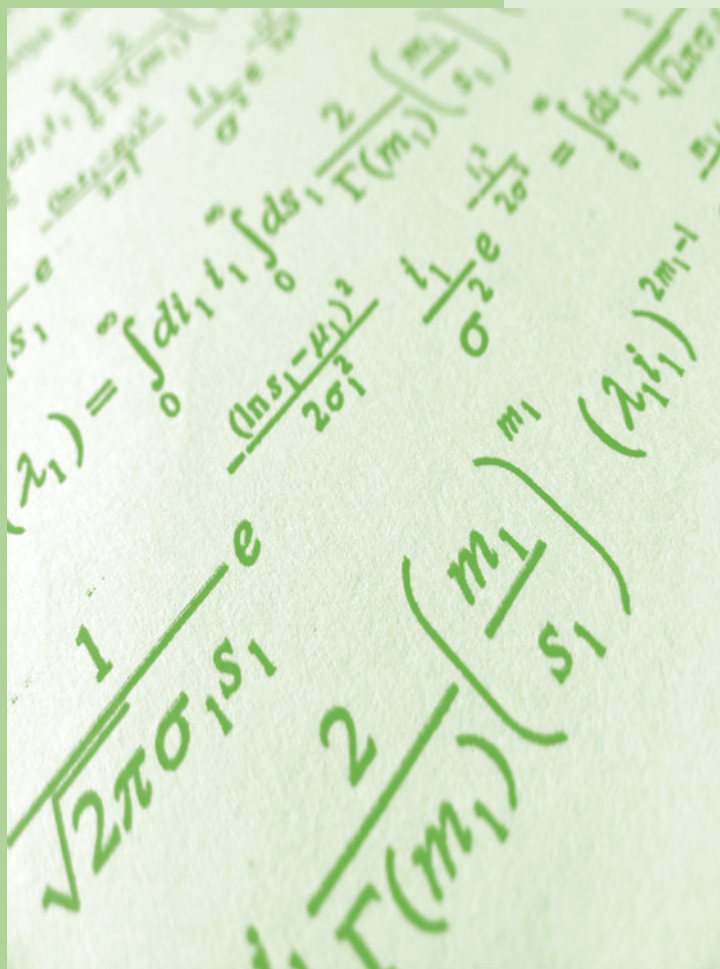


Chapter 6

Sets, Relations and Functions



REMEMBER

Before beginning this chapter, you should be able to:

- Know the use of sets, and some definitions of sets
- Apply fundamental operations on sets
- Understand the term Venn diagram
- Define terms such as relations, functions, etc.

KEY IDEAS

After completing this chapter, you would be able to:

- Learn representation of sets using different methods
- Apply advanced operations on sets such as union, intersection, etc.
- Represent sets using Venn diagrams and apply operations on it
- Define terms such as relations, functions, domain, range, etc.
- Present relations and functions by methods such as using arrow diagram, tree diagram, graphs, etc.

INTRODUCTION

In everyday life we come across different collections of objects. For example: A herd of sheep, a cluster of stars, a posse of policemen, etc. In mathematics, we call such collections as sets. The objects are referred to as the elements of the sets.

SET

A set is a well-defined collection of objects.

Let us understand what we mean by a well-defined collection of objects.

We say that a collection of objects is well-defined if there is some reason or rule by which we can say, whether a given object of the universe belongs to or does not belong to the collection.

We usually denote the sets by capital letters A , B , C or X , Y , Z , etc.

To understand the concept of a set, let us look at some examples.

Examples:

1. Let us consider the collection of odd natural numbers less than or equal to 15.

In this example, we can definitely say what the collection is. The collection comprises the numbers 1, 3, 5, 7, 9, 11, 13 and 15.

2. Let us consider the collection of students in a class who are good at painting. In this example, we cannot say precisely which students of the class belong to our collection. So, this collection is not well-defined.

Hence, the first collection is a set whereas the second collection is not a set. In the first example given above, the set of the odd natural numbers less than or equal to 15 can be represented as set $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$.

Elements of a Set

The objects in a set are called its elements or members.

If a is an element of a set A , then we say that a belongs to A and we write it as, $a \in A$.

If a is not an element of A , then we say that a does not belong to A and we write it as, $a \notin A$.

Some Sets of Numbers and Their Notations

N = Set of all natural numbers = $\{1, 2, 3, 4, 5, \dots\}$

W = Set of all whole numbers = $\{0, 1, 2, 3, 4, 5, \dots\}$

Z or I = Set of all integers = $\{0, \pm 1, \pm 2, \pm 3, \dots\}$

Q = Set of all rational numbers = $\left\{ \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0 \right\}$

Cardinal Number of a Set

The number of elements in a set A is called its cardinal number. It is denoted by $n(A)$. A set which has finite number of elements is a finite set and a set which has infinite number of elements is an infinite set.

Examples:

1. Set of English alphabets is a finite set.
2. Set of number of days in a month is a finite set.

3. The set of all even natural numbers is an infinite set.
4. Set of all the lines passing through a point is an infinite set.
5. The cardinal number of the set $X = \{a, c, c, a, b, a\}$ is $n(X) = 3$, as in sets distinct elements only are counted.
6. If $A = \{a, \{a, b\}, b, c, \{c, d\}\}$, then $n(A) = 5$.

Representation of Sets

We represent sets by the following methods:

Roster or List Method

In this method, a set is described by listing out all the elements in the set.

Examples:

1. Let W be the set of all letters in the word JANUARY.
Then we represent W as, $W = \{A, J, N, R, U, Y\}$.
2. Let M be the set of all multiples of 3 less than 20. Then we represent the set M as, $M = \{3, 6, 9, 12, 15, 18\}$.

Set-builder Method

In this method, a set is described by using a representative and stating the property or properties which the elements of the set satisfy, through the representative.

Examples:

1. Let D be the set of all days in a week. Then we represent D as, $D = \{x/x \text{ is a day in a week}\}$.
2. $N = \{x/10 < x < 20 \text{ and } x \in N\}$.

Types of Sets

Empty Set or Null Set or Void Set A set with no elements in it is called an empty set (or) void set (or) null set. It is denoted by $\{ \}$ or ϕ . (read as phi).

Note $n(\phi) = 0$.

Examples:

1. Set of all positive integers less than 1 is an empty set.
2. Set of all mango trees with apples.

Singleton Set

A set consisting of only one element is called a singleton set.

Examples:

1. The set of all vowels in the word MARCH is a singleton, as A is the only vowel in the word.
2. The set of whole numbers which are not natural numbers is a singleton, as 0 is the only whole number which is not a natural number.
3. The set of all SEVEN WONDERS in India is a singleton, as Tajmahal is the only wonder in the set.

Equivalent Sets

Two sets A and B are said to be equivalent if their cardinal numbers are equal. We write this symbolically as $A \sim B$ or $A \leftrightarrow B$.

Examples:

1. Sets, $X = \{2, 4, 6, 8\}$ and $Y = \{a, b, c, d\}$ are equivalent as $n(X) = n(Y) = 4$.
2. Sets, $X = \{\text{Dog, Cat, Rat}\}$ $Y = \{\Delta, \bigcirc, \square\}$ are equivalent.
3. Sets, $X = \{-1, -7, -5\}$ and $B = \{\text{Delhi, Hyderabad}\}$ are not equivalent, as $n(X) \neq n(B)$.

Note If the sets A and B are equivalent, we can establish a one-to-one correspondence between the two sets, i.e., we can pair up elements in A and B such that every element of A is paired with a distinct element of set B and every element of set B is paired with a distinct element of set A .

Equal Sets

Two sets A and B are said to be equal if they have the same elements.

Examples:

1. Sets, $A = \{a, e, i, o, u\}$ and $B = \{x/x \text{ is a vowel in the English alphabet}\}$ are equal sets.
2. Sets, $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$ are not equal sets.
3. Sets, $A = \{1, 2, 3, 4, \dots\}$ and $B = \{x/x \text{ is a natural number}\}$ are equal sets.

Note If A and B are equal sets, then they are equivalent but the converse need not be true.

Disjoint Sets

Two sets A and B are said to be disjoint, if they have no elements in common.

Examples:

1. Sets $X = \{3, 6, 9, 12\}$ and $Y = \{5, 10, 15, 20\}$ are disjoint as they have no element in common.
2. Sets $A = \{a, e, i, o, u\}$ and $B = \{e, i, j\}$ are not disjoint as they have common elements e and i .

Subset and Superset

Let A and B be two sets. If every element of set A is also an element of set B , then A is said to be a subset of B or B is said to be a superset of A . If A is a subset of B , then we write $A \subseteq B$ or $B \supseteq A$.

Examples:

1. Set $A = \{2, 4, 6, 8\}$ is a subset of set $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
2. Set of all primes except 2 is a subset of the set of all odd natural numbers.
3. Set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ is a superset of set $B = \{1, 3, 5, 7\}$.

Notes

1. Empty set is a subset of every set.
2. Every set is a subset of itself.
3. If a set A has n elements, then the number of subsets of A is 2^n .
4. If a set A has n elements, then the number of non-empty subsets of A is $2^n - 1$.

Proper Subset

If $A \subseteq B$ and $A \neq B$, then A is called a proper subset of B and is denoted by $A \subset B$. When $A \subset B$ then B is called a superset of A and is denoted as $B \supset A$, if $A \subset B$ then $n(A) < n(B)$ and if $B \supset A$ then $n(B) > n(A)$.

Power Set

The set of all subsets of a set A is called its power set. It is denoted by $P(A)$.

Example: Let $A = \{x, y, z\}$. Then the subsets of A are ϕ , $\{x\}$, $\{y\}$, $\{z\}$, $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x, y, z\}$.

So, $P(A) = \{\phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$.

We observe that the cardinality of $P(A)$ is $8 = 2^3$.

Notes

1. If a set A has n elements, then the number of subsets of A is 2^n , i.e., the cardinality of the power set is 2^n .
2. If a set A has n elements, then the number of proper subsets of A is $2^n - 1$.

Universal Set

A set which consists of all the sets under consideration or discussion is called the universal set. It is usually denoted by U or μ .

Example: Let $A = \{a, b, c\}$, $B = \{c, d, e\}$ and $C = \{a, e, f, g, h\}$. Then, the set $\{a, b, c, d, e, f, g, h\}$ can be taken as the universal set here.

$$\therefore \mu = \{a, b, c, d, e, f, g, h\}.$$

Complement of a Set

Let μ be the universal set and $A \subseteq \mu$. Then, the set of all those elements of μ which are not in set A is called the complement of the set A . It is denoted by A' or A^c . $A' = \{x/x \in \mu \text{ and } x \notin A\}$.

Examples:

1. Let $\mu = \{3, 6, 9, 12, 15, 18, 21, 24\}$ and $A = \{6, 12, 18, 24\}$. Then, $A' = \{3, 9, 15, 21\}$.
2. Let $\mu = \{x/x \text{ is a student and } x \in \text{Class X}\}$
and $B = \{x/x \text{ is a boy and } x \in \text{Class X}\}$.
Then, $B' = \{x/x \text{ is a girl and } x \in \text{Class X}\}$.

Notes

1. A and A' are disjoint sets.
2. $\mu' = \phi$ and $\phi' = \mu$.

EXAMPLE 6.1

If $X = \left\{ p : \text{where } p = \frac{(n+2)(2n^5 + 3n^4 + 4n^3 + 5n^2 + 6)}{n^2 + 2n} \text{ and } n, p \in \mathbb{Z}^+ \right\}$, then find the number of elements in the set X .

- (a) 2 (b) 3 (c) 4 (d) 6

HINTS

Divide each term by 'n' and find the positive factors of 6.

Operations on Sets

Union of Sets

Let A and B be two sets. Then, the union of A and B , denoted by $A \cup B$, is the set of all those elements which are either in A or in B or in both A and B . That is, $A \cup B = \{x/x \in A \text{ or } x \in B\}$.

Examples:

1. Let $A = \{-1, -3, -5, 0\}$ and $B = \{-1, 0, 3, 5\}$. Then, $A \cup B = \{-5, -3, -1, 0, 3, 5\}$.
2. Let $A = \{x/5 \leq 5x < 25 \text{ and } x \in \mathbb{N}\}$ and $B = \{x/5 \leq (10x) \leq 20 \text{ and } x \in \mathbb{N}\}$. Then, $A \cup B = \{x/5 \leq 5x \leq 20 \text{ and } x \in \mathbb{N}\}$.

Notes

1. If $A \subseteq B$, then $A \cup B = B$.
2. $A \cup \mu = \mu$ and $A \cup \phi = A$.
3. $A \cup A' = \mu$.

Intersection of Sets

Let A and B be two sets. Then the intersection of A and B , denoted by $A \cap B$, is the set of all those elements which are common to both A and B . That is, $A \cap B = \{x/x \in A \text{ and } x \in B\}$.

Examples:

1. Let $A = \{1, 2, 3, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 7\}$. Then, $A \cap B = \{1, 3, 5, 7\}$.
2. Let A be the set of all English alphabet and B be the set of all consonants, then, $A \cap B$ is the set of all consonants in the English alphabet.
3. Let E be the set of all even natural numbers and O be the set of all odd natural numbers. Then $E \cap O = \{ \}$ or ϕ .

Notes

1. If A and B are disjoint sets, then $A \cap B = \phi$.
2. If $A \subseteq B$, then $A \cap B = A$.
3. $A \cap \mu = A$ and $A \cap \phi = \phi$.
4. $A \cap A' = \phi$.

Difference of Sets

Let A and B be two sets. Then the difference $A - B$ is the set of all those elements which are in A but not in B .

That is, $A - B = \{x/x \in A \text{ and } x \notin B\}$.

Example: Let $A = \{3, 6, 9, 12, 15, 18\}$ and $B = \{2, 6, 8, 10, 14, 18\}$. $A - B = \{3, 9, 12, 15\}$ and $B - A = \{2, 8, 10, 14\}$.

Notes

1. $A - B \neq B - A$ unless $A = B$.
2. For any set A , $A' = \mu - A$.

Symmetric Difference of Sets

Let A and B be two sets. Then the symmetric difference of A and B , denoted by $A \Delta B$, is the set of all those elements which are either in A or in B but not in both. That is, $A \Delta B = \{x/x \in A \text{ and } x \notin B \text{ or } x \in B \text{ and } x \notin A\}$.

Note $A \Delta B = (A - B) \cup (B - A)$ (or) $A \Delta B = (A \cup B) - (A \cap B)$.

Example: Let $A = \{1, 2, 4, 6, 8, 10, 12\}$ and $B = \{3, 6, 12\}$. Then,
 $A \Delta B = (A - B) \cup (B - A)$
 $= \{1, 2, 4, 8, 10\} \cup \{3\}$
 $= \{1, 2, 3, 4, 8, 10\}$.

Some Results For any three sets A , B and C , we have the following results.

1. Commutative Law

- (i) $A \cup B = B \cup A$
- (ii) $A \cap B = B \cap A$
- (iii) $A \Delta B = B \Delta A$

2. Associative Law

- (i) $(A \cup B) \cup C = A \cup (B \cup C)$
- (ii) $(A \cap B) \cap C = A \cap (B \cap C)$
- (iii) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

3. Distributive Law

- (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. De Morgan's Law

- (i) $(A \cup B)' = A' \cap B'$
- (ii) $(A \cap B)' = A' \cup B'$
- (iii) $A - (B \cup C) = (A - B) \cap (A - C)$
- (iv) $A - (B \cap C) = (A - B) \cup (A - C)$

5. Identity Law

- (i) $A \cup \phi = \phi \cup A = A$
- (ii) $A \cap \mu = \mu \cap A = A$

6. Idempotent Law

(i) $A \cup A = A$

(ii) $A \cap A = A$

7. Complement Law

(i) $(A')' = A$

(ii) $A \cup A' = \mu$

(iii) $A \cap A' = \phi$

Dual of an Identity

An identity obtained by interchanging $\cup$ and $\cap$, and ϕ and μ in the given identity is called the dual of the identity.

Examples:

1. Consider the identity, $A \cup B = B \cup A$.

Dual of the identity is, $A \cap B = B \cap A$.

2. Consider the identity, $A \cup \mu = \mu$.

Dual of the identity is, $A \cap \phi = \phi$.

Venn Diagrams

We also represent sets pictorially by means of diagrams called Venn diagrams. In Venn diagrams, the universal set is usually represented by a rectangular region and its subsets by closed regions inside the rectangular region. The elements of the set are written in the closed regions and the elements which belong to the universal set are written in the rectangular region.

Example:

Let $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 4, 6, 7, 8\}$ and $B = \{2, 3, 4, 5, 9\}$.

We represent these sets in the form of Venn diagram as follows (see Fig. 6.1):

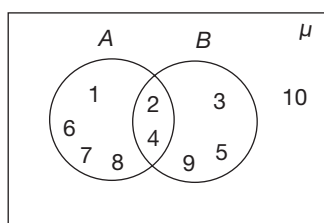


Figure 6.1

We can also represent the sets in Venn diagrams by shaded regions:

Examples:

1. Venn diagram of $A \cup B$, where A and B are two overlapping sets, is as shown in the Fig. 6.2.

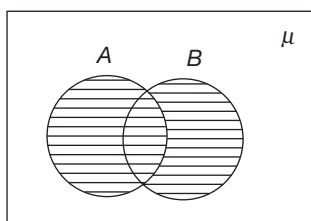


Figure 6.2

2. Let A and B be two overlapping sets. Then, the Venn diagram of $A \cap B$ is as shown in the Fig. 6.3.

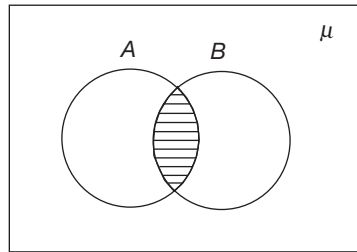


Figure 6.3

3. For a non-empty set A , Venn diagram of A' is as shown in the Fig. 6.4.

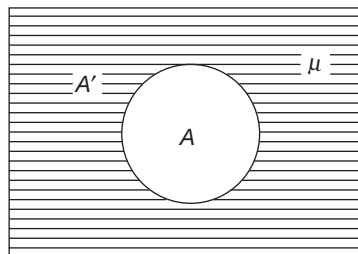


Figure 6.4

4. Let A and B be two overlapping sets. Then, the Venn diagram of $A - B$ is as shown in the Fig. 6.5.

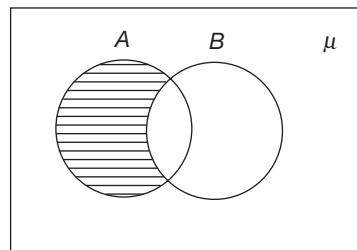


Figure 6.5

5. Let A and B be two sets such that $A \subseteq B$. We can represent this relation using Venn diagram as follows (see Fig. 6.6).

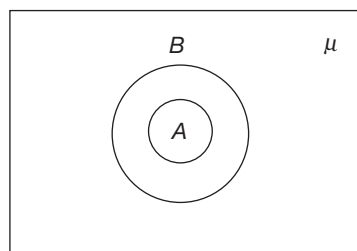


Figure 6.6

Some Formulae on the Cardinality of Sets

Let $A = \{1, 2, 3, 5, 6, 7\}$ and $B = \{3, 4, 5, 8, 10, 11\}$.

Then, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11\}$ and $A \cap B = \{3, 5\}$.

In terms of the cardinal numbers, $n(A) = 6$, $n(B) = 6$, $n(A \cap B) = 2$ and $n(A \cup B) = 10$.

So, $n(A) + n(B) - n(A \cap B) = 6 + 6 - 2 = 10 = n(A \cup B)$.

We have the following formulae:

For any three sets A , B and C

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
2. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

EXAMPLE 6.2

If $n(A) = 7$, $n(B) = 5$ and $n(A \cup B) = 10$, then find $n(A \cap B)$.

SOLUTION

Given, $n(A) = 7$, $n(B) = 5$ and $n(A \cup B) = 10$.

We know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$10 = 7 + 5 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 2.$$

EXAMPLE 6.3

If $n(A) = 8$ and $n(B) = 6$ and the sets A and B are disjoint, then find $n(A \cup B)$.

SOLUTION

Given, $n(A) = 8$, $n(B) = 6$.

A and B are disjoint

$$\Rightarrow A \cap B = \phi \Rightarrow n(A \cap B) = 0$$

$$\begin{aligned} \therefore n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 8 + 6 - 0 = 14. \end{aligned}$$

Note If A and B are two disjoint sets then, $n(A \cup B) = n(A) + n(B)$. The Venn diagram and the summary of it when three overlapping sets are given, are as follows (see Fig. 6.7):

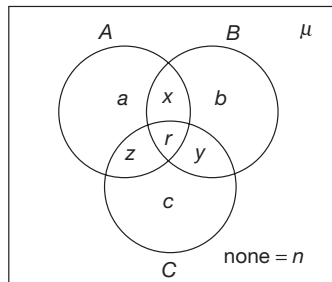


Figure 6.7

$$A = a + x + r + z.$$

Only A (or) exactly $A = a$.

$$B = b + x + r + y.$$

Only B (or) exactly $B = b$.

$$C = c + y + r + z.$$

Only C (or) exactly $C = c$.

$$A \cap B = x + r.$$

$A \cap B \cap C'$ (or) only $A \cap B = x$.

$$B \cap C = y + r.$$

$B \cap C \cap A'$ (or) only $B \cap C = y$; $C \cap A = z + r$; $C \cap A \cap B'$ (or) only $C \cap A = z$.

Only two sets (or) exactly two sets $= x + y + z$.

$A \cap B \cap C$ (or) all the three $= r$

Atleast one set = Exactly one + Exactly two + Exactly three

$$= a + b + c + x + y + z + r$$

$$= \text{Total} - \text{None}.$$

Atleast two sets = Exactly two + Exactly three

$$= x + y + z + r.$$

Atleast three sets = Exactly three $= r$.

Atmost two sets = Exactly two + Exactly one + Exactly zero

$$= x + y + z + a + b + c + n$$

$$= n(\mu) - r.$$

Atmost three sets = Exactly three + Exactly two + Exactly one + Exactly zero

$$= r + x + y + z + a + b + c + n$$

$$= n(\mu).$$

EXAMPLE 6.4

There are 40 students in a class. Each student speaks at least one of the languages Tamil, English and Hindi. Ten students speak exactly one of the languages. Twenty five students speak atmost two languages. How many students speak atleast two languages?

(a) 15

(b) 25

(c) 30

(d) 5

SOLUTION

Number of students who speak at least one of the languages

Tamil, English and Hindi $= 40$

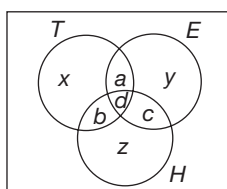
$$= (x + y + z + a + b + c + d)$$

Number of students who speak only one language $= 10$

$$= (x + y + z)$$

$\therefore$ No of students who speak at least two languages $= (a + b + c + d)$

$$= 40 - 10 = 30.$$



ORDERED PAIR

Let A be a non-empty set and $a, b \in A$. The elements a and b written in the form (a, b) is called an ordered pair. In the ordered pair (a, b) , a is called the first coordinate and b is called the second coordinate.

Note Two ordered pairs are said to be equal only when their first as well as the second coordinates are equal, i.e., $(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$.

So, $(1, 2) \neq (2, 1)$ and if $(a, 5) = (3, b) \Rightarrow a = 3$ and $b = 5$.

Cartesian Product of Sets

Let A and B be two non-empty sets. The Cartesian product of A and B , denoted by $A \times B$ is the set of all ordered pairs (a, b) , such that $a \in A$ and $b \in B$. That is, $A \times B = \{(a, b) / a \in A, b \in B\}$.

Notes

1. $A \times B \neq B \times A$, unless $A = B$.
2. For any two sets A and B , $n(A \times B) = n(B \times A)$.
3. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$.

$A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\}$ and

$B \times A = \{(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)\}$.

We observe that, $A \times B \neq B \times A$ and $n(A \times B) = 6 = n(B \times A)$.

Some Results on Cartesian Product

1. $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (or) $(A \cup B) \times (A \cup C)$
2. $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (or) $(A \cap B) \times (A \cap C)$
3. If $A \times B = \phi$, then either,
 - (i) $A = \phi$ or
 - (ii) $B = \phi$ or
 - (iii) both $A = \phi$ and $B = \phi$.

Cartesian product of sets can be represented in following ways:

1. Arrow diagram
2. Tree diagram and
3. Graphical representation

Representation of $A \times B$ using Arrow Diagram

EXAMPLE 6.5

If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$, then find $A \times B$.

SOLUTION

In order to find $A \times B$, represent the elements of A and B as shown in the Fig. 6.8.

Now draw an arrow from each element of A to each element of B , as shown in the figure.

Now represent all the elements related by arrows in ordered pairs in a set, which is the required $A \times B$.

That is, $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$.

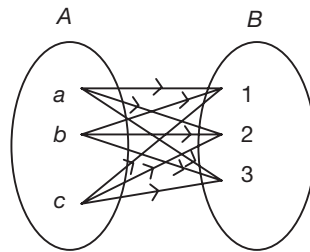


Figure 6.8

Representation of $A \times B$ Using a Tree Diagram

EXAMPLE 6.6

If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$, then find $A \times B$.

SOLUTION

To represent $A \times B$ using tree diagram, write all the elements of A vertically and then for each element of A , write all the elements of B and draw arrows as shown in the Fig. 6.9.

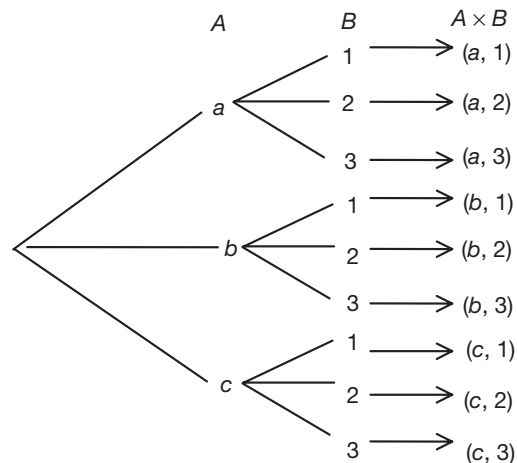


Figure 6.9

$\therefore A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$.

Graphical Representation of $A \times B$

EXAMPLE 6.7

If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then find $A \times B$.

SOLUTION

Consider the elements of A on the X -axis and the elements of B on the Y -axis and mark the points (see Fig. 6.10).

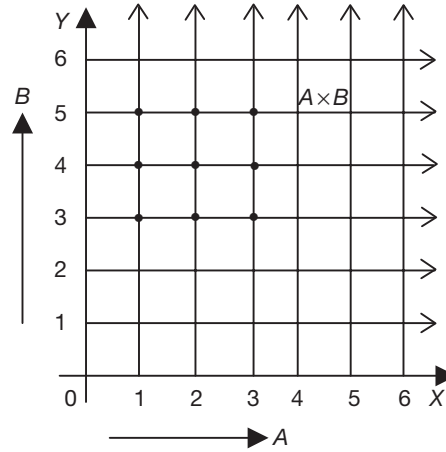


Figure 6.10

$$\therefore A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}.$$

RELATION

We come across certain relations in real life and also in basic geometry, like *is father of*, *is a student of*, *is parallel to*, *is similar to*, etc. We now define the mathematical form of the sum.

Definition

Let A and B be two non-empty sets and $R \subseteq A \times B$. R is called a relation from the set A to B . (Any subset of $A \times B$ is called a relation from A to B).

$\therefore$ A relation contains ordered pairs as elements. Hence, 'A relation is a set of ordered pairs'.

Examples:

1. Let $A = \{1, 2, 4\}$ and $B = \{2, 3\}$

Then, $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$

Let $R_1 = \{(1, 2), (1, 3), (2, 3)\}$

Clearly, $R_1 \subseteq A \times B$ and we also notice that, for every ordered pair $(a, b) \in R_1$, $a < b$.

So, R_1 is the relation *is less than* from A to B .

2. In the previous example, let $R_2 = \{(4, 2), (4, 3)\}$

Clearly, $R_2 \subseteq A \times B$ and we also notice that, for every ordered pair $(a, b) \in R_2$, $a > b$.

So, R_2 is the relation *is greater than* from A to B .

Notes

1. If $n(A) = p$ and $n(B) = q$, then the number of relations possible from A to B is 2^{pq} .
2. If $(x, y) \in R$, then we write $x R y$ and read as x is related to y .

Domain and Range of a Relation

Let A and B be two non-empty sets and R be a relation from A to B , we note that:

1. The set of first coordinates of all ordered pairs in R is called the domain of R .
2. The set of second coordinates of all ordered pairs in R is called the range of R .

Example:

Let $A = \{1, 2, 4\}$, $B = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (4, 3)\}$ be a relation from A to B .

Then, domain of $R = \{1, 2, 4\}$ and range of $R = \{1, 2, 3\}$.

Representation of Relations

We represent the relations by the following methods:

Roster Method (or) List Method

In this method we list all the ordered pairs that satisfy the rule or property given in the relation.

Example: Let $A = \{1, 2, 3\}$. If R is a relation on the set A having the property *is less than*, then the roster form of R is, $R = \{(1, 2), (1, 3), (2, 3)\}$.

Set-builder Method

In this method, a relation is described by using a representative and stating the property or properties, which the first and second coordinates of every ordered pair of the relation satisfy, through the representative.

Example: Let $A = \{1, 2, 3\}$. If R is a relation on the set A having the property *is greater than or equal to*, then the set-builder form of R is,

$$R = \{(x, y) / x, y \in A \text{ and } x \geq y\}$$

Arrow Diagram

In this method, a relation is described by drawing arrows between the elements which satisfy the property or properties given in the relation.

Example: Let $A = \{1, 2, 4\}$ and $B = \{2, 3\}$. Let R be a relation from A to B with the property *is less than*.

Then, the arrow diagram of R is as shown in the Fig. 6.11.

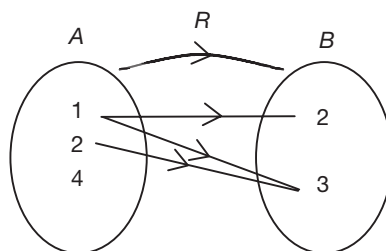


Figure 6.11

Inverse of a Relation

Let R be a relation from A to B . The inverse relation of R , denoted by R^{-1} , is defined as, $R^{-1} = \{(y, x) / (x, y) \in R\}$.

Example: Let $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (4, 3)\}$ be a relation from A to B , where $A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$. Then, $R^{-1} = \{(1, 1), (1, 2), (2, 1), (3, 2), (3, 4)\}$.

Notes

1. Domain of $R^{-1} = \text{Range of } R$.
2. Range of $R^{-1} = \text{Domain of } R$.
3. If R is a relation from A to B , then R^{-1} is a relation from B to A .
4. If $R \subseteq A \times A$, then R is called a binary relation or simply a relation on the set A .
5. For any relation R , $(R^{-1})^{-1} = R$.

Types of Relations

1. **One-One relation:** A relation $R: A \rightarrow B$ is said to be one-one relation if different elements of A are paired with different elements of B . That is, $x \neq y$ in $A \Rightarrow f(x) \neq f(y)$ in B .

Example: From the given Fig. 6.12, the relation $R = \{(1, 3), (2, 4), (3, 5)\}$.

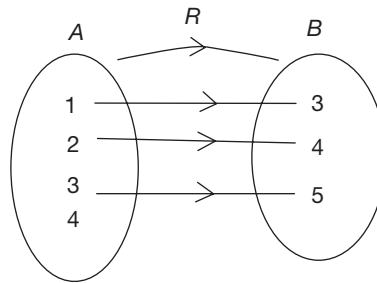


Figure 6.12

2. **One-many relation:** A relation $R: A \rightarrow B$ is said to be one-many relation if at least one element of A is paired with two or more elements of B .

Example: From the given Fig. 6.13, the relation $R = \{(1, 3), (1, 4), (3, 5)\}$.

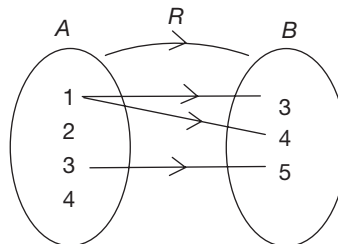


Figure 6.13

3. **Many-one relation:** A relation $R: A \rightarrow B$ is said to be many-one relation if two or more elements of A are paired with an element of B .

Example: From the given Fig. 6.14, the relation $R = \{(1, 2), (2, 4), (3, 2), (5, 3)\}$.

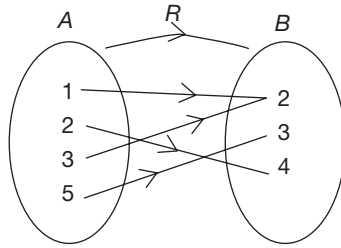


Figure 6.14

- 4. Many-many relation:** A relation $R: A \rightarrow B$ is said to be many-many relation if two or more elements of A are paired with two or more elements of B .

Example: From the given Fig. 6.15, the relation $R = \{(1, 3), (1, 4), (2, 3), (2, 5), (3, 4), (3, 5)\}$.

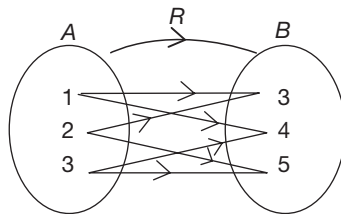


Figure 6.15

Properties of Relations

- 1. Reflexive relation:** A relation R on a set A is said to be reflexive if for every $x \in A$, $(x, x) \in R$.

Examples:

- Let $A = \{1, 2, 3\}$ then, $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (2, 3)\}$ is a reflexive on A .
- Let $A = \{1, 2, 3\}$ then, $R = \{(1, 1), (2, 3), (1, 2), (1, 3), (2, 2)\}$ is not a reflexive relation as $(3, 3) \notin R$.

Note Number of reflexive relations defined on set having n elements is 2^{n^2-n} .

- 2. Symmetric relation:** A relation R on a set A is said to be symmetric, if for every $(x, y) \in R$, $(y, x) \in R$.

Examples:

- Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$ is a symmetric relation on A .
- Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (1, 2), (3, 1), (2, 2), (3, 3)\}$ is not a symmetric relation as $(1, 2) \in R$ but $(2, 1) \notin R$.

Note A relation R on a set A is symmetric, iff $R = R^{-1}$, i.e., R is symmetric, iff $R = R^{-1}$.

- 3. Transitive relation:** A relation R on a set A is said to be transitive, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$. That is, R is said to be transitive, whenever $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

Examples:

- If $A = \{1, 2, 3\}$, then the relation on set A , $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (3, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$ is a transitive relation.

2. Let $A = \{1, 2, 3\}$, then the relation on set A , $R = \{(1, 2), (2, 2), (2, 1), (3, 3), (1, 3)\}$ is not a transitive relation as $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.
4. **Anti-symmetric relation:** A relation R on a set A is said to be anti-symmetric, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$. That is, R is said to be anti-symmetric if for $x \neq y$, $(x, y) \in R \Rightarrow (y, x) \notin R$.

Examples:

1. Let $A = \{1, 2, 3\}$, then $R = \{(1, 1), (1, 2), (3, 3)\}$ is an anti-symmetric relation.
2. Let $A = \{1, 2, 3\}$, then $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (3, 3)\}$ is not an anti-symmetric relation as $(1, 2) \in R$ and $(2, 1) \in R$ but $1 \neq 2$.
5. **Equivalence relation:** A relation R on a set A is said to be an equivalence relation if it is,
- (i) reflexive
 - (ii) symmetric and
 - (iii) transitive.

Note For any set A , $A \times A$ is an equivalence relation. In fact it is the largest equivalence relation.

Identity Relation

A relation R on a set A defined as, $R = \{(x, x)/x \in A\}$ is called an identity relation on A . It is denoted by I_A .

Example: Let $A = \{1, 2, 3\}$. Then, $R = \{(1, 1), (2, 2), (3, 3)\}$ is the identity relation on A .

Note Identity relation is the smallest equivalence relation on a set A .

EXAMPLE 6.8

If $A = \{a, b, c, x, y, z\}$, then the maximum number of elements in any relation on A is

- (a) 12 (b) 16 (c) 32 (d) 36

HINT

$A \times A$ has maximum number of elements.

FUNCTION

Let A and B be two non-empty sets. f is a relation from A to B . If f is such that

- for every $a \in A$, there is $b \in B$ such that $(a, b) \in f$ and
- no two ordered pairs in f have the same first element, then f is called a function from set A to set B and is denoted as $f: A \rightarrow B$.

Notes

- If $(a, b) \in f$, then $f(a) = b$ and b is called the f image of a . a is called the pre-image of b .
- If $f: A \rightarrow B$ is a function, then A is called the domain of f and B is called the co-domain of f .
- The set $f(A)$ which is all the images of elements of A under the mapping f is called the range of f .

Examples:

1.

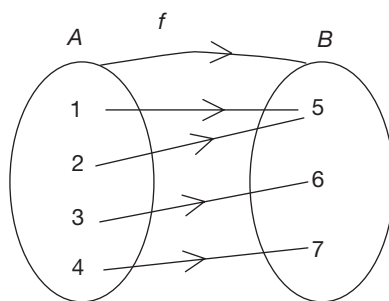


Figure 6.16

2.

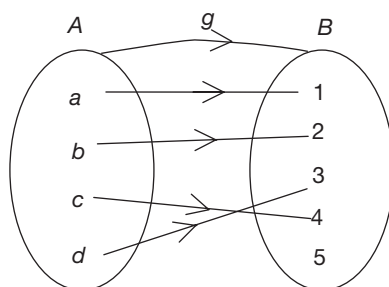


Figure 6.17

Let A and B be two non-empty sets. A relation f from A to B is said to be a function, if every element in A is associated with exactly one element in B . It is denoted by $f: A \rightarrow B$ (read as f is mapping from A to B). If $(a, b) \in f$, then b is called the f image of a and is written as $b = f(a)$. a is called the pre-image of b . Also in $f(a) = b$, a is called the independent variable and b is called the dependent variable.

Domain and Co-domain

If $f: A \rightarrow B$ is a function, then A is called domain and B is the co-domain of the function.

Range

If $f: A \rightarrow B$ is a function, then the set of all images of elements in its domain is called the range of f and is denoted by $f(A)$.

That is, $f(A) = \{f(a)/a \in A\}$.

Note Range of a function is always subset of its co-domain, i.e., $f(A) \subseteq B$.

If $f: A \rightarrow B$ is a function, and $n(A) = m$, $n(B) = p$, then the number of functions that can be defined from A to B is p^m .

Examples:

1. $A = \{1, 2, 3, 4\}$; $B = \{2, 3, 4, 5, 6\}$ are two sets. A relation f is defined as $f(x) = x + 2$. The relation $f: A \rightarrow B$ is a function and $f = \{(1, 3), (2, 4), (3, 5), (4, 6)\}$.
2. $A = \{-2, 2, 3, 4\}$, $B = \{4, 9, 16\}$ are two sets. The relation f , defined as $f(x) = x^2$, is a function from A to B , since every element in A is associated with exactly one element in B . The function $f = \{(-2, 4), (2, 4), (3, 9), (4, 16)\}$.

3. $A = \{-1, 1, 2, 5\}$, $B = \{1, 8, 125\}$ are two sets. The relation f defined as $f(x) = x^3$ is not a function from A to B . The relation $f = \{(1, 1), (2, 8), (5, 125)\}$. The number -1 is an element in A but it has no image in B .
4. $A = \{1, 2, 3, 4\}$, $B = \{x, y, z, t, u, v\}$ are two sets. A relation f is defined as follows:
 $f = \{(1, x), (2, y), (3, z), (4, t), (1, u)\}$. Here f is not a function, because the element 1 in A is associated with two elements x, u in B . Therefore, f is not a function.

Arrow Diagram

Functions can be represented by arrow diagrams.

Example: $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 4, 9, 16, 25, 36\}$ are two sets. A relation f is defined as $f(n) = n^2$. The arrow diagram of this function is shown in the Fig. 6.18.

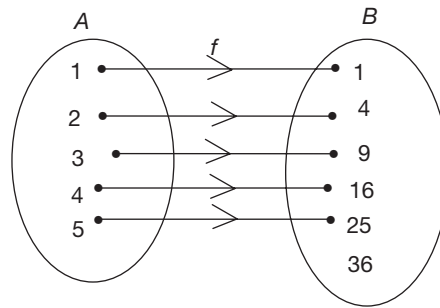


Figure 6.18

Every element in A is associated with exactly one element in B . So, $f: A \rightarrow B$ is a function.

Example:

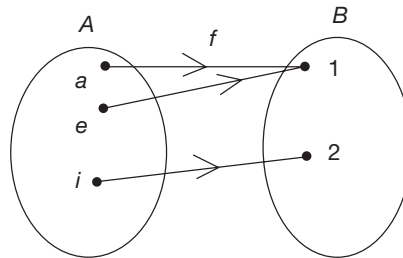


Figure 6.19

Every element in A is associated with exactly one element in B (see Fig. 6.19). So, $f: A \rightarrow B$ is a function.

Example:

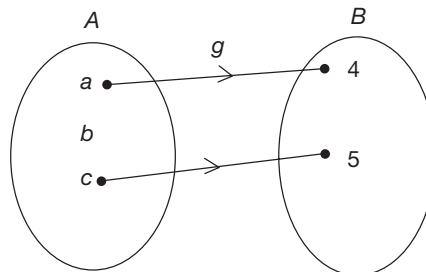


Figure 6.20

b is an element in A and it is not associated with any element in B (see Fig. 6.20). So, $g: A \rightarrow B$ is not a function.

Example:

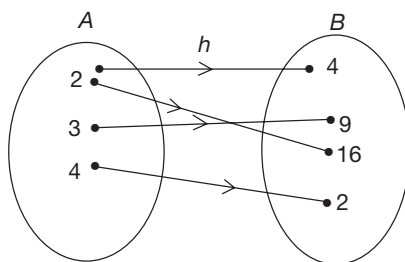


Figure 6.21

2 is an element in A . It is associated with two different elements in B (see Fig. 6.21). That is, 2 has two different images. So, $h: A \rightarrow B$ is not a function.

Difference Between Relations and Functions

Every function is a relation but every relation need not be a function. A relation f from A to B is called a function, if

1. $\text{Dom}(f) = A$,
2. no two different ordered pairs in f have the same first coordinate.

Example:

Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d, e\}$.

Some relations f, g, h are defined as follows:

$$f = \{(1, a), (2, b), (3, c), (4, d)\}$$

$$g = \{(1, a), (2, b), (3, c)\}$$

$$h = \{(1, a), (1, b), (2, c), (3, d), (4, e)\}.$$

In the relation f the domain of f is A and all first coordinates are different. So f is a function. In the relation g the domain of g is not A . So g is not a function. In the relation h the domain of h is A , but two of the first coordinates are equal, i.e., 1 has two different images. So h is not a function.

EXAMPLE 6.9

Find the domain of the function $f(x) = \frac{1}{x} + \frac{1}{\log(2-x)}$.

(a) $x > 2$

(b) $x \in \mathbb{R} - \{2\}$

(c) $x < 2, x \neq 0, x \neq 1$

(d) $x < 2, x \neq 0.$

HINTS

$\frac{1}{x}$ is not defined for $x = 0$ and logarithmic function takes only positive values.

Types of Functions

One-one Function or Injection

Let $f: A \rightarrow B$ be a function. If different elements in A are assigned to different elements in B , then the function $f: A \rightarrow B$ is called a one-one function or an injection.

That is, $a_1, a_2 \in A$ and $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$ then $f: A \rightarrow B$ is a one-one function (or) if $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$, then $f: A \rightarrow B$ is a one-one function.

Example:

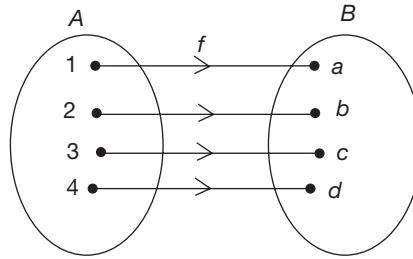


Figure 6.22

Different elements in A are assigned to different elements in B . $f: A \rightarrow B$ is a one-one function (see Fig. 6.22).

Example:

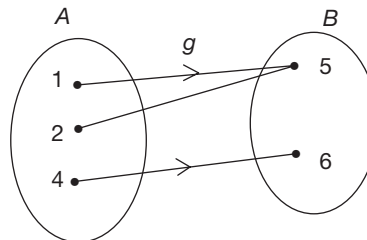


Figure 6.23

1 and 2 are two different elements in A , but they are assigned to the same element in B . So $g: A \rightarrow B$ is not a one-one function (see Fig. 6.23).

Note A and B are finite sets and $f: A \rightarrow B$ is one-one. Then $n(A) \leq n(B)$.

Many to One Function

If the function $f: A \rightarrow B$ is not one-one, then it is called a many to one function; i.e., two or more elements in A are assigned to the same element in B .

Example:

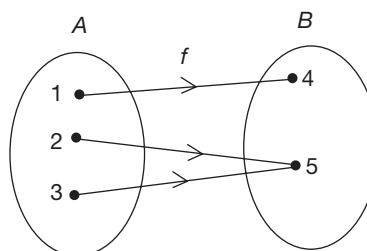


Figure 6.24

2, 3 are different elements in A , and they are assigned to the same element, i.e., 5 in B . So, $f: A \rightarrow B$ is a many to one function (see Fig. 6.24).

Onto Function or Surjection

$f: A \rightarrow B$ is said to be an onto function, if every element in B is the image of at least one element in A . That is, for every $b \in B$, there exists at least one element $a \in A$, such that $f(a) = b$.

Note If $f: A \rightarrow B$ is an onto function, then the co-domain of f must be equal to the range of f , i.e., $f(A) = B$.

Example:

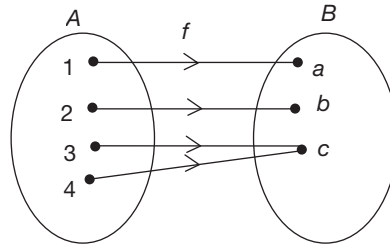


Figure 6.25

Range = $\{a, b, c\}$

Co-domain = $\{a, b, c\}$

Range = Co-domain

In the Fig. 6.25, every element in B is the image of at least one element in A . Therefore, it is an onto function.

Example: In the Fig. 6.26, d is an element in B , but it is not the image of any element in A . Therefore, it is not an onto function.

Note A and B are finite sets and $f: A \rightarrow B$ is onto. Then, $n(B) \leq n(A)$.

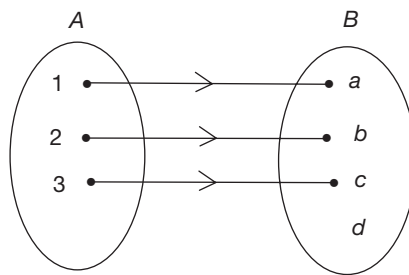


Figure 6.26

Into Function

If a function is not onto, then it is an into function, i.e., at least one element in B is not the image of any element in A , or the range is a subset of the co-domain.

Bijjective Function

If the function $f: A \rightarrow B$ is both one-one and onto then it is called a bijective function.

Example:

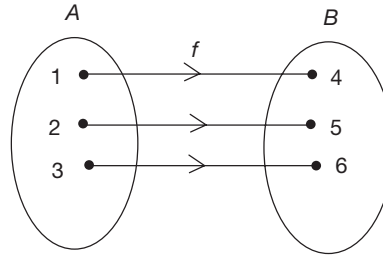


Figure 6.27

In the Fig. 6.27, $f: A \rightarrow B$ is both one-one and onto. So f is a bijective function.

Example:

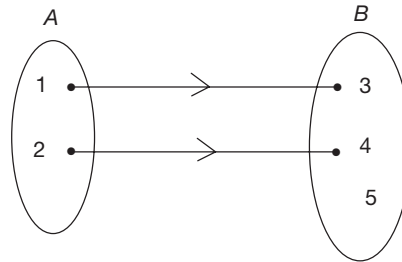


Figure 6.28

In the Fig. 6.28, it is only one-one but not onto, so it is not bijective.

Note A and B are finite sets and $f: A \rightarrow B$ is one-one and onto then $n(A) = n(B)$.

Inverse of a Function

If $f: A \rightarrow B$ is a function, then the set of ordered pairs obtained by interchanging the first and second coordinates of each ordered pair in f is called the inverse of f and is denoted by f^{-1} . That is, $f: A \rightarrow B$ is a function then its inverse is $f^{-1}: B \rightarrow A$.

Examples:

1. $f = \{(1, 2), (2, 3), (3, 4)\}$
 $f^{-1} = \{(2, 1), (3, 2), (4, 3)\}.$
2. $g = \{(1, 4), (2, 4), (3, 5), (4, 6)\}$
 $g^{-1} = \{(4, 1), (4, 2), (5, 3), (6, 4)\}.$
3. $A = \{1, 2, 3, 4\}$, $B = \{x, y, z, a, b\}$ are two sets, the function $h: A \rightarrow B$ is defined as follows:
 $h = \{(1, a), (2, b), (3, x), (4, z)\}$
 $h^{-1} = \{(a, 1), (b, 2), (x, 3), (z, 4)\}.$

In the above examples only f^{-1} is a function, but g^{-1} , h^{-1} are not functions.

$\therefore f: A \rightarrow B$ a function, then $f^{-1}: B \rightarrow A$ need not be a function.

Inverse Function If $f: A \rightarrow B$ is a bijective function, then $f^{-1}: B \rightarrow A$ is also a function. That is, the inverse of a function is also a function, only when the given function is bijective.

Example:

$A = \{1, 2, 3, 4, 5\}$; $B = \{a, e, i, p, u\}$. A function f is defined as follows:

$f = \{(1, a), (2, e), (3, i), (4, u), (5, p)\}$. Clearly, f is a bijective function.

Now $f^{-1} = \{(a, 1), (e, 2), (i, 3), (u, 4), (p, 5)\}$. Clearly f^{-1} is also a function and it is also bijective.

EXAMPLE 6.10

If $f(x) = 2x + 3$ and $g(x) = 3x - 1$, then find $f^{-1} \circ g^{-1}$.

(a) $\frac{x+8}{6}$ (b) $\frac{x-8}{6}$ (c) $\frac{8-x}{6}$ (d) $\frac{x-8}{2}$

HINTS

(i) Find $g \circ f(x)$.

(ii) We know that, $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Identity Function

$f: A \rightarrow A$ is said to be an identity function on A , if $f(a) = a$ for every $a \in A$, it is denoted by IA .

Example: $A = \{1, 2, 3, 4\}$. The identity function on A is $IA = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.

Notes

1. Identity function is always bijective function.
2. The inverse of the identity function is the identity function itself.

Constant Function

A function $f: A \rightarrow B$ is a constant function if there is an element $b \in B$, such that $f(a) = b$, for all $a \in A$.

That is, in a constant function the range has only one element.

Example: $A = \{1, 2, 3, 4\}$; $B = \{a, e, i, u\}$ are two sets and a function from A to B is defined as follows:

$f = \{(1, a), (2, a), (3, a), (4, a)\}$, therefore f is a constant function.

Note The range of a constant function is a singleton.

Equal Functions

Two functions f and g , defined on the same domain D are said to be equal, if $f(x) = g(x)$ for all $x \in D$.

Example:

Let $f: R - \{2\} \rightarrow R$ be defined by $f(x) = x + 2$; and $g: R - \{2\} \rightarrow R$ be defined by $g(x) = \frac{x^2 - 4}{x - 2}$.

$\therefore f$ and g have the same domain $R - \{2\}$.

Given $f(x) = x + 2$;

$$g(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2} = x + 2.$$

$\therefore f(x) = g(x)$ for all $x \in R - \{2\}$.

$\Rightarrow f$ and g are equal functions.

Composite Function (or) Product Function

Let f and g be two functions such that $f: A \rightarrow B$ and $g: B \rightarrow C$. Let a be an arbitrary element in A .

Since f is a function from A to B , there exists an element $b \in B$, such that $f(a) = b$.

Since g is a function from B to C , there exists an element $c \in C$, such that $g(b) = c$.

$$\therefore g(f(a)) = c \Rightarrow g \circ f(a) = c.$$

$\therefore g \circ f$ is a function from A to C .

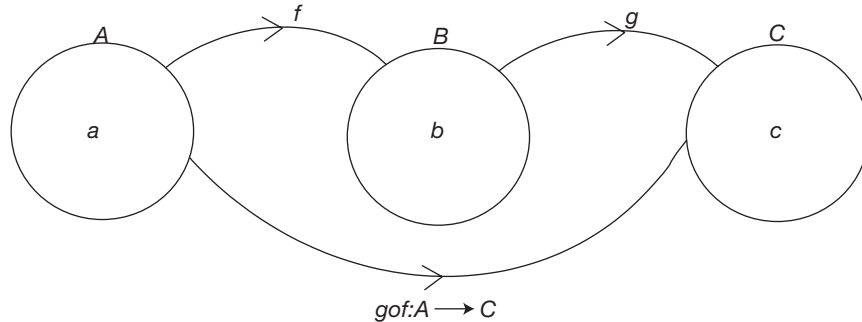


Figure 6.29

If $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions, then the function $g[f(x)] = g \circ f$ from A to C , denoted by $g \circ f$ is called the composite function of f and g .

In the composite function $g \circ f$,

1. the co-domain of f is the domain of g .
2. the domain of $g \circ f$ is the domain of f , the co-domain of $g \circ f$ is the co-domain of g .
3. composite function does not satisfy commutative property, i.e., $g \circ f \neq f \circ g$.
4. if $f: A \rightarrow B$; $g: B \rightarrow C$; $h: C \rightarrow D$ are three functions, $h \circ (g \circ f) = (h \circ g) \circ f$, i.e., the composite function satisfies associative property.

Real Function

If $f: A \rightarrow B$ such that $A \subseteq \mathbb{R}$, then f is said to be a real variable function.

If $f: A \rightarrow B$ such that $B \subseteq \mathbb{R}$, then f is said to be a real valued function.

If $f: A \rightarrow B$, and A and B are both subsets of the set of real numbers ($\mathbb{R}$), then f is called a real function.

Even and Odd Functions

1. If $f(-x) = f(x)$, then the function $f(x)$ is called an even function.

Example:

$$f(x) = x^2$$

$$f(-x) = x^2$$

$$\text{Here, } f(-x) = f(x)$$

$\therefore f(x) = x^2$ is an even function.

2. If $f(-x) = -f(x)$, then the function $f(x)$ is called an odd function.

Example:

$$f(x) = x^3$$

$$f(-x) = -x^3$$

$$\text{Here, } f(x) = -f(-x)$$

$f(x) = x^3$ is an odd function.

Notes

1. There are functions which are neither even nor odd.

Example: $2x + 3$, a^x , etc.

2. If $f(x)$ is a real function, then $\frac{f(x) + f(-x)}{2}$ is always even and $\frac{f(x) - f(-x)}{2}$ is always odd.
3. Product of two even functions is even.
4. Product of two odd functions is even.
5. Product of an even function and an odd function is odd.
6. $f(x) = 0$ is both even and odd.

Domain and Range of Some Functions are Listed below

Function	Domain	Range
$\frac{1}{x}$	$R - \{0\}$	R
$\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$ x $	R	$[0, \infty)$
$\log x$	$(0, \infty)$	R
$a^x (a > 0)$	R	$(0, \infty)$

Graphs of Functions

A graph does not represent a function, if there exists a vertical line which meets the graph in two or more points, i.e., a vertical line meets the graph at only one point, then the graph represents a function.

Example:

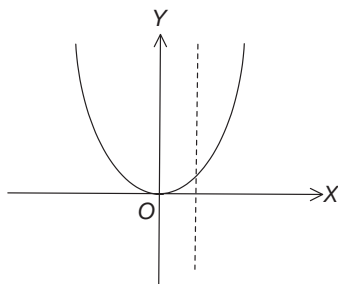


Figure 6.30

In the Fig. 6.30, the vertical dotted line meets the graph at only one point. So, the graph represents a function.

Example:

In the Fig. 6.31, the vertical dotted line meets the graph at two points. So, it is not a function.

Notes

1. The Y-axis does not represent a function.
2. The X-axis represents a many-one function.

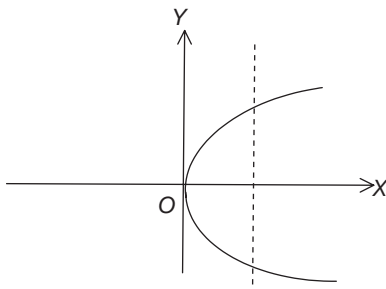


Figure 6.31

Zeroes of a Function

If $f: A \rightarrow R$ ($A \subset R$), then the points $k \in A$, such that $f(k) = 0$ are called the zeroes of the function f .

If k is a zero of $f: A \rightarrow R$, then $(k, 0)$ is the corresponding point on the graph of f . k is called the x -intercept of the graph.

Example:

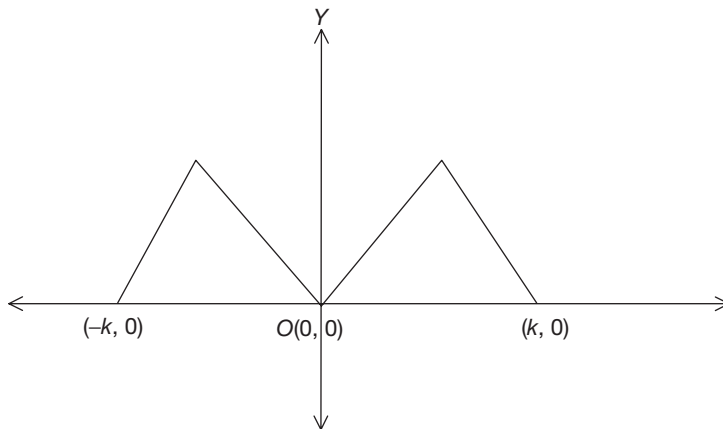


Figure 6.32

Graph of the Fig. 6.32, represents a function, zeroes of the graph are $-k$, 0 and k .

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If A is a non-empty set, then $((A')')'$ is _____.
2. The number of non-empty proper subsets of a set A is 0, then $n(A) =$ _____.
3. The number of non-empty proper subsets of a set containing 7 elements is _____.
4. If A and B are disjoint sets, then $A \Delta B =$ _____.
5. $n(A \cup B \cup C) =$ _____.
6. If A and B are disjoint, then $(A \cap B)' =$ _____.
7. In the given figure, if A and B are any two non-empty sets and μ is an universal set, then the shaded region represents _____.

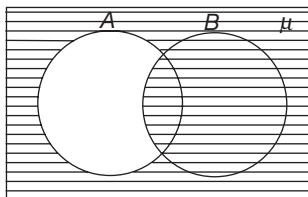


Figure 6.33

8. If any two of the sets $A_1, A_2, \dots, A_n$ are disjoint, then $A_1 \cap A_2 \cap \dots \cap A_n =$ _____.
9. If $n(A) = 25$, $n(B) = 10$ and also $B \subset A$, then $n(B - A) =$ _____.
10. If $n(A) = 15$, $n(B) = 13$ and $n(A \cap B) = 10$, then the symmetric difference of A and B is _____.
11. The ordered pair (x, y) is a subset of $\{x, y\}$. (True/False)
12. If $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$, then $R = \{(a, 2), (b, 1), (d, 3), (2, c)\}$ is a relation from A to B . (True/False)
13. $A = \{a, b, c\}$ and R is an identity relation on set A . Then the ordered pairs of R are _____.
14. $n(P \times Q) = 200$ and $n(P) = 100$, then $n(Q) =$ _____.
15. Relation $R = \{(x, y) : x > y \text{ and } x + y = 8 \text{ where } x, y \in N\}$, then write R^{-1} in roster form.
16. If $R = R^{-1}$, then the relation R is _____.
17. If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 3, 5, 7\}$, then $n(A \times B) =$ _____.
18. If A and B are two equivalent sets and $n(B) = 6$, then $n(A \times B) =$ _____.
19. The domain of the relation $R = \{(x, y) : x, y \in N \text{ and } x + y \leq 9\}$ is _____.
20. The range of the relation $R = \{(x, y) : 3x + 2y = 15 \text{ and } x, y \in N\}$ is _____.
21. $A = \{x, y, z, p\}$; $B = \{7, 8, 9, 10\}$ and a rule f is given by $f(x) = 7, f(y) = 7, f(z) = 7, f(p) = 7$. The relation f is a function. [True/False]
22. Range of the function $|x - 5|$ is _____.
23. If $f(x) = 2x + \frac{3}{2}$ then find $f(3)$ and $f\left(\frac{3}{2}\right)$.
24. If the function $f: A \rightarrow \{a, b, c, d\}$ is an onto function, then the minimum number of elements in A must be equal to _____.
25. In a bijection, the number of elements of the domain is equal to the number of elements of the co-domain. [True/False]
26. Domain of the function $\frac{1}{\sqrt{x}}$ is _____.
27. Number of elements of an identity function defined on a set containing four elements is _____.
28. If f is a constant function and $f(100) = 100$, then $f(2007) =$ _____.
29. $f: R \rightarrow R$ be defined by $f(x) = 7x + 6$. What is f^{-1} ?
30. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are such that $g \circ f$ is onto then g is necessarily onto. [True/false]

Short Answer Type Questions

31. If $A = \{1, 2, 3, 6, 8, 9\}$, $B = \{3, 4, 5, 6\}$, and $\mu = \{1, 2, 3, \dots, 10\}$, then find $(A \cup B)'$.
32. Write the following sets in the roster form.
 $X = \{a/30 \leq a \leq 40 \text{ and } a \text{ is a prime}\}$
33. If $n(P - Q) = x + 37$, $n(Q - P) = 30 + 3x$, $n(P \cup Q) = 120 + 2x$ and $n(P \cap Q) = 35$, then find x .
34. In a club of 70 members 30 play Tennis but not cricket and 55 play Tennis. How many members



play cricket but not Tennis? (Each member plays either Tennis or Cricket).

35. Find the value of $n(A \cap B \cap C)$, if $n(A) = 35$, $n(A \cap B \cap C') = 8$, $n(A \cap C \cap B') = 10$ and $n(A \cap B' \cap C') = 6$.
36. $R = \{(a, 2a - b) / a, b \in N \text{ and } 0 < a, b < 3\}$, then find the domain and range of the relation R .
37. If $n(X \cap Y') = 9$, $n(Y \cap X') = 10$ and $n(X \cup Y) = 24$, then find $n(X \times Y)$.
38. In a gathering, two persons are related 'if they have the same bike', then find the properties that are satisfied by the relation.
39. If $A = \{3, 5, 6, 9\}$ and R is a relation in A defined as $R = \{(x, y) \in R \text{ and } x + y < 18\}$, then write R in roster form.
40. Given $R = \{(a, a), (a, b), (b, c), (a, c), (b, b), (b, a), (c, a), (c, b)\}$ on set $A = \{a, b, c\}$. What are the properties that R satisfies?

41. If $f(x) = \frac{2x+3}{4}$, then find $f^{-1}\left(\frac{3}{4}\right)$.

42. If $f(x)$ is a polynomial function of 4th degree, $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(2) = 17$, then find $f(3)$.

43. If $f(x) = (1 - x^3)^{\frac{1}{3}}$, then find $f \circ f(x)$.

44. If $f(x) = \frac{x^2 - 1}{3}$ for $x \in \{-2, -1, 0, 1, 2\}$, then find $f^{-1}(x)$.

45. Find the range of the function $f(x) = \frac{1}{2x^2 + 1}$.

Essay Type Questions

46. In a colony of 125 members, 70 members watch Telugu channel, 80 members watch Hindi channel and 95 watch English channel, 20 watch only Telugu and Hindi, 35 watch only English and Hindi and 15 watch only Telugu and English. How many members watch all the three channels, if each watches either of the channels?
47. If $f(a) = \log\left(\frac{1+a}{1-a}\right)$, then find $f\left(\frac{a_1 + a_2}{1 + a_1 a_2}\right)$ in terms of $f(a_1), f(a_2)$.
48. If $f(x) = 2x + 1$ and $g(x) = 3x - 5$, then find $(f \circ g)^{-1}(0)$.
49. If $f(x) = x^3 - 1$, $x < 0 = x^2 - 1$, $x \geq 0$ and $g(x) = (x+1)^{\frac{1}{3}}$, $x < 1 = (x+1)^{\frac{1}{2}}$, $x \geq 1$, then find $g \circ f(x)$.
50. If $f = \{(1, 3), (2, 1), (3, 4), (4, 2)\}$ and $g = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$, then find $f \circ g$.

CONCEPT APPLICATION

Level 1

1. A, B and C are three non-empty sets. If $A \subset B$ and $B \subset C$, then which of the following is true?
 - (a) $B - A = C - B$
 - (b) $A \cap B \cap C = B$
 - (c) $A \cup B = B \cap C$
 - (d) $A \cup B \cup C = A$
2. If S is the set of squares and R is the set of rectangles, then $(S \cup R) - (S \cap R)$ is
 - (a) S .
 - (b) R .
 - (c) set of squares but not rectangles.
 - (d) set of rectangles but not squares.
3. If $A = \{1, 2, 3, 4, 5, 6\}$, then how many subsets of A contain the elements 2, 3 and 5?
 - (a) 4
 - (b) 8
 - (c) 16
 - (d) 32

4. If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$ and $C = \{b, d, e, g\}$, then which of the following is true?
- (a) $C \subset (A \cup B)$
 (b) $C \subset (A \cap B)$
 (c) $A \cup B = A \cup C$
 (d) Both (a) and (c)
5. If $A_1 \subset A_2 \subset A_3 \subset \dots \subset A_{50}$ and $n(A_x) = x - 1$, then find $n\left[\bigcap_{x=11}^{50} A_x\right]$.
- (a) 49 (b) 50
 (c) 11 (d) 10
6. A group of 30 people take either tea or coffee. If 12 people do not take tea and 15 people take coffee, then how many people take tea?
- (a) 18 (b) 16
 (c) 15 (d) 12
7. If P is the set of parallelograms, and T is the set of trapeziums, then $P \cap T$ is
- (a) P (b) T
 (c) ϕ (d) None of these
8. If X , Y and Z are three sets such that $X \supset Y \supset Z$, then $(X \cup Y \cup Z) - (X \cap Y \cap Z) = \underline{\hspace{2cm}}$.
- (a) $X - Y$ (b) $Y - Z$
 (c) $X - Z$ (d) None of these
9. If $n(A_x) = x + 1$ and $A_1 \subset A_2 \subset A_3 \subset \dots \subset A_{99}$, then $n\left[\bigcup_{x=1}^{99} A_x\right] = \underline{\hspace{2cm}}$.
- (a) 99 (b) 98
 (c) 100 (d) 101
10. In a class every student can speak either English or Telugu. The number of students who can speak only English, the number of students who can speak only Telugu and the number of students who can speak both English and Telugu are equal. Then which of the following can represent the number of students of the class?
- (a) 20 (b) 25
 (c) 45 (d) 50
11. If $(2x - y, x + y) = (1, 11)$, then the values of x and y respectively are
- (a) 6, 5. (b) 7, 4.
 (c) 4, 7. (d) 7, 3.
12. A relation between two persons is defined as follows:
 $a R b$ 'if a and b born in different months', R is
- (a) reflexive. (b) symmetric.
 (c) transitive. (d) equivalence.
13. If A is a non-empty set, then which of the following is false?
- p : There is atleast one reflexive relation on A .
 q : There is atleast one symmetric relation on A .
- (a) p alone (b) q alone
 (c) Both p and q (d) Neither p nor q
14. In a set of teachers of a school, two teachers are said to be related if 'they teach the same subject', then the relation is
- (a) reflexive and symmetric.
 (b) symmetric and transitive.
 (c) reflexive and transitive.
 (d) equivalence.
15. If $A = \{x, y, z\}$, then the relation $R = \{(x, x), (y, y), (z, z), (z, x), (z, y)\}$ is
- (a) symmetric. (b) anti symmetric.
 (c) transitive. (d) Both (b) and (c)
- Direction for questions 16 and 17: In the set of animals, a relation R is defined in each question.**
16. $a R b$ if ' a and b are in different zoological parks', then R is
- (a) only reflexive. (b) only symmetric.
 (c) only transitive. (d) equivalence.
17. On the set of human beings a relation R is defined as follows:
 $a R b$ if ' a and b have the same brother', then R is
- (a) only reflexive. (b) only symmetric.
 (c) only transitive. (d) equivalence.
18. Consider the following statements:
- p : Every reflexive relation is a symmetric relation.
 q : Every anti-symmetric relation is reflexive.



Which of the following is/are true?

- (a) p alone (b) q alone
(c) Both p and q (d) Neither p nor q

19. In a set of ants in a locality, two ants are said to be related if they walk on a same straight line, then the relation is

- (a) reflexive and symmetric.
(b) symmetric and transitive.
(c) reflexive and transitive.
(d) equivalence.

20. If $n(A) = 4$ and $n(B) = 4$, then find the number of subsets of $A \times B$.

- (a) 65636 (b) 65536
(c) 65532 (d) None of these

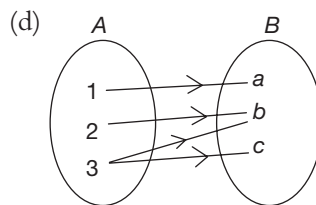
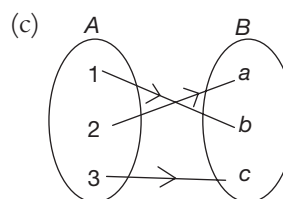
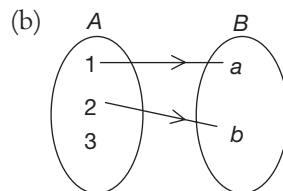
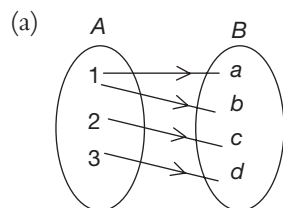
21. A function f is constant from set $A = \{1, 2, 3\}$ onto set $B = \{a, b, c\}$ such that $f(1) = a$, then the range of f is

- (a) $\{a, c\}$. (b) $\{a\}$.
(c) $\{a, b\}$. (d) $\{a, b, c\}$.

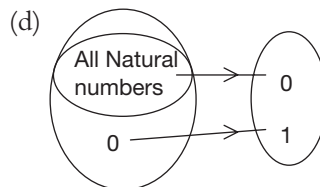
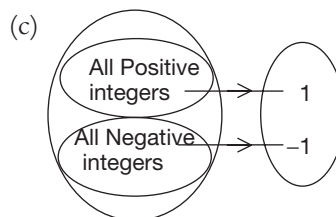
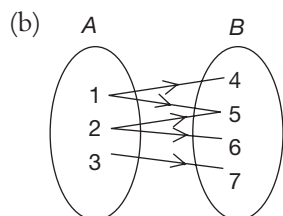
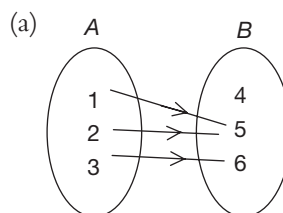
22. Which of the following is an odd function?

- (a) $x + x^3$ (b) $x^3 - x^2 - 5$
(c) $x^2 + x^4$ (d) $\frac{3x^2}{x^2 + 1}$

23. Which of the following relations is a function?



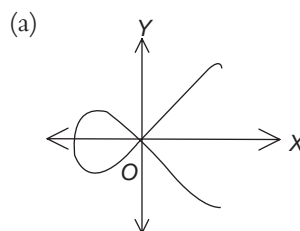
24. Which of the following relation is not a function?

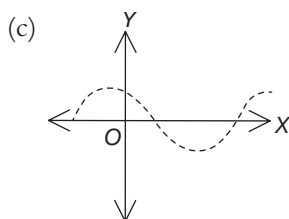
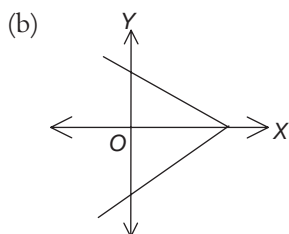


25. If $f: A \rightarrow B$ is an onto function defined by $f(x) = 3x - 4$ and $A = \{0, 1, 2, 3\}$, then the co-domain of f is

- (a) $\{-4, 0, 2, 5\}$. (b) $\{-1, 2, 5, 6\}$.
(c) $\{-4, -1, 2, 5\}$. (d) None of these

26. Which of the following graphs represents a function?





(d) None of these

27. If $f(x) = 2x - 3x^2 - 5$ and $g(x) = \frac{f(x) + f(-x)}{2}$, then $g(x)$ is
- (a) odd.
 (b) even.
 (c) even as well as odd.
 (d) Neither (a) nor (b)

28. Domain of the function $f(x) = \frac{5-x}{|3-x|}$ is

(a) $x \in R$ (b) $x \in Z$
 (c) $R - \{3\}$ (d) $R - \{5\}$

29. $A = \{-1, 0, 1, 2\}$, $B = \{0, 1, 2\}$ and $f: A \rightarrow B$ defined by $f(x) = x^2$, then f is

(a) only one-one function.
 (b) only onto function.
 (c) bijective.
 (d) not a function.

30. If two sets A and B have p and q number of elements respectively and $f: A \rightarrow B$ is one-one, then the relation between p and q is

(a) $p \geq q$ (b) $p > q$
 (c) $p \leq q$ (d) $p = q$

Level 2

31. If $n(A \cap B) = 10$, $n(B \cap C) = 20$ and $n(A \cap C) = 30$, then find the greatest possible value of $n(A \cap B \cap C)$.
- (a) 15 (b) 20
 (c) 10 (d) 4
32. If X , Y and Z are any three non-empty sets such that any two of them are disjoint, then $(X \cup Y \cup Z) \cap (X \cap Y \cap Z)$ is
- (a) X (b) Y
 (c) Z (d) ϕ
33. A and B are any two non-empty sets and A is proper subset of B . If $n(A) = 5$, then find the minimum possible value of $n(A \Delta B)$.
- (a) 1
 (b) 5
 (c) Cannot be determined
 (d) None of these
34. If $n(A \cap B) = 5$, $n(A \cap C) = 7$ and $n(A \cap B \cap C) = 3$, then the minimum possible value of $n(B \cap C)$ is _____.
- (a) 0 (b) 1
 (c) 3 (d) 2
35. If a set contains n elements, then which of the following cannot be the number of reflexive relations in the set?
- (a) 2^n (b) 2^{n-1}
 (c) 2^{n^2-1} (d) 2^{n+1}
36. If $A = \{4, 6, 10, 12\}$ and R is a relation defined on A as 'two elements are related if they have exactly one common factor other than 1'. Then the relation R is
- (a) anti symmetric
 (b) only transitive
 (c) only symmetric
 (d) equivalence



37. X is the set of all members in a colony and R is a relation defined on X as 'two persons are related if they speak same language'. The relation R is

- (a) only symmetric.
 (b) only reflexive.
 (c) both symmetric and reflexive but not transitive.
 (d) equivalence.

38. The relation 'is a factor of' on the set of natural numbers is not

- (a) reflexive. (b) symmetric.
 (c) anti-symmetric. (d) transitive.

39. If $f(x) = \log x$, then $\frac{f(xy) + f\left(\frac{x}{y}\right)}{f(x)f(y)} =$

- (a) $\frac{2}{\log x}$ (b) $2\log y$
 (c) $2\log x$ (d) $\frac{2}{\log y}$

40. If $f(x) = \frac{x-1}{x+1}$, $x \neq -1$; then find $f\left(\frac{x-1}{x+1}\right)$.

- (a) x (b) $-\frac{1}{x}$
 (c) $f(x)$ (d) $f\left(\frac{1}{x}\right)$

41. If $f: R \rightarrow R$ defined by $f(x) = 3x - 5$, then $f^{-1}(\{-1, -2, 1, 2\}) =$

- (a) $\left\{1, \frac{4}{3}, \frac{7}{3}\right\}$ (b) $\left\{-1, 2, \frac{-4}{3}\right\}$
 (c) $\left\{1, 2, \frac{4}{3}, \frac{7}{3}\right\}$ (d) $\{1, 2, -1, -2\}$

42. If $f: R \rightarrow R$ is a function defined as $f(\alpha - f(\alpha)) = 5f(\alpha)$ and $f(1) = 7$, then find $f(-6)$.

- (a) 37 (b) 35
 (c) 7 (d) 21

Level 3

43. All the students of a class like Horlicks, Maltova or Viva. Number of students who like only Horlicks and Maltova, only Maltova and Viva and only Horlicks and Viva are all equal to twice the number of students who like all the three foods. Number students who like only Horlicks, only Maltova and only Viva are all equal to thrice the number of students who like all the three foods. If four students like all the three, then find the number of students in the class.

- (a) 64 (b) 48
 (c) 68 (d) 52

44. The inverse of the function $f(x) = (x^3 - 1)^{\frac{1}{4}} - 12$ is

- (a) $[1 + (x + 12)^3]^{\frac{1}{4}}$
 (b) $[1 - (x + 12)^4]^{\frac{1}{4}}$
 (c) $[(x + 12)^3 - 1]^{\frac{1}{4}}$
 (d) $[1 + (x + 12)^4]^{\frac{1}{4}}$

45. The relation, $R = \{(1, 3), (3, 5)\}$ is defined on the set with minimum number of elements of

natural numbers. The minimum number of elements to be included in R so that R is equivalence, is

- (a) 5 (b) 6
 (c) 7 (d) 8

46. If $f(2x + 3) = 4x^2 + 12x + 15$, then the value of $f(3x + 2)$ is

- (a) $9x^2 - 12x + 36$
 (b) $9x^2 + 12x + 10$
 (c) $9x^2 - 12x + 24$
 (d) $9x^2 - 12x - 5$

47. If a relation $f: A \rightarrow B$ is defined by $f(x) = x + 2$, where $A = \{-1, 0, 1\}$ and $B = \{1, 2, 3\}$, then f is

- (a) only one-one function.
 (b) only onto function.
 (c) bijective.
 (d) None of these

48. The domain of the function, $f(x) = \frac{|x| - 2}{|x| - 3}$ is _____.

- (a) R (b) $R - \{2, 3\}$
 (c) $R - \{2, -2\}$ (d) $R - \{-3, 3\}$

49. If $f(x + y) = f(xy)$ and $f(1) = 5$, then find the value of $\sum_{k=0}^6 f(k)$.
- (a) 25 (b) 35
(c) 36 (d) 24
50. If $f(x) = (x + 1)$ and $g(x) = (x - 1)$, then find $(f \circ g)((g \circ f)(2))$.
- (a) 1 (b) 2
(c) 3 (d) 4
51. If $f(x) = x$, $g(x) = x^2$ and $h(x) = x^3$, then find $[(h \circ g) \circ f](x)$.
- (a) x (b) x^2
(c) x^3 (d) x^6
52. If $f = \{(2, 4), (3, 6), (4, 8)\}$ and $g = \{(4, 3), (6, 4), (8, 2)\}$, then find $f \circ g$.
- (a) $\{(2, 3), (3, 4), (4, 2)\}$
(b) $\{(4, 6), (6, 8), (8, 4)\}$
(c) $\{(3, 2), (4, 3), (2, 4)\}$
(d) $\{(6, 4), (8, 6), (4, 8)\}$
53. Find the domain of the function $f(x) = \frac{1}{\sqrt{2x^2 + 5x + 2}}$.
- (a) R
(b) $\left(-2, \frac{-1}{2}\right)$
(c) $(-\infty, -2] \cup \left[\frac{-1}{2}, \infty\right)$
(d) $(-\infty, -2) \cup \left(\frac{-2}{2}, \infty\right)$
54. Find the domain of function, $\sum_{p=1}^{10} \frac{1}{|2x - p|}$.
- (a) R
(b) $R - \left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \dots, 10\right\}$
(c) $R - \left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \dots, 5\right\}$
(d) $R - \{1, 2, \dots, 10\}$
55. Find the inverse function of $f(x) = 2x - 3$.
- (a) $3x + 2$ (b) $3x - 2$
(c) $(x + 3)/2$ (d) $(x - 3)/2$
56. If $f(x) = x + 1$ and $g(x) = x - 2$, then find $(f^{-1} \circ g^{-1})(x)$.
- (a) $x - 1$ (b) $x + 2$
(c) $g(x)$ (d) $f(x)$
57. If $f(x) + f(1 - x) = 10$, then the value of $f\left(\frac{1}{10}\right) + f\left(\frac{2}{10}\right) + \dots + f\left(\frac{9}{10}\right)$
- (a) is 45.
(b) is 50.
(c) is 90.
(d) Cannot be determined
58. There are 60 students in a class. The number of students who passed in Mathematics is 45 and the number of students who passed in Physics is 40. The number of students who failed in both the subjects is 5. Find the number of students who passed in exactly one of the subjects.
- (a) 35
(b) 25
(c) 15
(d) Cannot be determined
59. If $X = \{2, 3, 5, 7, 11\}$ and $Y = \{4, 6, 8, 9, 10\}$, then find the number of one-one functions from X to Y .
- (a) 720 (b) 120
(c) 24 (d) 12
- Directions for questions 60 and 61: These questions are based on the following data.**
For any two sets A and B , $n(A) = 15$, $n(B) = 12$, $A \cap B \neq \phi$ and $B \not\subset A$.
60. Find the maximum possible value of $n(A \Delta B)$.
- (a) 27 (b) 26
(c) 24 (d) 25
61. Find the minimum possible value of $n(A \Delta B)$.
- (a) 3 (b) 4
(c) 5 (d) 6



Directions for questions 62 and 63: These questions are based on the following data.

A and B are two finite sets. The difference of the number of elements of the power sets is 96. (Assume $n(A) > n(B)$)

62. Find $n(A) + n(B)$.

- (a) 11 (b) 12
(c) 13 (d) 14

63. Find $n(A) - n(B)$.

- (a) 2 (b) 3
(c) 4 (d) 5

Directions for questions 64 and 65: These questions are based on the following data.

The relation R is defined on a set $P = \{a, b, c, d, e\}$ and R is a reflexive relation.

64. Which of the following is true about the number of elements of R ?

- (a) $1 \leq n(R) \leq 5$ (b) $1 \leq n(R) \leq 2^5$
(c) $5 \leq n(R) < 2^5$ (d) $5 \leq n(R) \leq 25$

65. How many reflexive relations are possible on P ?

- (a) 2^5 (b) 2^{25}
(c) 2^{20} (d) 2^{18}



TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. A
2. 1
3. 126
4. $A \cup B$
5. $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
6. μ
7. A'
8. ϕ
9. 0
10. 8
11. False
12. False
13. $R = \{(a, a), (b, b), (c, c)\}$
14. 2
15. $R^{-1} = \{(1, 7), (2, 6), (3, 5)\}$
16. symmetric
17. 24
18. $n(A \times B) = 36$
19. $\{1, 2, 3, 4, 5, 6, 7, 8\}$
20. $\{3, 6\}$
21. True
22. $(0, \infty)$
23. $\frac{15}{2}, \frac{9}{2}$
24. 4
25. True
26. R^+
27. 4
28. 100
29. $f^{-1}(y) = \frac{y-6}{7}$
30. True

Short Answer Type Questions

31. $\{7, 10\}$
32. $X = \{31, 37\}$
33. 9
34. 15
35. 11
36. Domain = $\{1, 2\}$, Range = $\{1, 2, 0, 3\}$
37. 210.
38. It is reflexive, symmetric, and transitive.
39. $R = \{(3, 3), (3, 5), (3, 6), (3, 9), (5, 3), (5, 5), (5, 6), (5, 9), (6, 3), (6, 5), (6, 6), (6, 9), (9, 3), (9, 5), (9, 6)\}$
40. R is only symmetric.
41. 0
42. 82
43. x
44. Inverse does not exist
45. $(0, 1)$

Essay Type Questions

46. 25
47. $f(a_1) + f(a_2)$
48. $\frac{3}{2}$
49. When $x < 0$ or $x \geq 1$, $g \circ f(x) = x$ and when $0 \leq x < 1$, $g \circ f(x) = x^{2/3}$
50. $\{(1, 1), (2, 4), (3, 2), (4, 3)\}$



CONCEPT APPLICATION

Level 1

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (d) | 3. (b) | 4. (d) | 5. (d) | 6. (a) | 7. (a) | 8. (c) | 9. (c) | 10. (c) |
| 11. (c) | 12. (b) | 13. (d) | 14. (d) | 15. (d) | 16. (b) | 17. (d) | 18. (d) | 19. (d) | 20. (b) |
| 21. (b) | 22. (a) | 23. (c) | 24. (b) | 25. (c) | 26. (c) | 27. (b) | 28. (c) | 29. (d) | 30. (c) |

Level 2

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 31. (c) | 32. (d) | 33. (a) | 34. (c) | 35. (d) | 36. (c) | 37. (d) | 38. (b) | 39. (d) | 40. (b) |
| 41. (c) | 42. (b) | | | | | | | | |

Level 3

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 43. (a) | 44. (d) | 45. (c) | 46. (c) | 47. (c) | 48. (d) | 49. (b) | 50. (b) | 51. (d) | 52. (b) |
| 53. (d) | 54. (c) | 55. (c) | 56. (d) | 57. (a) | 58. (b) | 59. (b) | 60. (d) | 61. (c) | 62. (b) |
| 63. (a) | 64. (d) | 65. (c) | | | | | | | |



CONCEPT APPLICATION

Level 1

- Using the concept of subset, verify the options.
- Every square is a rectangle.
- Find $A \cup B$, $A \cap B$ and $A \cup C$.
- If $A_1 \subset A_2 \subset A_3, \dots$, then $n(A_1 \cap A_2 \cap A_3 \dots) = n(A_1)$.
- Use Venn diagram concept.
- Recall the properties of parallelogram and trapezium.
- Recall the concept of superset, union of sets and intersection.
- Recall the concepts of subset and union of sets.
- $n(A \cup B) = n(\text{only } A) + n(\text{only } B) + n(A \cap B)$.
- Equate the corresponding coordinates.
- Use the definitions of reflexive, symmetric, anti-symmetric and transitive.
- Recall the different types of relations.
- Use the definitions of reflexive, symmetric, anti-symmetric and equivalence.
- Recall the properties of relations.
- Recall the properties of relations.
- Use the definitions of reflexive, symmetric, anti-symmetric, transitive and equivalence.
- Recall the definitions of reflexive, symmetric and anti-symmetric relations.
- Recall the properties of relations.
- Number of subsets of $A \times B$ is $2^{n(A).n(B)}$.
- Constant function contains only one element in the range.
- If $f(x)$ is an odd function, then $f(-x) = -f(x)$.
- Recall the definition of function.
- Recall the definition of function.
- Substitute domain values in $f(x)$.
- Use the definition of a function.
- Find $\frac{f(x) + f(-x)}{2}$ and verify even or odd.
- Function is defined when denominator is not equal to zero.
- Substitute the domain values in $f(x)$ and find range.
- Recall the definition of one-one function.

Level 2

- The greatest possible value of $n(A \cap B \cap C)$ is the least value amongst the values of $n(A \cap B)$, $(B \cap C)$ and $n(A \cap C)$.
- A and B are disjoint $\Rightarrow A \cap B = \phi$ and $A \cap \phi = \phi$.
- (i) A is proper subset of B , $A - B = \phi$, i.e., $n(A - B) = 0$.
(ii) Given $n(A) = 5$, the minimum number of elements in B is 6.
(iii) The minimum possible value of $n(A \Delta B)$ is $n(B) - n(A)$.
- Number of elements in $(A \cap B \cap C)$ becomes the minimum number of elements in $B \cap C$.
- Number of reflexive relations in a set containing 'n' elements is 2^{n^2-n} .
- (i) Write the elements in R .
(ii) Apply definition of symmetric relation.
- Recall the properties of relations.
- Use the definitions of reflexive, symmetric, anti-symmetric and equivalence.
- (i) Substitute $f(xy)$, $f(x/y)$ in the given equation.
(ii) $\log ab = \log a + \log b$.
(iii) $\log \left(\frac{a}{b} \right) = \log a - \log b$.



(iv) Replace x by xy and also by $\frac{x}{y}$ in $f(x)$ and simplify.

40. (i) Replace x by $\frac{x-1}{x+1}$ in $f(x)$.

(ii) Then simplify.

41. (i) Find $f^{-1}(x)$.

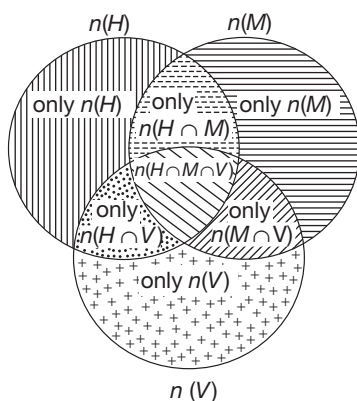
(ii) Put $x = -1, -2, 1, 2$ in $f^{-1}(x)$ and find their values.

42. Put $\alpha = 1$ in the given equation and simplify.

Level 3

43. (i) Let the number of students who like all the three be 'x'.

(ii)



(iii) Substitute the given values in the above figure.

44. (i) Let $f(x) = y$.

(ii) Write the value of x in terms of y .

45. Recall the properties of equivalence relation.

46. (i) Put $2x + 3 = t$.

(ii) Replace x by t .

(iii) Again put $t = 3x + 2$.

47. Given, $f: A \rightarrow B$, $f(x) = x + 2$, A

$= \{-1, 0, 2\}$ and $B = \{1, 2, 3\}$, $f(A) = B$ and $\forall x \in A, f(x_1) \neq f(x_2)$.

$\therefore f$ is bijective.

48. $F(x) = \frac{|x| - 2}{|x| - 3}$. Here, x cannot be ± 3 .

(f is not defined when $x = \pm 3$)

$\therefore$ Domain $= R - \{-3, 3\}$

49. Given $f(x + y) = f(xy)$

and $f(1) = 5$

Put $x = 1$ and $y = 0$

$$f(1 + 0) = f(0) \Rightarrow f(0) = 5$$

Put $x = 1, y = 1$ in Eq. (1)

$$f(1 + 1) = f(1)$$

$$\therefore f(2) = 5$$

Put $x = 2, y = 1$ in Eq. (1)

$$f(2 + 1) = f(2 \times 1)$$

$$\therefore f(3) = 5$$

Similarly, $f(4) = f(5) = f(6) = 5$

$$\therefore \sum_{k=0}^6 f(k) = f(0) + f(1) + \dots + f(6) = 7 \times 5 = 35.$$

50. Given $f(x) = x + 1, g(x) = x - 1$

$$(g \circ f)(x) = g[f(x)] = g[x + 1] = (x + 1) - 1 = x$$

$$\therefore (g \circ f)(2) = 2 \quad (1)$$

$$(g \circ f)(x) = f[g(x)]$$

$$= f(x - 1) = (x - 1) + 1$$

$$= x$$

$$(2)$$

$$\therefore (g \circ f)[(g \circ f)(2)]$$

$$= (g \circ f)(2) \text{ (from Eq. (1))}$$

$$= 2.$$

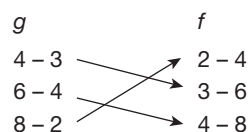
51. Given $f(x) = x, g(x) = x^2$ and $h(x)$

$$= x^3 [(h \circ g) \circ f](x) = (h \circ g)[f(x)]$$

$$= (h \circ g)(x) = h[g(x)] = h(x^2)$$

$$= (x^2)^3 = x^6.$$

52. For $f \circ g$:



$$\therefore f \circ g = \{(4, 6), (6, 8), (8, 4)\}.$$

$$\begin{aligned}
 53. \quad f(x) &= \frac{1}{\sqrt{2x^2 + 5x + 2}} \\
 \therefore 2x^2 + 5x + 2 &> 0 \\
 \Rightarrow (2x + 1)(x + 2) &> 0 \\
 \Rightarrow 2x + 1 > 0 \text{ and } x + 2 > 0 \\
 (\text{Or}) \\
 \Rightarrow 2x + 1 < 0 \text{ and } x + 2 < 0 \\
 \Rightarrow x < -2 \text{ or } x > -\frac{1}{2} \\
 \Rightarrow x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right).
 \end{aligned}$$

$$\begin{aligned}
 54. \quad \sum_{p=1}^{10} \frac{1}{|2x - p|} \\
 = \frac{1}{|2x - 1|} + \frac{1}{|2x - 2|} + \cdots + \frac{1}{|2x - 10|}
 \end{aligned}$$

Here, for $x = \frac{1}{2}, 1, \frac{3}{2}, \dots, 5$, it is not defined.

$$\therefore \text{Domain} = \mathbb{R} - \left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \dots, 5\right\}.$$

$$\begin{aligned}
 55. \quad \text{Let } y = f(x) = 2x - 3 \\
 \Rightarrow y = 2x - 3 \Rightarrow x = \frac{y + 3}{2} \\
 y = f(x) \Rightarrow x = f^{-1}(y) \\
 f^{-1}(y) = \frac{y + 3}{2} \\
 \text{Replace } y \text{ by } x. \\
 f^{-1}(x) = \frac{x + 3}{2}.
 \end{aligned}$$

$$\begin{aligned}
 56. \quad f(x) &= x + 1 & g(x) &= x - 2 \\
 \Rightarrow x &= yx - 1 & z &= x - 2 \\
 & & x &= z + 2 \\
 \therefore f^{-1}(y) &= y - 1 & g^{-1}(z) &= z + 2 \\
 \therefore f^{-1}(x) &= x - 1 & g^{-1}(x) &= x + 2
 \end{aligned}$$

Now, $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$

$$\begin{aligned}
 &= f^{-1}(x + 2) = (x + 2) - 1 \\
 &= x + 1 = f(x).
 \end{aligned}$$

$$57. \text{ Given } f(x) + f(1 - x) = 10 \quad (1)$$

$$\begin{aligned}
 &f\left(\frac{1}{10}\right) + f\left(\frac{2}{10}\right) + \cdots + f\left(\frac{9}{10}\right) \\
 &= f\left(\frac{1}{10}\right) + f\left(\frac{9}{10}\right) + f\left(\frac{2}{10}\right) \\
 &\quad + f\left(\frac{8}{10}\right) + \cdots + f\left(\frac{5}{10}\right) \\
 &= \left[f\left(\frac{1}{10}\right) + f\left(1 - \frac{1}{10}\right)\right] + \left[f\left(\frac{2}{10}\right) + f\left(1 - \frac{2}{10}\right)\right] \\
 &\quad + \cdots + f\left(\frac{5}{10}\right) \\
 &= 4(10) + f\left(\frac{1}{2}\right)
 \end{aligned}$$

Put $x = \frac{1}{2}$ in Eq. (1)

$$\begin{aligned}
 \therefore f\left(\frac{1}{2}\right) + f\left(1 - \frac{1}{2}\right) &= 10 \\
 \Rightarrow 2\left[f\left(\frac{1}{2}\right)\right] &= 10 \Rightarrow f\left(\frac{1}{2}\right) = 5 \\
 \therefore f\left(\frac{1}{10}\right) + f\left(\frac{1}{10}\right) + \cdots + f\left(\frac{9}{10}\right) &= 40 + 5 = 45.
 \end{aligned}$$

$$58. \text{ Number of students} = 60.$$

Number of students who passed in Mathematics = 45.

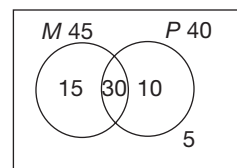
Number of students who passed in Physics = 40.

Number of students who failed in both the subjects = 5.

$\therefore$ Number of students who passed in one of the subjects = $60 - 5 = 55$.

$\therefore$ Number of students who passed in both the subjects = $45 + 40 - 55 = 30$.

$\therefore$ Number of students who passed in one of the subjects exactly = $(45 - 30) + (40 - 30) = 25$.



$$59. \text{ Given } X = \{2, 3, 5, 7, 11\} \quad Y = \{4, 6, 8, 9, 10\}.$$

$$\therefore n(x) = 5 \text{ and } n(y) = 5.$$

Number of one to one functions from X to $Y = 5!$
 $= 120$.



Solutions 65 and 66:

$$A \cap B \neq \phi, B \not\subset A$$

$$n(A) = 15 \text{ and } n(B) = 12$$

$$\text{As } A \cap B \neq \phi, n(A \cap B) \neq 0.$$

60. For the maximum value of $n(A \Delta B)$, $n(A \cap B)$ should be minimum.

The minimum value of $n(A \cap B) = 1$,
($\because n(A \cap B) \neq 0$).

$$\begin{aligned} \text{Now, } n(A \Delta B) &= n(A \cup B) - n(A \cap B) \\ &= n(A) + n(B) - 2[n(A \cap B)] \\ &= 15 + 12 - 2 = 25. \end{aligned}$$

61. For the minimum value of $n(A \Delta B)$, $n(A \cap B)$ should be maximum.

The maximum value of $n(A \cap B) = 11$, ($\because B \not\subset A$ and $n(B) = 12$).

$$\begin{aligned} \text{Now } n(A \Delta B) &= n(A \cup B) - n(A \cap B) \\ &= n(A) + n(B) - 2[n(A \cap B)] \\ &= 15 + 12 - 2(11) = 5. \end{aligned}$$

Solutions 62 and 63:

$$\text{Let } n(A) = x \text{ and } n(B) = y.$$

$$\text{Given, } n[p(A)] - n[p(B)] = 96.$$

$$\therefore 2^x - 2^y = 96$$

$$\Rightarrow 2^x - 2^y = 2^5 (3)$$

$$\Rightarrow 2^{x-5} - 2^{y-5} = 3$$

$$\Rightarrow 2^{x-5} - 2^{y-5} = 2^2 - 2^0$$

$$\Rightarrow x - 5 = 2 \text{ and } y - 5 = 0$$

$$\Rightarrow x = 7 \text{ and } y = 5.$$

$$62. n(A) + n(B) = x + y = 7 + 5 = 12.$$

$$63. n(A) - n(B) = x - y = 7 - 5 = 2.$$

Solutions 64 and 65:

$$64. R \text{ is reflexive relation defined on } P = \{a, b, c, d, e\}.$$

$$\text{And also } R \subseteq P \times P.$$

$$\therefore 5 \leq n(R) \leq 25.$$

$$65. \text{ Number of possible reflexive relations}$$

$$= 2^{5^2-5} = 2^{20}.$$