JEE Type Solved Examples :
Single Option Correct Type Questions

- This section contains **10 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Ex. 1 *The probability that in a year of 22nd century chosen at random, there will be 53 Sundays, is*

(a) $\frac{3}{28}$ (b) $\frac{2}{28}$ (c) $\frac{7}{28}$ (d) $\frac{5}{28}$

Sol. (d) In the 22nd century, there are 25 leap years viz. 2100, 2104, 2108,..., 2196 and 75 non-leap years.

Consider the following events:

E_1 = Selecting a leap year from 22nd century

E_2 = Selecting a non-leap year from 22nd century

E = There are 53 Sundays in a year of 22nd century

We have,

$$P(E_1)=\frac{25}{100},\ P(E_2)=\frac{75}{100},\ P\left(\frac{E}{E_1}\right)=\frac{2}{7} \text{ and } P\left(\frac{E}{E_2}\right)=\frac{1}{7}$$

Required probability $= P(E) = P((E\cap E_1)\cup(E\cap E_2))$

$$= P(E\cap E_1)+P(E\cap E_2)$$

$$= P(E_1).P\left(\frac{E}{E_1}\right)+P(E_2).P\left(\frac{E}{E_2}\right)$$

$$= \frac{25}{100}\times\frac{2}{7}+\frac{75}{100}\times\frac{1}{7}=\frac{5}{28}$$

Ex. 2 *In a convex hexagon two diagonals are drawn at random. The probability that the diagonals intersect at an interior point of the hexagon, is*

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

Sol. (a) We have,

Number of diagonals of a hexagon $= {}^6C_2 - 6 = 9$

$\therefore$ $n(s)$ = Total number of selections of two diagonals $= {}^9C_2 = 36$

and $n(E)$ = The number of selections of two diagonals which intersect at an interior point

= The number of selections of four vertices $= {}^6C_4 = 15$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

Ex. 3 *If three integers are chosen at random from the set of first 20 natural numbers, the chance that their product is a multiple of 3, is*

(a) $\frac{1}{57}$ (b) $\frac{13}{19}$

(c) $\frac{2}{19}$ (d) $\frac{194}{285}$

Sol. (d) $n(S)$ = Total number of ways of selecting 3 integers from 20 natural numbers $= {}^{20}C_3 = 1140.$

Their product is multiple of 3 means atleast one number is divisible by 3. The number which are divisible by 3 are 3, 6, 9, 12, 15 and 16.

$\therefore$ $n(E)$ = The number of ways of selecting atleast one of them multiple of 3

$$= {}^6C_1\times{}^{14}C_2+{}^6C_2\times{}^{14}C_1\times{}^6C_3=776$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)}$

$$= \frac{776}{1140} = \frac{194}{285}$$

Ex. 4 *If three numbers are selected from the set of the first 20 natural numbers, the probability that they are in GP, is*

(a) $\frac{1}{285}$ (b) $\frac{4}{285}$

(c) $\frac{11}{1140}$ (d) $\frac{1}{71}$

Sol. (c) $n(S)$ = Total number of ways of selecting 3 numbers from first 20 natural numbers $= {}^{20}C_3 = 1140$

Three numbers are in GP, the favourable cases are 1, 2, 4; 1, 3, 9; 1, 4, 16; 2, 4, 8; 2, 6, 18; 3, 6, 12; 4, 8, 16; 5, 10, 20; 4, 6, 9; 8, 12, 18; 9, 12, 16

$\therefore$ $n(E)$ = The number of favourable cases = 11

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{11}{1140}$

Ex. 5 *Two numbers b and c are chosen at random with replacement from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that $x^2+bx+c>0$ for all $x\in R$, is*

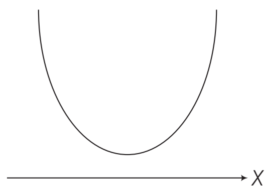
(a) $\frac{17}{123}$ (b) $\frac{32}{81}$

(c) $\frac{82}{125}$ (d) $\frac{45}{143}$

Sol. (b) Here, $x^2+bx+c>0, \forall\ x\in R$

$\therefore$ $D<0$

$\Rightarrow$ $b^2<4c$



Value of b	Possible values of c		
1	$1 < 4c$	$\Rightarrow c > \frac{1}{4} \Rightarrow$	{1, 2, 3, 4, 5, 6, 7, 8, 9}
2	$4 < 4c$	$\Rightarrow c > 1 \Rightarrow$	{2, 3, 4, 5, 6, 7, 8, 9}
3	$9 < 4c$	$\Rightarrow c > \frac{9}{4} \Rightarrow$	{3, 4, 5, 6, 7, 8, 9}
4	$16 < 4c$	$\Rightarrow c > 4 \Rightarrow$	{5, 6, 7, 8, 9}
5	$25 < 4c$	$\Rightarrow c > 6.25 \Rightarrow$	{7, 8, 9}
6	$36 < 4c$	$\Rightarrow c > 9 \Rightarrow$	Impossible
7	Impossible		
8	Impossible		
9	Impossible		

$n(E)$ = Number of favourable cases $= 9 + 8 + 7 + 5 + 3 = 32$

$n(S)$ = Total ways $= 9 \times 9 = 81$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{32}{81}$

Ex. 6 *Three dice are thrown. The probability of getting a sum which is a perfect square, is*

(a) $\frac{2}{5}$ (b) $\frac{9}{20}$

(c) $\frac{1}{4}$ (d) None of these

Sol. (d) $n(S)$ = Total number of ways $= 6 \times 6 \times 6 = 216$

The sum of the numbers on three dice varies from 3 to 18 and among these 4, 9 and 16 are perfect squares.

$\therefore$ $n(E)$ = Number of favourable ways

= Coefficient of x^4 in $(x + x^2 + \ldots + x^6)^3$ + Coefficient of x^9 in $(x + x^2 + \ldots + x^6)^3$ + Coefficient of x^{16} in $(x + x^2 + \ldots + x^6)^3$

= Coefficient of x in $(1 + x + \ldots + x^5)^3$ + Coefficient of x^6 in $(1 + x + x^2 + \ldots + x^5)^3$ + Coefficient of x^{13} in $(1 + x + x^2 + \ldots + x^5)^3$

= Coefficient of x in $(1 - x^6)^3(1 - x)^{-3}$ + Coefficient of x^6 in $(1 - x^6)^3(1 - x)^{-3}$ + Coefficient of x^{13} in $(1 - x^6)^3(1 - x)^{-3}$

= Coefficient of x in $(1)(1 + {}^3C_1x + \ldots)$ + Coefficient of x^6 in $(1 - 3x^6)(1 + {}^3C_1x + \ldots)$ + Coefficient of x^{13} in $(1 - 3x^6 + 3x^{12} + \ldots); (1 + {}^3C_1x + \ldots)$

$= {}^3C_1 + ({}^8C_6 - 3) + ({}^{15}C_{13} - 3 \times {}^9C_7 + 9)$

$= {}^3C_1 + ({}^8C_2 - 3) + ({}^{15}C_2 - 3 \times {}^9C_2 + 9)$

$= 3 + 25 + 6$

$= 34$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{34}{216} = \dfrac{17}{108}$

Ex. 7 *A quadratic equation is chosen from the set of all quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots, is*

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

Sol. (a) Let α and β be the roots of the quadratic equation.

According to question,

$\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2 \quad \Rightarrow \quad \alpha\beta(\alpha\beta - 1) = 0$

$\Rightarrow \quad \alpha\beta = 1$ or $\alpha\beta = 0$

$\Rightarrow \quad \alpha = 1, \beta = 1; \alpha = \omega, \beta = \omega^2$ [cube roots and unity]

$\alpha = 1, \beta = 0; \alpha = 0, \beta = 0$

$\therefore n(S)$ = Number of quadratic equations which are unchanged by squaring their roots = 4

and $n(E)$ = Number of quadratic equations have equal roots = 2

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

Ex. 8 *Three-digit numbers are formed using the digits* 0, 1, 2, 3, 4, 5 *without repetition of digits. If a number is chosen at random, then the probability that the digits either increase or decrease, is*

(a) $\frac{1}{10}$ (b) $\frac{2}{11}$ (c) $\frac{3}{10}$ (d) $\frac{4}{11}$

Sol. (c) $n(S)$ = Total number of three digit numbers

$= {}^6P_3 - {}^5P_2 = 120 - 20 = 100$

$n(E)$ = Number of numbers with digits either increase or decrease

= Number of numbers with increasing digits + Number of numbers with decreasing digits

$= {}^5C_3 + {}^6C_3 = 10 + 20 = 30$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{30}{100} = \dfrac{3}{10}$

Ex. 9 *If X follows a binomial distribution with parameters $n = 8$ and $p = \frac{1}{2}$, then $p(|x - 4| \le 2)$ is equal to*

(a) $\frac{121}{128}$ (b) $\frac{119}{128}$ (c) $\frac{117}{128}$ (d) $\frac{115}{128}$

Sol. (b) Here, $p = \frac{1}{2}, n = 8$

$\therefore \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ The binomial distribution is $\left(\frac{1}{2} + \frac{1}{2}\right)^8$

Also, $|x - 4| \le 2$

$\Rightarrow \quad -2 \le x - 4 \le 2 \Rightarrow 2 \le x \le 6$

$\therefore\ p(|x-4| \le 2) = p(x=2) + p(x=3) + p(x=4) + p(x=5) + p(x=6)$

$$= {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 + {}^8C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 + {}^8C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4 + {}^8C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + {}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2$$

$$= \frac{{}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6}{2^8}$$

$$= \frac{238}{256} = \frac{119}{128}$$

● ***Ex. 10*** *A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighbourhood are sick with the flue, denoted by F, while 10% are sick with the measles, denoted by M. A well-known symptom of measles is a rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.95. However, occasionally children with the flue also develop a rash with conditional probability 0.08. Upon examination the child, the doctor finds a rash, then the probability that the child has the measles, is*

(a) $\frac{89}{167}$ (b) $\frac{91}{167}$ (c) $\frac{93}{167}$ (d) $\frac{95}{167}$

Sol. (d) $\because P(F) = 0\cdot 90,\ P(M) = 0\cdot 10,$

$$P\left(\frac{R}{F}\right) = 0\cdot 08,\ P\left(\frac{R}{M}\right) = 0\cdot 95$$

$$\therefore\quad P\left(\frac{M}{R}\right) = \frac{P(M)\cdot P\left(\frac{R}{M}\right)}{P(M)\cdot P\left(\frac{R}{M}\right) + P(F)\cdot P\left(\frac{R}{F}\right)}$$

$$= \frac{0\cdot 10 \times 0\cdot 95}{0\cdot 10\times 0\cdot 95 + 0\cdot 90 \times 0\cdot 08} = \frac{0\cdot 095}{0\cdot 167} = \frac{95}{167}$$

JEE Type Solved Examples :
More than One Correct Option Type Questions

- This section contains **5 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of **which more than one** may be correct.

● ***Ex. 11*** *Let p_n denote the probability of getting n heads, when a fair coin is tossed m times. If p_4, p_5, p_6 are in AP, then values of m can be*

(a) 5 (b) 7 (c) 10 (d) 14

Sol. (b, d) $\because p_4 = {}^mC_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{m-4} = {}^mC_4\left(\frac{1}{2}\right)^m$

$$p_5 = {}^mC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{m-5} = {}^mC_5\left(\frac{1}{2}\right)^m$$

and $$p_6 = {}^mC_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{m-6} = {}^mC_6\left(\frac{1}{2}\right)^m$$

According to the question, p_4, p_5, p_6 are in AP

$$\therefore\quad 2p_5 = p_4 + p_6$$

$$\Rightarrow\quad 2\times {}^mC_5\left(\frac{1}{2}\right)^m = {}^mC_4\left(\frac{1}{2}\right)^m + {}^mC_6\left(\frac{1}{2}\right)^m$$

or $$2\times {}^mC_5 = {}^mC_4 + {}^mC_6$$

or $$2 = \frac{{}^mC_4}{{}^mC_5} + \frac{{}^mC_6}{{}^mC_5} \Rightarrow 2 = \frac{5}{m-5+1} + \frac{m-6+1}{6}$$

$$\Rightarrow\quad 2 = \frac{5}{m-4} + \frac{m-5}{6} \Rightarrow (m^2-2)\,m + 98 = 0$$

$$\Rightarrow\quad (m-14)(m-7) = 0$$

$$\Rightarrow\quad m = 7 \text{ or } 14$$

● ***Ex. 12*** *A random variable X follows binomial distribution with mean a and variance b. Then,*

(a) $a > b > 0$ (b) $\frac{a}{b} > 1$

(c) $\frac{a^2}{a-b}$ is an integer (d) $\frac{a^2}{a+b}$ is an integer

Sol. (a, b, c) Suppose, $X \sim B(n, p)$ i.e. $(q+p)^n$

Here, $np = a$ and $npq = b$

$$\therefore\quad q = \frac{b}{a}, \text{ then } p = 1 - q = 1 - \frac{b}{a}$$

Now, $0 < q < 1 \Rightarrow 0 < \frac{b}{a} < 1 \Rightarrow a > b > 0$ [alternate (a)]

and $\frac{a}{b} > 1$ [alternate (b)]

Also, $\frac{a^2}{a-b} = \frac{(np)^2}{np - npq} = \frac{np}{1-q} = \frac{np}{p} = n = \text{Integer}$ [alternate (c)]

● ***Ex. 13*** *If $A_1, A_2, \ldots, A_n$ are n independent events, such that $P(A_i) = \frac{1}{i+1}$, $i = 1, 2, \ldots, n$, then the probability that none of $A_1, A_2, A_3, \ldots, A_n$ occur, is*

(a) $\frac{n}{n+1}$ (b) $\frac{1}{n+1}$

(c) less than $\frac{1}{n}$ (d) greater than $\frac{1}{n+2}$

Sol. (b, c, d) $\because A_1, A_2, A_3, \ldots, A_n$ are n independent, then

Required probability $= P(A_1' \cap A_2' \cap A_3' \cap \ldots \cap A_n')$

$= P(A_1').P(A_2').P(A_3') \ldots P(A_n')$

$= (1 - P(A_1))(1 - P(A_2))(1 - P(A_3)) \ldots (1 - P(A_n))$

$= \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \ldots \left(1 - \frac{1}{n+1}\right)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{n}{n+1} = \frac{1}{n+1}$

$\because n + 2 > n + 1 > n; \quad \therefore \quad \frac{1}{n+2} < \frac{1}{n+1} < \frac{1}{n}$

● **Ex. 14** *A and B are two events, such that* $P(A \cup B) \geq \frac{3}{4}$ *and* $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$, *then*

(a) $P(A) + P(B) \leq \frac{11}{8}$ (b) $P(A) \cdot P(B) \leq \frac{3}{8}$

(c) $P(A) + P(B) \geq \frac{7}{8}$ (d) None of these

Sol. (a, c) $\because \frac{3}{4} \leq P(A \cup B) \leq 1$

$\Rightarrow \quad \frac{3}{4} \leq P(A) + P(B) - P(A \cap B) \leq 1$

As the minimum value of $P(A \cap B) = \frac{1}{8}$, we get

$P(A) + P(B) - \frac{1}{8} \geq \frac{3}{4} \Rightarrow P(A) + P(B) \geq \frac{7}{8}$ [alternate (c)]

As the maximum value of $P(A \cap B) = \frac{3}{8}$, we get

$P(A) + P(B) - \frac{3}{8} \leq 1 \Rightarrow P(A) + P(B) \leq \frac{11}{8}$ [alternate (a)]

● **Ex. 15** *A, B, C and D cut a pack of* 52 *cards successively in the order given. If the person who cuts a spade first receives* ₹ 350, *then the expectations of*

(a) B is ₹ 96 (b) D is ₹ 54

(c) $(A + C)$ is ₹ 200 (d) $(B - D)$ is ₹ 56

Sol. (a, b, c) Let E be the event of any one cutting a spade in one cut and let S be the sample space, then

$n(E) = {}^{13}C_1 = 13$ and $n(S) = {}^{52}C_1 = 52$

$\therefore \quad p = P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$ and $q = p(\bar{E}) = 1 - p = \frac{3}{4}$

The probability of A winning (when A starts the game)

$$= p + q^4 p + q^8 p + \ldots \infty = \frac{p}{1 - q^4} = \frac{\frac{1}{4}}{1 - \left(\frac{3}{4}\right)^4} = \frac{64}{175}$$

$\therefore \quad E(A) = ₹\, 350 \times \frac{64}{175} = ₹\, 128$

$E(B) = ₹128 \times q = ₹128 \times \frac{3}{4} = ₹\, 96$

$E(C) = ₹96 \times q = ₹96 \times \frac{3}{4} = ₹\, 72$

and $E(D) = ₹72 \times q = ₹72 \times \frac{3}{4} = ₹\, 54$

$\therefore$ $E(A + C) = ₹\, 200$ and $E(B - D) = ₹\, 42$.

JEE Type Solved Examples : Passage Based Questions

- This section contains **3 solved passages** based upon each of the passage **3 multiple choice** examples have to be answered. Each of these examples has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I

(Ex. Nos. 16 to 18)

Each coefficient in the equation $ax^2 + bx + c = 0$ *is determined by throwing an ordinary die.*

16. *The probability that roots of quadratic are real and distinct, is*

(a) $\frac{5}{216}$ (b) $\frac{19}{108}$ (c) $\frac{173}{216}$ (d) $\frac{17}{108}$

Sol. (b) For roots of $ax^2 + bx + c = 0$ to be real and distinct,

$$b^2 - 4ac > 0$$

Value of b	Possible values of a and c
1, 2	No values of a and c
3	(1, 1), (1, 2), (2, 1)
4	(1, 1), (1, 2), (2, 1), (1, 3), (3, 1)
5	(1, 1), (1, 2), (2, 1), (1, 3), (3, 1), (2, 2), (1, 4), (4, 1), (1, 5), (5, 1), (2, 3), (3, 2), (1, 6), (6, 1)
6	(1, 1), (1, 2), (2, 1), (1, 3), (3, 1), (2, 2), (1, 4), (4, 1), (1, 5), (5, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 4), (4, 2)

If E be the event of favourable cases, then $n(E) = 38$

Total ways, $n(S) = 6 \times 6 \times 6 = 216$

Hence, the required probability, $p_1 = \frac{n(E)}{n(S)} = \frac{38}{216} = \frac{19}{108}$

17. *The probability that roots of quadratic are equal, is*

(a) $\frac{5}{216}$ (b) $\frac{7}{216}$ (c) $\frac{11}{216}$ (d) $\frac{17}{216}$

Sol. (a) For roots of $ax^2 + bx + c = 0$ to be equal $b^2 = 4ac$

i.e. b^2 must be even.

Value of b	Possible values of a and c
2	(1, 1)
4	(2, 2), (1, 4), (4, 1)
6	(3, 3)

If E be the event of favourable cases, then $n(E) = 5$

Total ways, $n(S) = 6 \times 6 \times 6 = 216$

Hence, the required probability, $p_2 = \frac{n(E)}{n(S)} = \frac{5}{216}$

18. *The probability that roots of quadratic are imaginary, is*

(a) $\frac{103}{216}$ (b) $\frac{133}{216}$ (c) $\frac{157}{216}$ (d) $\frac{173}{216}$

Sol. (d) Let p_3 = Probability that roots of $ax^2 + bx + c = 0$ are imaginary

$= 1 -$ (Probability that roots of $ax^2 + bx + c = 0$ are real)

$= 1 - (p_1 + p_2)$ [from above]

$= 1 - \frac{43}{216} = \frac{173}{216}$

Passage II

(Ex. Nos. 19 to 21)

A box contains n coins. Let $P(E_i)$ be the probability that exactly i out of n coins are biased. If $P(E_i)$ is directly proportional to $i(i+1); 1 \le i \le n$.

19. *Proportionality constant k is equal to*

(a) $\frac{3}{n(n^2+1)}$ (b) $\frac{1}{(n^2+1)(n+2)}$

(c) $\frac{3}{n(n+1)(n+2)}$ (d) $\frac{1}{(n+1)(n+2)(n+3)}$

Sol. (c) $\because P(E_i) \propto i(i+1)$

$\Rightarrow P(E_i) = k\,i(i+1)$, where k is proportionality constant.

We have, $P(E_1) + P(E_2) + P(E_3) + \dots + P(E_n) = 1$

($\because E_1, E_2, \dots, E_n$ are mutually exclusive and exhaustive events)

$$\Rightarrow \sum_{i=1}^{n} P(E_i) = 1$$

$$\Rightarrow k \sum_{i=1}^{n} (i^2 + i) = 1$$

$$\Rightarrow k\left[\sum n^2 + \sum n\right] = 1$$

$$\Rightarrow k\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right] = 1$$

$$\therefore \quad k = \frac{3}{n(n+1)(n+2)} \quad \text{...(i)}$$

20. *If P be the probability that a coin selected at random is biased, then $\lim\limits_{x \to \infty} P$ is*

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{7}{8}$

Sol. (b) $\because P = P(E) = \sum_{i=1}^{n} \cdot P(E_i)\, P\left(\frac{E}{E_i}\right)$ ...(ii)

$$= \sum_{i=1}^{n} k\, i(i+1) \cdot \frac{i}{n}$$

$$= \frac{k}{n} \sum_{i=1}^{n} (i^3 + i^2) = \frac{k}{n}\left[\sum n^3 + \sum n^2\right]$$

$$= \frac{k}{n}\left[\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{k\,(n+1)(n+2)(3n+1)}{12}$$

$$= \frac{3}{n(n+1)(n+2)} \cdot \frac{(n+1)(n+2)(3n+1)}{12} \quad \text{[from Eq. (i)]}$$

$$= \frac{3n+1}{4n} = \frac{3}{4} + \frac{1}{4n}$$

$$\therefore \lim_{n \to \infty} P = \lim_{n \to \infty}\left[\frac{3}{4} + \frac{1}{3n}\right] = \frac{3}{4} + 0 = \frac{3}{4}$$

21. *If a coin is selected at random is found to be biased, the probability that it is the only biased coin the box, is*

(a) $\frac{1}{(n+1)(n+2)(n+3)(n+4)}$ (b) $\frac{12}{n(n+1)(n+2)(3n+1)}$

(c) $\frac{24}{n(n+1)(n+2)(2n+1)}$ (d) $\frac{24}{n(n+1)(n+2)(3n+1)}$

Sol. (d) $P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{\sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{E}{E_i}\right)} = \frac{2k \times \frac{1}{n}}{P(E)}$ [from Eq. (ii)]

$$= \frac{\frac{2k}{n}}{\left(\frac{3n+1}{4n}\right)} = \frac{8k}{(3n+1)}$$

$$= \frac{24}{n(n+1)(n+2)(3n+1)} \quad \text{[from Eq. (i)]}$$

Passage III

(Ex. Nos. 22 to 24)

Let S be the set of the first 21 natural numbers, then the probability of

22. *Choosing $\{x, y\} \subseteq S$, such that $x^3 + y^3$ is divisible by 3, is*

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Sol. (d) $\because S = \{1, 2, 3, 4, 5, \dots, 21\}$

Total number of ways choosing x and y is

$$^{21}C_2 = \frac{21 \cdot 20}{1 \cdot 2} = 210$$

Now, arrange the given numbers as below:

1	4	7	10	13	16	19
2	5	8	11	14	17	20
3	6	9	12	15	18	21

We see that, $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ will be divisible by 3 in the following cases:

One of two numbers belongs to the first row and one of the two numbers belongs to the second row or both numbers occurs in third row.

$\therefore$ Number of favourable cases $= ({}^7C_1)({}^7C_1) + {}^7C_2 = 70$

$\therefore$ Required probability $= \dfrac{70}{210} = \dfrac{1}{3}$

23. *Choosing $\{x, y, z\} \subseteq S$, such that x, y, z are in AP, is*

(a) $\dfrac{5}{133}$ (b) $\dfrac{10}{133}$ (c) $\dfrac{3}{133}$ (d) $\dfrac{2}{133}$

Sol. (b) Given, x, y, z are in AP

$\therefore$ $2y = x + z$

It is clear that sum of x and z is even.

$\therefore$ x and z both are even or odd out of set S.

i.e., 11 numbers (1, 3, 5,..., 21) are odd and 10 numbers (2, 4, 6,..., 20) are even.

$\therefore$ Number of favourable cases

$$= {}^{21}C_2 + {}^{10}C_2 = \frac{11 \cdot 10}{1 \cdot 2} + \frac{10 \cdot 9}{1 \cdot 2} = 100$$

and total number of ways choosing x, y and z is

$${}^{21}C_3 = \frac{21 \cdot 20 \cdot 19}{1 \cdot 2 \cdot 3} = 1330$$

$\therefore$ Required probability $= \dfrac{100}{1330} = \dfrac{10}{133}$

24. *Choosing $\{x, y, z\} \subseteq S$, such that x, y, z are not consecutive, is*

(a) $\dfrac{17}{70}$ (b) $\dfrac{34}{70}$ (c) $\dfrac{51}{70}$ (d) $\dfrac{34}{35}$

Sol. (c) Given, x, y and z are not consecutive.

$\therefore$ Number of favourable ways $= {}^{21-3+1}C_3$

$$= {}^{19}C_3 = \frac{19 \cdot 18 \cdot 17}{1 \cdot 2 \cdot 3} = 969$$

and total number of ways $= {}^{21}C_3 = \dfrac{21 \cdot 20 \cdot 19}{1 \cdot 2 \cdot 3} = 1330$

$\therefore$ Required probability $= \dfrac{969}{1330} = \dfrac{51}{70}$

JEE Type Solved Examples :
Single Integer Answer Type Questions

- This section contains **2 examples.** The answer to each example is a **single digit integer** ranging from **0** to **9** (both inclusive).

● **Ex. 25** *The altitude through A of ΔABC meets BC at D and the circumscribed circle at E. If $D \equiv (2, 3)$, $E \equiv (5, 5)$, the ordinate of the orthocentre being a natural number. If the probability that the orthocentre lies on the lines $y = 1;\ y = 2;\ y = 3\ldots\ldots y = 10$ is $\dfrac{m}{n}$, where m and n are relative primes, the value of $m + n$ is*

Sol. (8) Let the orthocentre be $O(x, y)$.

It is clear from the OE is perpendicular bisector of line BC.

$\therefore$ $OD = DE$

$\Rightarrow \sqrt{(x-2)^2 + (y-3)^2} = \sqrt{(5-2)^2 + (5-3)^2}$

$\Rightarrow$ $(x-2)^2 + (y-3)^2 = (5-2)^2 + (5-3)^2$

$\Rightarrow$ $x^2 + y^2 - 4x - 6y = 0 \Rightarrow x = 2 \pm \sqrt{13 - (y-3)^2}$

$\Rightarrow y$ can take the values as 1, 2, 3, 4, 5, 6

$\therefore$ Required probability $= \dfrac{6}{10} = \dfrac{3}{5} = \dfrac{m}{n}$ [given]

$\Rightarrow$ $m = 3$ and $n = 5$

$\therefore$ $m + n = 8$

● **Ex. 26** *The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order to form a nine digit number. Let p be the probability that this number is divisible by 36, the value of $9p$ is*

Sol. (2) $\because$ $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$, a number consisting all these digits will be divisible by 9. Thus, the number will be divisible by 36, if and only if it is divisible by 4. The number formed by its last two digits must be divisible by 4. The possible values of the last pair to the following:

12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96.

i.e., There are 16 ways of choosing last two digits.

The remaining digits can be arranged in ${}^7P_7 = 7!$ ways.

Therefore, number of favourable ways $= 16 \times 7!$

and number of total ways $= 9!$

$\therefore$ Required probability, $p = \dfrac{16 \times 7!}{9!} = \dfrac{16}{9 \times 8} = \dfrac{2}{9}$

$\Rightarrow$ $9p = 2$

JEE Type Solved Examples :
Matching Type Questions

- This section contains **2 examples.** Examples 27 and 28 have four statements (A, B, C and D) given in **Column I** and four statements (p, q, r and s) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II.**

● ***Ex. 27*** *If n positive integers taken at random are multiplied together.*

	Column I		Column II
(A)	The probability that the last digit is 1, 3, 7 or 9 is $P(n)$, then 100 $P(2)$ is divisible by	(p)	3
(B)	The probability that the last digit is 2, 4, 6 or 8 is $Q(n)$, then 100 $Q(2)$ is divisible by	(q)	4
(C)	The probability that the last digit is 5 is $R(n)$, then 100 $R(2)$ is divisible by	(r)	6
(D)	The probability that the last digit is zero is $S(n)$, then 100 $S(2)$ is divisible by	(s)	9

Sol. A → (q); B → (p, q, r); C → (p, s); D → (p, s)

Let n positive integers be $x_1, x_2, x_3, \ldots, x_n$

Let $a = x_1 . x_2 . x_3 \ldots x_n$

Since, the last digit in each of the numbers $x_1, x_2, \ldots, x_n$ can be any one of the digits

0, 1, 2,..., 9 (total 10)

$\therefore \quad n(S) = 10^n$

Let E_1, E_2, E_3 and E_4 are the events given in A, B, C and D, respectively.

(A) $n(E_1) = 4^n \Rightarrow P(E_1) = \left(\frac{4}{10}\right)^n = P(n)$ [given]

$\therefore \quad 100\,P(2) = 16$

(B) $n(E_2) = n$ (last digit is 1 or 2 or 3 or 4 or 6 or 7 or 8 or 9) $- n(E_1) = 8^n - 4^n$

$\Rightarrow \quad P(E_2) = \frac{8^n - 4^n}{10^n} = Q(n)$ [given]

$\therefore \quad 100\,Q(2) = 64 - 16 = 48$

(C) $n(E_3) = n$ (last digit is 1 or 3 or 5 or 7 or 9) $- n(E_1)$

$= 5^n - 4^n$

$\Rightarrow \quad P(E_3) = \frac{5^n - 4^n}{10^n} = R(n)$ [given]

$\therefore \; 100\,R(2) = 25 - 16 = 9$

(D) $n(E_4) = n(S) - n$ (last digit is 1 or 2 or 3 or 4 or 6 or 7 or 8 or 9)$- n(E_3) = 10^n - 8^n - (5^n - 4^n)$

$\therefore \quad P(E_4) = \frac{10^n - 8^n - 5^n + 4^n}{10^n} = S(n)$ [given]

$\therefore \quad 100\,S(2) = 27$

● ***Ex. 28*** *If A and B are two independent events, such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$.*

	Column I		Column II
(A)	If $P\left(\frac{A}{B}\right) = \lambda_1$, then $12\lambda_1$ is	(p)	a prime number
(B)	If $P\left(\frac{A}{A \cup B}\right) = \lambda_2$, then $9\lambda_2$ is	(q)	a composite number
(C)	If $P[(A \cap \overline{B}) \cup (\overline{A} \cap B)] = \lambda_3$, then $12\lambda_3$ is	(r)	a natural number
(D)	If $P(\overline{A} \cup B) = \lambda_4$, then $12\lambda_4$ is	(s)	a perfect number

Sol. A → (q, r); B (q, r, s); C → (p, r); D → (q, r)

$\because$ A and B are independent events.

$\therefore \quad P(A \cap B) = P(A) \cdot P(B) = \frac{1}{12}$,

$P(A \cap \overline{B}) = P(A) \cdot P(\overline{B}) = \frac{1}{3} \times \left(1 - \frac{1}{4}\right) = \frac{1}{4}$,

$P(\overline{A} \cap B) = P(\overline{A}) \cdot P(B) = \left(1 - \frac{1}{3}\right) \cdot \frac{1}{4} = \frac{1}{6}$

(A) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{1}{3} = \lambda_1$ [given]

$\therefore \quad 12\lambda_1 = 4$ [natural number and composite number]

(B) $P\left(\frac{A}{A \cup B}\right) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$

$= \frac{P(A)}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)} - P(A \cap B)$

$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{12}} = \frac{2}{3} = \lambda_2$ [given]

$\therefore \quad 9\lambda_2 = 6$

[natural number, composite number and perfect number]

(C) $P(A \cap \overline{B}) \cup (\overline{A} \cap B)) = P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12} = \lambda_3$ [given]

$\therefore \quad 12\lambda_3 = 5$ [prime number and natural number]

(D) $P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$

$= \left(1 - \frac{1}{3}\right) + \frac{1}{4} - \frac{1}{6} = \frac{3}{4} = \lambda_4$ [given]

$\therefore \quad 12\lambda_4 = 9$ [natural number and composite number]

JEE Type Solved Examples :
Statement I and II Type Questions

- **Directions** Example numbers 29 and 30 are Assertion-Reason type examples. Each of these examples contains two statements:
  Statement-1 (Assertion) and **Statement-2** (Reason)
  Each of these examples also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
  (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
  (b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
  (c) Statement-1 is true, Statement-2 is false
  (d) Statement-1 is false, Statement-2 is true

Ex. 29. *A man P speaks truth with probability p and another man Q speaks truth with probability* $2p$.

Statement-1 *If P and Q contradict each other with probability* $\frac{1}{2}$, *then there are two values of p.*

Statement-2 *A quadratic equation with real coefficients has two real roots.*

Sol. (c) Let E_1 be the event that P speaks the truth, then $P(E_1) = p$ and let E_2 be the event that Q speaks the truth, then $P(E_2) = 2p$.

Statement-1 If P and Q contradict each other with probability $\frac{1}{2}$, then $P(E_1)\cdot P(E_2') + P(E_1')\cdot P(E_2) = \frac{1}{2}$

$$\Rightarrow p\cdot(1-2p) + (1-p)\cdot 2p = \frac{1}{2} \Rightarrow 8p^2 - 6p + 1 = 0$$

$$\Rightarrow (2p-1)(4p-1) = 0 \Rightarrow p = \frac{1}{2} \text{ and } p = \frac{1}{4}$$

$\therefore$ Statement-1 is true.

Statement-2 Let quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$

If $b^2 - 4ac < 0$

then, roots are imaginary.

$\therefore$ Statement-2 is false.

Ex. 30 *A fair die thrown twice. Let* (a, b) *denote the outcome in which the first throw shows a and the second shows b. Let A and B be the following two events:*

$A = \{(a, b) | a \text{ is even}\}$, $B = \{(a, b) | b \text{ is even}\}$

Statement-1 If $C = \{(a, b) | a + b \text{ is odd}\}$, then $P(A \cap B \cap C) = \frac{1}{8}$.

Statement-2 If $D = \{(a, b) | a + b \text{ is even}\}$, then

$$P[(A \cap B \cap D) | (A \cup B)] = \frac{1}{3}$$

Sol. (c) If a and b are both even, then $a + b$ is even, therefore $P(A \cap B \cap C) = 0$

$\therefore$ Statement-1 is false.

Also, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$$

$$\therefore P[(A \cap B \cap D) | (A \cup B)] = \frac{P((A \cap B \cap D) \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P(A \cap B)}{P(A \cup B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \quad [\because A \cap B \subseteq D]$$

$\therefore$ Statement-2 is true.

Subjective Type Examples

- In this section, there are **24 subjective** solved examples.

Ex. 31 *Three critics review a book. Odds in favour of the book are* $5:2$, $4:3$ *and* $3:4$ *respectively for the three critics. Find the probability that majority are in favour of the book.*

Sol. Let the critics be E_1, E_2 and E_3. Let $P(E_1)$, $P(E_2)$ and $P(E_3)$ denotes the probabilities of the critics E_1, E_2 and E_3 to be in favour of the book. Since, the odds in favour of the book for the critics E_1, E_2 and E_3 are $5:2$, $4:3$ and $3:4$, respectively.

$$\therefore \quad P(E_1) = \frac{5}{5+2} = \frac{5}{7}, P(E_2) = \frac{4}{4+3} = \frac{4}{7}$$

and $$P(E_3) = \frac{3}{3+4} = \frac{3}{7}$$

Clearly, the event of majority being in favour = the event of atleast two critics being in favour

$\therefore$ The required probability

$$= P(E_1 E_2 \overline{E}_3) + P(\overline{E}_1 E_2 E_3) + P(E_1 \overline{E}_2 E_3) + P(E_1 E_2 E_3)$$
$$= P(E_1)\cdot P(E_2)\cdot P(\overline{E}_3) + P(\overline{E}_1)\cdot P(E_2)\cdot P(E_3)$$
$$+ P(E_1)\cdot P(\overline{E}_2)\cdot P(E_3) + P(E_1)\cdot P(E_2)\cdot P(E_3)$$

$[\because E_1, E_2 \text{ and } E_3 \text{ are independent}]$

$$= \frac{5}{7}\cdot\frac{4}{7}\cdot\left(1-\frac{3}{7}\right)+\left(1-\frac{5}{7}\right)\cdot\frac{4}{7}\cdot\frac{3}{7}+\frac{5}{7}\cdot\left(1-\frac{4}{7}\right)\cdot\frac{3}{7}+\frac{5}{7}\cdot\frac{4}{7}\cdot\frac{3}{7}$$

$$= \frac{1}{7^3}[80+24+45+60] = \frac{209}{343}$$

Ex. 32 *A has 3 shares is a lottery containing 3 prizes and 9 blanks; B has 2 shares in a lottery containing 2 prizes and 6 blanks. Compare their chances of success.*

Sol. Let E_1 and E_2 be the events of success of A and B, respectively. Therefore, E'_1 and E'_2 are the events of unsuccess of A and B, respectively.

Since, A has 3 shares in a lottery containing 3 prizes and 9 blanks, therefore A will draw 3 tickets out of 12 tickets (3 prizes + 9 blanks). Then, A will get success if he draws atleast one prize out of 3 draws. Similarly, B will get success if he draws atleast one prize out of 2 draws.

$$\therefore \quad P(E'_1) = \frac{{}^9C_3}{{}^{12}C_3} = \frac{\frac{9\cdot 8\cdot 7}{1\cdot 2\cdot 3}}{\frac{12\cdot 11\cdot 10}{1\cdot 2\cdot 3}} = \frac{21}{55}$$

$$\therefore \quad P(E_1) = 1 - P(E'_1) = 1 - \frac{21}{55} = \frac{34}{55}$$

Again, $$P(E'_2) = \frac{{}^6C_2}{{}^8C_2} = \frac{\frac{6\cdot 5}{1\cdot 2}}{\frac{8\cdot 7}{1\cdot 2}} = \frac{15}{28}$$

$$\therefore \quad P(E_2) = 1 - P(E'_2) = 1 - \frac{15}{28} = \frac{13}{28}$$

Hence, $$\frac{P(E_1)}{P(E_2)} = \frac{\frac{34}{55}}{\frac{13}{28}} = \frac{952}{715}$$

$$\therefore \quad P(E_1) : P(E_2) = 952 : 715$$

Ex. 33 *A bag contains a white and b black balls. Two players A and B alternately draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. If A begins the game and the probability of A winning the game is three times that of B, show that $a : b = 2 : 1$.*

Sol. Let E_1 denote the event of drawing a white ball at any draw and E_2 that for a black ball and let E be the event for A winning the game

$$\therefore P(E_1) = \frac{a}{a+b} \text{ and } P(E_2) = \frac{b}{a+b}$$

$$\therefore P(E) = P(E_1 \text{ or } E_2E_2E_1 \text{ or } E_2E_2E_2E_2E_1 \text{ or } \ldots)$$

$$= P(E_1) + P(E_2E_2E_1) + P(E_2E_2E_2E_2E_1) + \ldots$$

$$= P(E_1) + P(E_2)\,P(E_2)\,P(E_1) + P(E_2)\,P(E_2)\,P(E_2)\,P(E_2)\,P(E_1) + \ldots$$

[$\because E_1$ and E_2 are independent]

$$= \frac{P(E_1)}{1 - \{P(E_2)\}^2} \quad \text{[sum of infinite GP]}$$

$$= \frac{\frac{a}{a+b}}{1 - \left(\frac{b}{a+b}\right)^2} = \frac{a(a+b)}{a^2 + 2ab} \quad \therefore \; P(E) = \frac{a+b}{a+2b}$$

Then, $P(E')$ is the probability for B winning the game

$$\therefore \; P(E') = 1 - P(E) = 1 - \frac{a+b}{a+2b} = \frac{b}{a+2b}$$

According to the problem, $P(E) = 3P(E')$

$$\Rightarrow \quad \frac{a+b}{a+2b} = \frac{3b}{a+2b} \quad \Rightarrow \alpha + \beta = 3\beta \Rightarrow \alpha = 2\beta$$

$$\therefore \quad \frac{a}{b} = \frac{2}{1} \quad \Rightarrow \alpha : \beta = 2 : 1$$

Ex. 34 *Five persons entered the lift cabin on the ground floor of an 8 floors house. Suppose that each of them, independently and with equal probability can leave the cabin at any floor beginning with the first. Find out the probability of all five persons leaving at different floors.*

Sol. Let S be the sample space and E be the event that the five persons get down at different floors.

Total number of floors excluding the ground floor = 7

Since, each of the 5 persons can get down at any one of the 7 floors in 7 ways.

$\therefore$ $n(S)$ = Total number of ways in which the 5 persons can get down = 7^5

and $n(E)$ = number of ways in which the 5 persons can get down at 5 different floors out of 7 floors = ${}^7P_5$

$$\therefore \text{ Required probability, } P(E) = \frac{n(E)}{n(S)} = \frac{{}^7P_5}{7^5}$$

Ex. 35 *Let X be a set containing n elements. Two subsets A and B of X are chosen at random. Find the probability that $A \cup B = X$.*

Sol. Let $X = \{x_1, x_2, \ldots, x_n\}$

For each $x_i \in X$ $(1 \le i \le n)$, we have the following four choices

(i) $x_i \in A$ and $x_i \in B$ (ii) $x_i \in A$ and $x_i \notin B$

(iii) $x_i \notin A$ and $x_i \in B$ (iv) $x_i \notin A$ and $x_i \notin B$

Let S be the sample space and E be the event favourable for the occurrence of $A \cup B = X$.

$$\therefore \quad n(S) = 4^n$$

and $$n(E) = 3^n \quad [\because \text{case (iv)} \notin X]$$

Hence, the required probability,

$$P(E) = \frac{n(E)}{n(S)} \Rightarrow = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$$

● **Ex. 36** *Two persons each makes a single throw with a pair of dice. Find the probability that the throws are unequal.*

Sol. Let E be the event that the throws of the two persons are unequal. Then, E' be the event that the throws of the two persons are equal.

$\therefore$ The total number of cases for E' is $(36)^2$

i.e., $n(S) = (36)^2$ [$\because S$ be the sample space]

We now proceed to find out the number of favourable cases for E'. Suppose

$$(x + x^2 + x^3 + \ldots + x^6)^2 = a_2 x^2 + a_3 x^3 + \ldots + a_{12} x^{12}$$

The number of favourable ways of $E' = a_2^2 + a_3^2 + \ldots + a_{12}^2$

$\therefore$ $n(E')$ = coefficient of constant term in

$$(a_2 x^2 + a_3 x^3 + \ldots + a_{12} x^{12}) \times \left(\frac{a_2}{x^2} + \frac{a_3}{x^3} + \ldots + \frac{a_{12}}{x^{12}}\right)$$

$$= \text{coefficient of constant term in } \frac{(1 - x^6)^2}{(1 - x)^2} \times \frac{\left(1 - \frac{1}{x^6}\right)^2}{\left(1 - \frac{1}{x}\right)^2}$$

$= \text{coefficient of } x^{10} \text{ in } (1 - x^6)^4 (1 - x)^{-4}$

$= \text{coefficient of } x^{10} \text{ in } (1 - 4x^6 + \ldots)$

$$(1 + {}^4C_1 x + {}^5C_2 x^2 + \ldots + {}^{13}C_{10} x^{10} + \ldots)$$

$= {}^{13}C_{10} - 4 \cdot {}^7C_4$

$$= {}^{13}C_3 - 4 \cdot {}^7C_3 = \frac{13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3} - 4 \cdot \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3} = 146$$

$$\therefore \quad P(E') = \frac{n(E')}{n(S)} = \frac{146}{(36)^2} = \frac{73}{648}$$

Hence, required probability,

$$P(E) = 1 - P(E') = 1 - \frac{73}{648} = \frac{575}{648}$$

● **Ex. 37** *If X and Y are independent binomial variates* $B(5, 1/2)$ *and* $B(7, 1/2)$*, find the value of* $P(X + Y = 3)$.

Sol. We have,

$$P(X + Y = 3) = P(X = 0, Y = 3) + P(X = 1, Y = 2) + P(X = 2, Y = 1) + P(X = 3, Y = 0)$$

$$= P(X = 0)\, P(Y = 3) + P(X = 1)\, P(Y = 2) + P(X = 2)\, P(Y = 1) + P(X = 3)\, P(Y = 0)$$

[$\because X$ and Y are independent]

$$= {}^5C_0 \left(\frac{1}{2}\right)^5 \cdot {}^7C_3 \left(\frac{1}{2}\right)^7 + {}^5C_1 \left(\frac{1}{2}\right)^5 {}^7C_2 \left(\frac{1}{2}\right)^7 + {}^5C_2 \left(\frac{1}{2}\right)^5 {}^7C_1 \left(\frac{1}{2}\right)^7 + {}^5C_3 \left(\frac{1}{2}\right)^5 {}^7C_0 \left(\frac{1}{2}\right)^7$$

$$= \left(\frac{1}{2}\right)^{12} [(1)(35) + (5)(21) + (10)(7) + (10)(1)]$$

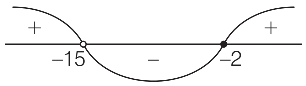
$$= \frac{220}{2^{12}} = \frac{55}{1024}$$

● **Ex. 38** *If* $a \in [-20, 0]$*, find the probability that the graph of the function* $y = 16x^2 + 8(a + 5)x - 7a - 5$ *is strictly above the X-axis.*

Sol. Since, the graph $y = 16x^2 + 8(a + 5)x - 7a - 5$ is strictly above the X-axis, therefore $y > 0$ for all x

$\Rightarrow \quad 16x^2 + 8(a + 5)x - 7a - 5 > 0, \forall x$

$\therefore$ Discriminant < 0



$\Rightarrow \quad 64(a + 5)^2 - 4 \cdot 16(-7a - 5) < 0$

$\Rightarrow a^2 + 17a + 30 < 0 \Rightarrow (a + 15)(a + 2) < 0$

$\Rightarrow \quad -15 < a < -2$

$$\therefore \text{ Required probability} = \frac{\int_{-15}^{-2} dx}{\int_{-20}^{0} dx} = \frac{13}{20}$$

● **Ex. 39** 3 *distinct integers are selected at random from* 1, 2, 3,..., 20. *Find out the probability that the sum is divisible by* 5.

Sol. The number of wayds choosing 3 distinct integers from 1, 2, 3, ..., 20 is

$${}^{20}C_3 = \frac{20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3} = 20 \times 57 = 1140$$

Now, arrange the given numbers as below:

1	6	11	16
2	7	12	17
3	8	13	18
4	9	14	19
5	10	15	20

We see that the sum of three digits divisible by 5 in the following cases :

Two number from 1st row and one number from 3rd row or one number from 2nd row and two numbers from 4th row or three numbers from 5th row or one number from each (1st row, 4th row, 5th row) or one number from each (2nd row, 3rd row, 5th row).

Then, the number of favourable ways

$$= {}^4C_2 \times {}^4C_1 + {}^4C_1 \times {}^4C_2 + {}^4C_3 + {}^4C_1 \times {}^4C_1 \times {}^4C_1 + {}^4C_1 \times {}^4C_1 \times {}^4C_1$$

$= 24 + 24 + 4 + 64 + 64 = 180$

Hence, the required probability $= \frac{180}{1140} = \frac{3}{19}$

Note

If divisible by 4, then take four rows and if divisible by 3, then take three rows, etc.

Ex. 40 5 *girls and* 10 *boys sit at random in a row having* 15 *chairs numbered as 1 to* 15. *Find the probability that end seats are occupied by the girls and between any two girls odd number of boys sit.*

Sol. There are four gaps in between the girls where the boys can sit. Let the number of boys in these gaps be $2a+1$, $2b+1$, $2c+1$, $2d+1$, then

$$2a+1+2b+1+2c+1+2d+1=10$$

or $$a+b+c+d=3$$

The number of solutions of above equation

$$= \text{coefficient of } x^3 \text{ in } (1-x)^{-4} = {}^6C_3 = 20$$

Thus, boys and girls can sit in $20 \times 10! \times 5!$ ways. Total ways $= 15!$

Hence, the required probability $= \dfrac{20 \times 10! \times 5!}{15!}$

Ex. 41 *A four digit number* (*numbered from* 0000 *to* 9999) *is said to be lucky if sum of its first two digits is equal to the sum of its last two digits. If a four-digit number is picked up at random, find the probability that it is lucky number.*

Sol. The total number of ways of choosing a four digit number is $10^4 = 10000$. Let a_k denote the number of distinct non-negative integral solutions of the equation $x+y=k$ $(0 \le k \le 18)$

$\therefore$ The number of favourable cases $= a_0^2 + a_1^2 + \dots + a_{18}^2$

Suppose, $(1+x+x^2+\dots+x^9)^2$

$$= a_0 + a_1x + a_2x^2 + \dots + a_{18}x^{18}$$

Thus, $a_0^2 + a_1^2 + \dots + a_{18}^2 =$ coefficient of constant term in

$$(a_0 + a_1x + \dots + a_{18}x^{18}) \times \left(a_0 + \frac{a_1}{x} + \dots + \frac{a_{18}}{x^{18}}\right)$$

$=$ coefficient of constant term in

$$(1+x+x^2+\dots+x^9)^2 \times \left(1 + \frac{1}{x} + \frac{1}{x^2} + \dots + \frac{1}{x^9}\right)^2$$

$=$ coefficient of x^{18} in $(1+x+x^2+\dots+x^9)^4$

$=$ coefficient of x^{18} in $(1-x^{10})^4(1-x)^{-4}$

$=$ coefficient of x^{18} in $(1-4x^{10})(1+{}^4C_1x+{}^5C_2x^2+\dots)$

$= {}^{21}C_{18} - 4 \cdot {}^{11}C_8 = 1330 - 660 = 670$

Hence, the required probability $= \dfrac{670}{10000} = 0.067$

Ex. 42

(i) *If four squares are chosen at random on a chess board, find the probability that they lie on a diagonal line.*

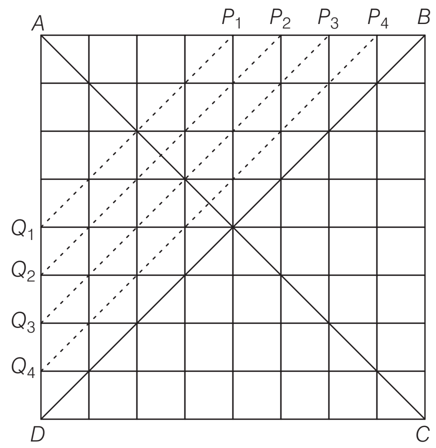
(ii) *If two squares are chosen at random on a chess board, what is the probability that they have exactly one corner in common?*

(iii) *If nine squares are chosen at random on a chess board, what is the probability that they form a square of size* 3×3?

Sol. (i) Total number of ways $= {}^{64}C_4$

The chess board can be divided into two parts by a diagonal line BD. Now, if we begin to select four squares from the diagonal P_1Q_1, P_2Q_2, …, BD, then we can find number of squares selected

$$= 2({}^4C_4 + {}^5C_4 + {}^6C_4 + {}^7C_4) = 112$$



Similarly, number of squares for the diagonals chosen parallel to $AC = 112$

$\therefore$ Total favourable ways $= 364$

$\therefore$ Required probability $= \dfrac{364}{{}^{64}C_4}$.

(ii) Total ways $= 64 \times 63$

Now, if first square is in one of the four corners, then the second square can be chosen in just one way $= (4)(1) = 4$

If the first square is one of the 24 non-corner squares along the sides of the chess board, the second square can be chosen in two ways $= (24)(2) = 48$.

Now, if the first square is any of the 36 remaining squares, the second square can be chosen in four ways

$$= (36)(4) = 144$$

$\therefore$ Favourable ways $= 4 + 48 + 144 = 196$

$\therefore$ Required probability $= \dfrac{196}{64 \times 63} = \dfrac{7}{144}$.

(iii) Total ways $= {}^{64}C_9$

A chess board has 9 horizontal and 9 vertical lines. We see that a square of size 3×3 can be formed by choosing four consecutive horizontal and vertical lines.

Hence, favourable ways $= ({}^6C_1)({}^6C_1) = 36$

$\therefore$ Required probability $= \dfrac{36}{{}^{64}C_9}$.

Ex. 43 *Out of $(2n+1)$ tickets consecutively numbered, three are drawn at random. Find the chance that the numbers on them are in AP.*

Sol. Let us consider first $(2n+1)$ natural numbers as $(2n+1)$ consecutive numbers.

Let S be the sample space and E be the event of favourable cases.

$$\therefore \quad n(S) = {}^{2n+1}C_3$$

$$= \frac{(2n+1)\,2n(2n-1)}{1\cdot 2\cdot 3} = \frac{n(4n^2-1)}{3}$$

Let the three numbers drawn be a, b, c where $a < b < c$.

Common difference d	Triplets (a, b, c)	Number of Triplets
1	$(1, 2, 3), (2, 3, 4), \ldots, (2n-1, 2n, 2n+1)$	$2n-1$
2	$(1, 3, 5), (2, 4, 6), \ldots, (2n-3, 2n-1, 2n+1)$	$2n-3$
3	$(1, 4, 7), (2, 5, 8), \ldots, (2n-5, 2n-2, 2n+1)$	$2n-5$
...	....	...
...	....	...
...	....	...
$n-1$	$(1, n, 2n-1), (2, n+1, 2n), (3, n+2, 2n+1)$	3
n	$(1, n+1, 2n+1)$	1

$$\therefore \ n(E) = 1 + 3 + \ldots + (2n-5) + (2n-3) + (2n-1)$$

$$= \frac{n}{2}\{1 + 2n - 1\} = n^2$$

$$\therefore \text{ Required probability, } P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{n^2}{\frac{n(4n^2-1)}{3}} = \frac{3n}{4n^2-1}$$

Aliter Let S be the sample space and E be the event of favourable cases.

$$\therefore \quad n(S) = {}^{2n+1}C_3 = \frac{(2n+1)\,2n(2n-1)}{1\cdot 2\cdot 3} = \frac{n(4n^2-1)}{3}$$

Let the three numbers a, b, c are drawn where $a < b < c$ and given a, b, c are in AP.

$$\therefore \quad b = \frac{a+c}{2} \text{ or } 2b = a + c \qquad \ldots\text{(i)}$$

It is clear from Eq. (i), a and c are both odd or both even.

Out of $(2n+1)$ tickets consecutively numbers either $(n+1)$ of them will be odd and n of them will be even (if the numbers begin with an odd number) or $(n+1)$ of them will be even and n of them will be odd (if the number begin with an even number).

$$\therefore \quad n(E) = {}^{n+1}C_2 + {}^{n}C_2$$

$$= \frac{(n+1)\,n}{2} + \frac{n(n-1)}{2} = n^2$$

$\therefore$ Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{n^2}{\frac{n(4n^2-1)}{3}} = \frac{3n}{4n^2-1}$$

Ex. 44 *Out of $3n$ consecutive integers, three are selected at random. Find the chance that their sum is divisible by 3.*

Sol. Let $3n$ consecutive integers (start with the integer m) are

$$m, m+1, m+2, \ldots, m+3n-1$$

Now, we write these $3n$ numbers in 3 rows as follows :

$$m, m+3, m+6, \ldots, m+3n-3$$
$$m+1, m+4, m+7, \ldots, m+3n-2$$
$$m+2, m+5, m+8, \ldots, m+3n-1$$

The total number of ways of choosing 3 integers out of $3n$ is

$${}^{3n}C_3 = \frac{3n(3n-1)(3n-2)}{1\cdot 2\cdot 3}$$

$$= \frac{n(3n-1)(3n-2)}{2}$$

The sum of the three numbers shall be divisible by 3 if and only if either all the three numbers are from the same row or all the three numbers are from different rows.

Therefore, the number of favourable ways are

$$3({}^{n}C_3) + ({}^{n}C_1)({}^{n}C_1)({}^{n}C_1)$$

$$= \frac{3n(n-1)(n-2)}{1\cdot 2\cdot 3} + n^3 = \frac{3n^3 - 3n^2 + 2n}{2}$$

$\therefore$ The required probability

$$= \frac{\text{Favourable ways}}{\text{Total ways}}$$

$$= \frac{\frac{3n^3 - 3n^2 + 2n}{2}}{\frac{n(3n-1)(3n-2)}{2}}$$

$$= \frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$$

Ex. 45 *If $6n$ tickets numbered 0, 1, 2, ..., $6n-1$ are placed in a bag and three are drawn out, show that the chance that the sum of the numbers on them is equal to $6n$ is $\dfrac{3n}{(6n-1)(6n-2)}$.*

Sol. Total number of ways to selecting 3 tickets from $6n$ tickets

$$= {}^{6n}C_3 = n(6n-1)(6n-2) \quad \text{...(i)}$$

For the sum of these tickets of be $6n$, we have the following pattern :

Lowest number	Numbers	Ways
0	$(0, 1, 6n-1), (0, 2, 6n-2) \dots (0, 3n-1, 3n+1)$	$(3n-1)$
1	$(1, 2, 6n-3), (1, 3, 6n-4) \dots (1, 3n-1, 3n)$	$(3n-2)$
2	$(2, 3, 6n-5), (2, 4, 6n-6) \dots (2, 3n-2, 3n)$	$(3n-4)$
3	$(3, 4, 6n-7), (3, 5, 6n-8) \dots (3, 3n-2, 3n-1)$	$(3n-5)$
4		
5		
:		
$(2n-2)$	$(2n-2, 2n-1, 2n+3), (2n-2, 2n, 2n+2)$	2
$(2n-1)$	$(2n-1, 2n, 2n+1)$	1

Lowest number cannot be greater than $(2n-1)$ as their sum will become $> 6n$.

$\therefore$ Favourable ways $= 1 + 2 + \dots + (3n-5) + (3n-4) + (3n-2) + (3n-1)$

Adding Ist with last, 2nd with last one, respectively

$$= [1 + 3n - 1] + [2 + 3n - 2] + \dots \text{ upto } n \text{ terms}$$

$$= 3n + 3n + \dots n \text{ terms}$$

$$= 3n(n) = 3n^2$$

Hence, $\quad$ probability $= \dfrac{3n^2}{n(6n-1)(6n-2)}$

$$= \frac{3n}{(6n-1)(6n-2)}$$