leftrightarrow s$ is always true only if both r and s have same truth values.

EXAMPLE 5.8

Show that $p \wedge q \equiv q \wedge p$.

SOLUTION

Truth Table

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

We observe that $p \wedge q$ and $q \wedge p$ have the same truth values. Hence, $p \wedge q \equiv q \wedge p$.

EXAMPLE 5.9

Show that $p \Rightarrow q \equiv \sim p \vee q$.

SOLUTION

Truth Table

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that $p \Rightarrow q$ and $\sim p \vee q$ have the same truth values.

Hence, $p \Rightarrow q \equiv \sim p \vee q$.

Laws of Algebra of Statements

Some logical equivalences are listed under the following laws:

1. Commutative Laws

(i) $p \vee q \equiv q \vee p$

(ii) $p \wedge q \equiv q \wedge p$

2. Associative laws

(i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

3. Distributive Laws

(i) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

4. Idempotent Laws

(i) $p \vee p \equiv p$

(ii) $p \wedge p \equiv p$

5. De Morgan's laws

(i) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(ii) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

6. Identity Laws

(i) $p \vee f \equiv p$, $p \vee t \equiv t$

(ii) $p \wedge f \equiv f$, $p \wedge t \equiv p$

7. Complement Laws

(i) $p \vee (\sim p) \equiv t$

(ii) $p \wedge (\sim p) \equiv f$

(iii) $\sim(\sim p) \equiv p$

(iv) $\sim t \equiv f$

(v) $\sim f \equiv t$

List of Equivalences Based on Implications

1. $p \Rightarrow q \equiv \sim p \vee q$
2. $\sim(p \Rightarrow q) \equiv p \wedge \sim q$
3. $p \Rightarrow q \equiv \sim q \Rightarrow \sim p$

   (i.e., a conditional and its contrapositive are logically equivalent)
4. $q \Rightarrow p \equiv \sim q \Rightarrow \sim p$

   (i.e., converse and inverse of a conditional are logically equivalent)
5. $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$
6. $\sim(p \Leftrightarrow q) \equiv (\sim p) \Leftrightarrow q$ or $p \Leftrightarrow (\sim q)$

OPEN SENTENCE

A sentence involving one or more variables is called an open sentence, if it becomes TRUE or FALSE when the variables are replaced by some specific values from the given set. The set from which the values of a variable can be considered is called the replacement set or domain of the variable.

Examples:

1. $x + 2 = 9$ is an open sentence.

   For $x = 7$, it becomes True and for other real values of x it becomes False.
2. $x^2 + 1 > 0$ is an open sentence.

   For all real values of x it is True.

Quantifiers

A quantifier is a word or phrase which quantifies a variable in the given open sentence.

There are two types of quantifiers.

1. Universal quantifier
2. Existential quantifier

Universal Quantifier

The quantifiers like *for all*, *for every*, *for each* are called universal quantifiers. A universal quantifier is denoted by '$\forall$'.

Examples:

1. Consider an open sentence, $|x| \geq 0$.

   This is true for all $x \in R$. So, we write $|x| \geq 0, \forall x \in R$.
2. Consider the sentence,

   $$1+2+3+\cdots+n=\frac{n(n+1)}{2}, \forall n \in N.$$

Existential Quantifier

The quantifiers like *for some*, *not all*, *there is/exists at least one* are called existential quantifiers.

An existential quantifier is denoted by '$\exists$'.

Examples:

1. Not all prime numbers are odd.
2. $\exists x \in R$ such that $x + 4 = 11$.

Negation of Statements Involving Quantifiers

1. p: All odd numbers are prime.

   $\sim p$: Not all odd numbers are prime

   (or)

Some odd numbers are not prime.
(or)
There is an odd number which is not prime.

2. p: All questions are difficult.
$\sim p$: Not all questions are difficult.
(or)
Some questions are not difficult.
(or)
There is at least one question which is not difficult.

3. p: All birds can fly.
$\sim p$: Not all birds can fly.
(or)
There are some birds which cannot fly.
(or)
There is at least one bird which cannot fly.

METHODS OF PROOF

Statements in mathematics are usually examined for their validity. The various steps involved in the process is referred as proof.

There are two important types of proofs.

Direct Proof

In this method, we begin with the hypothesis and end up with the desired result through a logical sequence of steps.

Example:
If x is odd, then x^2 is odd.
Given, x is an odd number.
Conclusion: x^2 is an odd number.

Proof:
$\Rightarrow \quad x = 2k + 1$ (for some $k \in Z$)
$\Rightarrow \quad x^2 = (2k + 1)^2$
$= 4k^2 + 4k + 1$
$= 2(2k^2 + 2k) + 1$
$= 2m + 1$, where $m = 2k^2 + 2k \in Z$ ($\because k \in Z \Rightarrow 2k^2 + 2k \in Z$).
$\therefore x^2 = 2m + 1, m \in Z$
$\Rightarrow \quad x^2$ is an odd number.
Hence, if x is an odd number, then x^2 is an odd number.

Indirect Proof

In this method, we proceed by assuming that the conclusion is false. Then we arrive at a contradiction. This implies that the desired result must be true.

Example:
If $a + b = 0$, then $(a + b)^2 = 0$. where $a, b \in Z - \{0\}$
Given: $a + b = 0$
Conclusion: $(a + b)^2 = 0$

Proof: Let us assume that $(a + b)^2 \neq 0$

$\Rightarrow \quad (a + b)^2 > 0$ (As $(a + b)^2$ cannot be negative)

$\Rightarrow \quad a + b > 0$ or $a + b < 0$

Which is a contradiction to the hypothesis, i.e., $a + b = 0$.

$\therefore$ Our assumption, i.e., $(a + b)^2 \neq 0$ is false.

Hence, if $a + b = 0$, then $(a + b)^2 = 0$.

METHODS OF DISPROOF

To disprove a given statement there are two methods.

Counter Example Method

In this method, we look for a counter example which disproves the given statement.

Examples:

1. Every odd number is a prime number.

   This statement is false, as 9 is an odd number but it is not a prime.

2. $x^2 - x - 6 = 0$, for all real values of x.

   This statement is false, as for $x = 2$, $x^2 - x - 6 = (2)^2 - 2 - 6 = -4 \neq 0$.

   $\therefore$ $x = 2$ is a counter example here.

Method of Contradiction

In this method, we assume that the given statement is true. Then we arrive at a contradiction. This implies that the given statement is false.

EXAMPLE 5.10

Disprove the statement, 'There can be two right angles in a triangle'.

SOLUTION

Let ABC be a triangle.

If possible, let $\angle A = 90°$ and $\angle B = 90°$.

We know that, the sum of the three internal angles of a triangle is 180°.

That is, $\angle A + \angle B + \angle C = 180°$.

$\Rightarrow \quad 90° + 90° + \angle C = 180°$

$\Rightarrow \quad \angle C = 0°$ which is a contradiction.

Hence, there cannot be two right angles in a triangle.

APPLICATION TO SWITCHING NETWORKS

Now we consider the statements p and p^1 as switches with the property that if one is on, then the other is off and vice versa.

Further, a switch allows only two possibilities.

1. It is either open (F), in which case there is no flow of current.

(Or)

2. It is closed (T), in which case there is a flow of current.

Hence, every switch has two truth values T or F only.

Let p and q denote two switches. We can connect p and q by using a wire in a series or parallel combination as shown in Fig. 5.1.

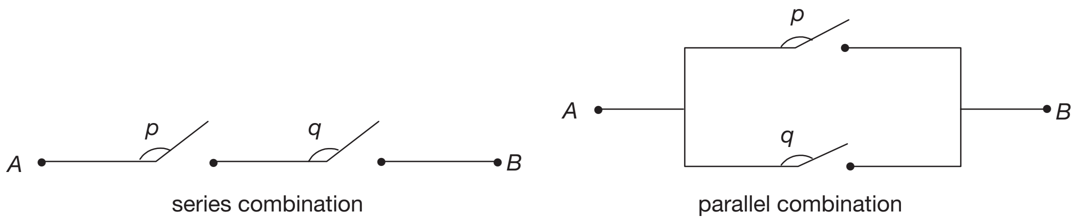


Figure 5.1

Note $p \wedge q$ denote the series combination and $p \vee q$ denote the parallel combination.

Switching Network

A switching network is a repeated arrangement of wires and switches in series and parallel combinations.

So, such a network can be described by using the connectives $\wedge$ and $\vee$.

EXAMPLE 5.11

Describe the behaviour of flow of current from A to B in the following circuit network (see Fig. 5.2).

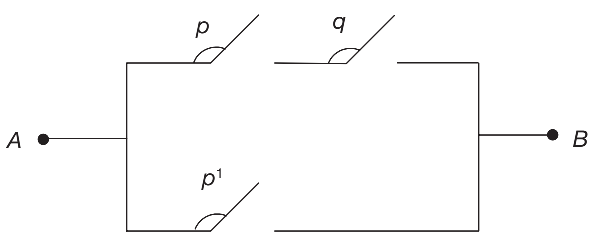


Figure 5.2

SOLUTION

The given network can be described by the compound statement $(p \wedge q) \vee p^1$. Truth table of $(p \wedge q) \vee p^1$ is:

p	q	p^1	$p \wedge q$	$(p \wedge q) \vee p^1$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

So, current flows from A to B if

1. p is closed, q is closed,
2. p is open, q is closed and
3. p is open, q is open.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If truth value of $(\sim P)$ is T, then truth value $\sim(\sim P)$ is _________.

2. $\sim(p \vee \sim q) \equiv$ _________.

3. Truth value of 'If x is even, then x^2 is even' is _________.

4. Counter example which disproves 'all primes are odd' is _________.

5. The negation of 'no dog barks' is _________.

6. The truth value of 'Hyderabad is the capital city of AP' is _________.

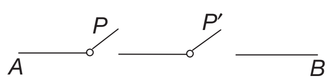
7. $\sim(\sim p \Rightarrow q) = \sim(p \vee q)$ (True/False)

8. If p is F and q is T, then $\sim q \Rightarrow p$ is _________.

9. The quantifier used in negation of 'every planet in the solar system has a satellite'. ($\forall$ or $\exists$)

10. Truth value of '$3 \times 7 = 28$, iff $3 + 7 = 12$' is _________.

11. The negation of '$a > b$' is $a = b$. (True/False)

12. 

    The current _________ (does/does not) flow from A to B.

13. The quantifier to be used to describe the statement, 'not all isosceles triangles are equilateral is _________. ($\forall$ or $\exists$)'.

14. If $p \vee \sim q$ is F, then q is _________.

15. The connective used in the negation of 'if the grass is green, then sky is blue' is _________.

16. Write the conjunction and implication of the statement: He is smart; He is intelligent.

17. Write the suitable quantifier for the sentence: there exists a real number x such that $x + 2 = 3$.

18. Find the truth value of 'Are you attending the meeting tomorrow?'.

19. The symbolic form of the statement, 'If p, then neither q nor r' is

20. Find the inverse of the conditional, 'If I am tired, then I will take rest'.

21. The converse of converse of the statement $p \Rightarrow \sim q$ is _________.

22. Find the truth value of the statement, 'The sum of any two odd numbers is an odd number'.

23. Find the negation of the statement, 'Some odd numbers are not prime'.

24. What is the truth value of the statement, $2 \times 3 = 6$ or $5 + 8 = 10$?

25. $p \Rightarrow q$ is logically equivalent to ______.

26. The counter example of the statement, 'All odd numbers are primes', is

27. Find the converse of the statement, 'If $ABCD$ is square, then it is a rectangle'.

28. Write the compound statement, 'If p, then q and if q, then p' in symbolic form.

29. The negation of the statement, 'I go to school everyday', is

30. The contrapositive of the statement $p \Rightarrow \sim q$ is.

Short Answer Type Questions

31. What is the converse of the statement $p \Rightarrow p \vee q$?

32. Write the converse, inverse and contrapositive of the conditional, 'If she is rich, then she is happy'.

33. Show that $p \Rightarrow p \vee q$ is a tautology.

34. Write the converse, inverse and contrapositive, of the statement 'In a ΔABC, if $AB \neq AC$, then $\angle B \neq \angle C$'.

35. Show that $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

36. Write the truth table of $(p \vee q) \vee \sim r$.

37. Show that $p \Rightarrow q \equiv \sim p \vee q$.

38. Write the truth table of $p \wedge \sim q$.

39. Show that $(p \wedge q) \vee \sim q \equiv p \vee \sim q$.

40. Write the truth table of $p \Rightarrow (p \wedge q)$.

41. Write the suitable quantifier for the following sentence:

(a) $x + 1 > x$ for all real values of x.

(b) There is no real number x such that $x^2 + 2x + 2 = 0$.

42. Prove that $(\sim p \wedge q) \wedge q$ is neither a tautology nor a contradiction.

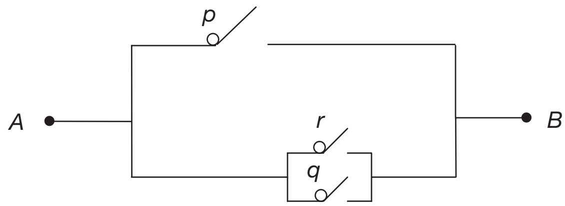
43. Write the truth table of $\sim p \vee (p \wedge q)$.

44. Show that $(p \wedge \sim p) \wedge (p \vee q)$ is a contradiction.

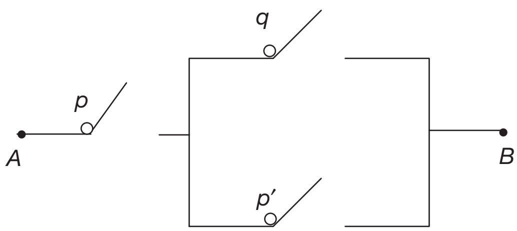
45. Negation of the compound statement $[(p \wedge q) \vee (p \wedge \sim q)]$ is ________.

Essay Type Questions

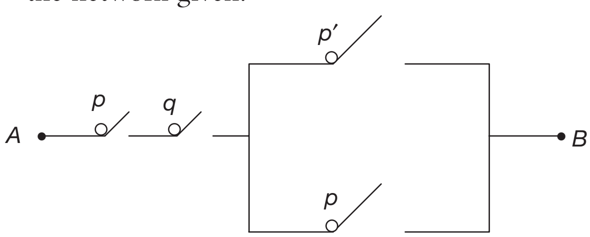
46. Discuss when does the current flow from A to B in the given network.



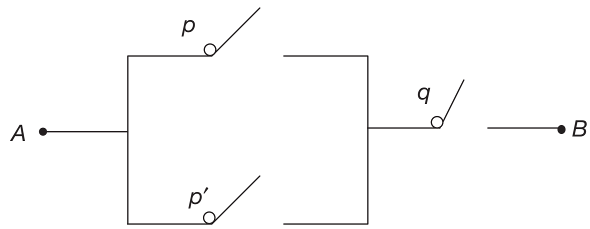
47. Discuss when does the current flow from A to B in the network given.



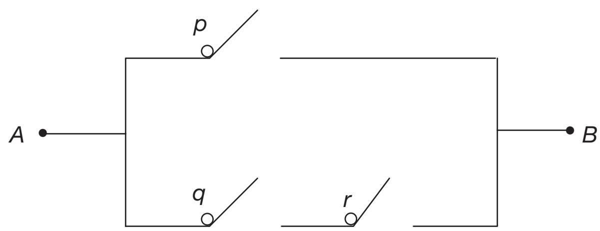
48. Discuss when does the current flow from A to B in the network given.



49. Discuss when does the current flow from A to B in the network given.



50. Discuss when does the current flow from the points A to B in the network given.



CONCEPT APPLICATION

Level 1

1. For which of the following cases does the statement $p \wedge \sim q$ take the truth value as true?

(a) p is true, q is true
(b) p is false, q is true
(c) p is false, q is false
(d) p is true, q is false

2. Which of the following sentences is a statement?

(a) Ramu is a clever boy.
(b) What are you doing?
(c) Oh! It is amazing.
(d) Two is an odd number.

3. Which of the following laws does the connective $\wedge$ satisfy?

(a) Commutative law

(b) Idempotent law

(c) Associative law

(d) All the above

4. The truth value of the statement, 'We celebrate our Independence day on August 15th', is

(a) T

(b) F

(c) Neither T nor F

(d) Cannot be determined

5. When does the inverse of the statement $\sim p \Rightarrow q$ results in T?

(a) $p = T, q = T$

(b) $p = T, q = F$

(c) $p = F, q = F$

(d) Both (b) and (c).

6. Which of the following is a contradiction?

(a) $p \vee q$ (b) $p \wedge q$

(c) $p \vee \sim p$ (d) $p \wedge \sim p$

7. In which of the following cases, $p \Leftrightarrow q$ is true?

(a) p is true, q is true

(b) p is false, q is true

(c) p is true, q is false

(d) None of these

8. Find the counter example of the statement 'Every natural number is either prime or composite'.

(a) 5 (b) 1

(c) 6 (d) None of these

9. Which of the following pairs are logically equivalent?

(a) Conditional, Contrapositive

(b) Conditional, Inverse

(c) Contrapositive, Converse

(d) Inverse, Contrapositive

10. The property $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ is called

(a) associative law.

(b) commutative law.

(c) distributive law.

(d) idempotent law.

11. Which of the following is contingency?

(a) $p \vee \sim p$ (b) $p \wedge q \Rightarrow p \vee q$

(c) $p \wedge (\sim q)$ (d) None of these

12. Which of the following pairs are logically equivalent?

(a) Converse, Contrapositive

(b) Conditional, Converse

(c) Converse, Inverse

(d) Conditional, Inverse

13. Which of the following connectives can be used for describing a switching network?

(a) $(p \wedge q) \vee p'$

(b) $(p \vee q) \wedge q'$

(c) Both (a) and (b)

(d) None of these

14. Find the quantifier which best describes the variable of the open sentence $x + 3 = 5$.

(a) Universal

(b) Existential

(c) Neither (a) nor (b)

(d) Cannot be determined

15. Which of the following is equivalent to $p \Leftrightarrow q$?

(a) $p \Rightarrow q$

(b) $q \Rightarrow p$

(c) $(p \Rightarrow q) \wedge (q \Rightarrow p)$

(d) None of these

16. The property $\sim (p \wedge q) \equiv \sim p \vee \sim q$ is called ________.

(a) associative law

(b) De Morgan's law

(c) commutative law

(d) idempotent law

17. The statement $p \vee q$ is

(a) a tautology.

(b) a contradiction.

(c) Neither (a) nor (b)

(d) Cannot say

18. Which of the following compound statement represents a series network?

(a) $p \vee q$ (b) $p \Rightarrow q$

(c) $p \wedge q$ (d) $p \Leftrightarrow q$

19. Which of the following is a tautology?

(a) $p \wedge q$ (b) $p \vee q$

(c) $p \vee \sim p$ (d) $p \wedge \sim p$

20. Find the truth value of the compound statement, 'If 2 is a prime number, then hockey is the national game of India'.

(a) T

(b) F

(c) Neither (a) nor (b)

(d) Cannot be determined

21. Find the truth value of the compound statement, 4 is the first composite number and $2 + 5 = 7$.

(a) T

(b) F

(c) Neither (a) nor (b)

(d) Cannot be determined

22. Find the inverse of the statement, 'If ΔABC is equilateral, then it is isosceles'.

(a) If ΔABC is isosceles, then it is equilateral.

(b) If ΔABC is not equilateral, then it is isosceles.

(c) If ΔABC is not equilateral, then it is not isosceles.

(d) If ΔABC is not isosceles, then it is not equilateral.

23. The statement $p \Rightarrow p \vee q$ is

(a) a tautology.

(b) a contradiction.

(c) Both (a) and (b)

(d) Neither (a) nor (b)

24. What is the truth value of the statement 'Two is an odd number, iff 2 is a root of $x^2 + 2 = 0$'?

(a) T

(b) F

(c) Neither (a) nor (b)

(d) Cannot be determined

25. Which of the following connectives satisfy commutative law?

(a) $\wedge$ (b) $\wedge$

(c) $\Leftrightarrow$ (d) All the above of these

26. $\sim [\sim p \wedge (p \Leftrightarrow q)] \equiv$

(a) $p \vee q$ (b) $q \wedge p$

(c) T (d) F

27. Write the negation of the statement 'If the switch is on, then the fan rotates'.

(a) 'If the switch is not on, then the fan does not rotate'.

(b) 'If the fan does not rotate, then the switch is not on'.

(c) 'The switch is not on or the fan rotates'.

(d) 'The switch is on and the fan does not rotate'.

28. If p: The number of factors of 20 is 5 and q : 2 is an even prime number, then the truth values of inverse and contrapositive of $p \Rightarrow q$ respectively are

(a) T, T. (b) F, F.

(c) T, F. (d) F, T.

29. 'No square of a real number is less than zero' is equivalent to

(a) for every real number a, a^2 is non-negative.

(b) $\forall\ a \in R$, $a^2 \geq 0$.

(c) Either (a) or (b)

(d) None of these

30. If a compound statement r is contradiction, then find the truth value of $(p \Rightarrow q) \wedge (r) \wedge [p \Rightarrow (\sim r)]$.

(a) T (b) F

(c) T or F (d) None of these

Level 2

31. Which of the following is/are counter example(s) of the statement $x^2 - 7x + 10 > 0$, for all real x?

(A) 2 (B) 3

(C) 4 (D) 5

(a) Only (A) and (D)
(b) Only (B) and (C)
(c) All (A), (B), (C) and (D)
(d) None of these

32. If p: 3 is an odd number and q: 15 is a prime number, then the truth value of $[\sim (p \Leftrightarrow q)]$ is equivalent to that of ________.

(A) $p \Leftrightarrow (\sim q)$

(B) $(\sim p) \Leftrightarrow q$

(C) $\sim (p \wedge q)$

(a) Only (A)
(b) Only (C)
(c) Both (A) and (B)
(d) All (A), (B) and (C)

33. The compound statement, 'If you want to top the school, then you do not study hard' is equivalent to

(a) 'If you want to top the school, then you need to study hard'.

(b) 'If you will not top in the school, then you study hard'.

(c) 'If you study hard, then you will not top the school'.

(d) 'If you do not study hard, then you will top in the school'.

34. If p: 25 is a factor of 625 and q: 169 is a perfect square then $\sim (p \Rightarrow q)$ is equivalent to

(a) $p \wedge q$. (b) $(\sim p) \wedge q$.

(c) $p \wedge (\sim q)$. (d) Both (b) and (c)

35. The compound statement, 'If you won the race, then you did not run faster than others' is equivalent to

(a) 'If you won the race, then you ran faster than others'.

(b) 'If you ran faster than others, then you won the race'.

(c) 'If you did not win the race, then you did not run faster than others'.

(d) 'If you ran faster than others, then you did not win the race'.

36. 'If x is a good actor, then y is bad actress' is

(a) a tautology. (b) a contradiction.

(c) a contingency. (d) None of these

37. Which of the following is negation of the statement 'All birds can fly'.

(a) 'Some birds cannot fly'.

(b) 'All the birds cannot fly'.

(c) 'There is at least one bird which can fly'.

(d) All of these

38. What are the truth values of $(\sim p \Rightarrow \sim q)$ and $\sim (\sim p \Rightarrow q)$ respectively, when p and q always speak true in any argument?

(a) T, T (b) F, F

(c) T, F (d) F, T

39. If p: 4 is an odd number and q: 4^3 is an even number, then which of the following is equivalent to $\sim (p \Rightarrow q)$?

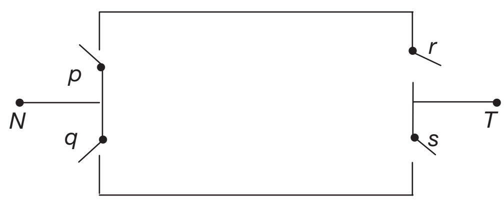
(a) '4 is an odd number and 4^3 is an even number'.

(b) 'The negation of the statement '4 is not an odd number or 4^3 is not an even number'.

(c) Both (a) and (b)

(d) None of these

40.



In the above network, current flows from N to T, when

(a) p closed, q closed, r opened and s opened.

(b) p closed, q opened, s closed and r opened.

(c) q closed, p opened, r opened and s closed.

(d) p opened, q opened, r closed and s closed.

PRACTICE QUESTIONS

41. If p: In a triangle, the centroid divides each median in the ratio 1 : 2 from the vertex and q: In an equilateral triangle, each median is perpendicular bisector of one of its sides. The truth values of inverse and converse of $p \Rightarrow q$ are respectively

(a) T, T. (b) F, F.
(c) T, F. (d) F, T.

42. If p always speaks against q, then $p \Rightarrow p \vee \sim q$ is

(a) a tautology. (b) contradiction.
(c) contingency. (d) None of these

43. If p: Every equilateral triangle is isosceles and q: Every square is a rectangle, then which of the following is equivalent to $\sim (p \Rightarrow q)$?

(a) The negation of 'Every equilateral triangle is not isosceles or every square is rectangle'.
(b) 'Every equilateral triangle is not isosceles, then every square is not a rectangle'.
(c) 'Every equilateral triangle is isosceles, then every square is a rectangle'.
(d) None of these

44. When does the value of the statement $(p \wedge r) \Leftrightarrow (r \wedge q)$ become false?

(a) p is T, q is F
(b) p is F, r is F
(c) p is F, q is F and r is F
(d) None of these

45. If the truth value of $p \vee q$ is true, then truth value $\sim p \wedge q$ is

(a) false if p is true.
(b) true if p is true.
(c) false if q is true.
(d) true if q is true.

46. The compound statement, 'If you want to win the gold medal in Olympics, then you need to work hard' is equivalent to

(a) 'If you will not win the gold medal in Olympics, then you need not work hard.'
(b) 'If you do not work hard, then you will not win the gold medal in Olympics.'
(c) Both (a) and (b)
(d) None of these

47. If p: sum of the angles in a triangle is 180°and q: every angle in a triangle is more than 0°, then $(p \Rightarrow q) \vee p$ is equivalent to

(a) $p \vee \sim q$. (b) $(p \wedge q) \Rightarrow p$.
(c) $\sim p \wedge \sim q$. (d) Both (a) and (b)

48. Which of the following is the negation of the statement, 'All animals are carnivores'?

(a) Some animals are not carnivores.
(b) Not all animals are carnivores.
(c) There is atleast one animal which is not carnivores.
(d) All of these

49. $(p \Leftrightarrow q) \Leftrightarrow (\sim p \Leftrightarrow \sim q)$ is a

(a) tautology. (b) contradiction.
(c) contingency. (d) None of these

50. Which of the following is a counter example of $x^2 - 6x + 8 \le 0$?

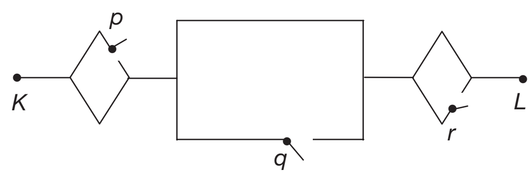
(a) $x = 2$ (b) $x = 3$
(c) $x = 4$ (d) $x = 5$

Level 3

51. If p and q are two statements, then $p \vee \sim(p \Rightarrow \sim q)$ is equivalent to

(a) $p \wedge \sim q$ (b) p
(c) q (d) $\sim p \wedge q$

52.



In the above network, current does not flow, when

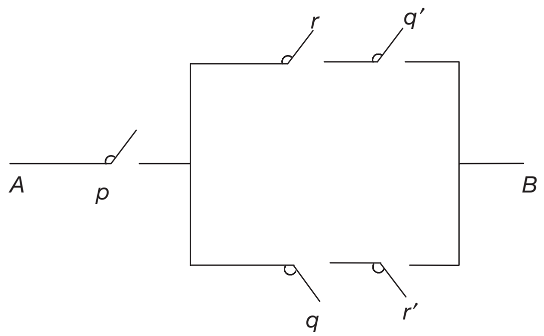
(a) p opened, q opened and r opened.
(b) p closed, q opened and r opened.
(c) p opened, q closed and r closed.
(d) None of these

53. The contrapositive of the statement $\sim p \Rightarrow (p \wedge \sim q)$ is

(a) $p \Rightarrow (\sim p \vee q)$ (b) $p \Rightarrow (p \wedge q)$

(c) $p \Rightarrow (\sim p \wedge q)$ (d) $(\sim p \vee q) \Rightarrow p$

54.



In the above circuit, the current flows from A to B when

(a) p is closed, q is open r is open.

(b) p is closed, q is closed and r is open.

(c) p is closed, q is closed and r is closed.

(d) All of these

55. If p: $5x + 6 = 8$ is an open sentence and q: 3, 4 are the roots of the equation $x^2 - 7x + 12 = 0$, then which of following is equivalent to $\sim [\sim p \vee q]$?

(a) The negation of 'If $5x + 6 = 8$ is an open sentence, then 3, 4 are the roots of the equation $x^2 - 7x + 12 = 0$'.

(b) $5x + 6 = 8$ is an open sentence or 3, 4 are not roots of the equation $x^2 - 7x + 12 = 0$.

(c) $5x + 6 = 8$ is not an open sentence and 3, 4 are the roots of the equation $x^2 - 7x + 12 = 0$.

(d) None of these

56. If the truth value of $p \vee q$ is T, then the truth value $p \wedge q$ is

(a) T

(b) F

(c) Neither (a) nor (b)

(d) Cannot be determined

57. $(p \wedge \sim q) \wedge q$ is equivalent to

(a) $p \vee q$ (b) $p \wedge q$

(c) $p \vee \sim q$ (d) $p \wedge \sim q$

58. Which of the following is a tautology?

(a) $p \Rightarrow (q \vee \sim q)$ (b) $p \Leftrightarrow (\sim q \wedge p)$

(c) $p \Leftrightarrow (p \wedge \sim p)$ (d) $(p \wedge \sim p) \wedge q$

59. $\sim[\sim p \vee (\sim p \Leftrightarrow q)]$ is equivalent to

(a) $p \wedge (p \Leftrightarrow q)$.

(b) $p \wedge q$.

(c) Both (a) and (b)

(d) None of these

60. When does the truth value of the statement $(p \Rightarrow q) \vee (r \Leftrightarrow s)$ become true?

(a) p is true (b) q is true

(c) r is true (d) s is true

61. Given that p: x is a prime number and q: x^2 is a composite number. Then $\sim(\sim p \Rightarrow q)$ is equivalent to

(a) x is not a prime number and x^2 is not a composite number.

(b) x is not a prime number and x^2 is a composite number.

(c) Both (a) and (b)

(d) None of these

62. Which of the following is a contradiction?

(a) $(p \vee q) \Leftrightarrow (p \wedge q)$

(b) $(p \vee q) \Rightarrow (p \wedge q)$

(c) $(p \Rightarrow q) \vee (q \Rightarrow p)$

(d) $(\sim q) \wedge (p \wedge q)$

63. When does the truth value of the statement $[(p \Leftrightarrow q) \Rightarrow (q \Leftrightarrow r)] \Rightarrow (r \Leftrightarrow p)$ become false?

(a) p is true, q is false, r is true.

(b) p is false, q is true, r is true.

(c) p is false, q is false, r is false.

(d) p is true, q is true, r is true.

64. If p is negation of q, then $(p \Rightarrow q) \vee (q \Rightarrow p)$ is a

(a) tautology.

(b) contradiction.

(c) contingency.

(d) None of these

65. Which of the following is a contingency?

(a) $\sim p \wedge q$ (b) $p \vee \sim q$

(c) $\sim p \wedge \sim q$ (d) All of these

66. Which among the following is negation of $\sim(p \wedge q) \vee r$?

(a) $(p \wedge q) \vee r$

(b) $(p \vee q) \vee \sim r$

(c) $(p \vee q) \vee r$

(d) $(p \wedge q) \wedge \sim r$

67. The inverse of a conditional statement is, 'If ABC is a triangle, then $\angle A + \angle B + \angle C = 180°$'. Then the contrapositive of the conditional statement is

(a) 'If $\angle A + \angle B + \angle C = 180°$, then ABC is a triangle'.

(b) 'If $\angle A + \angle B + \angle C \neq 0$, then ABC is not a triangle'.

(c) 'If ABC is not a triangle, then $\angle A + \angle B + \angle C \neq 180°$'.

(d) 'If ABC is a triangle, then $\angle A + \angle B + \angle C = 180°$'.

68. 'There is atleast one prime number which is not odd number'. The quantifier used in the above statement is _______.

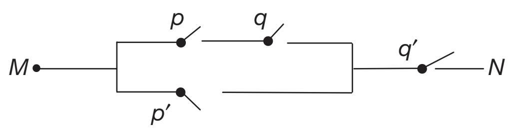
(a) universal quantifier

(b) existential quantifier

(c) Both (a) and (b)

(d) No quantifier used

69.



In the above network, current flows from M to N, when

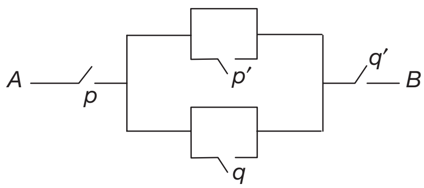
(a) p closed, q closed.

(b) p closed, q opened.

(c) p opened, q closed.

(d) p opened, q opened.

70.



In the above network, current flows from A to B, when

(a) p opened, q opened.

(b) p closed, q closed.

(c) p closed, q opened.

(d) p opened, q closed.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. False
2. $\sim p \wedge q$
3. True
4. 2 is even prime
5. some dogs bark.
6. True
7. True
8. True
9. $\exists$
10. True
11. False
12. does not
13. $\exists$
14. True
15. and
16. He is smart and he is intelligent.

    If he is smart, then he is intelligent.
17. $\exists$
18. Neither true nor false
19. $p \Rightarrow \sim q \wedge \sim r$.
20. 'If I am not tired, then I will not take rest'.
21. $p \Rightarrow \sim q$
22. False
23. All odd numbers are primes.
24. True
25. $\sim p \vee q$
26. 9
27. If $ABCD$ is a rectangle, then it is square.
28. $(p \Rightarrow q) \wedge (q \Rightarrow p)$.
29. Some days I do not go to school.
30. $q \Rightarrow \sim p$

Short Answer Type Questions

31. $p \vee q \Rightarrow p$
32. Converse: If she is happy, then she is rich.

    Inverse: If she is not rich, then she is not happy.

    Contrapositive: If she is not happy, then she is not rich.
34. Converse: In a ΔABC, if $\angle B \neq \angle C$, then $AB \neq AC$.

    Inverse: In a ΔABC, if $AB = AC$, then $\angle B = \angle C$.

    Contrapositive: In a ΔABC, if $\angle B = \angle C$, then $AB = AC$.
41. $\forall$

    $\forall$
45. $\sim p \vee [-q \vee q]$

Essay Type questions

46. (i) p is closed, q is closed, r is closed

    (ii) p is closed, q is closed, r is open

    (iii) p is closed, q is open, r is closed

    (iv) p is open, q is closed, r is closed

    (v) p is closed, q is open, r is open

    (vi) p is open, q is closed, r is open

    (vii) p is open, q is open, r is closed
47. p is closed, q is closed
48. p is closed, q is closed.
49. (i) p is closed, q is closed

    (ii) p is open, q is closed
50. (i) p is closed, q is closed, r is closed

    (ii) p is closed, q is closed, r is open

    (iii) p is closed, q is open, r is closed

    (iv) p is open, q is closed, r is closed

    (v) p is closed, q is open, r is open

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)
11. (c)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (a)
21. (a)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (c)	**30.** (b)

Level 2

31. (c)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (d)	**36.** (d)	**37.** (a)	**38.** (c)	**39.** (d)	**40.** (c)
41. (b)	**42.** (a)	**43.** (a)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (d)	**48.** (d)	**49.** (a)	**50.** (d)

Level 3

51. (b)	**52.** (d)	**53.** (d)	**54.** (b)	**55.** (a)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (c)	**60.** (b)
61. (a)	**62.** (d)	**63.** (b)	**64.** (a)	**65.** (d)	**66.** (d)	**67.** (a)	**68.** (b)	**69.** (d)	**70.** (c)

CONCEPT APPLICATION

Level 1

1. Check from options.

2. Recall the definition of statement.

3. Recall the properties.

5. Check from options.

6. Verify through truth tables.

7. Check from truth tables.

10. Recall the properties.

11. Converse of $p \Rightarrow q$ is $q \Rightarrow p$.

13. Recall the concept of switching network.

14. For only one value of 'x' the equation is true.

15. Check from truth tables.

16. Recall the properties.

17. Verify through truth tables.

18. Recall the concept of switching networking.

19. Check from truth tables.

20. If $p \Rightarrow q$ is false only p is true, q is false.

21. Conjunction is true only when both the statements are true.

22. Inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$.

23. Use truth table.

24. Recall bi-conditional truth table.

25. Recall the properties.

26. Inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$.

27. $\sim(p \Rightarrow q) \equiv \sim(\sim p \vee q) \equiv (p \wedge \sim q)$.

28. (i) The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$.

The contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$.

(ii) $p \Rightarrow q$ is false only when p is true and q is false.

29. Square of a real number is always non-negative.

30. Conjunction is false if atleast one of the statements is false.

Level 2

31. (i) Check from the options.

(ii) Go contrary to the given statement by substituting the value of x in it.

32. (i) Use truth table of double implication.

(ii) Apply the identity, $\sim(p \Leftrightarrow q) \equiv (\sim p \Leftrightarrow q) \equiv (p \Leftrightarrow \sim q)$.

33. $p \Rightarrow$ q is equivalent to $\sim q \Rightarrow \sim p$.

34. (i) Both p and q are true.

(ii) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$.

35. A conditional is equivalent to its contrapositive. That is, $p \Rightarrow q \equiv \sim q \Rightarrow \sim p$.

36. Given sentence is not a statement.

37. Contradict the given statement by using different quantifiers.

38. $p \Rightarrow q$ is false only when p is true and q is false.

39. $\sim(p \Rightarrow q) \equiv p \wedge \sim q \equiv \sim(p \vee q)$.

40. Current flows only when p and r are closed or q and s are closed.

41. (i) p is false and q is true.

(ii) The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$. The converse of $p \Rightarrow q$ is $q \Rightarrow p$.

(iii) $p \Rightarrow q$ is false only when p is true and q is false.

42. Verify through truth table.

43. $\sim(p \Rightarrow q) \equiv p \wedge \sim q \equiv \sim(p \vee q)$.

44. (i) $x \wedge y$ is always false when y is false.

(ii) Compare with truth table of double implication.

45. Cheek from options.

46. We know that, a conditional statement is equivalent to its contrapositive.

Let the given statement be $p \Rightarrow q$.

$\therefore$ Option (a) represents $\sim p \Rightarrow \sim q$

Option (b) represents $\sim q \Rightarrow \sim p$.

$\therefore$ Option (b) follows.

47. Given that p is true and q is true.

$\therefore$ $(p \Rightarrow q) \vee p \equiv t$.

Option (a): $p \vee \sim q \equiv t$.

Option (b): $(p \wedge q) \Rightarrow p \equiv t$.

Option (c): $\sim p \wedge \sim q \equiv f$.

Option (d) follows.

48. Negation of all is all not follows.

49. Let $(p \Leftrightarrow q) \Leftrightarrow (\sim p \Leftrightarrow \sim q) \equiv r$

p	q	$p \Leftrightarrow q$	$\sim p$	$\sim q$	$\sim p \Leftrightarrow \sim q$	r
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

$\therefore$ Given statement is tautology.

50. $x^2 - 6x + 8 \leq 0$

$(x - 2)(x - 4) \leq 0$

$x \in [2, 4]$

$\therefore$ $x = 5 \notin [2, 4]$.

Level 3

51. (i) $\sim(p \Rightarrow q) = \sim p \vee q$.

(ii) $p \vee (q \vee r) = (p \vee q) \vee r$.

52. Current flows irrespective of p, q and r whether closed or open.

53. The contrapositive of $p \Rightarrow q \Rightarrow \sim q \Rightarrow \sim p$.

54. (i) Parallel connection is represented by $x \wedge y$.

(ii) Series connection is represented by $x \vee y$.

55. $\sim(\sim p \vee q) = p \wedge \sim q$.

56. Given $p \vee q$ is true, i.e., any one of p or q is true, so we cannot say the truth value of $p \wedge q$.

57. $(p \vee \sim q) \wedge q$

$\equiv (p \wedge q) \vee (\sim q \wedge q)$ (distributive property).

$\equiv (p \wedge q) \vee f \equiv p \wedge q$.

58. Consider Option (a)

Since $q \vee \sim q$ is true

$p \Rightarrow (q \vee\sim q)$ is always true.

$\therefore$ $p \Rightarrow (q \vee \sim q)$ is tautology.

59. $\sim[\sim p \vee (\sim p \Leftrightarrow q)]$

$\equiv [p \wedge \sim(\sim p \Leftrightarrow q)]$

$\equiv p \wedge (p \Leftrightarrow q)$ or $p \wedge(\sim p \Leftrightarrow \sim q)$

From truth tables,

$p \wedge (p \Leftrightarrow q) \equiv p \wedge q$.

60. When q is true, $p \Rightarrow q$ is always true.

$\therefore$ $(p \Rightarrow q) \vee (r \Leftrightarrow s)$ is true. When q is true.

61. $\sim(\sim p \Rightarrow q) \equiv \sim p \wedge \sim q$.

$\therefore$ $\sim p \wedge \sim q = x$ is not a prime number and x^2 is not a composite number.

62. From the truth tables, first three options are not contradictions.

Option (d): $(\sim q) \wedge (p \wedge q)$

$\equiv p \wedge (\sim q \wedge q)$

$\equiv p \wedge (f) \equiv f$.

63. From the options,

$(p \Leftrightarrow q) \Rightarrow (q \Leftrightarrow r)$

$\Rightarrow (r \Leftrightarrow p)$ is false, when p is false, q is true and r is true.

64. Given, p is negation of $q \Rightarrow p \equiv \sim q$

$\therefore$ $(p \Rightarrow q) \vee (q \Rightarrow p)$

$\Rightarrow (\sim q \Rightarrow q) \vee (q \Rightarrow \sim q)$

$\therefore$ If $(\sim q \Rightarrow q)$ is T, then $(q \Rightarrow \sim q)$ is F.

If $(\sim q \Rightarrow q)$ is F, then $(q \Rightarrow \sim q)$ is T.

$\therefore$ T or $F \equiv F$ or $T \equiv T$.

65. From the truth tables,

All the options are contingencies.

66. $\sim[\sim(p \wedge q) \vee r] \equiv (p \wedge q) \wedge \sim r$.

67. Let inverse of a conditional statement be $p \Rightarrow q$.

$\therefore$ The conditional statement is $\sim p \Rightarrow \sim q$.

$\therefore$ The contrapositive of the conditional is $q \Rightarrow p$.

That is, if $\angle A + \angle B + \angle C = 180°$, then ABC is a triangle.

68. According to the statement, some prime numbers of are not odd numbers, the quantifier is Existential Quantifier.

69. For the given network, current flows from M to N when p is opened and q is opened.

70. For the given network, current flows from A to B when p is closed and q is opened.