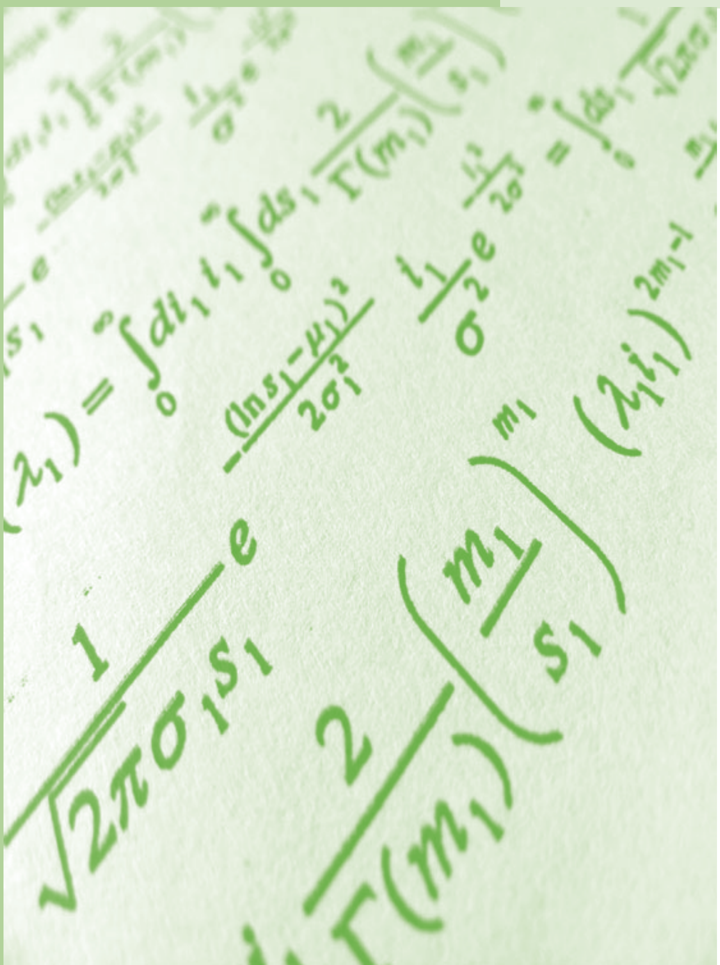
Chapter

26

Partial Fractions



REMEMBER

Before beginning this chapter, you should be able to:

- State expressions and equations
- Have knowledge of polynomials

KEY IDEAS

After completing this chapter, you would be able to:

- Learn about rational fractions and partial fractions
- Study various methods to resolve polynomial equations to partial fractions

INTRODUCTION

An expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$, where $a_0, a_1, \ldots, a_n$ are real numbers and $a_0 \neq 0$ is called a polynomial of degree n.

The quotient of any two polynomials $f(x)$ and $g(x)$ where $g(x) \neq 0$ is called a rational fraction.

In a rational fraction $\frac{f(x)}{g(x)}$, if the degree of $f(x)$ is less than the degree of $g(x)$, then the rational fraction $\frac{f(x)}{g(x)}$ is called a proper rational fraction or simply a proper fraction.

Example: $\frac{2x+3}{3x^2-5x+2}, \frac{3}{x-4}$ are some examples of proper fractions.

If the degree of $f(x)$ is greater than or equal to the degree of $g(x)$, then the rational fraction $\frac{f(x)}{g(x)}$ is called an improper fraction.

Example: $\frac{x^2+2x-1}{2x^2-5x+1}, \frac{3x^3+5x+7}{x^2+6x+5}, \frac{x^2+x+1}{2x+1}$ are some examples of improper fractions.

Note The sum of two proper rational fractions is a proper fraction.

Example: $\frac{2}{x+3}$ and $\frac{5}{2x+1}$ are two proper fractions.

Consider $\frac{2}{x+3} + \frac{5}{2x+1} = \frac{2(2x+1)+5(x+3)}{(x+3)(2x+1)} = \frac{9x+17}{2x^2+7x+3}$, which is a proper fraction.

That is, a proper fraction may be split into sum of two or more proper fractions, which are also called partial fractions.

The process of expressing a proper fraction as the sum of two or more proper fractions is called resolving it into partial fractions.

Note

If an improper fraction $\frac{f(x)}{g(x)}$ is to split into partial fractions, first we divide $f(x)$ by $g(x)$ till we obtain a remainder $r(x)$ which is of lower degree than $g(x)$ and then we express the given fraction as $\frac{f(x)}{g(x)} = \text{quotient} + \frac{r(x)}{g(x)}$. Then we resolve the proper fraction $\frac{r(x)}{g(x)}$ into partial fractions.

Now let us discuss about the different methods of resolving a given proper fraction $\frac{f(x)}{g(x)}$ into partial fractions.

Method 1

When $g(x)$ contains non-repeated linear factors only: For every non-repeated linear factor of the form $ax + b$ of $g(x)$, there exists a corresponding partial fraction of the form $\frac{A}{ax+b}$.

EXAMPLE 26.1

Resolve $\dfrac{3x+5}{(x+2)(3x-1)}$ into partial fractions.

SOLUTION

In the given fraction, the denominator has two linear, non-repeated factors.

$\therefore$ The given fraction can be written as the sum of two partial fractions.

Let $\dfrac{3x+5}{(x+2)(3x-1)} = \dfrac{A}{x+2} + \dfrac{B}{3x-1}$

$$\Rightarrow \quad 3x+5 = A(3x-1) + B(x+2) \tag{1}$$

Put $x = -2$ in Eq. (1)

$\Rightarrow \quad 3\,(-2) + 5 = A[3(-2) - 1] + B(-2+2) - 1 = -7A \quad \Rightarrow \quad A = \dfrac{1}{7}.$

Comparing the constant terms on both sides of Eq. (1), we have

$5 = -A + 2B$

$5 = -\dfrac{1}{7} + 2B$ or $B = \dfrac{18}{7}$.

$$\therefore \frac{3x+5}{(x+2)(3x-1)} = \frac{\frac{1}{7}}{x+2} + \frac{\frac{18}{7}}{3x-1} = \frac{1}{7}\left[\frac{1}{x+2} + \frac{18}{3x-1}\right].$$

EXAMPLE 26.2

Resolve $\dfrac{2x^2+5x-1}{x^2-3x-10}$ into partial fractions.

SOLUTION

Given $\dfrac{2x^2+5x-1}{x^2-3x-10}$

We can clearly notice that the given fraction is an improper fraction. So dividing the numerator by the denominator we can express $\dfrac{2x^2+5x-1}{x^2-3x-10}$ as $2 + \dfrac{11x+19}{x^2-3x-10}$.

Let $\dfrac{11x+19}{(x+2)(x-5)} = \dfrac{A}{x+2} + \dfrac{B}{x-5}$

$$\Rightarrow \quad 11x + 19 = A(x-5) + B(x+2) \tag{1}$$

Put $x = 5$ in Eq. (1) $\quad \Rightarrow \quad 55 + 19 = A(0) + B(7)$

$\therefore B = \dfrac{74}{7}$.

Again put $x = -2$ in Eq. (1) $\quad \Rightarrow \quad -22 + 19 = A(-7) + B\,(0) \quad \Rightarrow \quad A = \dfrac{3}{7}$.

$$\therefore \frac{2x^2+5x-1}{x^2-3x-10} = 2 + \frac{3}{7(x+2)} + \frac{74}{7(x-5)}.$$

Method 2

When $g(x)$ contains some repeated linear factors and the remaining are non-repeated linear factors: Let $g(x) = (px + q)^n (a_{1x} + \alpha_1)(a_{2x} + \alpha_2) \ldots (a_n x + \alpha_n)$, then there exist fractions of the form $\frac{A_1}{Px+q}, \frac{A_2}{(Px+q)^2}, \cdots, \frac{A_n}{(Px+q)^n}$ corresponding to every repeated linear factor and fractions of the form $\frac{B_1}{a_1x+\alpha_1}, \frac{B_2}{a_2x+\alpha_2}, \cdots, \frac{B_n}{a_nx+\alpha_n}$ corresponding to every non-repeated linear factors where $A_1, A_2, \ldots, A_n$, and $B_1, B_2, \ldots, B_n$ are real numbers.

EXAMPLE 26.3

Resolve $\frac{2x^2-5x+7}{(x+1)^2(x+3)(2x+1)}$ into partial fractions.

SOLUTION

Let $\frac{2x^2-5x+7}{(x+1)^2(x+3)(2x+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3} + \frac{D}{2x+1}$

$\Rightarrow \quad 2x^2 - 5x + 7 = A(x + 1)(x + 3)(2x + 1) + B(x + 3)(2x + 1) + C(x + 1)^2 (2x + 1)$
$+ D(x + 1)^2 (x + 3)$ (1)

Substituting $x = -1$ in Eq. (1), we have

$2(-1)^2 - 5(-1) + 7 = A(0) + B(-1 + 3)(-2 + 1) + C(0) + D(0)\ 14 = -2B$

$B = -7$.

Substituting $x = -3$ in Eq. (1), we get

$40 = A(0) + B(0) + C(-20) + D(0)\ -20C = 40$

$C = -2$.

Substituting $x = -\frac{1}{2}$ in Eq. (1), we have

$10 = \frac{5}{8}D \quad \Rightarrow \quad D = 16.$

Substituting $x = 0$, we have

$7 = 3A + 3B + C + 3D$

Substituting the values of B, C, D in the above equation, we get $A = -6$.

$$\therefore \quad \frac{2x^2-5x+7}{(x+1)^2(x+3)(2x+1)} = \frac{-6}{x+1} + \frac{-7}{(x+1)^2} + \frac{-2}{x+3} + \frac{16}{2x+1}.$$

Method 3

When $g(x)$ contains non-repeated irreducible factors of the form $px^2 + qx + c$: Corresponding to every non-reducible non-repeated quadratic factors of $g(x)$, there exists a partial fraction of the form $\frac{Ax+B}{px^2+qx+c}$, where p, q, A, and B are real numbers.

EXAMPLE 26.4

Resolve $\dfrac{2x-5}{(x+2)(x^2-x+5)}$ into partial fractions.

SOLUTION

Let $\dfrac{2x-5}{(x+2)(x^2-x+5)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2-x+5}$

$\Rightarrow \quad 2x - 5 = A(x^2 - x + 5) + (Bx + C)\ (x + 2)$ (1)

Put $x = -2$ in Eq. (1),

$\Rightarrow \quad -9 = A\ (11) + 0$

$A = \dfrac{-9}{11}$.

Again put $x = 0$ in Eq. (1), we have

$-5 = 5A + 2C$

$-5 = 5 \times \dfrac{-9}{11} + 2C$

$\Rightarrow \quad C = \dfrac{-5}{11}$.

Comparing the coefficients of x^2 on both sides of Eq. (1), we have $A + B = 0$

$\Rightarrow \quad B = -A = \dfrac{9}{11}$.

$$\therefore \frac{2x-5}{(x+2)(x^2-x+5)} = \frac{-9}{11(x+2)} + \frac{\frac{9}{11}x - \frac{5}{11}}{x^2-x+5} = \frac{1}{11}\left[\frac{9x-5}{x^2-x+5} - \frac{9}{x+2}\right].$$

Method 4

When $g(x)$ contains repeated and non-repeated irreducible quadratic factors of the form $(px^2 + qx + r)^n$: Corresponding to every repeated irreducible quadratic factor of $g(x)$ there exist partial fractions of the form $\dfrac{p_1x+q_1}{(px^2+qx+r)}, \dfrac{p_2x+q^2}{(px^2+qx+r)}, \ldots, \dfrac{p_nx+q_n}{(px^2+qx+r)^n}$, where p_1, p_2, …, p_n and q_1, q_2, …, q_n are real numbers.

EXAMPLE 26.5

Resolve $\dfrac{2x+1}{(x+3)(x^2+1)^2}$ into partial fractions.

SOLUTION

Let $\dfrac{2x+1}{(x+3)(x^2+1)^2} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$.

$\Rightarrow \quad 2x + 1 = A(x^2 + 1)^2 + (Bx + C)(x + 3)(x^2 + 1) + (Dx + E)(x + 3)$ (1)

Put $x = -3$ in Eq. (1),

$$\Rightarrow \quad -5 = 100A \quad \Rightarrow \quad A = \frac{-1}{20}.$$

Comparing the coefficients of x^4 on either sides of Eq. (1), we have

$$0 = A + B \quad \Rightarrow \quad B = \frac{1}{20}.$$

Comparing the coefficients of x^3 on either sides of Eq. (1), we have

$$0 = 3B + C \quad \Rightarrow \quad C = \frac{-3}{20}.$$

Put $x = 0$ in Eq. (1), we have

$$\Rightarrow \quad 1 = A + 3C + 3E \quad \Rightarrow \quad 3E = 1 + \frac{1}{20} + \frac{9}{20}$$

$$\Rightarrow \quad E = \frac{1}{2}.$$

By putting $x = 1$ in Eq. (1), we have

$$3 = 4A + 8B + 8C + 4D + 4E$$

$$3 = \frac{-4}{20} + \frac{8}{20} + 8\left(\frac{-3}{20}\right) + 4D + 4\left(\frac{1}{2}\right)$$

$$3 + \frac{4}{20} - \frac{8}{20} + \frac{24}{20} - 2 = 4D$$

$$D = \frac{2}{4} = \frac{1}{2}.$$

$$\therefore \frac{2x+1}{(x+3)(x^2+1)^2} = \frac{\frac{-1}{20}}{x+3} + \frac{\frac{x}{20} - \frac{3}{20}}{x^2+1} + \frac{\frac{1}{2}x + \frac{1}{2}}{(x^2+1)^2}$$

$$= \frac{-1}{20(x+3)} + \frac{(x-3)}{20(x^2+1)} + \frac{(x+1)}{2(x^2+1)^2}.$$

EXAMPLE 26.6

Resolve $\dfrac{3x^2 - 3x - 11}{(x+3)(3x+4)^2}$ into partial fractions.

(a) $\dfrac{1}{x+3} + \dfrac{2}{3x+4} - \dfrac{1}{(3x+4)^2}$

(b) $\dfrac{1}{x+3} - \dfrac{2}{3x+4} - \dfrac{1}{(3x+4)^2}$

(c) $\dfrac{1}{x+3} + \dfrac{2}{3x+4} - \dfrac{1}{2(3x+4)^2}$

(d) None of these

SOLUTION

Let $\dfrac{3x^2 - 3x - 11}{(x+3)(3x+4)^2} = \dfrac{A}{x+3} + \dfrac{B}{3x+4} + \dfrac{C}{(3x+4)^2}$.

Consider

$3x^2 - 3x - 11 = A(3x+4)^2 + B(x+3)(3x+4) + C(x+3)$

$= A(9x^2 + 16 + 24x) + B(3x^2 + 13x + 12) + C(x + 3)$

Comparing coefficient of x^2, coefficient of x and constant terms

We get
$$9A + 3B = 3 \quad (1)$$
$$24A + 13B + C = -3 \quad (2)$$
$$16A + 12B + 3C = -11 \quad (3)$$

Solving Eqs. (1), (2) and (3), we get

$A = 1,\ B = -2,\ C = -1$

$$\therefore \quad \frac{3x^2 - 3x - 11}{(x+3)(3x+4)^2} = \frac{1}{x+3} - \frac{2}{3x+4} - \frac{1}{(3x+4)^2}.$$

EXAMPLE 26.7

If $\dfrac{x}{(x-1)(x^2+1)^2} = \dfrac{P}{x-1} + \dfrac{Qx+R}{x^2+1} + \dfrac{Sx+T}{(x^2+1)^2}$, then $P + Q - R - S + T =$ ______.

(a) $\dfrac{5}{4}$

(b) $\dfrac{3}{2}$

(c) $\dfrac{9}{7}$

(d) $\dfrac{8}{9}$

SOLUTION

$$\frac{x}{(x-1)(x^2+1)^2}$$
$$= \frac{P}{x-1} + \frac{Qx+R}{(x^2+1)} + \frac{Sx+T}{(x^2+1)^2}.$$

Consider

$x = P(x^2 + 1)^2 + (Qx + R)(x^2 + 1)(x - 1) + (sx + T)(x - 1)$

Put $x = 1$,

$$\Rightarrow \quad 4P = 1 \quad \Rightarrow \quad P = \frac{1}{4}.$$

Equating the coefficient of x^4, the coefficient of x^3, constant term and the coefficient of x^2 from both sides.

$$P + Q = 0, \text{ i.e., } P = -Q \quad \therefore \quad Q = -\frac{1}{4}.$$

$$-Q + R = 0, \text{ i.e., } Q = R, \quad \therefore \quad R = \frac{-1}{4}.$$

$$P - R - T = 0, \text{ i.e., } T = P - R$$

$$\therefore \quad T = \frac{1}{2}.$$

$$2P + Q - R + S = 0$$

That is, $S = -2P = \dfrac{-1}{2}$.

$$P + Q - R - S + T = \frac{1}{4} - \frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \frac{1}{2} = \frac{5}{4}.$$

EXAMPLE 26.8

If $\dfrac{9x-13}{x^2-2x-15} = \dfrac{A}{x+3} + \dfrac{B}{x-5}$, then $A + B$ is ________.

(a) 9

(b) 13

(c) 12

(d) 8

SOLUTION

Given, $\dfrac{9x-13}{x^2-2x-15} = \dfrac{A}{x+3} + \dfrac{B}{x-5}$.

Consider $9x - 13 = A(x - 5) + B(x + 3)$

$9x - 13 = Ax - 5A + Bx + 3B$

Comparing the coefficients of x, we get

$A + B = 9$.

EXAMPLE 26.9

Resolve $\dfrac{x}{(x-2)(x^2+3)^2}$ into partial fractions.

(a) $\dfrac{1}{49}\left[\dfrac{2}{x-2} - \dfrac{2x+4}{x^2+3} + \dfrac{12-2x}{(x^2+3)^2}\right]$

(b) $\dfrac{1}{49}\left[\dfrac{2}{x-2} - \dfrac{2x+4}{x^2+3} - \dfrac{12-2x}{(x^2+3)^2}\right]$

(c) $\dfrac{1}{49}\left[\dfrac{2}{x-2} + \dfrac{2x+4}{x^2+3} - \dfrac{12-2x}{(x^2+3)^2}\right]$

(d) $\dfrac{1}{49}\left[\dfrac{2}{x-2} - \dfrac{2x+4}{x^2+3} + \dfrac{21-14x}{(x^2+3)^2}\right]$

SOLUTION

Let $\dfrac{x}{(x-2)(x^2+3)^2} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+3} + \dfrac{Dx+E}{(x^2+3)^2}$

Consider

$x = A\,(x^2 + 3)^2 + (Bx + C)\,(x - 2)\,(x^2 + 3) + (Dx + E)(x - 2)$

Put $x = 2$, we get

$A = \dfrac{2}{49}$.

Comparing the coefficients of x^4, x^3, x^2 and constant terms, we get

$A + B = 0 \quad \Rightarrow \quad \therefore \quad B = -\dfrac{2}{49}$.

$-2B + C = 0 \quad \Rightarrow \quad C = -\dfrac{4}{49}$.

$6A + 3B - 2C + D = 0$

$\Rightarrow \quad \therefore \quad D = \dfrac{-14}{49}$

and $-6C - 2E + 9A = 0$

$E = \dfrac{21}{49}$.

$$\frac{x}{(x-2)(x^2+3)^2}$$

$$= \frac{\frac{2}{49}}{x-2} + \frac{\frac{-2}{49}x - \frac{4}{49}}{x^2+3} + \frac{-\frac{14}{49}x + \frac{21}{49}}{(x^2+3)^2}$$

$$= \frac{2}{49(x-2)} - \frac{2x+4}{49(x^2+3)} + \frac{21-14x}{49(x^2+3)^2}.$$

EXAMPLE 26.10

If $\frac{1}{(1-x)(1-2x)(1-3x)} = \frac{A}{(1-x)} + \frac{B}{(1-2x)} + \frac{C}{(1-3x)}$, then $A + B + C =$ ________.

(a) 0 **(b)** 1

(c) -1 **(d)** 2

SOLUTION

$$\frac{1}{(1-x)(1-2x)(1-3x)} = \frac{A}{1-x} + \frac{B}{1-2x} + \frac{C}{1-3x}$$

$$= \frac{A(1-2x)(1-3x) + B(1-3x)(1-x) + C(1-x)(1-2x)}{(1-x)(1-2x)(1-3x)}.$$

Consider

$A(1-2x)(1-3x) + B(1-3x)(1-x) + C(1-x)(1-2x) = 1$

Comparing constant terms, we get $A + B + C = 1$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Improper fraction is a fraction in which the degree of the numerator is ________ than or equal to the degree of denominator.
2. Improper fraction can always be written as the ________ of a polynomial and a proper fraction.
3. The partial fractions of $\dfrac{Ax+B}{x^4}$ has ________ terms.
4. A proper fraction is one in which the degree of numerator is ________ than the degree of denominator.
5. A proper fraction may be written as the sum of ________.
6. The number of terms of the partial fraction of $\dfrac{x^3}{(x-1)(2x-3)(x-4)}$ is ________.
7. The partial fraction of $\dfrac{2(x^2+1)}{x^3-1}$ is $\dfrac{1}{x-1}-\dfrac{f(x)}{g(x)}$. Then the degree of $f(x)$ is ________.
8. There are three terms of partial fractions of the proper fraction $\dfrac{f(x)}{x^3}$. Then the degree of the polynomial $f(x)$ would be ________.
9. If $\dfrac{x^2-2x-9}{(x^2+x+6)(x+1)}=\dfrac{2x-3}{x^2+x+6}+\dfrac{A}{x+1}$, then A is ________.
10. If the partial fractions of $\dfrac{3x^2+4}{(x-1)^3}$ is $\dfrac{3}{x-1}+\dfrac{A}{(x-1)^2}+\dfrac{7}{(x-1)^3}$, then $A=$ ________.
11. If $\dfrac{f(x)}{x^2-x+1}-\dfrac{1}{f(x)}$ is the partial fraction of $\dfrac{3x}{x^3+1}$, then $f(x)$ is ________.
12. If $\dfrac{ax^2+bx+c}{(x+1)(x^2-3x-4)}=\dfrac{f(x)}{x+1}+\dfrac{g(x)}{x+4}+\dfrac{h(x)}{x-1}$, then the degree of $f(x)$, $g(x)$ and $h(x)$ is ________.
13. If $\dfrac{2x^2+3x+4}{(x+2)^4}=\dfrac{A}{(x+2)}+\dfrac{2}{(x+2)^2}-\dfrac{5}{(x+2)^3}+\dfrac{6}{(x+2)^4}$, then $A=$ ________.
14. If $\dfrac{x^2}{(x-a)(x-b)}=1+\dfrac{a^2}{(a-b)(x-a)}+\dfrac{B}{(x-b)}$, then B is ________.
15. If $\dfrac{x^4}{(x+1)(x+2)}=f(x)+\dfrac{A}{x+1}+\dfrac{B}{x+2}$, then the degree of the polynomial $f(x)$ is ________.

Short Answer Type Questions

16. Resolve into partial fractions: $\dfrac{1}{x^2-a^2}$.
17. Resolve into partial fractions: $\dfrac{2x+5}{x^2+3x+2}$.
18. Resolve into partial fractions: $\dfrac{3x+4}{x^2+x-12}$.
19. Resolve into partial fractions: $\dfrac{2x^2+5x+8}{(x-2)^3}$.
20. Resolve into partial fractions: $\dfrac{x+1}{(x+2)(x^2+4)}$.
21. Find the value of A, B and C if $\dfrac{2x^2-x-10}{(x-2)^3}=\dfrac{A}{(x-2)}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x-2)^3}$.
22. If $\dfrac{x+5}{(x-1)^2}=\dfrac{A}{(x-1)}+\dfrac{B}{(x-1)^2}$, then find the values of A and B.
23. Find the partial fractions of $\dfrac{ax+b}{(x-1)^2}$, where a and b are constants.
24. Resolve into partial fractions: $\dfrac{2x^2-3x+3}{x^3-2x^2+x}$.

25. Resolve into partial fractions: $\dfrac{x^2+5x+6}{x^3-7x-6}$.

26. Resolve into partial fractions: $\dfrac{x^3}{(x+1)(x+2)}$.

27. Resolve into partial fractions: $\dfrac{x+1}{x^3-1}$.

28. Resolve into partial fractions: $\dfrac{2x^2+4}{x^4+5x^2+4}$.

29. Find the partial fractions of $\dfrac{2x+3}{x^4+x^2+1}$.

30. Resolve into partial fractions: $\dfrac{1}{x^4+x}$.

Essay Type Questions

31. Resolve into partial fractions: $\dfrac{3x^2+4x+5}{x^3+9x^2+26x+24}$.

32. Resolve into partial fractions: $\dfrac{x^2-x-1}{x^4+x^2+1}$.

33. Resolve into partial fractions: $\dfrac{3x+4}{x^3-2x-4}$.

34. Resolve into partial fractions: $\dfrac{x^2-3}{x^3-2x^2-x+2}$.

35. Resolve into partial fractions: $\dfrac{x^4+1}{(x-2)(x+2)}$.

CONCEPT APPLICATION

Level 1

1. Resolve $\dfrac{1}{x^2-9}$ into partial fractions.

(a) $\dfrac{1}{3(x-3)}-\dfrac{1}{3(x+3)}$

(b) $\dfrac{1}{2(x-3)}-\dfrac{3}{2(x+3)}$

(c) $\dfrac{1}{6(x-3)}-\dfrac{1}{6(x+3)}$

(d) $\dfrac{1}{6(x-3)}+\dfrac{1}{6(x+3)}$

2. Resolve $\dfrac{2x+3}{x^2-6x+5}$ into partial fractions.

(a) $\dfrac{1}{4}\left(\dfrac{13}{x-5}-\dfrac{5}{x-1}\right)$

(b) $\dfrac{5}{x-5}-\dfrac{13}{4(x-1)}$

(c) $\dfrac{13}{5(x-1)}-\dfrac{4}{5(x-5)}$

(d) $\dfrac{5}{(x-5)}-\dfrac{4}{(x-1)}$

3. Resolve $\dfrac{x^2+x+1}{(x-1)^3}$ into partial fractions.

(a) $\dfrac{1}{x-1}+\dfrac{1}{(x-1)^2}+\dfrac{1}{(x-1)^3}$

(b) $\dfrac{1}{(x-1)}+\dfrac{2}{(x-1)^2}+\dfrac{1}{(x-1)^3}$

(c) $\dfrac{1}{x-1}+\dfrac{3}{(x-1)^2}+\dfrac{3}{(x-1)^3}$

(d) $\dfrac{2}{x-1}+\dfrac{3}{(x-1)^2}+\dfrac{3}{(x-1)^3}$

4. Resolve $\dfrac{3x+2}{x^2+5x+6}$ into partial fractions.

(a) $\dfrac{7}{(x+2)}+\dfrac{4}{(x+3)}$

(b) $\dfrac{4}{(x+3)}+\dfrac{8}{(x+2)}$

(c) $\dfrac{4}{(x+3)}-\dfrac{7}{(x+2)}$

(d) $\dfrac{7}{(x+3)}-\dfrac{4}{(x+2)}$

5. Resolve $\dfrac{1}{x^2+x+42}$ into partial fractions.

(a) $\dfrac{1}{13(x-6)}+\dfrac{1}{13(x+7)}$

(b) $\dfrac{1}{(x-7)}+\dfrac{1}{(x-6)}$

(c) $\dfrac{1}{(x-7)}-\dfrac{1}{(x-6)}$

(d) $\dfrac{1}{13(x-6)}-\dfrac{1}{13(x+7)}$

6. Resolve $\dfrac{x^2+4x+6}{(x^2-1)(x+3)}$ into partial fractions

(a) $\dfrac{11}{8(x-1)}-\dfrac{3}{21(x+1)}+\dfrac{3}{8(x+3)}$

(b) $\dfrac{11}{8(x-1)}-\dfrac{3}{4(x+1)}+\dfrac{3}{8(x+3)}$

(c) $\dfrac{11}{4(x+1)}-\dfrac{3}{8(x+1)}+\dfrac{3}{8(x+3)}$

(d) $\dfrac{11}{8(x-1)}-\dfrac{3}{4(x+1)}+\dfrac{3}{8(x+3)}$

7. Resolve $\dfrac{x+1}{x(x-1)(x+3)}$ into partial fractions.

(a) $\dfrac{-1}{3x}+\dfrac{1}{2(x-1)}-\dfrac{1}{6(x+3)}$

(b) $\dfrac{1}{3x}+\dfrac{1}{2(x-1)}-\dfrac{1}{6(x+3)}$

(c) $\dfrac{1}{3x}-\dfrac{1}{2(x-1)}+\dfrac{1}{6(x+3)}$

(d) $\dfrac{1}{2x}-\dfrac{1}{3(x-1)}+\dfrac{1}{6(x+3)}$

8. Resolve $\dfrac{1}{x^2-(a+b)x+ab}$ into partial fractions.

(a) $\dfrac{1}{a+b}\left[\dfrac{1}{x-a}+\dfrac{1}{x-b}\right]$

(b) $\dfrac{1}{a+b}\left[\dfrac{1}{x-a}-\dfrac{1}{x-b}\right]$

(c) $\dfrac{1}{a-b}\left[\dfrac{1}{x-a}+\dfrac{1}{x-b}\right]$

(d) $\dfrac{1}{a-b}\left[\dfrac{1}{x-a}-\dfrac{1}{x-b}\right]$

9. Resolve $\dfrac{1}{x^2-5x+6}$ into partial fractions.

(a) $\dfrac{1}{x-3}-\dfrac{1}{x-2}$

(b) $\dfrac{1}{x-2}+\dfrac{1}{x-3}$

(c) $\dfrac{1}{x-3}+\dfrac{1}{x-2}$

(d) $\dfrac{2}{x-3}+\dfrac{2}{x-2}$

10. Resolve $\dfrac{x+2}{x^2-2x-15}$ into partial fractions.

(a) $\dfrac{7}{8(x-5)}+\dfrac{1}{8(x+3)}$

(b) $\dfrac{8}{7(x-5)}-+\dfrac{1}{7(x+3)}$

(c) $\dfrac{5}{8(x-5)}+\dfrac{1}{8(x+3)}$

(d) None of these

11. Resolve $\dfrac{3x^2+2x+4}{(x-1)(x^2-4)}$ into partial fractions.

(a) $\dfrac{5}{x-2}+\dfrac{1}{x-1}+\dfrac{1}{x+2}$

(b) $\dfrac{5}{x-2}+\dfrac{2}{x-1}+\dfrac{1}{x+2}$

(c) $\dfrac{5}{x-2}+\dfrac{2}{x-1}-\dfrac{1}{x+2}$

(d) $\dfrac{5}{x-2}-\dfrac{3}{(x-1)}+\dfrac{1}{(x+2)}$

12. Resolve $\dfrac{2x+1}{x^2-2x-8}$ into partial fractions.

(a) $\dfrac{3}{2(x-4)}-\dfrac{1}{2(x+2)}$

(b) $\dfrac{3}{2(x-4)}+\dfrac{1}{2(x+2)}$

(c) $\dfrac{1}{2(x-4)}-\dfrac{3}{2(x+2)}$

(d) None of these

13. Resolve $\frac{2x^2+3x+18}{(x-2)(x+2)^2}$ into partial fractions.

(a) $\frac{2}{(x+2)} - \frac{5}{(x-1)^2}$

(b) $\frac{2}{x-2} + \frac{1}{(x+2)} + \frac{3}{(x+2)^2}$

(c) $\frac{2}{x-2} - \frac{5}{(x+2)^2}$

(d) $\frac{2}{x-2} - \frac{1}{(x+2)} + \frac{3}{(x+2)^2}$

14. Resolve $\frac{x^2-x+1}{x^3-1}$ into partial fractions.

(a) $\frac{1}{3(x-1)} + \frac{2x-2}{3(x^2+x+1)}$

(b) $\frac{1}{3(x-1)} - \frac{2x+2}{3(x^2+x+1)}$

(c) $\frac{1}{2(x-1)} + \frac{x+2}{2(x^2+x+1)}$

(d) $\frac{1}{3(x-1)} - \frac{x-2}{3(x^2+x+1)}$

15. Resolve $\frac{x-1}{x^3-3x^2+2x}$ into partial fractions.

(a) $\frac{1}{(x-2)} - \frac{1}{2x}$

(b) $\frac{1}{2(x-2)} - \frac{1}{2x}$

(c) $\frac{1}{(x-2)} + \frac{1}{x-1} + \frac{1}{2x}$

(d) $\frac{1}{x-2} - \frac{1}{x-1} + \frac{1}{2x}$

Level 2

16. Resolve $\frac{x-b}{x^2+(a+b)x+ab}$ into partial fractions.

(a) $\frac{2a}{(a+b)(x+a)} - \frac{2b}{(a+b)(x+b)}$

(b) $\frac{a+b}{(a-b)(x+a)} + \frac{2b}{(a-b)(x+b)}$

(c) $\frac{a+b}{(a-b)(x+a)} - \frac{2b}{(a-b)(x+b)}$

(d) None of these

17. Resolve $\frac{x+1}{x^2-4}$ into partial fractions.

(a) $\frac{3}{2(x-2)} + \frac{1}{2(x+2)}$

(b) $\frac{3}{2(x-2)} - \frac{1}{2(x+2)}$

(c) $\frac{3}{4(x-2)} + \frac{1}{4(x+2)}$

(d) $\frac{3}{4(x-2)} - \frac{1}{4(x+2)}$

18. Resolve $\frac{x}{(x+1)(x-1)^2}$ into partial fractions.

(a) $\frac{1}{2(x-1)^2} + \frac{1}{4(x-1)} + \frac{1}{4(x+1)}$

(b) $\frac{-1}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$

(c) $\frac{-1}{4(x-1)} + \frac{1}{4(x+1)} + \frac{1}{2(x-1)^2}$

(d) None of these

19. Resolve $\frac{3x-5}{x^2+3x+2}$ into partial fractions.

(a) $\frac{7}{x+2} - \frac{5}{x+1}$

(b) $\frac{-8}{x+2} - \frac{11}{(x+1)}$

(c) $\frac{11}{x+2} - \frac{8}{x+1}$

(d) $\frac{7}{x+2} + \frac{5}{x+1}$

20. Resolve $\frac{3x+5}{x^2+8x-20}$ into partial fractions.

(a) $\frac{11}{24(x-2)}+\frac{25}{24(x+10)}$

(b) $\frac{11}{6(x-2)}+\frac{25}{6(x+10)}$

(c) $\frac{11}{12(x+2)}+\frac{25}{12(x-10)}$

(d) $\frac{11}{12(x-2)}+\frac{25}{12(x+10)}$

21. Resolve $\frac{ax+b^2}{x^2-(a+b)^2}$ into partial fractions.

(a) $\frac{a^2+ab+b^2}{2(a+b)(x-(a+b))}+\frac{a^2+ab-b^2}{2(a+b)(x+(a+b))}$

(b) $\frac{a^2+b^2}{2(a+b)(x-(a+b))}+\frac{a^2-b^2}{2(a+b)(x+(a+b))}$

(c) $\frac{a^2-b^2}{2(a+b)(x-(a-b))}+\frac{a^2+b^2}{2(a+b)(x+(a+b))}$

(d) None of these

22. Find the constants a, b, c and d respectively, if $\frac{1}{x^4-x}=\frac{a}{x}+\frac{b}{x-1}+\frac{cx+d}{x^2+x+1}$.

(a) $-1, \frac{1}{3}, \frac{-1}{2}, \frac{-5}{6}$

(b) $-1, \frac{1}{3}, \frac{-1}{2}, \frac{5}{6}$

(c) $1, \frac{-1}{3}, \frac{1}{2}, \frac{5}{6}$

(d) None of these

23. If $\frac{x^3}{(x-1)(x-2)}=Ax+B+\frac{C}{x-1}+\frac{D}{x-2}$, then A, B, C and D respectively are

(a) 1, 3, 1, 8

(b) 1, −1, 3, 8

(c) 1, 3, −1, 8

(d) −1, −3, 1, 8

24. Resolve $\frac{2x^2+3}{x^4+8x^2+15}$ into partial fractions.

(a) $\frac{3}{2(x^2+3)}+\frac{5}{2(x^2+5)}$

(b) $\frac{3}{2(x^2-3)}+\frac{5}{2(x^2+5)}$

(c) $\frac{-3}{2(x^2+3)}+\frac{7}{2(x^2+5)}$

(d) $\frac{7}{2(x^2+3)}-\frac{3}{2(x^2+5)}$

25. Resolve $\frac{3x+1}{(x-1)^2}$ into partial fractions.

(a) $\frac{4}{(x-1)}-\frac{3}{(x-1)^2}$

(b) $\frac{3}{(x-1)}-\frac{4}{(x-1)^2}$

(c) $\frac{3}{(x-1)^2}-\frac{3}{(x-1)^2}$

(d) $\frac{3}{(x-1)}+\frac{4}{(x-1)^2}$

26. If $\frac{3x^2+14x+10}{x^2(x+2)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$, then A, B and C respectively are

(a) $\frac{9}{2}, -5, \frac{3}{2}$

(b) $\frac{9}{2}, 5, \frac{-3}{2}$

(c) $\frac{9}{2}, \frac{3}{2}, -5$

(d) $\frac{-9}{2}, \frac{-3}{2}, 5$

27. If $\frac{2x-1}{(x+1)(x^2-1)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$, then A, B, C respectively are

(a) $\frac{1}{4}, \frac{-1}{4}, \frac{3}{2}$

(b) $\frac{1}{4}, \frac{1}{4}, \frac{3}{2}$

(c) $\frac{1}{4}, \frac{-5}{2}, \frac{5}{4}$

(d) $\frac{1}{4}, \frac{5}{2}, \frac{5}{4}$

28. Resolve $\dfrac{ax-b}{(x+1)^2}$ into partial fractions.

(a) $\dfrac{a}{x+1}+\dfrac{a+b}{(x+1)^2}$

(b) $\dfrac{a}{x+1}+\dfrac{a-b}{(x+1)^2}$

(c) $\dfrac{a}{x+1}-\dfrac{a-b}{(x+1)^2}$

(d) $\dfrac{a}{x+1}-\dfrac{a+b}{(x+1)^2}$

29. Resolve $\dfrac{4x^2+3x+2}{(x-4)^3}$ into partial fractions.

(a) $\dfrac{25}{4(x-4)}+\dfrac{59}{2(x-4)}+\dfrac{78}{(x-4)^3}$

(b) $\dfrac{25}{4(x-4)}-\dfrac{69}{2(x-4)^2}+\dfrac{78}{(x-4)^3}$

(c) $\dfrac{35}{4(x-4)}+\dfrac{69}{2(x-4)^2}+\dfrac{78}{(x-4)^3}$

(d) $\dfrac{4}{x-4}+\dfrac{35}{(x-4)^2}+\dfrac{78}{(x-4)^3}$

30. Resolve $\dfrac{1}{x^2-7x-18}$ into partial fractions.

(a) $\dfrac{1}{11}\left(\dfrac{1}{x-9}-\dfrac{1}{x+2}\right)$

(b) $\dfrac{1}{11}\left(\dfrac{1}{x-9}+\dfrac{1}{x+2}\right)$

(c) $\dfrac{1}{11}\left(\dfrac{1}{x+9}-\dfrac{1}{x-2}\right)$

(d) $\dfrac{1}{11}\left(\dfrac{1}{x+9}+\dfrac{1}{x+2}\right)$

31. Resolve $\dfrac{10}{x^2-25}$ into partial fractions.

(a) $\dfrac{1}{x-5}-\dfrac{1}{x+5}$

(b) $\dfrac{1}{x-5}+\dfrac{2}{x+5}$

(c) $\dfrac{1}{x+5}+\dfrac{1}{x-5}$

(d) None of these

32. Resolve $\dfrac{2x}{3(x-1)(x-3)}$ into partial fractions.

(a) $\dfrac{1}{x+3}-\dfrac{1}{3(x+1)}$

(b) $\dfrac{1}{x-3}-\dfrac{1}{x+1}$

(c) $\dfrac{1}{x-3}-\dfrac{1}{3(x-1)}$

(d) $\dfrac{1}{x-3}-\dfrac{2}{3(x-1)}$

33. If $\dfrac{x}{(x+2)(x+3)}=\dfrac{A}{x+3}+\dfrac{B}{x+2}$, then $A-B$ is = ______.

(a) 4

(b) 5

(c) −6

(d) 2

34. Resolve $\dfrac{5}{(x+2)(x+3)}$ into partial fractions.

(a) $5\left(\dfrac{1}{x+2}+\dfrac{1}{x+3}\right)$

(b) $5\left(\dfrac{1}{x+2}-\dfrac{1}{x+3}\right)$

(c) $10\left(\dfrac{1}{x+2}-\dfrac{1}{x+3}\right)$

(d) $10\left(\dfrac{1}{x+2}+\dfrac{1}{x+3}\right)$

35. Resolve $\dfrac{1}{x^2-7x+12}$ into partial fractions.

(a) $\dfrac{1}{x-4}-\dfrac{1}{x-3}$

(b) $\dfrac{1}{x-3}+\dfrac{1}{x-4}$

(c) $\dfrac{2}{x-4}-\dfrac{1}{x-3}$

(d) $\dfrac{1}{x+3}-\dfrac{1}{x+4}$

Level 3

36. Resolve $\dfrac{1}{x^4 - 3x^2 - 4}$ into partial fractions.

(a) $\dfrac{1}{5(x^2-4)} + \dfrac{1}{5(x^2+1)}$

(b) $\dfrac{1}{5(x^2-4)} - \dfrac{1}{5(x^2+1)}$

(c) $\dfrac{1}{4(x^2+4)} - \dfrac{1}{4(x^2+1)}$

(d) $\dfrac{1}{4(x^2+4)} + \dfrac{1}{4(x^2+1)}$

37. Resolve $\dfrac{4x+3}{x^3 - 7x - 6}$ into partial fractions.

(a) $\dfrac{1}{4(x+1)} + \dfrac{3}{4(x-3)} - \dfrac{1}{(x+2)}$

(b) $\dfrac{-1}{4(x+1)} + \dfrac{3}{4(x-3)} + \dfrac{1}{(x+2)}$

(c) $\dfrac{1}{4(x+1)} - \dfrac{3}{4(x-3)} + \dfrac{1}{(x+2)}$

(d) $\dfrac{1}{4(x+1)} - \dfrac{3}{4(x-3)} - \dfrac{1}{x+2}$

38. If $\dfrac{1}{(a^2-bx)(b^2-ax)} = \dfrac{A}{a^2-bx} + \dfrac{B}{b^2-ax}$, then the value of A and B respectively would be

(a) $\dfrac{b}{b^3-a^3}, \dfrac{a}{b^3-a^3}$

(b) $\dfrac{b}{b^3-a^3}, \dfrac{a}{a^3-b^3}$

(c) $\dfrac{b}{a^3-b^3}, \dfrac{a}{a^3-b^3}$

(d) $\dfrac{-b}{b^3-a^3}, \dfrac{-a}{a^3-b^3}$

39. Resolve $\dfrac{3x-5}{(x-1)^4}$ into partial fractions.

(a) $\dfrac{1}{(x-1)} + \dfrac{2}{(x-1)^2} - \dfrac{3}{(x-1)^3} + \dfrac{4}{(x-1)^4}$

(b) $\dfrac{3}{(x-1)^2} + \dfrac{2}{(x-1)^3} - \dfrac{1}{(x-1)^4}$

(c) $\dfrac{3}{(x-1)^3} + \dfrac{2}{(x-1)^4}$

(d) $\dfrac{3}{(x-1)^3} - \dfrac{2}{(x-1)^4}$

40. Resolve $\dfrac{x^2+x+1}{x^3+1}$ into partial fractions.

(a) $\dfrac{1}{3(x+1)} + \dfrac{x+1}{3(x^2-x+1)}$

(b) $\dfrac{-1}{3(x+1)} + \dfrac{2x+2}{3(x^2-x+1)}$

(c) $\dfrac{1}{3(x+1)} + \dfrac{2x+2}{3(x^2-x+1)}$

(d) None of these

41. Resolve $\dfrac{2x^2+8x+13}{(x+1)^4}$ into partial fractions.

(a) $\dfrac{2}{(x+1)^2} + \dfrac{5}{(x+1)^3} + \dfrac{7}{(x+1)^4}$

(b) $\dfrac{1}{(x+1)^2} + \dfrac{5}{(x+1)^3} + \dfrac{7}{(x+1)^4}$

(c) $\dfrac{2}{(x+2)^2} + \dfrac{4}{(x+1)^3} + \dfrac{7}{(x+1)^4}$

(d) $\dfrac{4}{(x+1)^2} + \dfrac{5}{(x+1)^3} + \dfrac{7}{(x+1)^4}$

42. Resolve $\dfrac{6x^2-14x+6}{x(x-1)(x-2)}$ into partial fractions.

(a) $\dfrac{2}{x} + \dfrac{3}{x-1} + \dfrac{1}{x-2}$

(b) $\dfrac{3}{x} + \dfrac{2}{x-1} + \dfrac{1}{x-2}$

(c) $\dfrac{1}{x} + \dfrac{2}{x-1} + \dfrac{3}{x-2}$

(d) $\dfrac{1}{x} + \dfrac{3}{x-1} + \dfrac{2}{x-2}$

43. Resolve $\dfrac{1}{(x-4)(x^2+3)}$ into partial fractions.

(a) $\dfrac{1}{19}\left[\dfrac{1}{x-4} + \dfrac{x+4}{x^2+3}\right]$

(b) $\frac{1}{19}\left[\frac{1}{x-4}+\frac{x+3}{x^2+3}\right]$

(c) $\frac{1}{19}\left[\frac{1}{x-4}+\frac{x+3}{x^2+3}\right]$

(d) $\frac{1}{19}\left[\frac{1}{x-4}-\frac{x+4}{x^2+3}\right]$

44. Resolve $\frac{3x^2+7}{x^4-3x^2+2}$ into partial fractions.

(a) $\frac{10}{x^2-2}+\frac{5}{x-1}-\frac{5}{x+1}$

(b) $\frac{13}{x^2-2}+\frac{5}{x+1}-\frac{5}{x-1}$

(c) $\frac{5}{x^2-2}-\frac{10}{x-1}+\frac{10}{x+1}$

(d) $\frac{5}{x-1}-\frac{5}{x+1}-\frac{13}{x^2-2}$

45. If $\frac{4x^2+5x+6}{x^2(x+3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}$, then $2A+3B+4C=$ ________.

(a) 10 (b) 20

(c) 30 (d) 40

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. greater
2. sum
3. two terms
4. smaller
5. partial fractions
6. four
7. zero or 1.
8. 2
9. -1
10. 6
11. $x+1$
12. zero
13. zero
14. $\dfrac{b^2}{b-a}$
15. two

Short Answer Type Questions

16. $\dfrac{1}{2a(x-a)}-\dfrac{1}{2a(x+a)}$
17. $\dfrac{3}{x+1}-\dfrac{1}{x+2}$
18. $\dfrac{8}{7(x+4)}+\dfrac{13}{7(x-3)}$
19. $\dfrac{2}{x-2}+\dfrac{13}{(x-2)^2}+\dfrac{26}{(x-2)^3}$
20. $\dfrac{-1}{8(x+2)}+\dfrac{\frac{1}{8}x+\frac{3}{4}}{x^2+4}$
21. $A=2$, $B=7$ and $C=-4$
22. $A=1$ and $B=6$
23. $\dfrac{a}{(x-1)}+\dfrac{a+b}{(x-1)^2}$
24. $\dfrac{3}{x}-\dfrac{1}{(x-1)}+\dfrac{2}{(x-1)^2}$
25. $\dfrac{3}{2(x-3)}-\dfrac{1}{2(x+1)}$
26. $\dfrac{9}{2(x+2)}-\dfrac{20}{x+3}+\dfrac{37}{2(x+4)}$
27. $\dfrac{2}{3(x-1)}-\dfrac{(2x+1)}{3(x^2+x+1)}$
28. $\dfrac{2}{3(x^2+1)}+\dfrac{4}{3(x^2+4)}$
29. $\dfrac{\frac{-3x}{2}+\frac{5}{2}}{x^2-x+1}+\dfrac{\frac{3}{2}x+\frac{1}{2}}{x^2+x+1}$
30. $\dfrac{1}{x}-\dfrac{1}{3(x+1)}+\dfrac{\frac{-2}{3}x+\frac{1}{3}}{x^2-x+1}$

Essay Type Questions

31. $x-3-\dfrac{1}{x+1}+\dfrac{8}{x+2}$.
32. $\dfrac{x-1}{(x^2-x+1)}-\dfrac{x}{(x^2+x+1)}$
33. $\dfrac{1}{(x-2)}-\dfrac{x+1}{(x^2+2x+2)}$
34. $\dfrac{1}{3(x-2)}+\dfrac{1}{(x-1)}-\dfrac{1}{3(x+1)}$
35. $x^2+4+\dfrac{17}{4(x-2)}-\dfrac{17}{4(x+2)}$

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (a)
11. (d)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (b)					

Level 2

16. (c)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (d)	**21.** (a)	**22.** (d)	**23.** (c)	**24.** (c)	**25.** (d)
26. (b)	**27.** (a)	**28.** (d)	**29.** (d)	**30.** (a)	**31.** (a)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (a)

Level 3

36. (b)	**37.** (a)	**38.** (b)	**39.** (d)	**40.** (c)	**41.** (c)	**42.** (b)	**43.** (d)	**44.** (b)	**45.** (b)
46. (b)	**47.** (a)	**48.** (a)	**49.** (d)	**50.** (b)					

CONCEPT APPLICATION

Level 1

1. $\frac{1}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3}$.

2. $\frac{2x+3}{x^2-6x+5} = \frac{A}{(x-5)} + \frac{B}{(x-1)}$.

3. $\frac{x^2+x+1}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$.

4. $\frac{3x+2}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3}$.

5. $\frac{1}{x^2+x-42} = \frac{A}{x+7} + \frac{B}{x-6}$.

6. $\frac{x^2+4x+6}{(x^2-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3}$.

7. $\frac{x+1}{x(x-1)(x+3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+3}$.

8. $\frac{1}{x^2-(a+b)x+ab} = \frac{A}{x-a} + \frac{B}{x-b}$.

9. $\frac{1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$.

10. $\frac{x+2}{x^2-2x-15} = \frac{A}{x-5} + \frac{B}{x+3}$.

11. $\frac{3x^2+2x+4}{(x-1)(x^2-4)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+2}$.

12. $\frac{2x+1}{x^2-2x-8} = \frac{A}{x-4} + \frac{B}{x+2}$.

13. $\frac{2x^2+3x+18}{(x-2)(x+2)^2} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$.

14. Split the denominator into factors and use the relevant method to find its partial fractions.

15. (i) $x^3 - 3x^2 + 2x = x(x-1)(x-2)$.
 (ii) $\frac{x-1}{(x^3-3x^2+2x)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$.

Level 2

16. $\frac{x-b}{x^2+(a+b)x+ab} = \frac{A}{x+a} + \frac{B}{x+b}$.

17. $\frac{x+1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$.

18. $\frac{x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$.

19. $\frac{3x-5}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$.

20. $\frac{3x+5}{x^2+8x-20} = \frac{A}{x+10} + \frac{B}{x-2}$.

21. $\frac{ax+b^2}{x^2-(a+b)^2} = \frac{A}{x-(a+b)} + \frac{B}{x+(a+b)}$.

22. (i) Simplify LHS and RHS.
 (ii) Put $x = 0$ and 1 and obtain the values of a and b.
 (iii) Compare the like terms and obtain the values of c and d.

23. (i) Simplify LHS and RHS by taking LCM.
 (ii) Compare the like terms and obtain the required values.

24. $\frac{2x^2+3}{x^4+8x^2+15} = \frac{Ax+B}{x^2+5} + \frac{Cx+D}{x^2+3}$.

25. $\frac{3x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$.

26. (i) Simplify LHS and RHS by taking LCM.
 (ii) Put $x = 0$ and -2 and obtain the values of A and B.
 (iii) Compare the like terms and obtain the value of C.

27. (i) Simplify LHS and RHS by taking LCM.
 (ii) Compare the like terms and obtain the required values.

28. $\frac{ax-b}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$.

29. $\frac{4x^2+3x+2}{(x-4)^3} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{(x-4)^3}$.

30. $\dfrac{1}{x^2-7x-18}=\dfrac{A}{x-9}+\dfrac{B}{x+2}$.

31. Let $\dfrac{10}{x^2-25}=\dfrac{A}{x-5}+\dfrac{B}{x+5}$

$$\frac{10}{x^2-25}=\frac{A(x+5)+B(x-5)}{(x-5)(x+5)}.$$

Consider $A(x+5)+B(x-5)=10$

Put $x=5$, $10A=10 \Rightarrow A=1$

Put $x=-5$, $-10B=10 \Rightarrow B=-1$

$$\therefore \frac{10}{x^2-5}=\frac{1}{x-5}-\frac{1}{x+5}.$$

32. Let $\dfrac{\frac{2x}{3}}{(x-1)(x-3)}=\dfrac{A}{(x-1)}+\dfrac{B}{(x-3)}$

$$\frac{\frac{2x}{3}}{(x-1)(x-3)}=\frac{A(x-3)+B(x-1)}{(x-1)(x-3)}.$$

Consider $A(x-3)+B(x-1)=\dfrac{2x}{3}$

Put $x=3$, $2B=2 \Rightarrow B=1$.

Put $x=1, -2A=\dfrac{2}{3} \Rightarrow A=-\dfrac{1}{3}$.

$$\therefore \frac{2x}{3(x-1)(x-3)}=\frac{1}{x-3}-\frac{1}{3(x-1)}.$$

33. Let $\dfrac{x}{(x+2)(x+3)}=\dfrac{A}{x+3}+\dfrac{B}{x+2}$

$$\frac{x}{(x+2)(x+3)}=\frac{A(x+2)+B(x+3)}{(x+2)(x+3)}.$$

Consider $A(x+2)+B(x+3)=x$

Put $x=-3$, $-A=-3 \Rightarrow A=3$.

Put $x=-2$, $B=-2$.

$\therefore A-B=3-(-2)=3+2=5$.

34. Let $\dfrac{5}{(x+2)(x+3)}=\dfrac{A}{(x+2)}+\dfrac{B}{(x+3)}$

$$\frac{5}{(x+2)(x+3)}=\frac{A(x+3)+B(x+2)}{(x+2)(x+3)}.$$

Consider $A(x+3)+B(x+2)=5$

If $x=-3$, $-B=5 \Rightarrow B=-5$.

If $x=-2$, $A=5$.

$$\therefore \frac{5}{(x+2)(x+3)}=\frac{5}{x+2}-\frac{5}{x+3}.$$

35. $\dfrac{1}{x^2-7x+12}=\dfrac{1}{(x-3)(x-4)}$

Let $\dfrac{1}{(x-3)(x-4)}=\dfrac{A}{x-3}+\dfrac{B}{x-4}$ (1)

$$\frac{1}{(x-3)(x-4)}=\frac{A(x-4)+B(x-3)}{(x-3)(x-4)}.$$

Consider $A(x-4)+B(x-3)=1$

Put $x=4$, we get $B=1$.

Put $x=3$, we get $-A=1$

$\Rightarrow A=-1$.

Substitute these values in Eq. (1), we get

$$\frac{1}{(x-3)(x-4)}=\frac{1}{x-4}-\frac{1}{x-3}.$$

Level 3

36. $\dfrac{1}{x^4-3x^2-4}=\dfrac{Ax+B}{x^2-4}+\dfrac{Cx+D}{x^2+1}$.

37. (i) Use $x^3-7x-6=(x+1)(x+2)(x-3)$.

(ii) $\dfrac{4x+3}{x^3-7x-6}=\dfrac{A}{x+1}+\dfrac{B}{x+2}+\dfrac{C}{x-3}$.

38. (i) Simplify LHS and RHS by taking LCM.

(ii) Compare the like terms and obtain the required values.

39. $\dfrac{3x-5}{(x-1)^4}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{(x-1)^3}+\dfrac{D}{(x-1)^4}$.

40. $\dfrac{x^2+x+1}{x^3+1}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2-x+1}$.

41. Given, $\dfrac{2x^2+8x+13}{(x+1)^4}$.

Let $y=x+1 \Rightarrow x=y-1$

$$= \frac{2(y-1)^2 + 8(y-1) + 13}{y^4}$$

$$= \frac{2y^2 + 2 - 4y + 8y - 8 + 13}{y^4}$$

$$= \frac{2y^2 + 4y + 7}{y^4} = \frac{2}{y^2} + \frac{4}{y^3} + \frac{7}{y^4}$$

$$= \frac{2}{(x+1)^2} + \frac{4}{(x+1)^3} + \frac{7}{(x+1)^4}.$$

42. Let $\frac{6x^2 - 14x + 6}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$

$$\frac{6x^2 - 14x + 6}{x(x-1)(x+2)}$$

$$= \frac{A(x-1)(x-2) + Bx(x-2) + Cx(x-1)}{x(x-1)(x-2)}$$

$6x^2 - 14x + 6 = A\ (x - 1)\ (x - 2) + Bx(x - 2) + C(x - 1)x$

Put $x = 1$,

$6 - 14 + 6 = B1(1 - 2) \quad \Rightarrow \quad -2 = -B$

$\Rightarrow \quad B = 2.$

Put $x = 2$, $6(2)^2 - 14(2) + 6 = C(2 - 1)(2)$

$2 = 2C$

$1 = C.$

Put $x = 0$, $6 = A\ (-1)\ (-2) \quad \Rightarrow \quad A = 3.$

$$\therefore \frac{6x^2 - 14x + 6}{x(x-1)(x-2)} = \frac{3}{x} + \frac{2}{x-1} + \frac{1}{x+2}.$$

43. $\frac{1}{(x-4)(x^2+3)} = \frac{A}{x-4} + \frac{Bx+C}{x^2+3}$

$$\frac{1}{(x-4)(x^2+3)}$$

$$= \frac{A(x^2+3) + (x-4)(Bx+C)}{(x-4)(x^2+3)}.$$

Consider $Ax^2 + 3A + Bx^2 + Cx - 4Bx - 4C = 1$

$(A + B)x^2 + (C - 4B)x + 3A - 4C = 1$

Comparing the like terms, we get

$A + B = 0,\ C - 4B = 0$ and $(3A - 4C) = 1$

Solving the above equations, we get

$$A = \frac{1}{19}, B = \frac{-1}{19}\ C = \frac{-4}{19}.$$

$$\therefore \quad \frac{1}{(x-4)(x^2+3)} = \frac{1}{19}\left[\frac{1}{x-4} - \frac{x+4}{x^2+3}\right].$$

44. Let $x^2 = p$, then $\frac{3x^2+7}{x^4 - 3x^2 + 2} = \frac{3p+7}{p^2 - 3p + 2}$.

Let $\frac{3p+7}{p^2-3p+2} = \frac{A}{p-1} + \frac{B}{p-2}$

$$\frac{3p+7}{(p-1)(p-2)} = \frac{A(p-2) + B(p-1)}{(p-1)(p-2)}.$$

Consider

$3p + 7 = A(p - 2) + B(p - 1)$

Put $p = 1$, $-A = 10 \quad \Rightarrow \quad A = -10.$

$p = 2$, $B = 13$

$$\frac{3p+7}{p^2-3p+2} = \frac{13}{p-2} - \frac{10}{p-1}.$$

But $p = x^2$,

$$\therefore \quad \frac{3x^2+7}{x^4 - 3p^2 + 2} = \frac{13}{x^2-2} - \frac{10}{x^2-1}.$$

$$= \frac{13}{x^2-2} - 5\left[\frac{1}{x-1} - \frac{1}{x+1}\right]$$

$$= \frac{13}{x^2-2} + \frac{5}{x+1} - \frac{5}{x-1}.$$

45. $\frac{4x^2+5x+6}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

$$\frac{4x^2+5x+6}{x^2(x+3)} = \frac{Ax(x+3) + B(x+3) + Cx^2}{x^2(x+3)}$$

Consider

$4x^2 + 5x + 6 = A\ x(x + 3) + B(x + 3) + cx^2$

Put $x = 0$, $6 = 3B \quad \Rightarrow \quad B = 2.$

Put $x = -3$, $27 = 9C \quad \Rightarrow \quad C = 3.$

Comparing the coefficient of x^2

$A + C = 4 \quad \Rightarrow \quad A = 1.$

$2A + 3B + 4C = 2 + 6 + 12 = 20.$