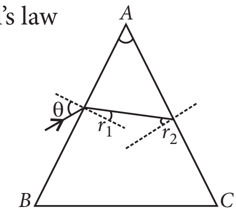
VERY SIMILAR
PRACTICE TEST 8

Hints and Explanations

1. **(c) :** According to Snell's law
$\sin\theta = \mu \sin r_1$



or $r_1 = \sin^{-1}\left(\dfrac{\sin\theta}{\mu}\right)$

Now, $A = r_1 + r_2$

$\therefore \quad r_2 = A - r_1$

$= A - \sin^{-1}\left(\dfrac{\sin\theta}{\mu}\right)$ ...(i)

For the ray to get transmitted through the face AC, r_2 must be less than critical angle,

i.e., $r_2 < \sin^{-1}\left(\dfrac{1}{\mu}\right)$ or $A - \sin^{-1}\left(\dfrac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\dfrac{1}{\mu}\right)$ (using (i))

$\Rightarrow \theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right)\right]$

2. **(c) :** Kinetic energy when the displacement of the particle is x, is given by

$$K = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

Potential energy at this instant is given by

$$U = \frac{1}{2} m\omega^2 x^2$$

According to the condition given in the question

$$K = \frac{1}{8} U$$

$$\Rightarrow \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{8}\cdot\frac{1}{2} m\omega^2 x^2$$

or $(A^2 - x^2) = \dfrac{x^2}{8}$

On rearranging, we get $x = \dfrac{2\sqrt{2}}{3} A$

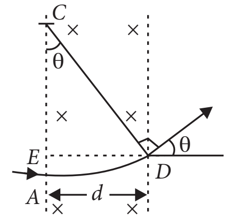
3. **(a) :** Current density $J = ar^2$

Current $I = \int J \cdot dA = \int_{R/3}^{R/2} ar^2 (2\pi r\, dr)$

[where $dA = 2\pi r dr$]

$$= 2\pi a \left[\frac{r^4}{4}\right]_{R/3}^{R/2} = \frac{65\pi a R^4}{2592}$$

4. **(a) :** A to D is part of circle with centre C and radius $CD = r$.



$mv = p = BQr$

or $r = \dfrac{p}{BQ}$

$$\sin\theta = \frac{ED}{CD} = \frac{d}{r} = \frac{BQd}{p}$$

5. **(d) :** As the rod rotates about A, therefore, from conservation of mechanical energy, decrease in potential energy = increase in rotational kinetic energy about A

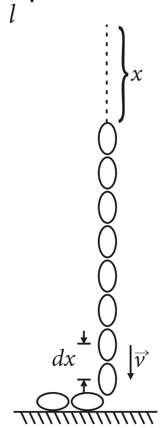
$$mg\left(\frac{l}{2}\right) = \frac{1}{2} I_A\, \omega^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 \text{ or } \omega^2 = \frac{3g}{l}$$

Centripetal force of centre of mass of the rod in this position is $= mr\omega^2 = m\dfrac{l}{2}\dfrac{3g}{l} = \dfrac{3mg}{2}$.

If F is the force exerted by the hinge on the rod (upwards), then $F - mg = \dfrac{3mg}{2}$

$$F = \frac{3mg}{2} + mg = \frac{5}{2} mg.$$

6. **(c) :** Mass per unit length, $\lambda = \dfrac{M}{l}$.



The descending part of the chain is in free fall, also its every point has descended by a distance x.

So, speed of each point,

$v = \sqrt{2gx}$

Assume a very small distance dx falls in a short internal of time dt.

Normal exerted on the falling part,

$$N = -\frac{dp_x}{dt} = \frac{-(0 - (\lambda dx)v)}{dt}$$

$$= \lambda v^2 = \lambda(2gx) = 2\lambda gx$$

Normal due to x part of the chain on the weighing machine,

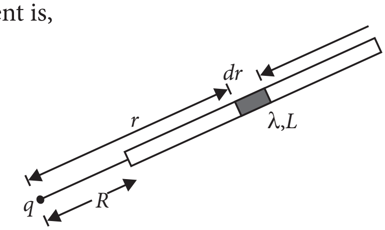
$N' = \lambda gx$

Reading of the scale $W = N + N' = 3\lambda gx$

$$= \frac{3Mgx}{l}$$

7. **(b) :** As coulomb's force is an action reaction pair, so force experienced by the linear charge is equal and opposite to the force experienced by point charge q. Here, we are computing electric force experienced by q due to the charge.

Considered a small element on line charge as shown, then force experienced by q due to this element is,



$$dF = \frac{q\lambda dr}{4\pi\varepsilon_0 r^2}$$

$$F = \int dF = \int_R^{R+L} \frac{q\lambda dr}{4\pi\varepsilon_0 r^2} = \frac{q\lambda L}{4\pi\varepsilon_0 R(R+L)}$$

8. **(a) :** Ideal gas equation $PV = nRT$

$(P_0 + (1-\alpha)V^2)V = nRT$

$$T = \frac{(P_0 + (1-\alpha)V^2)V}{nR}$$

$$\frac{dT}{dV} = \frac{P_0}{nR} + \frac{3V^2(1-\alpha)}{nR} = 0$$

[For maximum temperature]

$P_0 = 3(\alpha - 1)V^2$ or $V^2 = \dfrac{P_0}{3(\alpha-1)}$

$$\therefore \quad P = P_0 + (1-\alpha)\frac{P_0}{3(\alpha-1)} = P_0 - \frac{P_0}{3} = \frac{2P_0}{3}$$

9. **(a) :** From Moseley's law, as $\lambda \propto \dfrac{1}{(Z-1)^2}$

$$\frac{\lambda_{\text{Mo}}}{\lambda_{\text{Cu}}} = \frac{(Z_{\text{cu}} - 1^2)}{(Z_{\text{Mo}} - 1)^2} = \frac{(29-1)^2}{(42-1)^2} = 0.4663$$

or $\lambda_{\text{Cu}} = \dfrac{\lambda_{\text{Mo}}}{0.4663} = \dfrac{0.71\text{Å}}{0.4663} = 1.52\ \text{Å}$

10. **(b) :** Comparing $B = B_0 \sin(kz - \omega t)$ T with $B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T, we get $B_0 = 12 \times 10^{-8}$ T

Average intensity of the beam,

$$I_v = \frac{cB_0^2}{2\mu_0} = \frac{3\times10^8 \times (12\times10^{-8})^2}{2\times4\pi\times10^{-7}} = 1.72\ \text{W m}^{-2}$$

11. **(a) :** Taking the gravitational potential at a large distance from the earth as zero, the gravitational potential at the surface of the planet $= -\dfrac{GM}{R}$.

From law of conservation of energy, if v is the velocity of particle while reaching the surface of the planet and m is its mass, then

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0$$

or $v^2 = \dfrac{2GM}{R} = v_e^2 \qquad \left(\because v_e = \sqrt{\dfrac{2GM}{R}}\right)$

or $v = v_e$.

12. **(a) :** Pressure outside the bigger drop = P_1

Pressure inside the bigger drop = P_2

Radius of bigger drop, r_1 = 3 cm

Excess pressure $= P_2 - P_1 = \dfrac{4S}{r_1} = \dfrac{4S}{3}$

Pressure inside small drop = P_3

Excess pressure = $P_3 - P_2 = \dfrac{4S}{r_2} = \dfrac{4S}{1}$

Pressure difference between inner side of small drop and outer side of bigger drop

$$= P_3 - P_1 = \frac{4S}{3} + \frac{4S}{1} = \frac{16S}{3}$$

This pressure difference should exist in a single drop of radius r.

$\therefore \quad \dfrac{4S}{r} = \dfrac{16S}{3}$ or $r = \dfrac{3}{4}$ cm $= 0.75$ cm

13. **(c) :** Here, $A_2 = 2A_1$

$\because$ Intensity $\propto$ (Amplitude)^2

$$\therefore \quad \frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2A_1}{A_1}\right)^2 = 4$$

$I_2 = 4I_1$

Maximum intensity, $I_m = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$$= \left(\sqrt{I_1} + \sqrt{4I_1}\right)^2 = \left(3\sqrt{I_1}\right)^2 = 9I_1 \text{ or } I_1 = \frac{I_m}{9} \quad \text{...(i)}$$

Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$= I_1 + 4I_1 + 2\sqrt{I_1(4I_1)}\cos\phi$

$= 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi$

$= I_1 + 4I_1(1 + \cos\phi)$

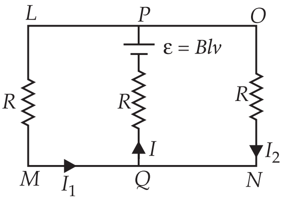
$= I_1 + 8I_1\cos^2\dfrac{\phi}{2} \qquad \left(\because 1 + \cos\phi = 2\cos^2\dfrac{\phi}{2}\right)$

$= I_1\left(1 + 8\cos^2\dfrac{\phi}{2}\right)$

Putting the value of I_1 from eq. (i), we get

$$I = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$$

14. (c) : Emf induced across PQ is $\varepsilon = Blv$.
The equivalent circuit diagram is as shown in the figure.



Applying Kirchhoff's first law at junction Q, we get

$I = I_1 + I_2$ ...(i)

Applying Kirchhoff's second law for the closed loop $PLMQP$, we get

$-I_1R - IR + \varepsilon = 0$

$I_1R + IR = Blv$ ...(ii)

Again, applying Kirchhoff's second law for the closed loop $PONQP$, we get

$-I_2R - IR + \varepsilon = 0$

$I_2R + IR = Blv$ ...(iii)

Adding equations (ii) and (iii), we get

$2IR + I_1R + I_2R = 2Blv$

$2IR + R(I_1 + I_2) = 2Blv$

$2IR + IR = 2Blv$ (Using (i))

$3IR = 2Blv$

$I = \frac{2Blv}{3R}$ ...(iv)

Substituting this value of I in equation (ii), we get

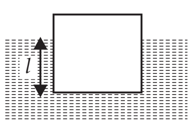
$I_1 = \frac{Blv}{3R}$

Substituting the value of I in equation (iii), we get

$I_2 = \frac{Blv}{3R}$

Hence, $I_1 = I_2 = \frac{Blv}{3R},\ I = \frac{2Blv}{3R}$

15. (c) :



Let l be the length of block immersed in liquid, when the block is floating.

$\therefore\ mg = Al\rho g$ ...(i)

If the block is given a vertical displacement y, then the effective restoring force is

$F = -[A(l + y)\rho g - mg]$

$= -[A(l + y)\rho g - Al\rho g] = -A\rho gy$ (Using (i))

i.e. $F \propto y$. –ve sign shows that F is directed towards its equilibrium position. Therefore, if the block is left free, it will execute SHM.

Acceleration, $a = \frac{F}{m} = -\frac{A\rho gy}{m} = -\omega^2 y$

$\therefore\ \omega = \sqrt{\frac{A\rho g}{m}}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{A\rho g}}$

i.e., $T \propto \frac{1}{\sqrt{A}}$.

16. (c) : $V = \frac{kQ}{2R^3}(3R^2 - r^2)$ (for $r < R$)

For $r = 0$, potential at the centre $= V_c = k\frac{3Q}{2R}$.

We require a point where $V = \frac{V_c}{2} = k\frac{3Q}{4R}$.

This point cannot lie inside the sphere where $V \geq k\frac{Q}{R}$.

Let the point lie outside the sphere, at a distance r from the centre. Then,

$V = k\frac{Q}{r} = \frac{3Q}{4R}$ or $r = \frac{4}{3}R$

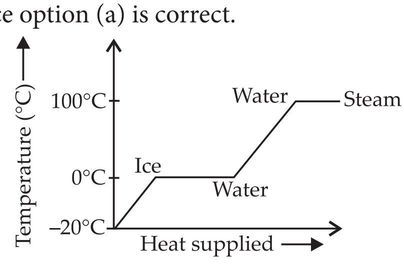
Distance from the surface $= r - R = \frac{4}{3}R - R = \frac{R}{3}$

17. (d) : Current in LR circuit is

$I = \frac{V_0}{\sqrt{R^2 + \omega^2L^2}}\sin\left(\omega t - \frac{\pi}{2}\right)$,

i.e., it is sinusoidal in nature.

18. (a) : When heat is supplied, the temperature of ice increases from –20°C to 0°C. It is represented by a straight line inclined to heat axis. At 0°C, the heat is used in converting ice into water. It is represented by horizontal straight portion. After that, heat is supplied to increase the temperature of water from 0°C to 100°C. It is represented by a straight line incline to heat axis. At 100°C, the heat is used in converting water into steam. It is represented by horizontal straight line.
Hence option (a) is correct.



19. (b) : Here,

$C_m(t) = 25\sin(300\pi t) + 15(\cos(200\pi t) - \cos(400\pi t))$

Compare this equation with standard equation of amplitude modulated wave,

$$C_m(t) = A_c \sin\omega_c t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$$

$A_c = 25$ V, $\omega_c = 300\pi \Rightarrow 2\pi f_c = 300\pi \Rightarrow f_c = 150$ Hz

$\omega_c - \omega_m = 200\pi \Rightarrow f_c - f_\omega = 100$ Hz

$\therefore\ f_\omega = 150 - 100 = 50$ Hz

20. (d) : Average speed $= \dfrac{\text{total distance}}{\text{total time}}$

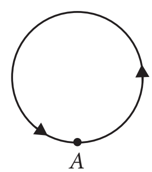
Average velocity $= \dfrac{\text{displacement}}{\text{time}}$

$\because$ distance $\geq$ displacement

$\therefore$ Average speed $\geq$ average velocity

$\dfrac{d|\vec{v}|}{dt} =$ tangential acceleration

$\left|\dfrac{d\vec{v}}{dt}\right| =$ net acceleration



In uniform circular motion,

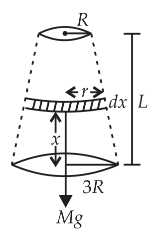
$\dfrac{d|\vec{v}|}{dt} = 0, \left|\dfrac{d\vec{v}}{dt}\right| \neq 0$

In circular motion, from point A to point A again, average velocity = 0

Instantaneous velocity $\neq 0$ (at any time)

21. (3) : Consider a uniform cross-section of wire of length dx and radius r at a vertical distance of x from the lower end.

Here, $r = 3R - \dfrac{R}{L}x$



$\therefore$ Extension in wire of length dx

$$dl = \frac{Fdx}{AY} = \frac{Mgdx}{\pi\left(3R - \frac{2R}{L}x\right)^2 Y}$$

Hence, extension in wire

$$l = \int dl = \int_0^L \frac{Mgdx}{\pi\left(3R - \frac{2R}{L}x\right)^2 Y}$$

$$= \frac{Mg}{\pi Y}\int_0^L \frac{dx}{\left(3R - \frac{2R}{L}x\right)^2} = \frac{MgL}{3\pi R^2 Y}$$

$\therefore$ Extended length of wire

$$= L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{Mg}{3\pi R^2 Y}\right)$$

22. (9995) : Resistance of the galvanometer,

$$G = \frac{\text{current sensitivity}}{\text{voltage sensitivity}} = \frac{10}{2} = 5\ \Omega$$

Number of divisions on the galvanometer scale, $n = 150$

Current required for full scale deflection,

$$I_g = \frac{n}{\text{current sensitivity}} = \frac{150}{10} = 15\text{ mA} = 15 \times 10^{-3}\text{ A}$$

Required range of voltmeter = 150 × 1 = 150 V

Required series resistance,

$$R = \frac{V}{I_g} - G = \frac{150}{15 \times 10^{-3}} - 5 = 9995\ \Omega$$

23. (98.01) : Here, the source (electric siren) is at rest and the observer is moving away from the source

$$\therefore\ \upsilon' = \frac{\upsilon(v - v_o)}{v}$$

where v is the speed of the sound

$\dfrac{\upsilon'}{\upsilon} = \dfrac{v - v_o}{v}$; $\dfrac{94}{100} = 1 - \dfrac{v_o}{v}$

$0.94 = 1 - \dfrac{v_o}{v}$

$\dfrac{v_o}{v} = 1 - 0.94 = 0.06$

$v_o = 0.06v = 0.06 \times 330 = 19.8$ m s^{-1}

$\therefore$ Distance covered, $s = \dfrac{v^2 - u^2}{2a} = \dfrac{(19.8)^2 - (0)^2}{2 \times 2}$

$= 98.01$ m

24. (15) : Here, $\sigma = 5$ mho cm^{-1}, $\mu_e = 3900$ cm^2 V^{-1} s^{-1}

As $\sigma = e(n_e\mu_e + n_h\mu_h)$

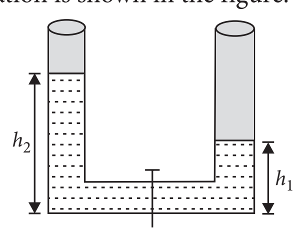
Since the density of holes is negligible, so $\sigma = en_e\mu_e$

$$\Rightarrow\ n_e = \frac{\sigma}{e\mu_e} = \frac{(5\text{ mho cm}^{-1})}{(1.6 \times 10^{-19}\text{ C})(3900\text{ cm}^2\text{ V}^{-1}\text{ s}^{-1})}$$

$= 8 \times 10^{15}$ cm^{-3}

The density of donor atoms is $n_d \approx n_e = 8 \times 10^{15}$ cm^{-3}

25. (0.635) : The situation is shown in the figure. The work done by the force of gravity in equalizing the levels when the vessels are interconnected is



$$W = \frac{1}{4}\rho g A(h_2 - h_1)^2$$

where ρ is the density of the liquid, A is the area of each base, g is the acceleration due to gravity and h_1 and h_2 are liquid heights in right and left vessels respectively.

Here, $\rho = 1.3 \times 10^3$ kg/m^3

$g = 9.8$ m/s^2, $A = 4.0$ cm$^2 = 4.0 \times 10^{-4}$ m^2

$h_1 = 0.854$ m, $h_2 = 1.560$ m

$$\therefore \quad W = \frac{1}{4}(1.3\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)$$
$$(4.0 \times 10^{-4} \text{ m}^2)(1.560 \text{ m} - 0.854 \text{ m})^2 = 0.635 \text{ J}$$

26. (b) : The solubilities of sulphates of alkaline earth metals decrease down the group because the hydration enthalpy decreases from Be^{2+} to Ba^{2+} appreciably as the size of the cation increases. As the sulphate ion is too big, the magnitude of lattice enthalpy almost remains constant on moving down the group.

So, the correct order of solubility in water is

$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$.

27. (b) : At low temperature and high pressure, the volume of the particles is not negligible as compared to the total volume of the gas. Also, the intermolecular forces start acting upon the molecules. Hence, they deviate from ideal behaviour.

28. (a) : The sharing of three corners *i.e.*, three oxygen of each tetrahedron (SiO_4^{4-}) results in an infinite two dimensional sheet structure of the formula $(Si_2O_5)_n^{2n-}$.

29. (a) : $\frac{1}{3}\frac{d[Br_2]}{dt} = -\frac{1}{5}\frac{d[Br^-]}{dt}$

$$\frac{d[Br_2]}{dt} = -\frac{3}{5}\frac{d[Br^-]}{dt}$$

30. (d) : $S + 3/2\, O_2 \rightarrow SO_3 + 2x$ kcal ...(i)

$SO_2 + 1/2\, O_2 \rightarrow SO_3 + y$ kcal ...(ii)

Now, subtract eq. (ii) from (i), we get

$S + O_2 \rightarrow SO_2 + 2x - y$ kcal

$\therefore$ Heat of formation of SO_2 is equal to $2x - y$ kcal.

31. (c) : From observations (i), (ii) and (iii), reducing character of metals are Pb > Cu, Pb > Ag and Cu > Ag. Hence, the order of decreasing reducing character is Pb > Cu > Ag.

32. (c) : Since water is a neutral molecule, the sum of the oxidation numbers of all the atoms in water is taken as zero.

$KAl(SO_4)_2 \cdot 12H_2O$

$+1 + 3 + 2x - 16 = 0 \Rightarrow x = +6$

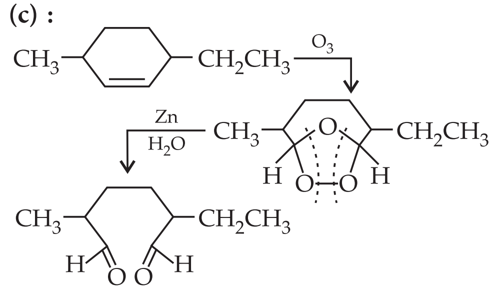
33. (c) : The vapour contains more of the volatile component (A) to that present in the solution, *i.e.*, $(x_A)_{liquid} < (x_A)_{vapour}$.

34. (d)

35. (a) : In (a), due to similar charges (two positive charges) on adjacent atoms, the structure is expected to be least stable.

36. (c) : In $[Fe(C_2O_4)_3]^{3-}$, iron is present in the highest oxidation state Fe^{3+} and $C_2O_4^{2-}$ is a chelating ligand. Chelates are always more stable complexes.

37. (c) :



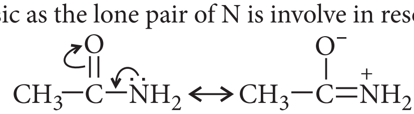
38. (c) : No. of moles of $A = \frac{x}{40}$

No. of atoms of $A = \frac{x}{40} \times N_A = y$

No. of moles of $B = \frac{2x}{80} = \frac{x}{40}$

Now, no. of atoms of $B = \frac{x}{40} \times N_A$, *i.e.*, y

39. (b) : In $CH_3-C(=NH)-NH_2$, lone pair on $-NH_2$ remains more available for donation and its conjugate acid is resonance stabilised thus, it is most basic. Between $CH_3CH_2NH_2$ and $(CH_3)_2NH$, the later is more basic because of the presence of two alkyl groups which facilitate the donation of lone pair of electrons. $CH_3-\overset{O}{\overset{\|}{C}}-NH_2$ is least basic as the lone pair of N is involve in resonance.



Thus, the correct order of basicity is I > III > II > IV.

40. (b) : As the obtained compound reduces Tollens' reagent, it must be an aldehyde. Thus, it is obvious that both the —Cl atoms are present at C – 1. Hence, the compound 'X' is 1, 1-dichloropropane and the reactions are as follows :

$$CH_3CH_2CHCl_2 \xrightarrow{\text{Hydrolysis}} CH_3CH_2CH\langle^{OH}_{OH}$$
1, 1-Dichloropropane 'X' — Unstable

$$\xrightarrow[-H_2O]{\text{Heat}} CH_3CH_2CHO \xrightarrow{\text{Tollens' reagent}} CH_3CH_2COOH + Ag\downarrow$$
Propanoic acid — Silver mirror

41. (c)

42. (d) : The equilibrium constant does not change at all with change in concentration, volume, pressure and presence of a catalyst. It changes only with change in temperature of the system.

43. (b)

44. (d) : According to Faraday's law of electrolysis
$m \propto It$ or $m = ZIt$
where I = current, t = time

$$Z = \frac{\text{Equivalent weight of substance}}{96500}$$

Eq. wt. of Cu = $\frac{63.5}{2}$ $(\because\ Cu^{2+} \rightarrow Cu)$

$$Z = \frac{63.5}{2 \times 96500}$$

$$\therefore\ m = \frac{63.5 \times I \times t}{2 \times 96500} = \frac{31.75 \times I \times t}{96500}$$

45. (b) : Since sp-hybridized carbon is more electronegative than a sp^2-hybridized carbon which in turn is more electronegative than sp^3-hybridized carbon, therefore, $CH \equiv \overset{sp}{C} - COOH$ is a stronger acid than $CH_2 = \overset{sp^2}{C}H - COOH$ which in turn, is a stronger acid than $CH_3 - \overset{sp^3}{C}H_2 - COOH$. Thus, the overall order of acid strength is (i) > (ii) > (iii).

46. (5) : Edge of the unit cell (a) = $2 \times Y^{1/3}$ nm
Formula mass, $M = 6.023\ Y\ g\ mol^{-1}$

$$= \frac{6.023Y}{1000}\ kg\ mol^{-1}$$

For rock salt type structure, $Z = 4$

$$\therefore\ \text{Density } (\rho) = \frac{Z \times M}{a^3 \times N_0}$$

$$= \frac{4 \times (6.023 \times 10^{-3}\, Y\, kg\, mol^{-1})}{(2 \times Y^{1/3} \times 10^{-9}\, m)^3 \times (6.023 \times 10^{23}\, mol^{-1})}$$

$= 5\ kg\ m^{-3}$

47. (1) : Volume strength = 11.2 × Molarity

$$\text{Molarity} = \frac{\text{Volume strength}}{11.2} = \frac{11.2}{11.2} = 1$$

48. (2) :

	X_3Y_2	$\rightleftharpoons$	$3X^{2+}$	+	$2Y^{3-}$
Initial conc.	1 mol		0		0
After ionization	1 – 0.25		3 × 0.25		2 × 0.25
	0.75		0.75		0.5

Total number of moles after dissociation
$= 0.75 + 0.75 + 0.5 = 2$
$\therefore\ i = 2$

49. (7) : Let the solubility of AgCl and CuCl be a mol litre^{-1} and b mol litre^{-1} respectively.

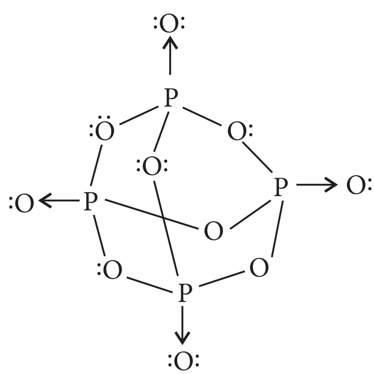
$$AgCl \rightleftharpoons \underset{a}{Ag^+} + \underset{a}{Cl^-};\ CuCl \rightleftharpoons \underset{b}{Cu^+} + \underset{b}{Cl^-}$$

$\therefore\ K_{sp}$ of AgCl = $[Ag^+]\ [Cl^-]$
$1.6 \times 10^{-10} = a(a + b)$ ...(i)
Similarly, K_{sp} of CuCl = $[Cu^+][Cl^-]$
$1.0 \times 10^{-6} = b(a + b)$ ...(ii)
Dividing eqn. (i) by (ii), we get

$$\frac{a}{b} = 1.6 \times 10^{-4} \text{ or } a = 1.6 \times 10^{-4} \times b$$

Substituting the value of a in eqn. (i), we get
$1.6 \times 10^{-10} = 1.6 \times 10^{-4}\ b(1.6 \times 10^{-4}\ b + b)$
$\Rightarrow\ 10^{-6} = b^2(1.6 \times 10^{-4} + 1)$
$\Rightarrow\ b = 10^{-3}\ [\because\ 1.6 \times 10^{-4} <<< 1] \Rightarrow a = 1.6 \times 10^{-7}$
$[Ag^+] = a = 1.6 \times 10^{-7}$; Comparing with 1.6×10^{-x}
$\Rightarrow\ x = 7$

50. (4) :



51. (a) : Since (3, 3), (6, 6), (9, 9), (12, 12) $\in R$
Hence R is reflexive
$\because$ (3, 6), (6, 12) and (3, 12) $\in R$. Therefore R is transitive.
$\because$ (3, 6) $\in R$ but (6, 3) $\notin R$, hence R is not symmetric.

52. (c) : Let common difference be d.
$a_p = a_1 + (p - 1)d,\ a_q = a_1 + (q - 1)d,$
$a_r = a_1 + (r - 1)d$
As a_p, a_q, a_r are in G.P.

$$\therefore\ \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q} \text{ (by law of proportions)}$$

or $\dfrac{a_q}{a_p}=\dfrac{a_r}{a_q}=\dfrac{a_1+(q-1)d-a_1-(r-1)d}{a_1+(p-1)d-a_1-(q-1)d}=\dfrac{q-r}{p-q}$

or $\dfrac{a_q}{a_p}=\dfrac{q-r}{p-q}=\dfrac{r-q}{q-p}$

53. (c) : $f(x)=\dfrac{4}{3}x^3-8x^2+16x+5$ ...(i)

Differentiating (i) with respect to x, we get

$f'(x)=\dfrac{4}{3}\times 3x^2-16x+16 = 4x^2-16x+16$

Now for maximum/minimum we put $f'(x) = 0$

$\Rightarrow\ x^2-4x+4=0 \ \Rightarrow\ (x-2)^2=0 \Rightarrow x=2$

$f''(x)=8x-16,\quad f''(x)|_{\text{at } x=2}=0$

$f'''(x)=8\neq 0$

$\therefore\ x=2$ is the point of inflection.

54. (c) : Let $a=\tan x-\tan y$, $b=\tan y-\tan z$ and $c=\tan z-\tan x$

$\therefore\ a+b+c=0$ ...(i)

From (i), $b^2=a^2+c^2+2ac$ ...(ii)

According to question, $2b^2=a^2+c^2$

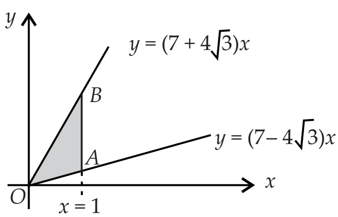
$\Rightarrow\ 2b^2=b^2-2ac$ [Using (ii)]

$\Rightarrow -b^2=2ac$

$\Rightarrow\ -b=\dfrac{2ac}{b}=\dfrac{2ac}{-(a+c)}\ \Rightarrow\ b=\dfrac{2ac}{a+c}$

$\therefore\ a, b, c$ are in H.P.

55. (a) :



$\sqrt{\dfrac{x}{y}}+\sqrt{\dfrac{y}{x}}=4$, let $t=\sqrt{\dfrac{y}{x}}$

$\Rightarrow t+\dfrac{1}{t}=4 \ \Rightarrow\ t^2-4t+1=0$

$\Rightarrow\ t=2\pm\sqrt{3}=\sqrt{\dfrac{y}{x}}\quad \therefore\ y=x\,(2\pm\sqrt{3})^2$

or $y=(7+4\sqrt{3})x,\ y=(7-4\sqrt{3})x$

Area $=\dfrac{1}{2}\times 1\times AB=\dfrac{1}{2}\left[7+4\sqrt{3}-(7-4\sqrt{3})\right]$

$=4\sqrt{3}.$

56. (a) : $\lim\limits_{x\to 0}\left(\dfrac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ $(1^\infty$ form$)$

$=\exp \lim\limits_{x\to 0}\left(\dfrac{1+5x^2}{1+3x^2}-1\right)\left(\dfrac{1}{x^2}\right)$

$=\exp \lim\limits_{x\to 0}\left(\dfrac{2}{1+3x^2}\right)=e^2.$

57. (c) : Here, $-1+\sqrt{-3}=re^{i\theta}$

$\Rightarrow\ -1+i\sqrt{3}=re^{i\theta}=r\cos\theta+ir\sin\theta$

Equating real and imaginary parts, we get

$r\cos\theta=-1$ and $r\sin\theta=\sqrt{3}$

Hence, $\tan\theta=-\sqrt{3}\Rightarrow \tan\theta=\tan\dfrac{2\pi}{3}$.

Hence $\theta=\dfrac{2\pi}{3}$.

58. (b) : $\Delta=\begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$

$=x!(x+1)!(x+2)!\begin{vmatrix} 1 & (x+1) & (x+2)(x+1) \\ 1 & (x+2) & (x+3)(x+2) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix}$

Applying $R_1\to R_2-R_1$, $R_2\to R_3-R_2$ we get

$=x!(x+1)!(x+2)!\begin{vmatrix} 0 & 1 & 2(x+2) \\ 0 & 1 & 2(x+3) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix}$

$=2(x!)(x+1)!(x+2)!$ (Expanding along C_1).

59. (c) :

$f(x)=\begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$

Applying $C_1\to C_1-C_2-C_3$, we get

$f(x)=\begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4\sin x \\ 0 & \sin x & 1 \end{vmatrix}=\sin x(3-4\sin^2 x)$

$=3\sin x-4\sin^3 x=\sin 3x$

Now, $\displaystyle\int_0^{\pi/2}\sin 3x\,dx=\left[\frac{-\cos 3x}{3}\right]_0^{\pi/2}=\left[\frac{-0+1}{3}\right]=\frac{1}{3}$

60. (a) : R.H.L. $(x=0)=\alpha+0=\alpha$

Now,

$\dfrac{\sin x-x}{x^3}=\dfrac{x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-......-x}{x^3}=\dfrac{-1}{3!}+\dfrac{x^2}{5!}-.....$

$$\therefore \quad \lim_{x\to 0}\frac{\sin x - x}{x^3} = \frac{-1}{6}$$

L.H.L. = $\beta - 1$

$f(x)$ is continuous at $x = 0 \Rightarrow \beta - 1 = 2 = \alpha$

$\Rightarrow \beta = 3, \alpha = 2$. So, $\beta - \alpha = 1$

61. (d) : Here range = r = largest value – smallest value

$$= \text{Max}\,|x_i - x_j|\ (i \neq j)$$

And $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Now, $(x_i - \bar{x})^2 = \left[x_i - \frac{x_1 + x_2 + \ldots + x_n}{n}\right]^2$

$= \frac{1}{n^2}\left[(x_i - x_1) + (x_i - x_2) + \ldots + (x_i - x_n)\right]^2$

$= \frac{1}{n^2}[(x_i - x_1) + (x_i - x_2) + \ldots + (x_i - x_{i-1})$
$+ (x_i - x_{i+1}) + \ldots + (x_i - x_n)^2]$

$\Rightarrow (x_i - \bar{x})^2 \le \frac{1}{n^2}[(n-1)r]^2 \quad (\because |x_i - x_j| \le r)$

$\Rightarrow \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \le \frac{1}{n^2(n-1)}\sum[(n-1)r]^2$

(summing up and dividing by $(n-1)$ both sides)

$= \frac{1}{n^2}\frac{1}{n-1}n(n-1)^2 r^2 = \frac{n-1}{n}r^2 < \frac{n}{n-1}r^2$

$$\left(\because \forall n > 1, n > \frac{1}{n}\right)$$

Therefore $S^2 < \frac{n}{n-1}\cdot r^2$ or $S < r\sqrt{\frac{n}{n-1}}$

62. (d) : The *d.r's* of the lines are $\frac{1}{2}, \frac{1}{3}, -1$ and $\frac{1}{6}, -1, -\frac{1}{4}$

or 3, 2, – 6 and 2, –12, –3

$3 \times 2 + 2 \times (-12) + (-6)(-3) = 0$.

$\therefore \quad \theta = \frac{\pi}{2}$

63. (c) : On putting $n = 1$ in $11^{n+2} + 12^{2n+1}$, we get $11^{1+2} + 12^{(2\times 1)+1} = 11^3 + 12^3 = 3059$ which is divisible by 133 only.

64. (c) : $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= \frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma}$

$= \frac{x}{1+x} + \frac{y}{1+y} + \frac{z}{1+z}$

(Here, $x = \tan^2\alpha, y = \tan^2\beta, z = \tan^2\gamma$)

$= \frac{(x+y+z)+(xy+yz+zx+2xyz)+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}$

$(\because xy + yz + zx + 2xyz = 1)$

$= \frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} = 1$

65. (b) : Given, $\tan y\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$= 2\sin x \cos y$

$\Rightarrow \tan y \sec y\, dy = 2\sin x\, dx$

$\Rightarrow \int \tan y \sec y\, dy = 2\int \sin x\, dx$

$\Rightarrow \sec y = -2\cos x + c \Rightarrow \sec y + 2\cos x = c$

66. (b) : Equation of normal to the curve $y^2 = 4x$ at $(m^2, 2m)$ is taken as

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(m^2,2m)}}(x - x_1)$$

$\Rightarrow (y - 2m) = -m(x - m^2)$

$\Rightarrow y + mx - 2m - m^3 = 0$ ...(i)

Similarly normal to $y^2 - 2x + 6 = 0$ at $\left(\frac{1}{2}t^2 + 3, t\right)$ is

$y + t(x-3) - t - \frac{1}{2}t^3 = 0$ ...(ii)

Shortest distance between two curves exist along the common normal.

Let (i) and (ii) are same

$\therefore \quad -2m - m^3 = -4m - \frac{1}{2}m^3 \Rightarrow m = 0, m = \pm 2$

Points on the parabolas $(m^2, 2m) = (4, 4)$

and $\left(\frac{1}{2}m^2 + 3, m\right) = (5, 2)$

$\therefore$ Shortest distance $= \sqrt{(5-4)^2 + (4-2)^2} = \sqrt{5}$

67. (c) : Given equation is $x^2 + x + 1 = 0$

$\therefore \quad \alpha + \alpha^2 = -1$ ...(i) and $\alpha^3 = 1$ ...(ii)

We have to find the equation whose roots are α^{31} and α^{62}.

Now, $\alpha^{31} + \alpha^{62} = \alpha^{31}(1 + \alpha^{31})$

$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha^{30}\cdot\alpha(1 + \alpha^{30}\cdot\alpha)$

$\Rightarrow \alpha^{31} + \alpha^{62} = (\alpha^3)^{10}\cdot\alpha\{1 + (\alpha^3)^{10}\cdot\alpha\}$

$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha(1 + \alpha)$ [From (ii)]

$\Rightarrow \alpha^{31} + \alpha^{62} = -1$ [From (i)]

Again, $\alpha^{31}\cdot\alpha^{62} = \alpha^{93} \Rightarrow \alpha^{31}\cdot\alpha^{62} = [\alpha^3]^{31} = 1$

Required equation is $x^2 - (\alpha^{31} + \alpha^{62})x + \alpha^{31}\cdot\alpha^{62} = 0$

$\Rightarrow x^2 + x + 1 = 0$

68. (a) : Let us define the events in the following way :

A : 4 being the minimum number

B : 8 being the maximum number
$A \cap B$: 4 being the minimum number and 8 being the maximum number

Therefore $P(A) = \frac{{}^6C_2}{{}^{10}C_3} = \frac{15}{120}$

$P(B) = \frac{{}^7C_2}{{}^{10}C_3} = \frac{21}{120}$

and $P(A \cap B) = \frac{{}^3C_2}{{}^{10}C_3} = \frac{3}{120}$

$\therefore$ The required probability,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{15}{120} + \frac{21}{120} - \frac{3}{120} = \frac{33}{120} = \frac{11}{40}$

69. (c) : $(x^2 - x - 2)^5 = (x - 2)^5(1 + x)^5$

$= [{}^5C_0x^5 - {}^5C_1x^4 \times 2 + ...][{}^5C_0 + {}^5C_1x + ...]$

$\therefore$ Coefficient of x^5:
$1 - 5\cdot5\cdot2 + 10\cdot10\cdot4 - 10\cdot10\cdot8 + 5\cdot5\cdot16 - 32$
$= 1 - 50 + 400 - 800 + 400 - 32 = -81$

70. (b) : Let $I = \int e^x(x^5 + 5x^4 + 1)\,dx$

$= \int e^x (x^5 + 5x^4)dx + \int e^x\,dx$

Now since $\int e^x (f(x) + f'(x))dx = e^x f(x) + C$

$\therefore$ $I = e^x x^5 + e^x + C = e^x(x^5 + 1) + C$

71. (77) : There are two possibilities :
(1) The digits used are 1, 1, 1, 1, 1, 2, 3.

The number of numbers formed = $\frac{\lfloor 7}{\lfloor 5} = 42$

(2) The digits used are 1, 1, 1, 1, 2, 2, 2
The number of numbers formed

$= \frac{\lfloor 7}{\lfloor 4 \lfloor 3} = \frac{7\cdot6\cdot5}{6} = 35$

The total number of numbers = 42 + 35 = 77.

72. (9) : $|\vec{a} + \vec{b} + \vec{c}| \geq 0$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 1 + 1 + 1 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) \geq 0$

$\therefore$ $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} \geq -\frac{3}{2}$ ...(i)

$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - a|^2$

$= 2|\vec{a}|^2 + 2|\vec{b}|^2 + 2|\vec{c}|^2 - 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$= 6 - 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) \leq 6 + 3$ [Using (i)]
$= 9$

73. (200) : Given, series, 2 + 5 + 8 + 11 + ... where $a = 2$, $d = 3$ and let number of terms is n

$\therefore$ Sum of A.P. $= \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 60100 = \frac{n}{2}\{2\times2 + (n-1)3\} \Rightarrow 120200 = n(3n+1)$

$\Rightarrow 3n^2 + n - 120200 = 0 \Rightarrow (n-200)(3n+601) = 0$

Hence $n = 200$.

74. (1.07) : The plane is $\begin{vmatrix} x-1 & y-1 & z-0 \\ 1-1 & 2-1 & 1-0 \\ -2-1 & 2-1 & -1-0 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow (x - 1)(-2) - (y - 1)(3) + z(3) = 0$
$\Rightarrow 2x + 3y - 3z = 5$

Distance of O is $\frac{5}{\sqrt{4+9+9}} = \frac{5}{\sqrt{22}} = 1.07$

75. (1) : $A^T = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

$\therefore$ $A^TA = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$= \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix}$

$\because$ $A^TA = I \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$

$3z^2 = 1 \Rightarrow z^2 = \frac{1}{3}$ and $6y^2 = 1 \Rightarrow y^2 = \frac{1}{6}$

$\therefore$ $x^2 + y^2 + z^2 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$