

Syllabus

- Electric charges; Conservation of charge; Coulomb's law—force between two point charges, forces between multiple charges; Superposition principle and continuous charge distribution.
- Electric field; Electric field due to a point charge; Electric field lines; Electric dipole, Electric field due to a dipole; Torque on a dipole in uniform electric field.
- Electric flux; Statement of Gauss's theorem and its applications to find the field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

Chapter Analysis

List of Topics	2016	2017	2018
Electric Field and Dipole	1 Q (3 marks)	2 Q (3 marks) 2 Q (5 marks)	1 Q (3 marks)
Gauss's Theorem and Its Applications	2 Q (1 mark)	1 Q (5 marks)	2 Q (3 marks) 1 Q (5 marks)



TOPIC-1

Electric Field and Dipole

TOPIC - 1

Electric Field and Dipole

.... P. 01

TOPIC - 2

Gauss's Theorem and Its Applications

... P. 18

Revision Notes

Electric Charge

- Electric Charge is one of the property of a matter due to which, it experiences a force when placed in an electromagnetic field.
- Point Charge is an accumulation of electric charges at a point, without spatial extent.
- Electrons are the smallest and lightest fundamental particles in an atom having negative charge as these are surrounded by invisible force field known as electrostatic field.
- Protons are larger and heavier than electrons with positive electrical charge which is similar in strength as electrostatic field in an electron with opposite polarity.
- Two electrons or two protons will tend to repel each other as they have negative and positive electrical charges respectively.

$$F \leftarrow \bullet^{e^-} \qquad \bullet^{e^-} \rightarrow F$$

$$F \leftarrow \bullet^P \qquad \bullet^P \rightarrow F$$

- The electron and proton will get attracted towards each other due to their unlike charges.

$$P \bullet \rightarrow F \qquad F \leftarrow \bullet^{e^-}$$

- The charge present on the electron is equal and opposite to charge on the proton.

Charge of on a proton = $+1.6 \times 10^{-19} \text{ C}$

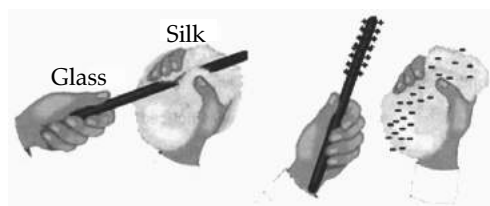
and Charge of on an electron = $-1.6 \times 10^{-19} \text{ C}$

Electrostatic Charge

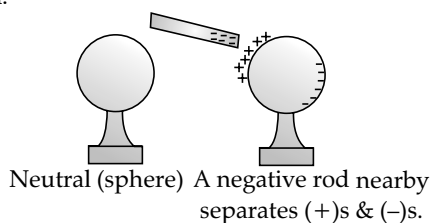
- Electrostatic charge means the charge is at rest.
- Electrostatic charge is a fundamental quantity like length, mass, and time.
- Charge on body is expressed as $q = \pm ne$
- The magnitude of charge is independent of the speed of the particle.
- Based on the flow of charge across them, materials are divided as :
 - Conductors - allow electric charge to flow freely - metals
 - Semi-conductors - behave as conductor or insulator - silicon
 - Insulators - do not allow electric charge to flow freely - rubber, wood, plastic, etc.
- Net charge is given by :
 - Charging by friction -charging insulators
 - Charging by conduction -charging metals and other conductors
 - Charging by induction -charging metals and other conductors.

Charging by Induction

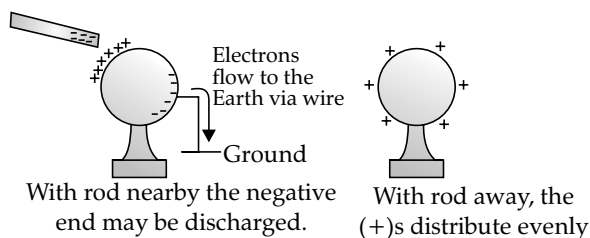
- Charging by induction means charging without contact.
- On rubbing a glass rod and silk cloth piece together, glass rod gets positively charged whereas silk cloth gets negatively charged.



- If a plastic rod is rubbed with wool, it becomes negatively charged.
- If a negatively charged rod is brought near neutral metal with insulator mounting, it repels free electrons and attracts positive charges on metal.



- If far end is connected to Earth by a wire, electrons will flow towards ground while positive charges are kept captive by the rod.



Properties of Electric Charge

Addition of charges

- If a system contains three point charges q_1 , q_2 and q_3 , then the total charge of the system will be the algebraic addition of q_1 , q_2 and q_3 , i.e., charges will add up.

$$q = q_1 + q_2 + q_3$$

Conservation of charges

- Electric charge is always conserved. It is the sum of positive and negative charges present in an isolated system, which remains constant.
- Charge cannot be created and destroyed in a process, but only in positive-negative pairs.

Quantization of charges

- Electric charge is always quantized i.e., electric charge is always an integral multiple of charge ' e '.

- Net charge q_{net} of an object having N_e electrons, N_p protons and N_n neutrons is :

$$q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = ne$$

- Neutron (n) : $m = 1.675 \times 10^{-27}$ kg; $q = 0$
- Proton (p) : $m = 1.673 \times 10^{-27}$ kg; $q = +1.6 \times 10^{-19}$ C
- Electron (e) : $m = 9.11 \times 10^{-31}$ kg; $q = -1.6 \times 10^{-19}$ C

Coulomb's Law

- The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as :

$$F = k \frac{|q_1| |q_2|}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{|q_1| |q_2|}{r^2}$$

where,

F = Force of attraction/repulsion between charges q_1 & q_2 .

q_1, q_2 = Magnitudes of charge 1 and charge 2

r = Distance between charges q_1, q_2

k = Constant whose value depends on medium where charges are kept.

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\text{As } \epsilon = k\epsilon_0, \quad k = \frac{1}{4\pi K'\epsilon_0}$$

ϵ_0 = permittivity of vacuum = 8.854×10^{-12} F/m

K' = relative permittivity of medium or dielectric constant.

- For vacuum, relative permittivity, $K' = 1$,
- As $\epsilon = \epsilon_0$, force of attraction/repulsion among two electric charges q_1, q_2 placed in vacuum and medium is :

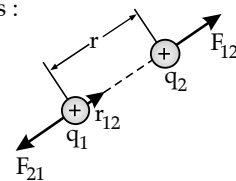
$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad (\text{vacuum})$$

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q_1 q_2}{r^2} \quad (\text{medium})$$

- The unit coulomb (C) is derived from the SI unit ampere (A) of the electric current.
- Current is the rate $\frac{dq}{dt}$ at which charge moves past a point or through a region, $i = \frac{dq}{dt}$, hence $1 \text{ C} = (1 \text{ A}) \times (1 \text{ s})$.
- The vector form of Coulomb force with $\hat{r}_{12}$ = unit vector from q_1 to q_2 is given as :

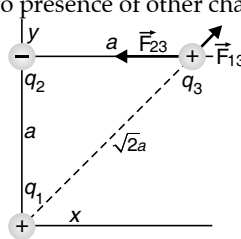
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \text{and} \quad \vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$\vec{F}_{21} = -\vec{F}_{12}$$



Principle of Superposition

- The force on any charge due to a number of other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- The individual forces are unaffected due to presence of other charges.



- force exerted by q_1 on $q_3 = \vec{F}_{13}$
- force exerted by q_2 on $q_3 = \vec{F}_{23}$
- net force exerted on q_3 is vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$

Electric field

- The space around a charge up to which its electric force can be experienced is called electric field.
- If a test charge q_0 is placed at a point where electric field is E , then force on the test charge is $F = q_0 E$
- The electric field strength due to a point source charge ' q ' at an observation point 'A' at a distance ' r ' from the source charge is given by

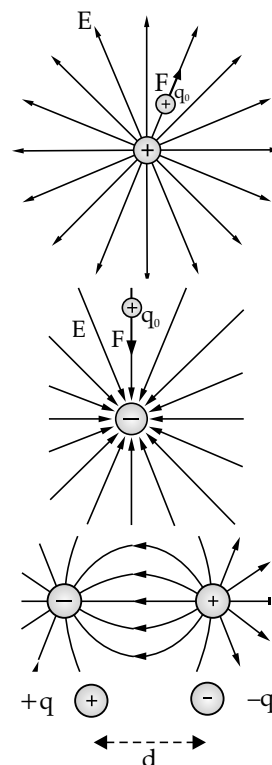
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^3} \vec{r}$$

$$\text{or } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^2}$$

- The unit of electric field is N/C
- Electric field inside the cavity of a charged conductor is zero.
- If a charged/uncharged conductor is placed in an external field, the field in conductor is zero.
- In case of charged conductor, electric field is independent of the shape of conductor.

Electric field lines

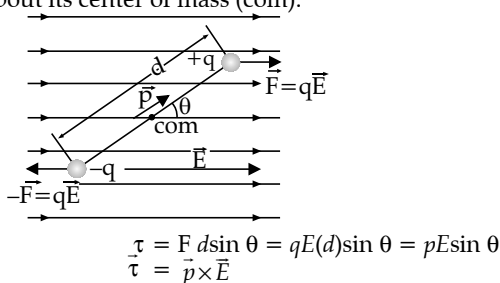
- Electric field lines are imaginary lines that extend from positive charge towards negative charge.
- Direction of electric field lines around positive charge is imagined by positive test charge q_0 located at points around source charge.
- Electric field has same direction as force on positive test charge.
- Electric field lines linked with negative charge are directed inward described by force on positive test charge q_0 .
- Field lines never cross each other.
- Strength of field is encoded in density of field lines.

**Electric Dipole**

- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.
- Dipoles make fields.
- The force on a dipole in an electric field is zero if both are in a stable as well as in an unstable equilibrium.
- The potential energy of a dipole in an electric field is minimum in a stable equilibrium and maximum in an unstable equilibrium.

Moment of Force

- In a dipole, when net force on dipole from field is zero and center of mass of dipole remains fixed, then forces on charged ends produce net torque τ about its center of mass (com).



- If $\theta = 0^\circ$ or 360° , dipole exists in stable state.
- If $\theta = 180^\circ$, dipole exists in unstable state.
- In uniform electric field, dipole experiences torque, net force on dipole is zero.
- In uniform electric field, dipole experiences rotatory motion.
- In non-uniform electric field, dipole experiences torque and net force.
- In non-uniform electric field, dipole experiences rotatory and translatory motion.
 - In this, torque aligns dipole with electric field and becomes zero.
 - In this, torque direction is normal to the plane going inward.

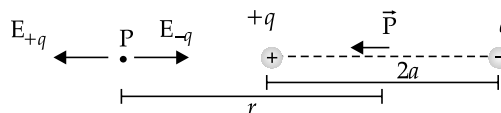
Electric Dipole Moment

- Dipole moment is a vector quantity whose unit is coulomb-meter (Cm).

- Dipole moment vector of electric dipole is $\vec{p} = \vec{q} \times 2a$ between pair of charges $q, -q$ along the line
- For point P at distance r from centre of dipole on charge q , for $r \gg a$, total field at point P is

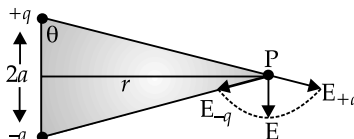
$$E = \frac{4qa}{4\pi\epsilon_0 r^3}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \quad (\text{if } a \ll r)$$



- For point P on the equatorial plane due to charges $+q$ and $-q$, electric field of dipole at a large distance

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$$



Know the Terms

- **1 coulomb** : When two point charges are placed at a distance of 1 m in vacuum, they repel/attract each other with force of 9×10^9 N, the charge on each is known as 1 coulomb.
- **Electric line of force** : It is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

Know the Formulae

- Coulomb's force : $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$;

where all alphabets have their usual meanings.

- Electric field due to point charge q : $E = \frac{k|q|}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

- Electric field due to a dipole at a point on the dipole axis : $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$ ($r \gg a$)

- Electric field at a point on equatorial plane : $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$ ($r \gg a$)

- Potential energy for dipole : $U = -pE \cos \theta$

- Potential energy of electric dipole in electric field :

$$P \cdot E = U = -(\vec{p}) \times \vec{E} = PE \sin \theta$$

- Work done by the rotating electric dipole in electric field :

$$W = (\vec{p}) \cdot \vec{E} (1 - \cos \theta)$$

? Objective Type Questions

(1 mark each)

Q. 1. Plastic rod rubbed with fur and glass rod rubbed with silk

- (a) repel each other. (b) mix up with each other.
(c) attract each other. (d) None of the above

Ans. Correct option : (c)

Explanation : Rubbing a rod with certain materials will cause the rod to become charged. If a plastic rod rubbed with fur becomes negatively charged and a glass rod rubbed with silk becomes positively charged.

Q. 2. Electric charge between two bodies can be produced by

- (a) sticking. (b) rubbing.
(c) oiling. (d) passing AC current.

Ans. Correct option : (b)

Explanation : The triboelectric effect is a type of contact electrification on which certain materials become electrically charged after they come into frictional contact with a different material.

Q. 3. Electric charges under action of electric forces is called

- (a) electrostatic. (b) electric flux.
(c) electric field. (d) electric field lines.

Ans. Correct option : (a)

Explanation : Coulomb force, also called electrostatic force or Coulomb interaction, attraction or repulsion of particles or objects because of their electric charges.

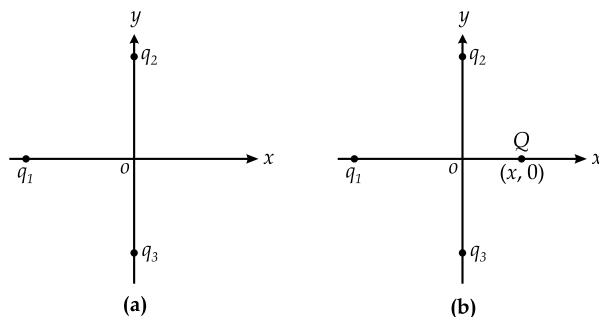
Q. 4. Law stating that “force is directly proportional to product of charges and inversely proportional to square of separation between them” is called

- (a) Newton’s law. (b) Coulomb’s law.
(c) Gauss’s law. (d) Ohm’s law.

Ans. Correct option : (b)

Explanation : Coulomb’s law states that : The magnitude of the electrostatic force of attraction or repulsion between two-point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distances between them.

Q. 5. In given figure, two positive charges q_2 and q_3 fixed along the y -axis, exert a net electric force in the $+x$ direction on a charge q_1 fixed along the x axis. If a positive charge Q is added at $(x, 0)$, the force on q_1



- (a) shall increase along the positive x -axis.
(b) shall decrease along the positive x -axis.
(c) shall point along the negative x -axis.
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3 .
[NCERT Exemp.]

Ans. Correct option : (a)

Explanation : Net force on charge q_1 , by other charges q_2 and q_3 is along the $+x$ -direction, so nature of forces between q_1 and q_2 and q_1 and q_3 are attractive. This is possible when charge q_1 is negative. Now, if a positive charge Q is placed at $(x, 0)$, then, the force on q_1 shall increase. Its direction will be along positive x -axis.

Commonly Made Error

- Some candidates do not understand about attraction and repulsion of net force. Here net force is the vector sum of all forces in x -direction.



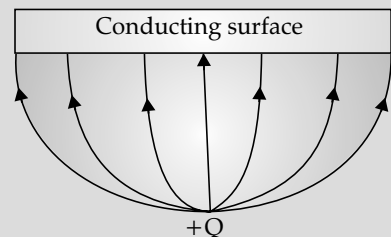
Very Short Answer Type Questions

(1 mark each)

Q. 1. A point charge $+Q$ is placed in the vicinity of a conducting surface. Draw the electric field lines between the surface and the charge.

[O.D. Comptt. I, II, III 2017]

Ans.



[CBSE Marking Scheme, 2017]

Q. 2. Why do the electrostatic field lines not form closed loops ?

[O.D. I, II, III 2014]

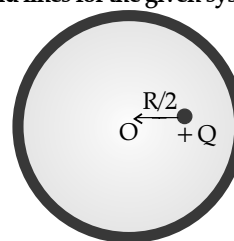
Ans. Electric field lines start from positive charge and end at infinity. Therefore, these lines do not form closed loops. [CBSE Marking Scheme, 2014] 1

Q. 3. Why do the electric field lines never cross each other ?

[O.D. I, II, III 2014]

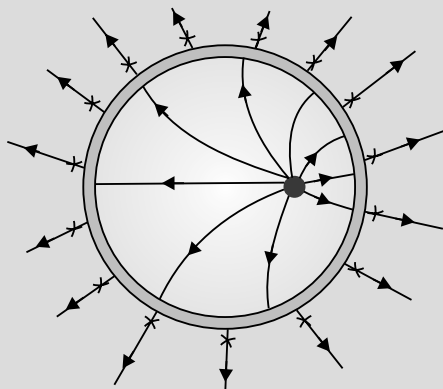
Ans. At the point of intersection, there will be two directions of electric field, which is not possible. 1

Q. 4. Figure shows a point charge $+Q$, located at a distance $R/2$ from the centre of a spherical metal shell. Draw the electric field lines for the given system.



[CBSE SQP 2016]

Ans.



Inside
Outside

$\frac{1}{2}$
 $\frac{1}{2}$

[CBSE Marking Scheme, 2016]

Q. 5. Why should electrostatic field be zero inside a conductor ? [U] [Delhi I, II, III, 2012]

Ans. In the static equilibrium, there is no current inside, or on the surface of the conductor, hence the electric field is zero everywhere inside the conductor.

Alternatively,

Since the charge inside the conductor is zero, the electric field is also zero. 1

OR

Alternatively,

Since the conductor is uncharged so the electric field inside it is zero. (or any other logically valid answer.) [CBSE Marking Scheme, 2012]

Q. 6. What is the value of the angle between the vectors $\vec{p}$ and $\vec{E}$ for which the potential energy of an electric dipole of dipole moment $\vec{p}$, kept in an external electric field $\vec{E}$, has the maximum value.

[A] [CBSE SQP 2014]

Ans. P.E. = $-\vec{p} \cdot \vec{E} = -pE \cos \theta$ $\frac{1}{2}$
 $\therefore$ P.E. is the maximum when $\cos \theta = -1$,
 i.e., $\theta = \pi$ ($= 180^\circ$). $\frac{1}{2}$

[CBSE Marking Scheme, 2014]

Q. 7. Two dipoles, made from charges $\pm q$ and $\pm Q$ respectively, have equal dipole moments. Give the (i) ratio between the 'separations' of these two pairs of charges, (ii) angle between the dipole axes of these two dipoles.

[U] [O.D. I, II, III 2013; CBSE SQP 2013]

Ans. As $qa = Qa'$, we have

$$\frac{a}{a'} = \frac{Q}{q} \quad \frac{1}{2}$$

and

$$\theta = 0^\circ. \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2013]

Detailed Answer :

If distance between two charges of dipoles be l_1 and l_2 respectively, then dipole moments of dipoles will be ql_1 and Ql_2

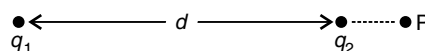
(i) ratio between the separations of two pairs of charges q, Q

$$ql_1 = Ql_2$$

$$\text{or, } \frac{l_1}{l_2} = \frac{Q}{q}$$

(ii) angle between dipole axes of two dipoles will be zero.

Q. 8. Two point charges ' q_1 ' and ' q_2 ' are placed at a distance ' d ' apart as shown in the figure. The electric field intensity is zero at a point ' P ' on the line joining them as shown. Write two conclusions that you can draw from this.



[Comptt. Delhi I, II, III 2014]

Ans. (i) The two point charges (q_1 and q_2) should be of opposite nature. $\frac{1}{2}$

(ii) Magnitude of charge q_1 must be greater than that of charge q_2 . $\frac{1}{2}$

[CBSE Marking Scheme, 2014]

Q. 9. Two equal balls having equal positive charge ' q ' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two ?

[O.D. I, II, III 2014]

Ans. The force decreases. 1

[CBSE Marking Scheme, 2014]

Detailed Answer :

The force given as $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(d-t+t\sqrt{K})}$ will

decrease when a plastic is introduced, as the dielectric constant K for plastic is more than 1.



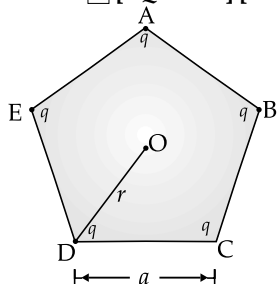
Short Answer Type Questions-I

(2 marks each)

Q. 1. Five charges, q each are placed at the corners of a regular pentagon of side a .

- What will be the electric field at O if the charge from one of the corners (say A) is removed ?
- What will be the electric field at O if the charge q at A is replaced by $-q$?

[SQP 2017] [NCERT Exemplar]



Ans. (i) $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$ along OA 1

(ii) $\frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{r^2}$ along OA 1

[CBSE Marking Scheme, 2017]

Commonly Made Error

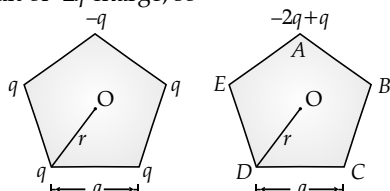
- Some candidates find resultant force F as a scalar sum of all forces, but resultant force is equal to vector sum of all forces.

Detailed Answer :

- If a charge q is removed from point A , a negative charge is developed at A where electric field will be

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \text{ which is along } OA. \quad 1$$

- If a charge q is replaced by charge $-q$ at point A , there generates a net electric field at point O as a result of $-2q$ charge, so

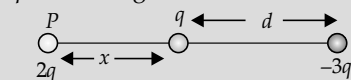


$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{-2q}{r^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{r^2} \text{ along } OA \quad 1$$

Q. 2. Two charges q and $-3q$ are placed on x -axis separated by distance d . Where a third charge $2q$ should be placed such that it will not experience any force ? [A] [SQP 2017] [NCERT Exemplar]

Ans. At P on $2q$, force due to q is to the left and that due to $-3q$ is to the right.



$$\therefore \frac{2q^2}{4\pi\epsilon_0 x^2} = \frac{6q^2}{4\pi\epsilon_0 (d+x)^2} \quad 1$$

$$\therefore (d+x)^2 = 3x^2$$

$$\therefore 2x^2 - 2dx - d^2 = 0$$

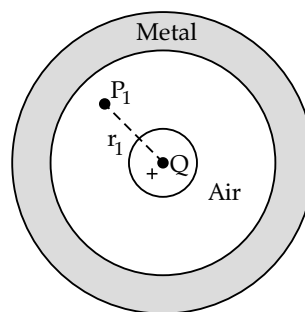
$$\therefore x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

As the negative charge will be in between q and $-3q$, hence it is unacceptable.

$$x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2} = \frac{d}{2}(1 + \sqrt{3}) \text{ to the left of } q. \quad 1$$

[CBSE Marking Scheme, 2017] 2

Q. 3. A small metallic sphere carrying charge $+Q$ is located at the centre of a spherical cavity in a large uncharged metallic spherical shell. Write the charges on the inner and outer surfaces of the shell. Write the expression for the electric field at the point P_1 . [R] [Comptt. Delhi I, II, III 2014]



Ans. Charge on inner surface : $-Q$ 1/2

Charge on outer surface : $+Q$ 1/2

Electric field at point P_1

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r_1^2} \quad 1$$

[CBSE Marking Scheme, 2014]

Q. 4. An electric dipole is placed in a uniform electric field $\vec{E}$ with its dipole moment $\vec{p}$ parallel to the field. Find

- the work done in turning the dipole till its dipole moment points in the direction opposite to $\vec{E}$.
- the orientation of the dipole for which the torque acting on it becomes maximum.

[A] [Comptt. O.D. I, II, III 2014]

Ans. (i) We have $W = \int_{\theta_1}^{\theta_2} \tau d\theta$

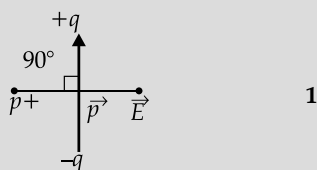
$\therefore W = \int_0^\pi pE \sin \theta d\theta$

$W = pE [-\cos \theta]_0^\pi$

$W = -2pE$ 1/2

$\therefore \tau = pE \sin \theta$

(ii) For $\theta = \frac{\pi}{2}$, τ is maximum. 1/2



[CBSE Marking Scheme, 2014]

Q. 5. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has charge ± 8 nC. [A] [Delhi I, II, III 2014]

Ans. Torque $\tau = pE \sin \theta$ 1/2

$4\sqrt{3} = pE \sin 60^\circ = pE \frac{\sqrt{3}}{2}$

$4\sqrt{3} = pE \frac{\sqrt{3}}{2}$

Hence, $pE = 8$ 1/2

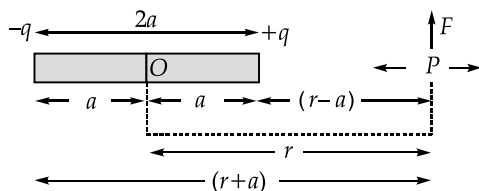
Potential energy, $U = -pE \cos \theta$ 1/2

or $U = -8 \times \cos 60^\circ = -4$ J 1/2

[CBSE Marking Scheme, 2014]

Q. 6. Find the expression for electric field intensity in an axial position due to electric dipole. [R] [O.D. Comptt. I, II, III, 2013]

Ans. Consider an electric dipole whose length is $2a$ and centre at O . From the mid-point O , consider a point P at a distance r , where the electric field intensity is to be determined.



We have $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

Case I, $E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-a)^2}$

Case II, $E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r+a)^2}$

Then $E = E_1 - E_2$

$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right]$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 \cdot (r+a)^2} \right]$$

$$E = \frac{q}{4\pi\epsilon_0} \cdot \frac{4ar}{(r^2 - a^2)^2}$$

for, $r^2 \gg a^2$

$\therefore E = \frac{q}{4\pi\epsilon_0} \left[\frac{4ar}{r^4} \right]$

$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2 \times q \times 2a \times r}{r^4}$

$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \text{ N/C} [\because p = 2qa]$ 2

Commonly Made Error

- Several candidates derive an expression for intensity of electric field E at a point in the broadside position i.e., coaxial position, instead of that in the end-on position as required. Some candidates do not understand which derivation to be given. Hence they write both the derivations. Many candidates are not able to draw the correctly labelled diagram.

Q. 7. An electric dipole is held in a uniform electric field.

(i) Show that the net force acting on it is zero.

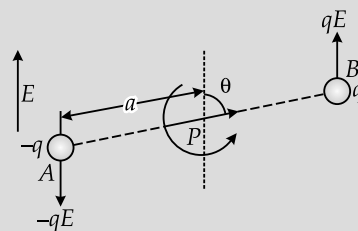
(ii) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of 180° .

[R] [O.D. I, II, III 2012]

Ans. (i) $\vec{F}_1 = +q\vec{E}$ and $\vec{F}_2 = -q\vec{E}$

$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$

$\therefore \vec{F}_{\text{net}} = \vec{0}$ 1



(ii) $W = -Ep(\cos \theta_1 - \cos \theta_2)$ 1/2

$W = 2Ep$ 1/2

[CBSE Marking Scheme, 2012]

Detailed Answer :

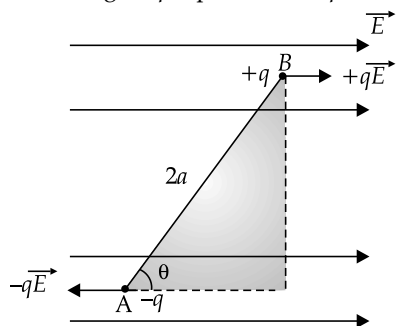
(i) In an electric dipole with two equal and opposite point charges, $-q$ at point A and $+q$ at point B with distance $2a$ as shown, so $AB = 2a$ with dipole moment $|\vec{P}| = q(2a)$

If dipole is kept in a uniform external electric field

$\vec{E}$ at angle θ with direction of $\vec{E}$, then :

(a) force on charge $-q$ at point $A = -q\vec{E}$, opposite to $\vec{E}$

(b) force on charge $+q$ at point $B = +q\vec{E}$, along $\vec{E}$



Hence, net force on dipole will be given as:

$$qE - qE = 0$$

(ii) Work done, $W = Ep(1 - \cos \theta)$
 $= Ep(1 - \cos 180^\circ)$
 $= 2Ep \text{ joules}$

or

Work done on dipole, $W = \Delta U$
 $= pE(\cos \theta_1 - \cos \theta_2)$
 $W = pE(\cos 0^\circ - \cos 180^\circ)$
 $W = 2pE$ 1

Q. 8. Two point charges q_1 and q_2 are located at points $(a, 0, 0)$ and $(0, b, 0)$ respectively. Find the electric field due to both these charges at the point $(0, 0, c)$. [A] [CBSE SQP 2013]

Ans. We have $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$
 $= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r_1^3} \vec{r}_1 + \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{r_2^3} \vec{r}_2$ 1/2

where, $\vec{r}_1 = -a\hat{i} + c\hat{k}$
 and $\vec{r}_2 = -b\hat{j} + c\hat{k}$ 1/2

$$\vec{E}_{net} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1(-a\hat{i} + c\hat{k})}{(a^2 + c^2)^{3/2}} + \frac{q_2(-b\hat{j} + c\hat{k})}{(b^2 + c^2)^{3/2}} \right]$$
 1
[CBSE Marking Scheme, 2013]

Q. 9. A particle of mass 10^{-3} kg and charge $5 \mu\text{C}$ enters into a uniform electric field of $2 \times 10^5 \text{ NC}^{-1}$, moving with a velocity of 20 ms^{-1} in a direction opposite to that of the field. Calculate the distance it would travel before coming to rest.

[A] [Delhi Comptt. I, II, III, 2012]

Ans. Here, $m = 10^{-3} \text{ kg}$, $q = 5 \times 10^{-6} \text{ C}$, $E = 2 \times 10^5 \text{ N/C}$,
 $u = 20 \text{ m/s}$, $v = 0$

As the particle enters opposite to the field, so it will retard.

Acceleration, $a = -\frac{qE}{m}$
 $= -\frac{5 \times 10^{-6} \times 2 \times 10^5}{10^{-3}}$
 $= -10^3 \text{ m/s}^2$ 1

Using, $v^2 = u^2 - 2as$
 $0 = (20)^2 - 2 \times 1000 \times s$
 $\Rightarrow s = \frac{400}{2000} = \frac{1}{5} = 0.2 \text{ m}$ 1

[CBSE Marking Scheme, 2012]

? Short Answer Type Questions-II

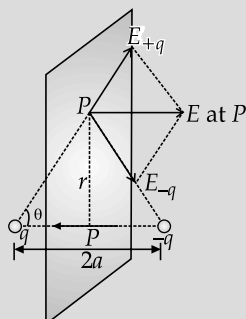
(3 marks each)

[AI] Q. 1. (i) Derive the expression for electric field at a point on the equatorial line of an electric dipole.

(ii) Depict the orientation of the dipole in (i) stable, (ii) unstable equilibrium in a uniform electric field. [A] [Delhi I 2017]

Ans. Derivation of expression of electric field on equatorial line of electric dipole 2

Depiction of orientation for stable and unstable equilibrium in electric field 1/2 + 1/2



(i) Let the point 'P' be at a distance 'r' from the mid point of the dipole.

$$|E_{+q}| = \frac{q}{4\pi\epsilon_0(r^2 + a^2)} \quad 1/2$$

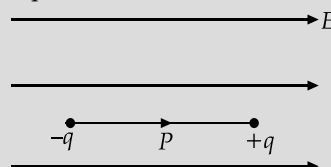
$$|E_{-q}| = \frac{q}{4\pi\epsilon_0(r^2 + a^2)} \quad 1/2$$

Both are equal and their directions are as shown in the figure, hence net electric field

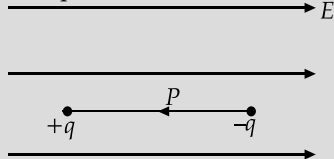
$$\vec{E} = [-(E_{+q} + E_{-q})\cos\theta]\hat{p} \quad 1/2$$

$$= -\frac{2qa}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}\hat{p} \quad 1/2$$

(ii) Stable equilibrium $\theta = 0^\circ$ 1/2



Unstable equilibrium $\theta = 180^\circ$ $\frac{1}{2}$



[CBSE Marking Scheme, 2017]

Q. 2. (i) Obtain the expression for the torque $\vec{\tau}$ experienced by an electric dipole of dipole moment $\vec{p}$ in a uniform electric field, $\vec{E}$.

(ii) What will happen if the field were not uniform?

[Delhi III 2017]

Ans. (i) Obtaining expression for torque $\vec{\tau}$ experienced by electric dipole in uniform electric field $\frac{1}{2}$

(ii) Effect of non-uniform electric field $\frac{1}{2}$

(i) Force on $+q$, $\vec{F} = q\vec{E}$ $\frac{1}{2}$

Force on $-q$, $\vec{F} = -q\vec{E}$ $\frac{1}{2}$

Magnitude of torque $\tau = qE \times 2a \sin \theta$ $\frac{1}{2}$

$$= 2qaE \sin \theta$$

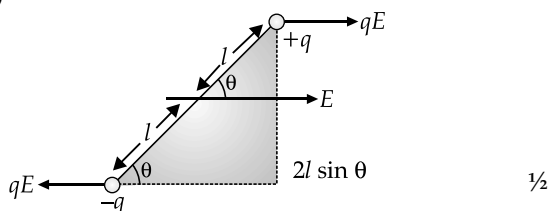
$$\vec{\tau} = \vec{p} \times \vec{E} \quad \frac{1}{2}$$

(ii) If the electric field is non-uniform, the dipole experiences a translatory force as well as a torque.

[CBSE Marking Scheme, 2017]

Detailed Answer :

(i)



Consider electric dipole kept in an uniform electric field at an angle θ where a dipole experience a torque, so, torque generated by parallel forces qE will act as couple as $\frac{1}{2}$

$$|\vec{\tau}| = qE 2l \sin \theta \quad \frac{1}{2}$$

$$= pE \sin \theta \quad [\text{as } p = 2ql] \quad \frac{1}{2}$$

$$|\vec{\tau}| = |\vec{p} \times \vec{E}|$$

(ii) When the field is non-uniform, force acting on both ends will not be equal, hence they result in mixture of couple and net force. With this, dipole experiences rotational as well as linear force. $\frac{1}{2}$

Q. 3. Derive an expression for electric field of a dipole at a point on the equatorial plane of the dipole. How does the field vary at large distances?

[A] [SQP 2017-18]

Ans. (i) Derivation $\frac{1}{2}$

(ii) At large distances, $\vec{E} \propto \frac{1}{r^3}$ $\frac{1}{2}$

[CBSE Marking Scheme, 2017]

Detailed Answer :

(i) Try yourself, similar to Q. 1 (i) Short Answer Type Questions-II $\frac{1}{2}$

(ii) Electric field, $E = \frac{-p}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}}$

If the dipole is at large distance $r \gg a$

$$E = \frac{-p}{4\pi\epsilon_0 r^3}$$

$$|\vec{E}| \propto \frac{1}{r^3} \quad \frac{1}{2}$$

Q. 4. A charge $+Q$, is uniformly distributed within a sphere of radius R . Find the electric field, due to this charge distribution, at a distant point r from the centre of the sphere where :

(i) $0 < r < R$

(ii) $r > R$ [A] [Foreign, 2016]

Ans. We have

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}, \text{ for a point charge}$$

Now (volume) charge density

$$\rho = \left(\frac{Q}{\frac{4\pi}{3} R^3} \right) \quad \frac{1}{2}$$

(i) $\therefore$ Charge contained within a sphere of radius r ($0 < r < R$)

$$Q' = \rho \cdot \frac{4\pi}{3} r^3 = Q \left(\frac{r^3}{R^3} \right) \quad \frac{1}{2}$$

$\therefore$ Electric Field,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q'}{r^2} = \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^3} \cdot r \right) \quad \frac{1}{2}$$

(ii) For $r > R$

Electric field = (Electric field due to a point charge Q at the centre) $\frac{1}{2}$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

[CBSE Marking Scheme, 2016]

Q. 5. Two thin concentric and coplanar spherical shells, of radii a and b ($b > a$) carry charges, q and Q , respectively. Find the magnitude of the electric field, at a point at distance x , from their common centre for :

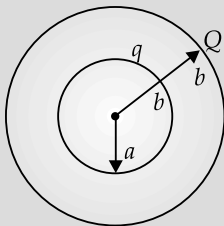
(i) $0 < x < a$

(ii) $a \leq x < b$

(iii) $b \leq x < \infty$

[A] [Comptt. Delhi, 2016]

Ans.

 $\frac{1}{2}$ (i) For $0 < x < a$

Point lies inside both the spherical shells. Hence,
 $E(x) = 0$ $\frac{1}{2}$

(ii) For $a \leq x < b$

Point is outside the spherical shell of radius 'a' but inside the spherical shell of radius 'b'.

$$\therefore E(x) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2} \quad 1$$

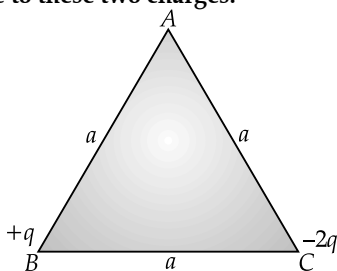
(iii) For $b \leq x \leq \infty$

Point is outside of both the spherical shells. Total effective charge at the centre equals $(Q + q)$.

$$\therefore E(x) = \frac{1}{4\pi\epsilon_0} \cdot \left(\frac{q + Q}{x^2} \right) \quad 1$$

[CBSE Marking Scheme, 2016]

Q. 6. Two point charges $+q$ and $-2q$ are placed at the vertices 'B' and 'C' of an equilateral triangle ABC of side 'a' as given in the figure. Obtain the expression for (i) the magnitude and (ii) the direction of the resultant electric field at the vertex A due to these two charges.

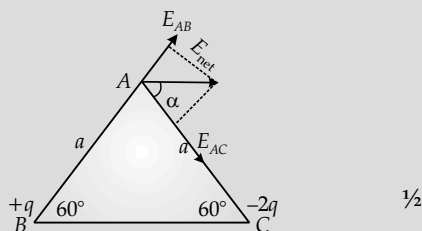


[A] [Comptt. O.D. I, II, III 2014]

Ans. (i) The magnitude

$$\left| \vec{E}_{AB} \right| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} = E \quad \frac{1}{2}$$

$$\left| \vec{E}_{AC} \right| = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{a^2} = 2E \quad \frac{1}{2}$$

 $\frac{1}{2}$

$$E_{net} = \sqrt{(2E)^2 + E^2 + 2 \times 2E \times E \cos 120^\circ}$$

$$E_{net} = \sqrt{(2E)^2 + E^2 + 2 \times 2E \times E \times \left(-\frac{1}{2}\right)}$$

$$E_{net} = \sqrt{4E^2 + E^2 - 2E^2}$$

$$E_{net} = E\sqrt{3} = \frac{1}{4\pi\epsilon_0} \frac{q\sqrt{3}}{a^2} \quad \frac{1}{2}$$

(ii) The direction of resultant electric field at vertex A

$$\tan \alpha = \frac{E_{AB} \sin 120^\circ}{E_{AC} + E_{AB} \cos 120^\circ} \quad \frac{1}{2}$$

$$\tan \alpha = \frac{E \times \frac{\sqrt{3}}{2}}{2E + E \times \left(-\frac{1}{2}\right)} = \frac{1}{\sqrt{3}}$$

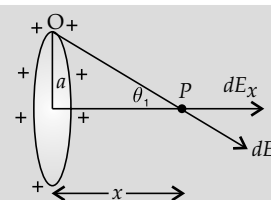
$$\alpha = 30^\circ \text{ (with side AC)} \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2014]

Q. 7. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence, show that for points at large distances from the ring, it behaves like a point charge.

[A] [Delhi I, II, III, 2016]

Ans.



$$OP = \sqrt{a^2 + x^2}, \text{ by Pythagoras theorem}$$

where, a is radius of ring x is the distance from centre to point 'P'

$$dE = \frac{k dQ}{a^2 + x^2}$$

$$dE_x = \frac{k dQ}{(a^2 + x^2)} \cos \theta \quad 1$$

and dE_y will be zero because all components will cancel out each other.

$$\therefore \cos \theta = \frac{x}{(a^2 + x^2)^{1/2}}$$

$$dE_x = \frac{k dQ}{a^2 + x^2} \cdot \frac{x}{(a^2 + x^2)^{1/2}}$$

$$E_x = \int \frac{k x dQ}{(a^2 + x^2)^{3/2}}$$

$$E_x = \frac{k x}{(a^2 + x^2)^{3/2}} \int dQ$$

$$E = \frac{k Q x}{(a^2 + x^2)^{3/2}} \quad 1$$

Now, if $x \gg a$

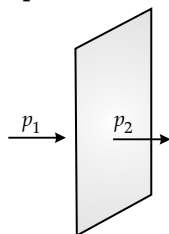
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{x^2}, \text{ which is equal to field produced by}$$

a point charge.

 $\therefore$ For large distance, ring behaves as a point charge. 1

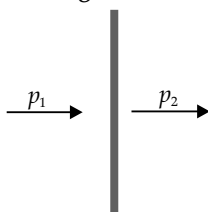
[CBSE Marking Scheme, 2016]

- Q. 8. (i) An electric dipole is kept first to the left and then to the right of a negatively charged infinite plane sheet having a uniform surface charge density. The arrows p_1 and p_2 show the directions of its electric dipole moments in the two cases.



Identify for each case, whether the dipole is in stable or unstable equilibrium. Justify each answer.

- (ii) Next, the dipole is kept in a similar way (as shown), near an infinitely long straight wire having uniform negative linear charge density.



Will the dipole be in equilibrium at these two positions? Justify your answer. [A] [SQP 2018-19]

Ans. (i) p_1 : stable equilibrium $\frac{1}{2}$

p_2 : unstable equilibrium

The electric field, on either side, is directed towards the negatively charged sheet and its magnitude is independent of the distance of the field point from the sheet. For position p_1 , dipole moment and electric field are parallel. For position p_2 , they are anti parallel. $\frac{1}{2} + \frac{1}{2}$

- (ii) The dipole will not be in equilibrium in any of the two positions. $\frac{1}{2}$

The electric field due to an infinite straight charged wire is non-uniform ($E \propto 1/r$). $\frac{1}{2}$

Hence, there will be a net non-zero force on the dipole in each case. $\frac{1}{2}$

[CBSE Marking Scheme, 2018-19]

- Q. 9. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Obtain the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. [U] [Comptt. Delhi I, II, III, 2015]

Ans. (i) Expression for torque experienced by dipole 2

- (ii) Identifying pairs of perpendicular vectors in expression 1

[CBSE Marking Scheme, 2015]

Detailed Answer :

- (i) Try yourself, Similar to Q. 2(i) Short Answer Type Questions-II

- (ii) From an expression $\vec{\tau} = \vec{p} \times \vec{E}$ two pairs of perpendicular vectors in expression are given as :

(i) $\vec{\tau}$ is perpendicular to $\vec{p}$ $\frac{1}{2}$

(ii) $\vec{\tau}$ is perpendicular to $\vec{E}$. $\frac{1}{2}$

- Q. 10. An electric dipole is placed in a uniform electric field.

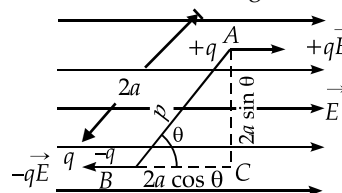
(i) Show that no translatory force acts on it.

(ii) Derive an expression for the torque acting on it.

- (iii) Find work done in rotating the dipole through 180° . [A] [Delhi Comptt. I, II, III, 2014,

O.D. I, II, III, 2013, 2012]

Ans. Electric dipole of charges $+q$ and $-q$ separated by distance $2a$ is shown in the figure.



It is placed in a uniform electric field at an angle θ with it.

- (i) Force on charge $+q$, $\vec{F}_1 = q\vec{E}$, in the direction of $\vec{E}$. $\frac{1}{2}$

Force on charge $-q$, $\vec{F}_2 = -q\vec{E}$, in the opposite direction of $\vec{E}$.

$$\therefore \text{Net translatory force on dipole} = \vec{F}_1 + \vec{F}_2$$

$$= +q\vec{E} - q\vec{E} = \vec{0}$$

Hence, no translatory force acts on it. $\frac{1}{2}$

- (ii) But the two equal, parallel and unlike forces form a couple in which a torque is given by

$\tau = \text{Force} \times \text{perpendicular distance between the two forces}$

$$\tau = qE \times 2a \sin \theta$$

$$\tau = pE \sin \theta$$

where, $p = q \times 2a = \text{dipole moment}$ 1

- (iii) Work done in rotating the dipole through 180° is

$$W = \int dW$$

$$W = \int_0^{180^\circ} \tau d\theta = pE \int_0^{180^\circ} \sin \theta d\theta$$

$$W = pE [-\cos \theta]_0^{180^\circ}$$

$$W = -pE [\cos 180^\circ - \cos 0^\circ]$$

$$W = pE [1 + 1] = 2pE$$

Q. 11. An electric dipole of moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Write the expression for the torque $\vec{\tau}$ experienced by the dipole.

Identify two pairs of perpendicular vectors in the expression.

Show diagrammatically the orientation of the dipole in the field for which the torque is (i) the maximum, (ii) half of the maximum value, (iii) zero.

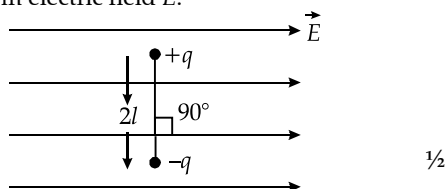
[O.D. Comptt. I, II, III; 2012, CBSE SQP II; 2009, 08, 07, 06]

Ans. Expression for torque experienced by dipole 1
Identifying two pairs of perpendicular vectors in expression $\frac{1}{2}$
Diagrammatic orientation of dipole in field for (i) maximum (ii) half of the maximum value (iii) zero torque $\frac{1}{2}$

[CBSE Marking Scheme, 2012]

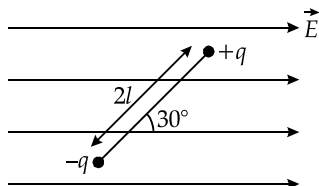
Detailed Answer :

- (i) Try yourself, Similar to Question 2(i) Short Answer Type Questions-II 1
(ii) Try yourself, Similar to Question 9(ii) Short Answer Type Questions-II 1
(iii) Diagrammatic orientation of dipole in field for (i) maximum (ii) half maximum value (iii) zero torque 1
(i) If θ is 90° , torque will be maximum, and is equal to pE which shows dipole is perpendicular to direction of uniform electric field E .

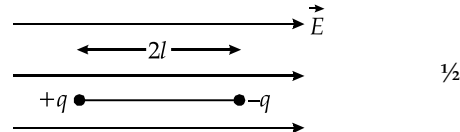


- (ii) If θ is 30° , torque will be $pE \sin \theta$ which is equal to $\frac{pE}{2}$

Since $\sin 30^\circ$ is $\frac{1}{2}$ that shows that torque will be half of the maximum value and dipole will be at 30° to the direction of uniform electric field E .

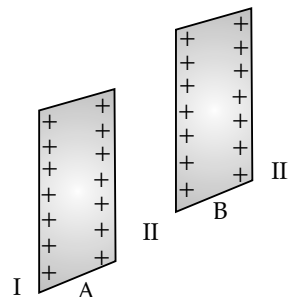


- (iii) If θ is 0° , torque will be minimum that is equal to 0 since $\sin 0^\circ$ is zero that shows that dipole to be parallel to direction of uniform electric field E .



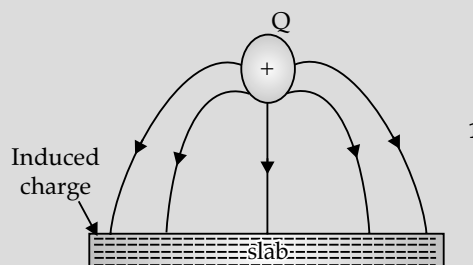
Q. 12. (i) A point charge (+Q) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines between the charge and the plate.

- (ii) Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the fields in the regions marked II and III.



[Foreign Set I, II, III 2014]

Ans. (i)



- (ii) (a) For region II,

$$E_{II} = \frac{1}{2\epsilon_0} (\sigma_1 - \sigma_2) \quad \frac{1}{2}$$

towards right side from sheet A to sheet B. $\frac{1}{2}$

- (b) For region III,

$$E_{III} = \frac{1}{2\epsilon_0} (\sigma_1 + \sigma_2) \quad \frac{1}{2}$$

towards right side away from the two sheets. $\frac{1}{2}$

[CBSE Marking Scheme, 2014]



Long Answer Type Questions

(5 marks each)

Q. 1. (i) Derive an expression for electric field E due to a dipole of length ' $2a$ ' at a distant point ' r ' from the centre of the dipole on an axial line.

- (ii) Draw a graph of E versus r for $r > a$

(iii) If this dipole were kept in a uniform external electric field E_0 , diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

[O.D. I, II, III 2017]

Ans. (i) Derivation of E along the axial line of dipole 2

- (ii) Graph of E versus r 1

(iii) (a) Diagrams for stable and unstable equilibrium of dipole $\frac{1}{2} + \frac{1}{2}$

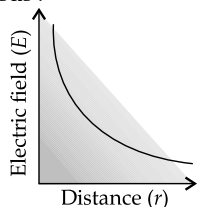
(b) Torque on dipole in both the cases $\frac{1}{2} + \frac{1}{2}$

[CBSE Marking Scheme, 2017]

Detailed Answer :

- (i) Derivation of E along the axial line of dipole
Try yourself, Similar to Question 6, Short Answer Type Questions-I 1

- (ii) Graph of E versus r



- (iii) (a) Diagrams for stable and unstable equilibrium of dipole. 1

Try yourself Similar to Question 1 (ii), Short Answer Type Questions-II 1

- (b) Torque on dipole in both the cases.

Torque on dipole = $pE \sin \theta$

for stable equilibrium 1

$$\theta = 0^\circ$$

$$\therefore \text{Torque} = 0$$

For unstable equilibrium

$$\theta = 180^\circ$$

Then new Torque = 0 1

- Q. 2. (i) Derive the expression for electric field at a point on the equatorial line of an electric dipole.

- (ii) Two point charges $+4 \mu\text{C}$ and $+1 \mu\text{C}$ are separated by distance of 2 m in air. Find the point on the line joining charges at which the net electric field of the system is zero? [O.D. (Comptt.) I, II, III 2017]

Ans. (i) Derivation of expression for electric field. 3

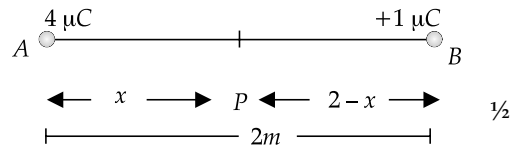
- (ii) Finding point on line joining charges 2

[CBSE Marking Scheme, 2017]

Detailed Answer :

- (i) Try yourself, Similar to Q. 1 (i), Short Answer Type Questions-II. 1/2

- (ii) Two point charges $q_A = +4 \mu\text{C}$ and $q_B = +1 \mu\text{C}$ are located 2 m apart in air as shown. P be a point, at a distance x from A , where electric field of the system is zero.



Distance between the two charges, $AB = 2 \text{ m}$

$\therefore PA = x \text{ m}, PB = (2 - x) \text{ m}$

Electric field at point P caused by $+4 \mu\text{C}$ charge,

$$E_1 = \frac{4 \times 10^{-6}}{4\pi\epsilon_0 (PA)^2} \text{ N/C along PA} \quad 1/2$$

Magnitude of electric field at point P caused by $+1 \mu\text{C}$ charge,

$$E_2 = \frac{1 \times 10^{-6}}{4\pi\epsilon_0 (PB)^2} \text{ N/C along PB}$$

$$\text{Now, } \frac{4 \times 10^{-6}}{4\pi\epsilon_0 (x)^2} = \frac{1 \times 10^{-6}}{4\pi\epsilon_0 (2-x)^2}$$

(Since net electric field at P is zero) 1

$$(2-x)^2 4 = (x)^2 1$$

$$16 - 16x + 4x^2 = x^2$$

$$4(4-x) - 3x(4-x) = 0$$

$$(4-3x)(4-x) = 0$$

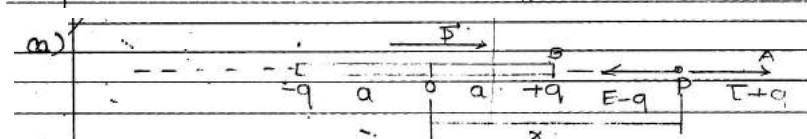
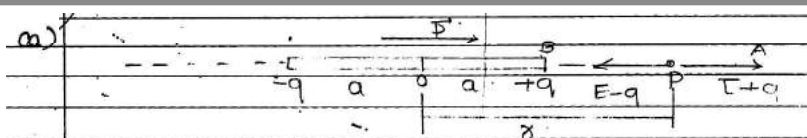
$$x = \frac{4}{3}, x = 4, \text{ possible point on the line will be } x = \frac{4}{3} \quad 1/2$$

- Q. 3. (a) Derive an expression for the electric field E due to a dipole of length ' $2a$ ' at a point distant r from the centre of the dipole on the axial line.

- (b) Draw a graph of E versus r for $r \gg a$.

- (c) If this dipole were kept in a uniform external electric field E_0 , diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases. [O.D. Set III 2017]

Ans. (a)



Consider a dipole having dipole moment p .
Electric field due to $-q$ at the point P along PB

$$E_{-q} = \frac{1}{4\pi\epsilon_0} \frac{q}{(a+x)^2} \text{ along PB}$$

Electric field due to $+q$ at the point P along PA

$$E_{+q} = \frac{1}{4\pi\epsilon_0} \frac{q}{(a-x)^2} \text{ along PA}$$

The net electric field

$$= E_q - E_{-q}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r+a)^2} - \frac{1}{(r-a)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4ra}{(r^2 - a^2)^2}$$

$$= \frac{2aq \times a}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

$$= \frac{2aq}{4\pi\epsilon_0 (r^2 - a^2)^2} \text{ along PA.}$$

Electric field at point D is

$$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{(r^2 - a^2)^2} \text{ in the direction of } \vec{D}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{D}}{(r^2 - a^2)^2}$$

ch) For $r \gg a$

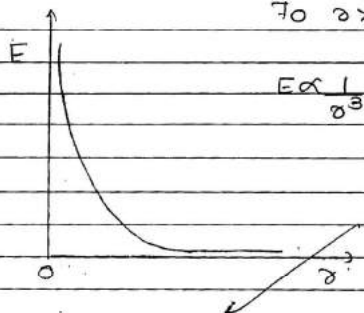
$$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

$$E \propto \frac{1}{r^3}$$

For $r \gg a$

$$E \propto \frac{1}{r^3}$$



ce) When $\theta = 0$

$$\text{energy} = -pE$$

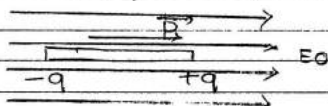
$$\text{Torque} = 0$$

It is stable equilibrium

When dipole is in stable equilibrium

$$\text{potential energy} = -pE$$

$$\text{Torque} = 0$$



$\vec{p}$ is parallel to $\vec{E}_0$

The dipole is placed parallel to the electric field.

In unstable equilibrium
 $\theta = 180^\circ$
 $p \cdot E = -PE \cos 180$
 $= -PE \times -1$
 $= PE$
 potential energy is maximum



The dipole is placed antiparallel to the field

Torque
 $\tau = p \times E$
 $PE \sin \theta$
 In 1st case $\tau = PE \sin \theta$
 In 2nd case $\tau = PE \sin 180$
 $= 0$

[Topper's Answer 2017]

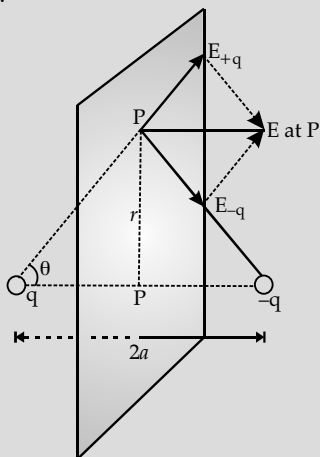
Q. 4. (i) Define electric dipole moment. Is it a scalar or a vector quantity? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

(ii) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero. [O.D. I, II, III 2013]

Ans. (i) Electric dipole moment : The strength of an electric dipole is measured by the quantity of electric dipole moment. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges.
 Electric dipole moment, $p = q \times 2a$
 It is a vector quantity.

In vector form, it is written as $\vec{p} = q \times 2a \hat{p}$, where the direction of $2a \hat{p}$ is from negative charge to positive charge.

Electric field of dipole at points on the equatorial plane :



The magnitudes of the electric field due to the two charges $+q$ and $-q$ are given by,

$$|\vec{E}_{+q}| = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{(r^2 + a^2)}$$

$$|\vec{E}_{-q}| = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{(r^2 + a^2)}$$

$$|\vec{E}_{+q}| = |\vec{E}_{-q}|$$

The direction of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ are shown in the figure. The components normal to the dipole axis cancel out. The components along the dipole axis add up.

$\therefore$ Total electric field

$$\vec{E} = -(E_{+q} + E_{-q}) \cos \theta \hat{p}$$

[Negative sign shows that field is opposite to $\hat{p}$]

$$\vec{E} = \frac{-2qa}{4\pi\epsilon_0(r^2 + a^2)^{3/2}} \hat{p} \quad \dots(i)$$

At large distances ($r \gg a$), this reduces to

$$\vec{E} = \frac{-2qa}{4\pi\epsilon_0 r^3} \hat{p} \quad \dots(ii)$$

$$|\vec{E}_{+q}| = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{(r^2 + a^2)} \quad \dots(iii)$$

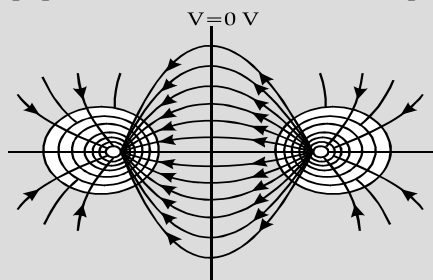
$$|\vec{E}_{-q}| = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{(r^2 + a^2)} \quad \dots(iv)$$

$$\therefore |\vec{E}_{+q}| = |\vec{E}_{-q}|$$

$$\therefore \vec{p} = 2qa\hat{p}$$

$$\therefore \vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3} \quad 3$$

(ii) Equipotential surface due to electric dipole :



The potential due to the dipole is zero at the line bisecting the dipole length. 2

[CBSE Marking Scheme, 2013]

Q. 5. A dipole is made up of two charges $+q$ and $-q$ separated by a distance $2a$. Derive an expression for the electric field $\vec{E}_e$ due to this dipole at a point distance r from the centre of the dipole on the equatorial plane. Draw the shape of the graph, between $|E_e|$ and r when $r \gg a$.

If this dipole were to be put in a uniform external electric field $\vec{E}$ obtain an expression for the torque acting on the dipole.

[R] [O.D. I, II, III, 2014; CBSE SQP 2014]

Ans. Derivation of expression for electric field $\vec{E}_e$ due to dipole 3
 Shape of the graph between $|E_e|$ and r when $r \gg a$ 1
 Expression for torque acting on dipole 1

[CBSE Marking Scheme, 2014]

Detailed Answer :

Derivation of expression for electric field $\vec{E}_e$ due to dipole :

Try yourself, Similar to Q. 4 Long Answer Type. ½

Shape of the graph between $|E_e|$ and r when $r \gg a$:

Try yourself, Similar to Q. 1 (ii), Long Answer Type Questions. 1½

Expression for torque acting on dipole : 2

Try yourself, Similar to Q. 2, Short Answer Type Questions-II.



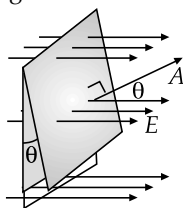
TOPIC-2

Gauss's Theorem and Its Applications

Revision Notes

Electric Flux

- Electric flux is proportional to algebraic number of electric field lines leaving the surface, outgoing lines with positive sign, incoming lines with negative sign.

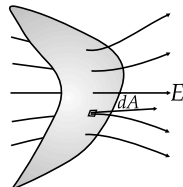


- Due to arbitrary arrangement of electric field lines, electric flux can be quantify as $\phi_E = EA$
- If vector A is perpendicular to surface, magnitude of vector A parallel to electric field is $A \cos \theta$

$$A_{\parallel} = A \cos \theta$$

$$\phi_E = EA_{\parallel} = EA \cos \theta$$

- In non-uniform electric field, the flux will be $\phi_E = \int E dA$



Continuous Charge Distribution

- It is a system in which the charge is uniformly distributed over the material. In this system, infinite number of charges are closely packed and have minor space among them. Unlike from the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the material. There are three types of continuous charge distribution system.

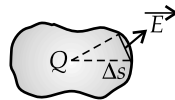
- For linear Charge Distribution (λ), $\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_x \frac{\lambda}{r^2} dX\hat{r}$ (Where, λ = linear charge density)

- For surface Charge Distribution (σ), $\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_s \frac{\sigma}{r^2} dS\hat{r}$ (Where, σ = surface charge density)

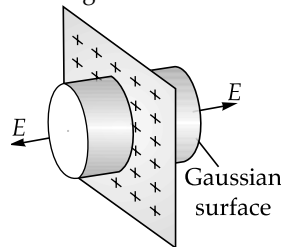
- For volume Charge Distribution (ρ), $\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_v \frac{\rho}{r^2} dV\hat{r}$ (Where, ρ = volume charge density)

Gauss law

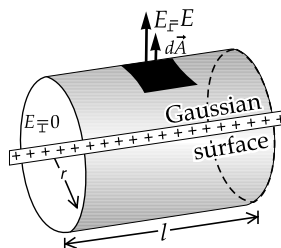
- The net outward normal electric flux through any closed surface of any shape is equal to $1/\epsilon_0$ times to net charge enclosed by the surface :



- The electric field flux at all points on Gaussian surface is $\phi = E \int dA = \frac{q}{\epsilon_0}$
- If there is a positive net flux, net positive charge is enclosed.
- If there is a negative net flux, net negative charge is enclosed.



- If there is zero net flux, no net charge is enclosed.
- The equation for electric field due to a point charge on Gaussian surface is $E = \frac{q}{4\pi r^2 \epsilon_0}$
- In an insulating sheet, charge remains in the sheet, so electric field, $E = \frac{\sigma}{2\epsilon_0}$



- Gauss' Law works in cases of cylindrical, spherical and rectangular symmetries.
- The field outside the wire points radially outward which depends on distance from wire, $\vec{E} = \frac{\lambda}{2\pi r \epsilon_0} \hat{n}$, where, λ is linear density of charge.
- **Closed surface** : It is a surface which divides the space in inside and outside region, where one can't move from one region to another without crossing the surface.
- **Gaussian surface** : It is a hypothetical closed surface having similar symmetry as problem on which we are working.
- **Electrostatic Shielding** : It is the phenomenon of protecting certain region of space from external electric field.
- **Dielectric** : The non-conducting material in which charges are easily produced on the application of electric field is called dielectric. e.g. Air, H_2 gas, glass, mica, paraffin wax, transformer oil etc.

Key Formulae

- Electric flux through an area A : $\phi = E.A = EA \cos \theta$
 ➤ Electric flux through a Gaussian surface : $\phi = \int E.dS$

- Gauss's Law : $\phi = \frac{q_{enc}}{\epsilon_0}$

- Electric Field due to an infinite line of charge : $E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$

where, E = electric field [N/C], λ = charge per unit length [C/m]

ϵ_0 = permittivity of free space = 8.85×10^{-12} [C²/Nm²], r = distance (m), $k = 9 \times 10^9$ Nm²C⁻²

- Electric field due to a ring at a distance x is : $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{(r^2 + x^2)^{3/2}}$

- When, $x \gg r$: $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$

- When $x \ll r$: $E = 0$

- Electric field due to a charged disc : $E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{R^2 + x^2}} \right]$

where ,

E = electric field [N/C]

$\epsilon_0 = 8.85 \times 10^{-12}$ [C²/Nm²]

R = radius of the disc [m]

σ = charge per unit area [C/m²]

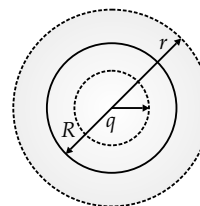
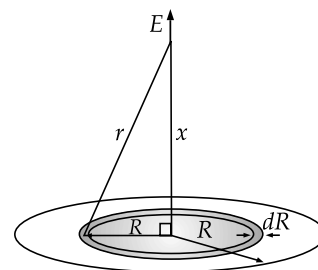
x = distance from charge [m]

- Electric field due to a thin infinite sheet : $\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$

- Electric field inside a spherical shell : $\vec{E} = \vec{0}$ for $r < R$

- Electric field outside a spherical shell $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$ or $E = \frac{kq}{r^2}$ for $r > R$

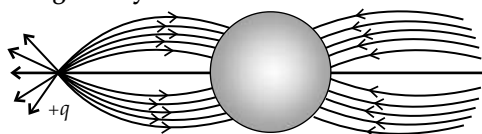
where, r = distance from the centre of shell to the point of measurement in m.



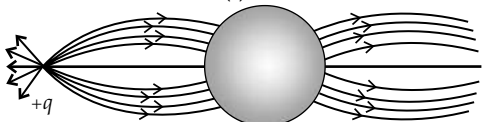
? Objective Type Questions

(1 mark each)

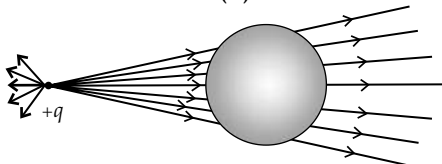
Q.1. A point positive charge is brought near an isolated conducting sphere in Figure. The electric field is best given by :



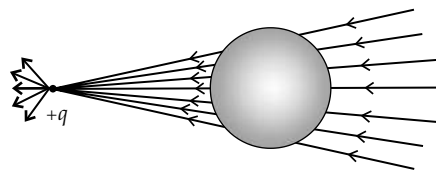
(i)



(ii)



(iii)



(iv)

(a) Fig (i)

(c) Fig (ii)

(b) Fig (iii)

(d) Fig (iv)

[NCERT Exemplar]

Ans. Correct option : (b)

Explanation : As given charge is $+q$ therefore lines of forces in positive charge must be outwards from positive charge q . Now, as the positive charge is kept near an isolated conducting sphere, due to induction left part of sphere gets accumulated negative charge and right part positive, and lines of force from right part of sphere must emerge outwards normally.

So, verifies the answer (b).

As lines of force are not perpendicular to the surface of sphere, so options (b) and (d) are not true again.

Q. 2. A point charge $+q$, is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is

- directed perpendicular to the plane and away from the plane.
- directed perpendicular to the plane but towards the plane.
- directed radially away from the point charge.
- directed radially towards the point charge.

[NCERT Exemplar]

Ans. Correct option : (a)

Explanation : When you place a positive charge near a conducting plane, then electric field lines from positive charges will enter into the conducting plane (from the side where positive charge is kept) and emerge from opposite side of the plane.

In both cases, the direction of electric field lines will always be perpendicular to the surface of the plane.

Q. 3. A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed

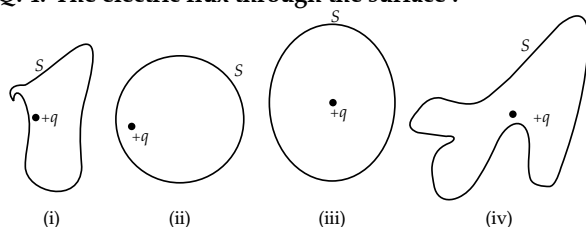
- perpendicular to the diameter.
- parallel to the diameter.
- at an angle tilted towards the diameter.
- at an angle tilted away from the diameter.

[NCERT Exemplar]

Ans. Correct option : (a)

Explanation : As the side or diameter of hemisphere is plane surface, and whole hemisphere is charged with positive charge so, the electric field line of forces emerging outward will be perpendicular to the plane surface or diameter.

Q. 4. The electric flux through the surface :



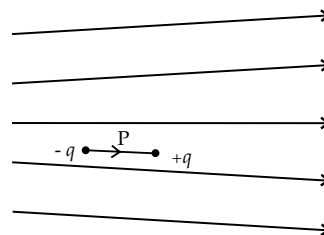
- in Figure (iv) is the largest.
- in Figure (iii) is the least.
- in Figure (ii) is same as Figure (iii) but is smaller than Figure (iv).
- is the same for all the figures.

[NCERT Exemplar]

Ans. Correct option : (d)

Explanation : Electric flux, through the closed surface (or space) depends only on the charge enclosed inside the surface. Here, charges inside all figures are same. So, electric flux will remain same.

Q. 5. Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct?



- The dipole will not experience any force.
- The dipole will experience a force towards right.
- The dipole will experience a force towards left.
- The dipole will experience a force upwards.

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : We know electric field emerges radially outward from positive point charge.

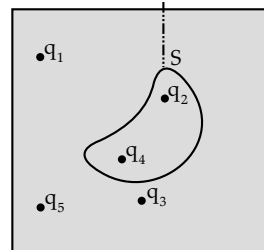
In the figure given above, space between field lines is increasing (or density of electric field line is decreasing). In other words, the electric force is decreasing while moving from left to right.

Thus, the force on charge $-q$ is greater than the force on charge $+q$ in turn dipole will experience a force towards left direction.

Q. 6. Five charges q_1, q_2, q_3, q_4 and q_5 are fixed at their positions as shown in Figure. S is a Gaussian surface. The Gauss's law is given by :

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{q}{\epsilon_0}$$

Gaussian Surface



Which of the following statements is correct?

- E on the LHS of the above equation will have a contribution from q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only.
- E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.
- E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only.
- Both E on the LHS and q on the RHS will have contributions from q_2 and q_4 only.

[NCERT Exemplar]

Ans. Correct option : (b)

Explanation : As all charges are positive (or of same signs) so electric field lines on R.H.S. of Gaussian surface will be due to q_2 and q_4 only.

On L.H.S. of Gaussian surface, the electric field lines on ' E ' will be due to q_1, q_2, q_3, q_4 and q_5 . So, answer (b) is verified.



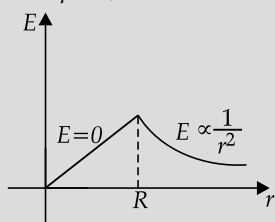
Very Short Answer Type Questions

(1 mark each)

Q. 1. Draw a plot showing variation of electric field with distance from centre of a solid conducting sphere of radius R having a charge of $+Q$ on its surface. [Delhi (Comptt.) I, II, III 2017]

Ans. Field at an inside point ($r < R$)

As $q = 0$, $E = 0$.



[CBSE Marking Scheme, 2017]

Q. 2. Define the term 'electric flux'. Write its S.I. unit. [Delhi Comptt. I, II, III 2013]

Ans. Electric flux through an area is the product of magnitude of area and the component of electric field vector normal to it.

$$\phi = \Delta S(E \cos \theta)$$

$$\phi = \vec{E} \cdot \vec{\Delta S}$$

Or, Electric flux is defined as the total number of electric field lines passing through an area normal to it.

Or, Electric flux is the property of an electric field which may be thought of as number of electric lines of force which intersects at given area.

The S.I. unit of 'electric flux' is NC^{-1}m^2 . 1

[CBSE Marking Scheme, 2013]

Q. 3. State Gauss's law in electrostatics. [Delhi I, II, III 2016]

Ans. Gauss's law in electrostatics : "The surface integral of electrostatic field $\vec{E}$ produced by any source over any closed surface S enclosing a volume V in vacuum, i.e., total electric flux over the closed surface S in vacuum, is $\frac{1}{\epsilon_0}$ times the total charge (Q) contained inside S , i.e.,

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \quad 1$$

[CBSE Marking Scheme, 2016]

Commonly Made Error

- Most of the candidates are unable in stating gauss law correctly. Key words like net charges, closed surface etc. are missed by students. Some candidates write magnetic flux instead of electric flux.

Q. 4. What is the electric flux through a cube of side 1 cm which encloses an electric dipole ?

[Delhi I, II, III 2015]

Ans. Zero (as net charge enclosed by the surface is zero.) 1

[CBSE Marking Scheme, 2015]

Detailed Answer :

In a cubic surface, the net electric charge will be zero since dipole carries equal and opposite charges. It is observed that the net electric flux through closed cubic surface will be

$$= \frac{\text{Charge enclosed}}{\epsilon_0}$$

and because the charge enclosed is zero,

$\therefore$ electric flux is also zero. 1

Q. 5. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased ?

[Delhi I, II, III 2016]

Ans. Electric flux remains unaffected. 1

[CBSE Marking Scheme, 2016]

Detailed Answer :

There will be no effect on electric flux, when the radius of Gaussian surface is increased because the charge enclosed by the gaussian surface remains the same.

The flux through Gaussian surface is given by

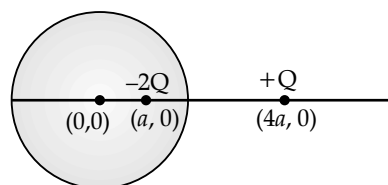
$$\phi = \frac{q_{\text{enc}}}{\epsilon_0} \text{ . As the flux is independent of radius, so}$$

there will be no effect on electric flux when radius of Gaussian surface increases.

Q. 6. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius ' $3a$ ' with its centre at the origin ? [O.D., I, 2013]

Ans. Gauss's theorem states that the electric flux through

a closed surface enclosing a charge is equal to $\left(\frac{1}{\epsilon_0}\right)$ times the magnitude of the charge enclosed.



The sphere encloses a charge of $-2Q$, so

$$\phi = \frac{2Q}{\epsilon_0} \quad 1$$

Q. 7. A charge q is placed at the centre of a cube of side l . What is the electric flux passing through each face of the cube ? [O.D. I, II, III 2012]

Ans. $\phi = q/6\epsilon_0$ 1
[CBSE Marking Scheme, 2012]

Detailed Answer :

If charge ' q ' is placed in centre of cube, electric flux

through the surface of cube will be 6ϕ , where ϕ is flux through each face. As per Gauss's law,

$$6\phi = \frac{q}{\epsilon_0}$$

$$\phi = \frac{q}{6\epsilon_0} \quad 1$$



Short Answer Type Questions-I

(2 marks each)

Q. 1. Define the term 'electric flux'. Write its SI unit.

What is the flux due to electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C through a square of side 10 cm, when it is held normal to $\vec{E}$? [R] [Comptt. O.D. I, II, III 2013]

Ans. Electric flux is the number of electric field lines passing through a given area.

Its SI unit : NC^{-1}m^2 or V-m

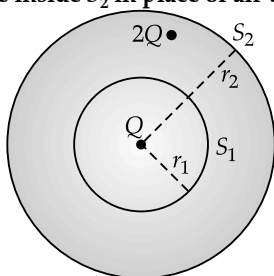
Given : $\vec{E} = 3 \times 10^3 \hat{i}$ N/C, $A = (10 \text{ cm})^2 = 0.01 \text{ m}^2$ and $\theta = 0^\circ$

$$\phi = EA \cos \theta$$

$$\phi = E.A = 3 \times 10^3 \times (0.1)^2 = 30 \text{ NC}^{-1}\text{m}^2 \quad 2$$

[CBSE Marking Scheme, 2013]

Q. 2. A sphere S_1 of radius r_1 encloses a net charge Q . If there is another concentric sphere S_2 of radius r_2 ($r_2 > r_1$) enclosing charge $2Q$, find the ratio of the electric flux through S_1 and S_2 . How will the electric flux through sphere S_1 change, if a medium of dielectric constant K is introduced in the space inside S_2 in place of air ?



[U] [Comptt. O.D. I, II, III 2014]

Ans. Ratio of electric flux through spheres S_1 and S_2 1

Change of electric flux through sphere S_1 1

Flux through S_1 , $\phi_1 = \frac{Q}{\epsilon_0}$ $\frac{1}{2}$

Flux through S_2 , $\phi_2 = \frac{Q+2Q}{\epsilon_0} = \frac{3Q}{\epsilon_0}$ $\frac{1}{2}$

Ratio of flux = $1 : 3$ $\frac{1}{2}$

No change in the flux through S_1 with dielectric medium inside the sphere S_2 . $\frac{1}{2}$

[CBSE Marking Scheme, 2014]

Q. 3. Given a uniform electric field $\vec{E} = 5 \times 10^3 \hat{i}$ N/C. Find the flux of this field through a square of side 10 cm on a side whose plane is parallel to the y - z plane. What would be the flux through the same square if the plane makes a 30° angle with the x -axis ? [A] [Delhi I, II, III, 2014]

Ans. Flux of this field through a square of 10 cm 1
Flux of the square with normal making 30° angle

$$\phi = EA \cos \theta \quad \frac{1}{2}$$

$$\phi = 5 \times 10^3 \times 10^{-2} \cos 0^\circ \text{ NC}^{-1} \text{ m}^2$$

$$\phi = 50 \text{ NC}^{-1} \text{ m}^2 \quad \frac{1}{2}$$

$$\phi = 5 \times 10^3 \times 10^{-2} \cos 60^\circ \text{ NC}^{-1} \text{ m}^2 \quad \frac{1}{2}$$

$$= 25 \text{ NC}^{-1} \text{ m}^2 \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2014]

Detailed Answer :

Given :

$E = 5 \times 10^3 \hat{i}$ N/C along (+) positive direction of x -axis.

$$\begin{aligned} \text{Surface area, } A &= 10 \text{ cm} \times 10 \text{ cm} \\ &= 0.10 \text{ m} \times 0.10 \text{ m} \\ &= 10^{-2} \text{ m}^2 \end{aligned}$$

(i) In case of plane parallel to y - z plane, normal to plane will be along x -axis, so $\phi = 0^\circ$

Electric flux will be calculated using $\phi = |\vec{E}| A \cos \theta$ $\frac{1}{2}$

$$= 5 \times 10^3 \times 10^{-2} \times \cos 0^\circ$$

$$= 50 \text{ Nm}^2/\text{C} \quad \frac{1}{2}$$

(ii) Since plane is making an angle of 30° with x -axis, so normal to its plane will make 60° with x -axis, so $\theta = 60^\circ$

Now finding Electric flux again with $\phi = |\vec{E}| A \cos \theta$

$$= 5 \times 10^3 \times 10^{-2} \times \cos 60^\circ \quad \frac{1}{2}$$

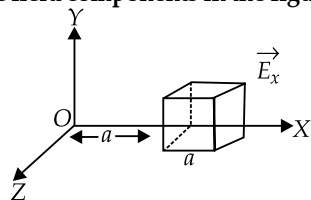
$$= 25 \text{ Nm}^2/\text{C} \quad \frac{1}{2}$$



Short Answer Type Questions-II

(3 marks each)

Q. 1. Define electric flux and write its SI unit. The electric field components in the figure shown are :



$E_x = \alpha x$, $E_y = 0$, $E_z = 0$ where $\alpha = \frac{100 \text{ N}}{\text{Cm}}$. Calculate the charge within the cube, assuming $a = 0.1 \text{ m}$.

[A&E] [OD Comptt. I, II, III 2018]

Ans. Definition of Electric flux	1
SI unit	$\frac{1}{2}$
Formula (Gauss's Law)	$\frac{1}{2}$
Calculation of Charge within the cube	1
Electric Flux is the dot product of electric field and area vector.	1
Also accept	

$$\phi = \oint \vec{E} \cdot d\vec{s}$$

SI Unit : Nm^2/C or volt-meter $\frac{1}{2}$

For a given case

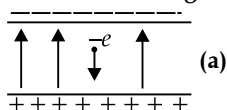
$$\begin{aligned} \phi &= \phi_1 + \phi_2 = [E_x(at x = 2a) - E_x(at x = a)]a^2 \\ &= [\alpha(2a) - \alpha(a)]a^2 \\ &= \alpha a^3 \\ &= 100 \times (0.1)^3 = 0.1 \text{ Nm}^2/\text{C} \quad \frac{1}{2} \end{aligned}$$

But $\phi = \frac{q}{\epsilon_0}$

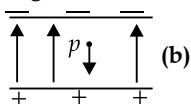
$$\therefore q = \epsilon_0 \phi = 8.854 \times 10^{-12} \times 0.1 \text{ C} = 0.8854 \text{ pC}$$

[CBSE Marking Scheme, 2018]

[AI] Q. 2. An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0 \times 10^4 \text{ N/C}$



Calculate the time it takes to fall through this distance starting from rest.



If the direction of the field is reversed (fig. b) keeping its magnitude unchanged, calculate the time taken by a proton to fall through this distance starting from rest. [A&E] [OD Comptt. I, II, III 2018]

Ans. Relevant formulae	1
Calculation of time taken by the electron	1
Calculation of time taken by the proton	1

We have
Force = qE
Acceleration

$$a = \frac{qE}{m}$$

Also

$$s = \frac{1}{2}at^2 \quad \text{as } u = 0$$

$$\therefore t = \sqrt{\frac{2s}{a}} \quad \frac{1}{2}$$

(i) For the electron

$$a = \frac{eE}{m} \quad \frac{1}{2}$$

$$\therefore t = \sqrt{\frac{3 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2.0 \times 10^4}} = 2.92 \text{ ns} \quad \frac{1}{2}$$

(ii) for proton

$$t = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 2 \times 10^4}} = 0.125 \mu\text{s} \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2018]

Q. 3. State Gauss law in electrostatics. Derive an expression for the electric field due to an infinitely long straight uniformly charged wire.

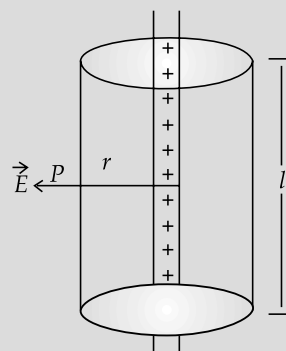
[A&E] [Delhi Comptt. I, II, III 2017]

Ans. Statement of Gauss Law	1
Derivation of electric field due to infinitely long straight uniformly charged wire	2
The surface integral of electric field over a closed surface is equal to $\frac{1}{\epsilon_0}$ times the charge enclosed	

by the surface.

Alternatively,

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \quad 1$$



Flux through the Gaussian surface

= flux through the curved cylindrical part of the surface

$$= E \times 2\pi r l \quad \frac{1}{2}$$

$$\begin{aligned}
 \text{Charge enclosed by the surface} &= \frac{\lambda}{2\pi\epsilon_0 r} \\
 &= E \times (2\pi r l) \\
 &= \frac{\lambda l}{\epsilon_0} \quad \frac{1}{2} \\
 E &= \frac{\lambda}{2\pi\epsilon_0 r} \quad \frac{1}{2}
 \end{aligned}$$

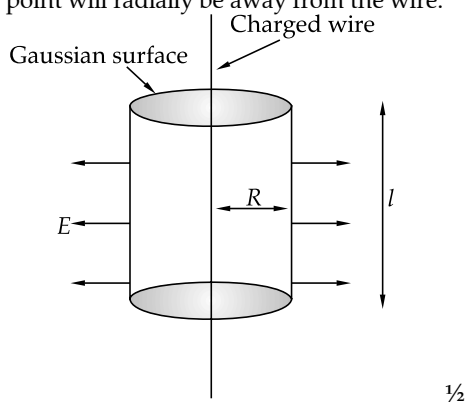
[CBSE Marking Scheme, 2017]

Detailed Answer :

Gauss Law states that total electric flux over the closed surface S will be $1/\epsilon_0$ times the total charge

$$\text{which is } \phi_E = \oint E ds = \frac{q}{\epsilon_0} \quad 1$$

In a long straight wire with uniform charge per unit length λ , there should be electric field generated by charge distribution for cylindrical symmetry. Also, field to point will radially be away from the wire.



In this, cylindrical gaussian surface is co-axial with the wire of radius R and length l where symmetry implies to electric field generated by wire that will be perpendicular to curved surface of cylinder, so as per Gauss' law,

$$E(R) \times 2\pi R l = \frac{\lambda l}{\epsilon_0} \quad \frac{1}{2}$$

where, $E(R)$ is electric field strength which acts at perpendicular distance R from the wire.

In figure, left part shows electric flux through Gaussian surface while right part shows total charge enclosed by cylinder which is divided by ϵ_0 .

$$\text{Further, } E(R) = \frac{\lambda}{2\pi\epsilon_0 R} \quad \frac{1}{2}$$

Here, the field points radially away from the wire when $\lambda > 0$, and radially towards the wire when $\lambda < 0$.

Commonly Made Error

- Some candidates do not know the correct expression. Few candidates are not sure about $\frac{1}{R}$ or $\frac{1}{R^2}$ (dependence of E on R).

Q. 4. A charge Q is distributed uniformly over a metallic sphere of radius R . Obtain the expression for the electric field (E) and electric potential (V) at a point $0 < x < R$.

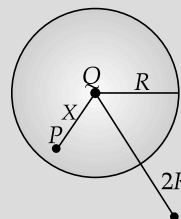
Show or plot the variation of E and V with x for $0 < x < 2R$. [A&E] [Foreign I, II, III 2017]

Ans. Expression for electric field 1½

Expression for potential ½

Plot of graph (E Vs r) ½

Plot of graph (V Vs r) ½



By Gauss law

$$\int \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \quad \frac{1}{2}$$

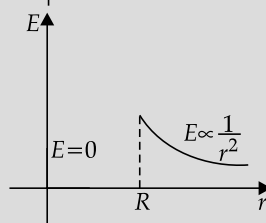
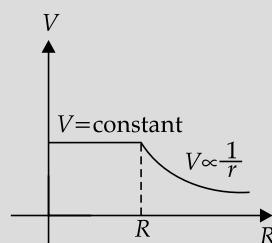
$q = 0$ in interval $0 < x < R$

$$E = 0 \quad \frac{1}{2}$$

or

$$E = -\frac{dV}{dr} \quad \frac{1}{2}$$

$$\text{Hence, } V = \text{constant} = V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$$



[Even if a student draws E and V for $0 < r < R$ award $\frac{1}{2} + \frac{1}{2}$ mark]

[CBSE Marking Scheme, 2017]

Q. 5. A long charged cylinder of linear charge density $+\lambda_1$ is surrounded by a hollow coaxial conducting cylinder of linear charge density $-\lambda_2$. Use Gauss's law to obtain expressions for the electric field at a point (a) in the space between the cylinders, and (b) outside the larger cylinder.

[A&E] [Foreign I 2017]

Ans. Gauss's law 1/2

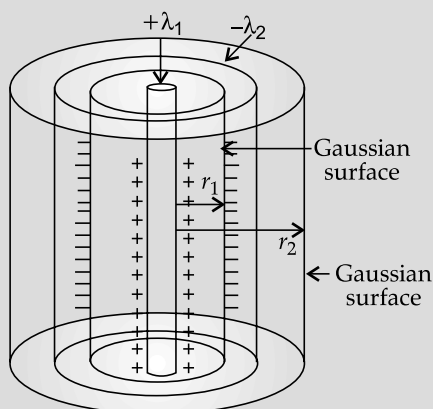
Diagram 1/2

Electric field between the cylinders 1

Electric field outside the cylinders 1

As Gauss's Law states :

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} \quad 1/2$$



(i) $\oint \vec{E} \cdot d\vec{s} = \frac{\lambda l}{\epsilon_0}$ 1/2

$$\vec{E}_1 = \frac{\lambda_1}{2\pi\epsilon_0 r_1} \hat{r}_1 \quad 1/2$$

(ii) $\oint \vec{E}_2 \cdot d\vec{s} = \frac{(\lambda_1 - \lambda_2)l}{\epsilon_0}$ 1/2

$$\vec{E}_2 = \frac{(\lambda_1 - \lambda_2)}{2\pi\epsilon_0 r_2} \hat{r}_2 \quad 1/2$$

[CBSE Marking Scheme, 2017]

Detailed Answer :

- (i) The charge density of long cylinder having length l and radius r_1 is λ_1 while other cylinder having similar length which surrounds the first cylinder having radius r_2 . If E is electric field in a space between the two cylinders, then as per Gauss law, electric flux through Gaussian surface will be : 1/2

$$\phi = 2\pi r l E$$

where,

r = distance of point from common axis of cylinders

Further, if q is the total charge on cylinder, so electric flux will be

$$\phi = \frac{q}{\epsilon_0} \quad 1/2$$

where,

q = charge on inner cylinder

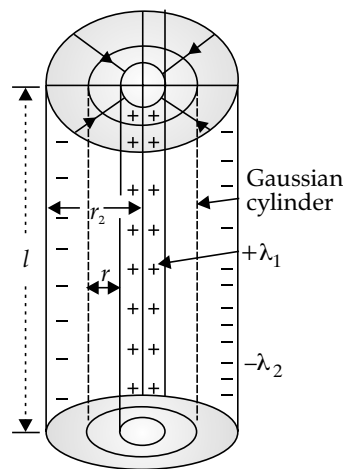
ϵ_0 = permittivity of free space

Further,

$$2\pi r l E = \frac{\lambda_1 l}{\epsilon_0} \quad 1/2$$

Hence, electric field between two cylinders is

$$E = \frac{\lambda_1}{2\pi\epsilon_0 r} \quad 1/2$$



- (ii) Also, when q is total charge on cylinder, then electric flux will be

$$\phi = 2\pi R l E$$

$$\phi = \frac{q}{\epsilon_0}$$

where, q = charge, ϵ_0 = permittivity of free space

Now, electric field at a point outside the larger cylinder will be

$$2\pi r_2 l E = \frac{(\lambda_1 - \lambda_2)l}{\epsilon_0} \quad 1/2$$

Electric field, $E = \frac{\lambda_1 - \lambda_2}{2\pi\epsilon_0 r_2}$ 1/2

Q. 6. Define electric flux. Write its SI unit.

Using Gauss's law, deduce an expression for electric field intensity due to an infinitely long straight uniformly charged wire.

[R] [Foreign II 2017]

Ans. Definition of electric flux 1/2

S.I. unit 1/2

Deducing the expression 2

- (i) Try yourself, Similar to Q. 2 Very Short Answer Type

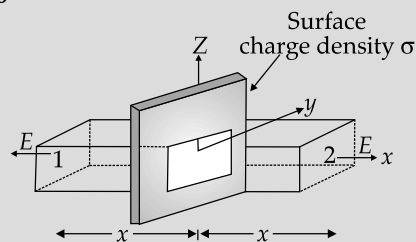
- (ii) Try yourself, Similar to Q. 1 Short Answer Type-II

[CBSE Marking Scheme, 2017]

Q. 7. Using Gauss's law in electrostatics, deduce an expression for electric field intensity due to a uniformly charged infinite plane sheet. If another identical sheet is placed parallel to it, show that there is no electric field in the region between the two sheets. [A&E] [Foreign III 2017]

Ans. Derivation of expression for electric field 2

Proving that there is no electric field between plates 1



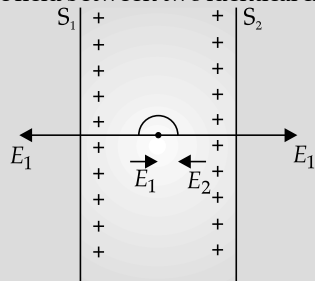
By Gauss's law $\oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$ 1/2

$\therefore 2EA = \frac{\sigma A}{\epsilon_0}$ 1/2

$\therefore E = \frac{\sigma}{2\epsilon_0}$

or $\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$ 1/2

Electric field between two identical charged sheets



$\therefore$ Both the sheets have same charge density, their electric fields will be equal and opposite in the region between the two sheets. 1/2
Hence the net field is zero. 1/2

[CBSE Marking Scheme, 2017]

Detailed Answer :

In the figure, if σ is uniform surface charge density of an infinite plane sheet with x -axis to be normal to the plane, then by symmetry, electric field will not depend on y and z coordinates and their directions. Considering Gaussian rectangular parallelepiped surface with cross-sectional area A , two faces 1 and 2 will contribute to flux where electric field lines result as parallel to other faces which do not contribute to total flux.

It is analysed that unit vector which is normal to first surface will be in negative x direction while unit vector which is normal to second surface will be in positive $+x$ direction, so flux $E \cdot \Delta S$ by the surfaces will result equal and will add up.

Hence,

net flux through Gaussian surface = $2EA$

charge enclosed by closed surface = σA

As per Gauss's law,

$$\oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0} \quad 1/2$$

$$\therefore 2EA = \frac{\sigma A}{\epsilon_0} \quad 1/2$$

Or, $E = \frac{\sigma}{2\epsilon_0}$ 1/2

This shows that the electric field around an infinite plane of charge does not vary with distance from the plane.

In terms of vector,

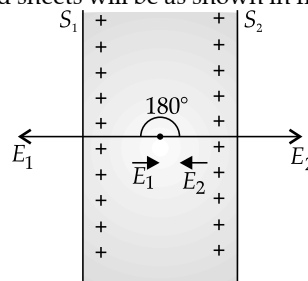
$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

where,

$\hat{n}$ = unit vector normal to plane and going away from it

$\vec{E}$ = directed away from plate if σ is (+) positive and toward the plate if σ is (-) negative.

Since the two charged infinite plates have identical charges, so electric field between two identical charged sheets will be as shown in figure :



Now electric field due to surface 1

$$\vec{E}_1 = \frac{\sigma}{2\epsilon_0}$$

Now electric field due to surface 2

$$\vec{E}_2 = -\frac{\sigma}{2\epsilon_0} \quad 1/2$$

Now the resultant electric field between the uniformly charged infinite plates = $\vec{E}_1 + \vec{E}_2$, $\theta = 180^\circ$

So,

$$\begin{aligned} E &= \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos \theta} \\ &= E_1 - E_2 \\ &= \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0 \end{aligned}$$

Q. 8. A point charge $+Q$ is placed at the centre O of an uncharged hollow spherical conductor of inner radius ' a ' and outer radius ' b '. Find the following :

(i) The magnitude and sign of the charge induced on the inner and outer surface of the conducting shell.

(ii) The magnitude of electric field vector at a distance

(i) $r = \frac{a}{2}$, and (ii) $r = 2b$, from the centre of the

shell. [A&E] [CBSE SQP 2017-18]

Ans. (i) As the electrostatic field inside a conductor is zero, using Gauss's law, charge on the inner surface of the shell = $-Q$ 1/2

Charge on the outer surface of the shell = $+Q$ 1/2

(ii) To show using Gauss's law

Expression for electric field for radius, $r = \frac{a}{2}$:

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Q}{a^2} \quad 1$$

Expression for electric field for radius, $r = 2b$:

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{b^2} \quad 1$$

[CBSE Marking Scheme, 2017]

Detailed Answer :

- (i) Charge placed at the centre of a shell is $+Q$. Hence, a charge of magnitude $-Q$ will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is $-Q$. $\frac{1}{2}$

A charge of $+Q$ is induced on the outer surface of the shell, so a charge of magnitude Q is placed on the outer surface of the shell. $\frac{1}{2}$

- (ii) The electric flux is electric field times the area of spherical surface

$$\phi = EA = 4E\pi r^2 = \frac{Q}{\epsilon_0}$$

So electric field outside the sphere ($r > R$) is seen to be identical to that of a point charge Q at center of sphere.

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Magnitude of electric field vector at a distance

$$r = \frac{a}{2}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Q}{a^2} \quad 1$$

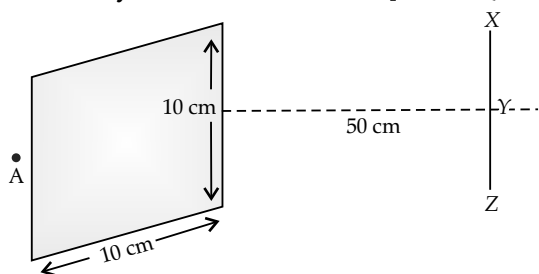
Magnitude of electric field vector at a distance

$$r = 2b$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Q}{4b^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{b^2} \quad 1$$

Q. 9. Given a uniformly charged plane/sheet of surface charge density $\sigma = 2 \times 10^{17} \text{ C/cm}^2$.

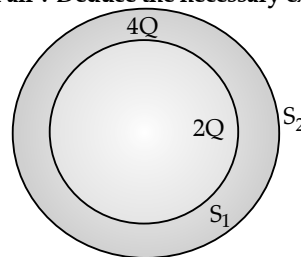
- (i) Find the electric field intensity at a point A, 5 mm away from the sheet on the left side.
 (ii) Given a straight line with three points X, Y and Z placed 50 cm away from the charged sheet on the right side. At which of these points, the field due to the sheet remains the same as that of point A and why ? [A&E] [CBSE SQP 2016]



Ans. (i) At point A, $E = \frac{\sigma}{2\epsilon_0} = \frac{2 \times 10^{17}}{2 \times 8.85 \times 10^{-12}} = 1.12 \times 10^{28} \text{ N/C} \quad 1\frac{1}{2}$

- (ii) At point Y, Because at 50 cm, the charge sheet acts as a finite sheet and thus the magnitude remains same towards the middle region of the plane sheet. [CBSE Marking Scheme, 2016] $1\frac{1}{2}$

Q. 10. Consider two hollow concentric spheres, S_1 and S_2 , enclosing charges $2Q$ and $4Q$ respectively as shown in the figure. (i) Find out the ratio of the electric flux through them. (ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant ' ϵ_r ' is introduced in the space inside S_1 in place of air ? Deduce the necessary expression.



[O.D. I, II, III 2014]

Ans. According to the Gauss's law, $\frac{1}{2}$

$$\phi = \oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\therefore \text{For sphere } S_1, \text{ flux enclosed} = \phi_1 = \frac{2Q}{\epsilon_0} \quad \frac{1}{2}$$

$$\text{and for sphere } S_2, \text{ flux enclosed} = \phi_2 = \frac{2Q + 4Q}{\epsilon_0} \quad \frac{1}{2}$$

$$\phi_2 = \frac{6Q}{\epsilon_0}$$

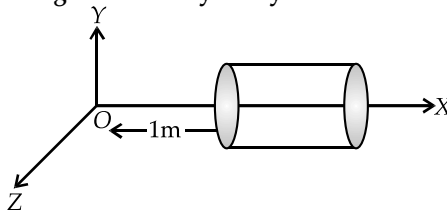
$$\therefore \frac{\phi_1}{\phi_2} = \frac{1}{3} \quad \frac{1}{2}$$

When a medium of dielectric constant ϵ_r is introduced in the space inside sphere S_1 , the flux through S_1 would be $\phi_1' = \frac{2Q}{\epsilon_r}$. 1

[CBSE Marking Scheme, 2014]

[AI] Q. 11. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50x \hat{i}$, where E is in NC^{-1} and x is in metres. Find

- (i) Net flux through the cylinder.
 (ii) Charge enclosed by the cylinder.



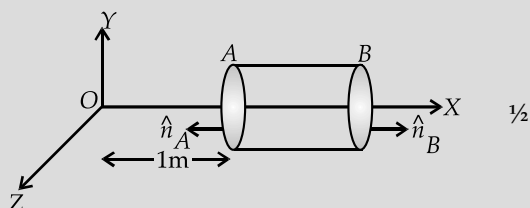
[A&E] [Delhi I, II, III 2013]

Ans. (i) Given :

$$\vec{E} = 50x \hat{i}$$

and $\Delta S = 25 \text{ cm}^2 \text{ or } 25 \times 10^{-4} \text{ m}^2$

As the electric field is only along the x-axis, hence, flux will pass only through the cross-section of cylinder.



Magnitude of electric field at cross-section A,

$$E_A = 50 \times 1 = 50 \text{ N/C.}$$

Magnitude of electric field at cross-section B,

$$E_B = 50 \times 2 = 100 \text{ N/C}$$

The corresponding electric fluxes are

$$\begin{aligned}\oint \phi_A &= \vec{E} \cdot \vec{\Delta S} \\ &= 50 \times 25 \times 10^{-4} \times \cos 180^\circ \\ &= -0.125 \text{ Nm}^2/\text{C}^2\end{aligned}\quad \frac{1}{2}$$

$$\begin{aligned}\oint \phi_B &= \vec{E} \cdot \vec{\Delta S} \\ &= 100 \times 25 \times 10^{-4} \times \cos 0^\circ \\ &= 0.25 \text{ Nm}^2/\text{C}^2\end{aligned}\quad \frac{1}{2}$$

So, the net flux through the cylinder,

$$\begin{aligned}\oint \phi &= \oint \phi_A + \oint \phi_B \\ &= 0.125 + 0.25 \\ &= 0.375 \text{ Nm}^2/\text{C}^2\end{aligned}\quad \frac{1}{2}$$

(ii) Using the Gauss's law,

$$\phi = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\Rightarrow 0.375 = \frac{q}{8.85 \times 10^{-12}}$$

$$\Rightarrow q = 8.85 \times 10^{-12} \times 0.375 = 3.3 \times 10^{-12} \text{ C.} \quad 1$$

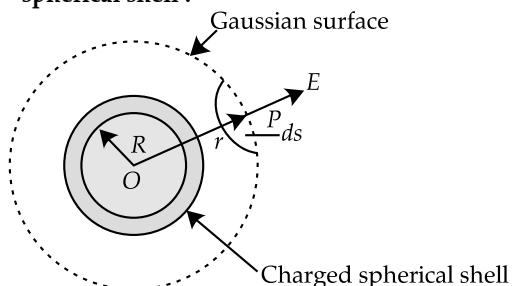
[CBSE Marking Scheme, 2013]

Q. 12. Using Gauss's law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside, and (ii) inside the shell.

Plot a graph showing variation of electric field as function of $r > R$ and $r < R$ (r being the distance from the centre of the shell).

[A] [O.D. & O.D. Comptt. I, II, III, 2013; Delhi I, II, III 2009]

Ans. Electric field due to a uniformly charged thin spherical shell :



(i) When point P lies outside the spherical shell :

Suppose that we have to calculate electric field at the point P at a distance r ($r > R$) from its centre. Draw the Gaussian surface through point P so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius r and centre O . 1

Let $\vec{E}$ be the electric field at point P , then the electric flux through area element $d\vec{S}$ is given by,

$$\Delta\phi = \vec{E} \cdot \vec{\Delta S}$$

Since $\vec{\Delta S}$ is also along normal to the surface,

$$\Delta\phi = E \cdot dS$$

$\therefore$ Total electric flux through the Gaussian surface is given by.

$$\phi = \oint_s E dS = E \oint_s dS$$

Now,

$$\oint_s dS = 4\pi r^2$$

$$\therefore \phi = E \times 4\pi r^2 \quad \dots(i)$$

Since the charge enclosed by the Gaussian surface is q , according to the Gauss's law,

$$\phi = \frac{q}{\epsilon_0} \quad \dots(ii)$$

From equations (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad (\text{for } r > R) \quad 1$$

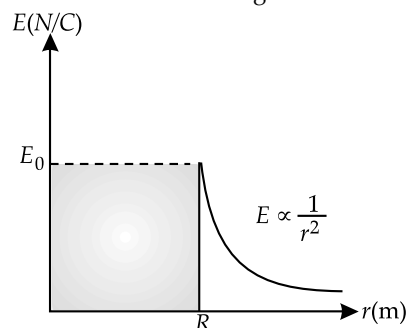
(ii) When point P lies inside the spherical shell :

In such a case, the Gaussian surface encloses no charge. According to the Gauss's law,

$$E \times 4\pi r^2 = 0$$

$$\text{i.e.,} \quad E = 0 \quad (r < R)$$

A graph showing the variation of electric field as a function of r is shown in figure.



Q. 13. Using Gauss's theorem, deduce an expression for the electric field intensity at any point due to a thin, infinitely long wire of charge/length λ C/m. 1

[A] [Delhi Comptt. I, II, III 2012; Delhi III, SQP II 2009]

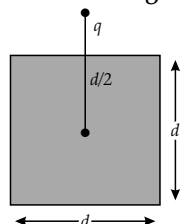
Ans. Try yourself, Similar to Q. 3 Short Answer Type Questions-II 3

? Long Answer Type Questions

(5 marks each)

Q. 1. (i) Define electric flux. Is it a scalar or a vector quantity ?

A point charge q is at a distance of $d/2$ directly above the centre of a square of side d , as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square.



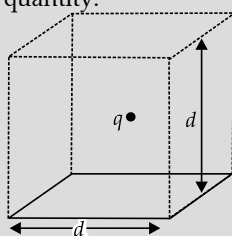
(ii) If the point charge is now moved to a distance ' d ' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected. **[R&A][CBSE Delhi OD 2018]**

Ans. (i) Definition of electric flux 1
Stating scalar/ vector 1/2
Gauss's law 1/2
Derivation of the expression for electric flux 1
(ii) Explanation of change in electric flux 2
(i) Electric flux through a given surface is defined as the dot product of electric field and area vector over that surface. 1
 Alternatively $\phi = \oint_s \vec{E} \cdot d\vec{S}$

Also accept

Electric flux, through a surface equals the surface integral of the electric field over that surface. 1/2

It is a scalar quantity.



Constructing a cube of side ' d ' so that charge ' q ' gets placed within of this cube (Gaussian surface)

According to Gauss's law the Electric flux

$$\phi = \frac{\text{Charge enclosed}}{\epsilon_0}$$

$$= \frac{q}{\epsilon_0} \quad 1/2$$

This is the total flux through all the six faces of the cube.

Hence electric flux through the square

$$\frac{1}{6} \times \frac{q}{\epsilon_0} = \frac{q}{6\epsilon_0} \quad 1/2$$

(ii) If the charge is moved to a distance d and the side of the square is doubled the cube will be constructed to have a side $2d$ but the total charge enclosed in it will remain the same. Hence the total flux through the cube and therefore the flux through the square will remain the same as before.

[Deduct 1 mark if the student just writes No change /not affected without giving any explanation.] **1+1**

[CBSE Marking Scheme 2018]

Q. 2. (i) Use Gauss' law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge density λ C/m.
(ii) Draw a graph to show the variation of E with perpendicular distance r from line of charge.
(iii) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$). **[A]**

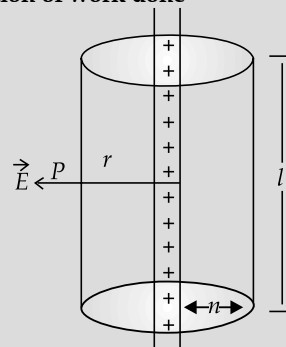
Ans. (i) Derivation of the expression for electric field $\vec{E}$

3

(ii) Graph to show the required variation of the electric field 1

(iii) Calculation of work done 1

(i)



1/2

To calculate the electric field, imagine a cylindrical Gaussian surface, since the field is everywhere radial, flux through two ends of the cylindrical Gaussian surface is zero. 1/2

At cylindrical part of the surface electric field $\vec{E}$ is normal to the surface at every point and its magnitude is constant.

Therefore flux through the Gaussian surface = Flux through the curved cylindrical part of the surface,

$$= E \times 2\pi r l \quad \dots(i) \quad 1/2$$

Applying Gauss's Law

$$\text{Flux,} \quad \phi = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\text{Total charge enclosed} = \text{Linear charge density} \times l$$

$$= \lambda l$$

$$\phi = \frac{\lambda l}{\epsilon_0} \quad \dots(ii) \quad 1/2$$

Using Equations (i) & (ii)

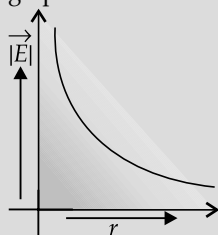
$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} \quad 1/2$$

$$\text{In vector notation,} \quad \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{n} \quad 1/2$$

(where $\hat{n}$ is a unit vector normal to the line charge)

(ii) The required graph is as shown :



1

(iii) Work done in moving the charge 'q' through a small displacement 'dr'

$$\begin{aligned} dW &= \vec{F} \cdot \vec{dr} \\ dW &= q\vec{E} \cdot \vec{dr} \\ &= qEdr \cos 0 \\ dW &= q \times \frac{\lambda}{2\pi\epsilon_0 r} dr \end{aligned} \quad \frac{1}{2}$$

Work done in moving the given charge from r_1 to r_2 ($r_2 > r_1$)

$$\begin{aligned} W &= \int_{r_1}^{r_2} dW = \int_{r_1}^{r_2} \frac{\lambda q dr}{2\pi\epsilon_0 r} \\ W &= \frac{\lambda q}{2\pi\epsilon_0} [\log_e r_2 - \log_e r_1] \quad \frac{1}{2} \\ W &= \frac{\lambda q}{2\pi\epsilon_0} \left[\log_e \frac{r_2}{r_1} \right] \end{aligned}$$

[CBSE Marking Scheme, 2018]

Q. 3. (i) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ .

(ii) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain an expression for the amount of work done in bringing a point charge q from infinity to a point, distant r , in front of the charged plane sheet. [R] [OD II, 2017]

Ans. (i) Similar to Q. 7, Short Answer Type II 3

(ii) Amount of work done :

$$\begin{aligned} W &= q \int_{\infty}^r \vec{E} \cdot d\vec{r} \quad \frac{1}{2} \\ W &= q \int_{\infty}^r (-E \cdot dr) \quad \frac{1}{2} \\ W &= -q \int_{\infty}^r \left(\frac{\sigma}{2\epsilon_0} \right) dr \quad \frac{1}{2} \\ W &= \frac{q\sigma}{2\epsilon_0} |_{\infty}^{-r} = \infty \quad \frac{1}{2} \end{aligned}$$

[CBSE Marking Scheme, 2017]

Q. 4. (i) State Gauss Law. Using this law, obtain the expression for the electric field due to an infinitely long straight conductor of linear charge density λ .

(ii) A wire of length L has linear charge density $\lambda = kx$, where x is measured from the end A of the wire. This wire is enclosed by Gaussian hollow surface. Find the expression for the electric flux through this Surface. [R] [O.D. (Comptt.) I, II 2017]

Ans. Statement of Gauss law	1
Derivation	2
Electric Flux Expression	2
[CBSE Marking Scheme, 2017]	

Detailed Answer :

(i) Try yourself, Similar to Q. 3, Short Answer Type Questions-II 1

(ii) Since the length of wire is l and linear charge density is $\lambda = kx$, so linear charge density $\lambda = dQ/dx$
Now,

$$dQ = \lambda dx = kx dx$$

$$Q = \int_0^l dQ = \int_0^l kx dx = \frac{1}{2} kl^2 \quad 2$$

As per Gauss law,

$$\phi = \frac{Q}{\epsilon_0} \quad 1$$

where, ϕ = electric flux, Q = net charge enclosed in Gaussian surface
 ϵ_0 = permittivity of medium

Hence,
$$\phi = \frac{kl^2}{2\epsilon_0} \quad 1$$

Q. 5. (i) Define electric flux. Write its S.I. unit.

(ii) Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(iii) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged ?

[R] [Delhi I, II, III 2012]

Ans. (i) Try yourself, Similar to Q. 2, Very Short Answer Type Questions 1

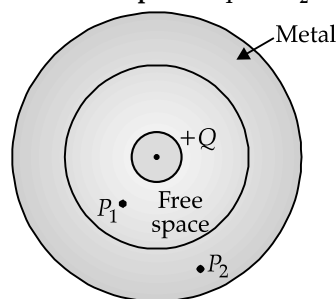
(ii) Try yourself, Similar to Q. 7, Short Answer Type Questions-II 3

(iii) For positively charged sheet, the field is directed away from the sheet. $\frac{1}{2}$
For negatively charged sheet, the field is directed towards the plane sheet. $\frac{1}{2}$

[CBSE Marking Scheme, 2012]

Q. 6. (i) Define electric flux. Write its S.I. unit. [R]

(ii) A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2 . [U]

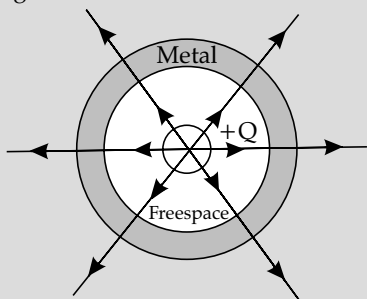


(iii) Draw the pattern of electric field lines in this arrangement. [Delhi, O.D. Comptt. I, II, III, 2012]

Ans. (i) Try yourself, Similar to Q. 2, Very Short Answer Type Questions 1

(ii) Try yourself, Similar to Q. 12, Short Answer Type Questions-II 3

(iii) The direction of electric field is shown in the figure. 1

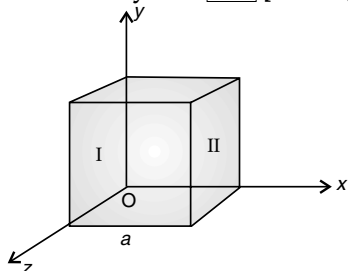


[CBSE Marking Scheme, 2012]

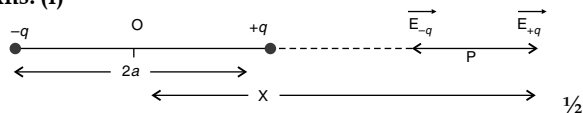
Q.7. (i) An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit

$$x \gg a, \vec{E} \rightarrow \frac{2\vec{p}}{(4\pi\epsilon_0 x^3)}.$$

(ii) Given the electric field in the region $\vec{E} = 2xi$, find the net electric flux through the cube and the charge enclosed by it. **R&U** [Delhi I, II, III 2015]



Ans. (i)



Electric field intensity at point P due to charge $-q$,

$$\vec{E}_{-q} = \frac{1}{4\pi\epsilon_0} \cdot \frac{-q}{(x+a)^2} (\hat{x})$$

Due to charge $+q$,

$$\vec{E}_{+q} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x-a)^2} (\hat{x})$$

Net electric field at point P,

$$\vec{E} = \vec{E}_{-q} + \vec{E}_{+q}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \times \left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right) (\hat{x}) \quad \frac{1}{2}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \times \left(\frac{4ax}{(x^2 - a^2)^2} \right) (\hat{x}) \quad \frac{1}{2}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \times \frac{(q \times 2a)2x}{(x^2 - a^2)^2} (\hat{x}) \quad \frac{1}{2}$$

$$= \frac{q}{4\pi\epsilon_0} \times \frac{2px}{(x^2 - a^2)^2} (\hat{x})$$

For $x \gg a, (x^2 - a^2)^2 \approx x^4$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \times \frac{2p}{x^3} (\hat{x}) = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{x^3} \quad \frac{1}{2}$$

(ii) From the given diagram,

Only the face perpendicular to the direction of x -axis, contribute to the electric flux. The remaining faces of the cube give zero contribution. 1

$$\begin{aligned} \text{Total flux } \phi &= \phi_I + \phi_{II} \\ &= \oint_I \vec{E} \cdot d\vec{s} + \oint_{II} \vec{E} \cdot d\vec{s} \quad \frac{1}{2} \\ &= 0 + 2(a) \cdot a^2 \quad \frac{1}{2} \\ &= 2a^3 \quad \frac{1}{2} \end{aligned}$$

Charge enclosed, $q = \epsilon_0 \phi = 2a^3 \epsilon_0$



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