05

Work, Energy and Power

TOPIC 1
Work and Energy

01 A particle is released from height S from the surface of the Earth. At a certain height its kinetic energy is three times its potential energy. The height from the surface of earth and the speed of the particle at that instant are respectively **[NEET 2021]**

(a) $\frac{S}{4}, \frac{3gS}{2}$ (b) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$
(c) $\frac{S}{2}, \frac{\sqrt{3gS}}{2}$ (d) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

Ans. *(d)*

Let the particle at height S from the surface of the Earth is as shown.

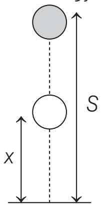
Given, at height x, kinetic energy
$= 3$ (potential energy)

$\Rightarrow \quad \text{KE} = 3\,mgx$

At height S, total energy, TE = PE + KE

TE = $mgS + 0 = mgS$

At height x, total energy,



$$\text{TE} = \text{PE} + \text{KE}$$
$$\text{TE} = mgx + 3mgx$$
$$mgS = 4mgx$$
$$\Rightarrow \quad x = \frac{S}{4} \quad \text{...(i)}$$

Now, we shall determine the speed of the particle at this height.

As, $\quad \text{KE} = 3 \times mgx$

$$\text{KE} = 3 \times mg\frac{S}{4} \quad \text{[from Eq. (i)]}$$
$$\frac{1}{2}mv^2 = \frac{3}{4}mgS$$
$$\Rightarrow \quad v = \sqrt{\frac{3}{2}gS}$$

02 The energy required to break one bond in DNA is 10^{-20} J. This value (in eV) is nearly **[NEET (Sep.) 2020]**

(a) 0.6 (b) 0.06
(c) 0.006 (d) 6

Ans. *(b)*

Given, $E = 10^{-20}$ J

In terms of eV, we get

$$E = \frac{10^{-20}}{1.6 \times 10^{-19}}\ \text{eV} = 0.06\ \text{eV}$$

Hence, correct option is (b).

03 A force $F = 20 + 10y$ acts on a particle in y-direction, where F is in newton and y in meter. Work done by this force to move the particle from $y = 0$ to $y = 1$ m is **[NEET (National) 2019]**

(a) 5 J (b) 25 J
(c) 20 J (d) 30 J

Ans. *(b)*

Work done by a force F, which is variable in nature in moving a particle from y_1 to y_2 is given by

$$W = \int_{y_1}^{y_2} F \cdot dy \quad \text{... (i)}$$

Here, force, $F = 20 + 10y$, $y_1 = 0$

and $\quad y_2 = 1$ m

Substituting the given values in Eq. (i), we get

$$\Rightarrow W = \int_0^1 (20 + 10y)dy = \left[20y + \frac{10y^2}{2}\right]_0^1$$
$$= 20(1-0) + 5(1-0)^2 = 25\ \text{J}$$

$\therefore$ Work done will be 25 J.

04 A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration, if the kinetic energy of the particle becomes equal to 8×10^{-4} **J** by the end of the second revolution after the beginning of the motion? **[NEET 2016]**

(a) 0.15 m/s^2 (b) 0.18 m/s^2
(c) 0.2 m/s^2 (d) 0.1 m/s^2

Ans. *(d)*

Given, mass of particle $m = 0.01$ kg.

Radius of circle along which particle is moving,

$r = 6.4$ cm.

$\because$ Kinetic energy of particle,

$$\text{KE} = 8 \times 10^{-4}\ \text{J}$$
$$\Rightarrow \quad \frac{1}{2}mv^2 = 8 \times 10^{-4}\ \text{J}$$
$$\Rightarrow \quad v^2 = \frac{16 \times 10^{-4}}{0.01} = 16 \times 10^{-2} \quad \text{...(i)}$$

As it is given that KE of particle is equal to 8×10^{-4} J by the end of second revolution after the beginning of motion of particle. It means, its initial velocity (u) is 0 m/s at this moment.

$\because$ By Newton's 3rd equation of motion,

$$v^2 = u^2 + 2a_t s$$

$$\Rightarrow \quad v^2 = 2a_t S \quad \text{or} \quad v^2 = 2a_t\,(4\pi r)$$

($\because$ particle covers 2 revolutions)

$$\Rightarrow a_t = \frac{v^2}{8\pi r} = \frac{16\times 10^{-2}}{8\times 3.14\times 6.4\times 10^{-2}}$$

($\because$ from Eq. (i), $v^2 = 16\times 10^{-2}$)

$$\therefore a_t = 0.1\,\text{m/s}^2$$

05 A particle moves from a point $(-2\hat{\mathbf{i}}+5\hat{\mathbf{j}})$ to $(4\hat{\mathbf{j}}+3\hat{\mathbf{k}})$ when a force of $(4\hat{\mathbf{i}}+3\hat{\mathbf{j}})$ N is applied. How much work has been done by the force? **[NEET 2016]**

(a) 8 J (b) 11 J (c) 5 J (d) 2 J

Ans. *(c)*

Position vectors of the particles are $\mathbf{r}_1 = -2\hat{\mathbf{i}}+5\hat{\mathbf{j}}$ and $\mathbf{r}_2 = 4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

$\therefore$ Displacement of the particle,

$$\Delta\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$$

$$= 4\hat{\mathbf{j}}+3\hat{\mathbf{k}}-(-2\hat{\mathbf{i}}+5\hat{\mathbf{j}}) = 2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$

Force on the particle, $\mathbf{F} = 4\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ N

$\therefore$ Work done, $W = \mathbf{F}\cdot\Delta\mathbf{s}$

$$= (4\hat{\mathbf{i}}+3\hat{\mathbf{j}})\cdot(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}) = 8-3 = 5\,\text{J}$$

06 Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. They are stretched, first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively **[CBSE AIPMT 2015]**

(a) $W_P = W_Q$; $W_P > W_Q$
(b) $W_P = W_Q$; $W_P = W_Q$
(c) $W_P > W_Q$; $W_Q > W_P$
(d) $W_P < W_Q$; $W_Q < W_P$

Ans. *(c)*

Given, $K_P > K_Q$

In case (a), the elongation is same

i.e. $x_1 = x_2 = x$

So, $W_P = \frac{1}{2}K_P x^2$ and $W_Q = \frac{1}{2}K_Q x^2$

$$\therefore \quad \frac{W_P}{W_Q} = \frac{K_P}{K_Q} > 1 \quad \Rightarrow W_P > W_Q$$

In case (b), the spring force is same

i.e. $F_1 = F_2 = F$

So, $x_1 = \frac{F}{K_P}, x_2 = \frac{F}{K_Q}$

$$\therefore \quad W_P = \frac{1}{2}K_P x_1^2 = \frac{1}{2}K_P\frac{F^2}{K_P^2} = \frac{1}{2}\frac{F^2}{K_P}$$

$$\text{and} \quad W_Q = \frac{1}{2}K_Q x_2^2 = \frac{1}{2}K_Q\cdot\frac{F^2}{K_Q^2} = \frac{1}{2}\frac{F^2}{K_Q}$$

$$\therefore \quad \frac{W_P}{W_Q} = \frac{K_Q}{K_P} < 1$$

$$\Rightarrow \quad W_P < W_Q$$

07 A block of mass 10 kg, moving in x-direction with a constant speed of 10 ms^{-1}, is subjected to a retarding force $F = 0.1x$ J/m during its travel from $x = 20$ m to 30 m. Its final KE will be **[CBSE AIPMT 2015]**

(a) 475 J (b) 450 J (c) 275 J (d) 250 J

Ans. *(a)*

From work-energy theorem,

Work done = Change in KE

$$\Rightarrow \quad W = K_f - K_i$$

$$\Rightarrow \quad K_f = W + K_i = \int_{x_1}^{x_2} F x dx + \frac{1}{2}mv^2$$

$$= \int_{20}^{30} -0.1.x dx + \frac{1}{2}\times 10\times 10^2$$

$$= -0.1\left[\frac{x^2}{2}\right]_{20}^{30} + 500$$

$$= -0.05[30^2 - 20^2] + 500$$

$$= -0.05[900 - 400] + 500$$

$$\Rightarrow \quad K_f = -25 + 500 = 475\,\text{J}$$

08 Two particles of masses m_1, m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy ε. If final velocities of particles be v_1 and v_2, then we must have **[CBSE AIPMT 2015]**

(a) $m_1^2 u_1 + m_2^2 u_2 - \varepsilon = m_1^2 v_1 + m_2^2 v_2$

(b) $\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \varepsilon$

(c) $\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 - \varepsilon = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$

(d) $\frac{1}{2}m_1^2 u_1^2 + \frac{1}{2}m_2^2 u_2^2 + \varepsilon = \frac{1}{2}m_1^2 v_1^2 + \frac{1}{2}m_2^2 v_2^2$

Ans. *(c)*

Total initial energy $= \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2$

Since, after collision one particle absorb energy ε.

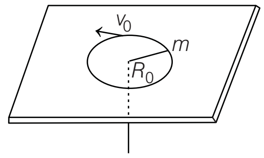
$\therefore$ Total final energy $= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_1^2 + \varepsilon$

From conservation of energy,

$$\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \varepsilon$$

$$\Rightarrow \quad \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 - \varepsilon = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

09 A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hole in the plane as shown.



The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is **[CBSE AIPMT 2015]**

(a) mv_0^2 (b) $\frac{1}{4}mv_0^2$ (c) $2\,mv_0^2$ (d) $\frac{1}{2}mv_0^2$

Ans. *(c)*

Conserving angular momentum

$$L_i = L_f$$

$$\Rightarrow \quad mv_0 R_0 = mv'\left(\frac{R_0}{2}\right) \Rightarrow v' = 2v_0$$

So, final kinetic energy of the particle is

$$K_f = \frac{1}{2}mv'^2 = \frac{1}{2}m(2v_0)^2$$

$$= 4\frac{1}{2}mv_0^2 = 2mv_0^2$$

10 A ball is thrown vertically downwards from a height of 20m with an initial velocity v_0. It collides with the ground, loses 50% of its energy in collision and rebounds to the same height. The initial velocity v_0 is (Take, $g = 10\,\text{ms}^{-2}$) **[CBSE AIPMT 2015]**

(a) 14 ms^{-1} (b) 20 ms^{-1}
(c) 28 ms^{-1} (d) 10 ms^{-1}

Ans. *(b)*

Suppose a ball rebounds with speed v,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20}$$
$$= 20 \text{ m/s}$$

Energy of a ball just after rebound,

$$E = \frac{1}{2} mv^2 = 200 \text{ m}$$

As, 50% of energy loses in collision means just before collision energy is 400 m.

According to law of conservation of energy, we have

$$\frac{1}{2} mv_0^2 + mgh = 400 \text{ m}$$

$$\Rightarrow \frac{1}{2} mv_0^2 + m \times 10 \times 20 = 400 \text{ m}$$

$$\Rightarrow v_0 = 20 \text{ m/s}$$

11 A uniform force of $(3\hat{\mathbf{i}} + \hat{\mathbf{j}})$ N acts on a particle of mass 2 kg. Hence, the particle is displaced from position $(2\hat{\mathbf{i}} + \hat{\mathbf{k}})$ m to position $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ m. The work done by the force on the particle is **[NEET 2013]**

(a) 9 J (b) 6 J (c) 13 J (d) 15 J

Ans. *(a)*

Given, force $F = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\mathbf{r}_1 = (2\hat{\mathbf{i}} + \hat{\mathbf{k}}) \text{ m and } \mathbf{r}_2 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) \text{ m}$$

$$\therefore \mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{k}})$$
$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \text{ m}$$

$$\therefore W = \mathbf{F} \cdot \mathbf{s} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$
$$= 3 \times 2 + 3 + 0 = 6 + 3 = 9 \text{ J}$$

12 The potential energy of a particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is **[CBSE AIPMT 2012]**

(a) $B/2A$ (b) $2A/B$
(c) A/B (d) B/A

Ans. *(b)*

Given, the potential energy of a particle in a force field, $U = \frac{A}{r^2} - \frac{B}{r^1}$

For stable equilibrium, $F = -\frac{dU}{dr} = 0$

$$= \frac{dU}{dr} = -2Ar^{-3} + Br^{-2}$$

$$0 = -\frac{2A}{r^3} + \frac{B}{r^2} \qquad \left(\text{As } \frac{-dU}{dr} = 0\right)$$

or $\frac{2A}{r} = B$

The distance of particle from the centre of the field

$$r = \frac{2A}{B}$$

13 The potential energy of a system increases, if work is done **[CBSE AIPMT 2011]**

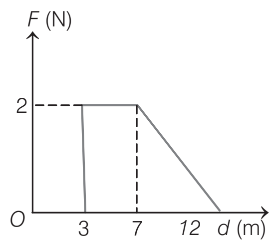
(a) by the system against a conservative force
(b) by the system against a non-conservative force
(c) upon the system by a conservative force
(d) upon the system by a non-conservative force

Ans. *(a)*

The potential energy of a system increases, if work is done by the system against a conservative force.

$$-\Delta U = W_{\text{conservative force}}$$

14 Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is **[CBSE AIPMT 2011]**



(a) 21 J (b) 26 J
(c) 13 J (d) 18 J

Ans. *(c)*

Concept Work done is equal to area under the curve in F-d graph.

Work done = Area under (F-d) graph

$$= 2 \times (7 - 3) + \frac{1}{2} \times 2 \times (12 - 7)$$
$$= 8 + \frac{1}{2} \times 10 = 8 + 5 = 13 \text{ J}$$

15 A block of mass M is attached to the lower end of a vertical spring. The spring is hung from a ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched. The maximum extension produced in the length of the spring will be **[CBSE AIPMT 2009]**

(a) Mg/k (b) $2\,Mg/k$
(c) $4\,Mg/k$ (d) $Mg/2k$

Ans. *(b)*

Let x be the extension in the spring.

Applying conservation of energy

$$Mgx - \frac{1}{2}kx^2 = 0 - 0$$

$$\Rightarrow x = \frac{2\,Mg}{k}$$

16 A body of mass 1 kg is thrown upwards with a velocity 20 ms^{-1}. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? (Take $g = 10 \text{ ms}^{-2}$) **[CBSE AIPMT 2009]**

(a) 20 J (b) 30 J (c) 40 J (d) 10 J

Ans. *(a)*

Concept Apply conservation of energy. Initially body posseses only kinetic energy and after attaining a height, the kinetic energy is zero.

Therefore, loss of energy = KE – PE

$$= \frac{1}{2} mv^2 - mgh$$
$$= \frac{1}{2} \times 1 \times 400 - 1 \times 18 \times 10$$
$$= 200 - 180 = 20 \text{ J}$$

17 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Taking $g = 10 \text{ m/s}^2$, work done against friction is **[CBSE AIPMT 2006]**

(a) 200 J (b) 100 J
(c) zero (d) 1000 J

Ans. *(b)*

Net work done in sliding a body up to a height h on inclined plane

= Work done against gravitational force + Work done against frictional force

$$\Rightarrow W = W_g + W_f \qquad \text{...(i)}$$

but $W = 300$ J

$$W_g = mgh = 2 \times 10 \times 10 = 200 \text{ J}$$

Putting in Eq. (i), we get

$$300 = 200 + W_f$$

$$\Rightarrow W_f = 300 - 200 = 100 \text{ J}$$

18 A body of mass 3 kg is under a constant force, which causes a displacement s in metre in it, given by the relation $s = \frac{1}{3}t^2$, where t is in second. Work done by the force in 2 s is **[CBSE AIPMT 2006]**

(a) $\frac{5}{19}$ J (b) $\frac{3}{8}$ J
(c) $\frac{8}{3}$ J (d) $\frac{19}{5}$ J

Ans. *(c)*

Work done by the force = force × displacement

or $W = F \times s$ ...(i)

But from Newton's 2nd law, we have

Force = mass × acceleration

i.e. $F = ma$ ...(ii)

Hence, from Eqs. (i) and (ii), we get

$$W = mas = m\left(\frac{d^2 s}{dt^2}\right)s \quad \left(\because a = \frac{d^2 s}{dt^2}\right) \text{...(iii)}$$

Now, we have,

$$s = \frac{1}{3}t^2$$

$$\therefore \quad \frac{d^2 s}{dt^2} = \frac{d}{dt}\left[\frac{d}{dt}\left(\frac{1}{3}t^2\right)\right]$$

$$= \frac{d}{dt} \times \left(\frac{2}{3}t\right)$$

$$= \frac{2}{3}\frac{dt}{dt} = \frac{2}{3}$$

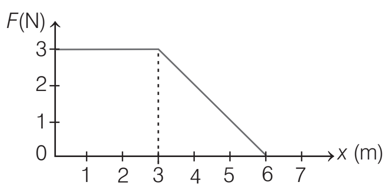
Hence, Eq. (iii) becomes

$$W = \frac{2}{3}ms = \frac{2}{3}m \times \frac{1}{3}t^2 = \frac{2}{9}mt^2$$

We have, $m = 3\,\text{kg}, t = 2\,\text{s}$

$$\therefore \quad W = \frac{2}{9} \times 3 \times (2)^2 = \frac{8}{3}\text{J}$$

19 A force F acting on an object varies with distance x as shown here. The force is in newton and x is in metre. The work done by the force in moving the object from $x = 0$ to $x = 6$ m is **[CBSE AIPMT 2005]**



(a) 4.5 J (b) 13.5 J
(c) 9.0 J (d) 18.0 J

Ans. *(b)*

Work done in moving the object from $x = 0$ to $x = 6$ m, is given by area under the curve.

W = Area of square + area of triangle

$$= 3 \times 3 + \frac{1}{2} \times 3 \times 3$$

$$= 9 + 4.5 = 13.5\,\text{J}$$

20 A bomb of mass 30 kg at rest explodes into two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is 6 ms^{-1}. The kinetic energy of the other mass is **[CBSE AIPMT 2005]**

(a) 256 J (b) 486 J
(c) 524 J (d) 324 J

Ans. *(b)*

Applying conservation of linear momentum, we write,

$$m_1 u_1 = m_2 u_2$$

Here, $m_1 = 18\,\text{kg}, \quad m_2 = 12\,\text{kg}$

$u_1 = 6\,\text{ms}^{-1}, u_2 = ?$

$$\therefore \quad 18 \times 6 = 12\,u_2$$

$$\Rightarrow \quad u_2 = \frac{18 \times 6}{12} = 9\,\text{ms}^{-1}$$

Thus, kinetic energy of 12 kg mass

$$K_2 = \frac{1}{2}m_2 u_2^2$$

$$= \frac{1}{2} \times 12 \times (9)^2$$

$$= 6 \times 81 = 486\,\text{J}$$

21 A particle of mass m_1 is moving with a velocity v_1 and another particle of mass m_2 is moving with a velocity v_2. Both of them have the same momentum, but their different kinetic energies are E_1 and E_2 respectively. If $m_1 > m_2$, then **[CBSE AIPMT 2004]**

(a) $E_1 < E_2$ (b) $\frac{E_1}{E_2} = \frac{m_1}{m_2}$
(c) $E_1 > E_2$ (d) $E_1 = E_2$

Ans. *(a)*

Kinetic energy is given by

$$E = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2$$

but, mv = momentum of the particle = p

$$\therefore \quad E = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mE}$$

Therefore, $\frac{p_1}{p_2} = \sqrt{\frac{m_1 E_1}{m_2 E_2}}$

but it is given that, $p_1 = p_2$

$$\therefore \quad m_1 E_1 = m_2 E_2$$

or $\frac{E_1}{E_2} = \frac{m_2}{m_1}$ ...(i)

Now, $m_1 > m_2$

or $\frac{m_1}{m_2} > 1$ ...(ii)

Thus, from Eqs. (i) and (ii), we get

$$\frac{E_1}{E_2} < 1 \quad \text{or} \quad E_1 < E_2$$

22 A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 ft tall building. After, a fall of 30 ft each towards earth, their respective kinetic energies will be in the ratio of **[CBSE AIPMT 2004]**

(a) $\sqrt{2}:1$ (b) 1 : 4
(c) 1 : 2 (d) $1:\sqrt{2}$

Ans. *(c)*

Concept Velocity of free falling body does not depend on its mass, it depends on the height from which it has been dropped.

$v_1 = v_2 = v$ at a 30 ft height from falling point.

Here, $m_1 = 2\,\text{kg}, \quad m_2 = 4\,\text{kg}$

Thus, $$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1 v^2}{\frac{1}{2}m_2 v^2} = \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$$

23 If kinetic energy of a body is increased by 300%, then percentage change in momentum will be **[CBSE AIPMT 2002]**

(a) 100% (b) 150%
(c) 265% (d) 73.2%

Ans. *(a)*

Kinetic energy

$$K = \frac{1}{2m}(p^2) \quad \text{or} \quad p = \sqrt{2mK}$$

If kinetic energy of a body is increased by 300%, let its momentum becomes p'.

New kinetic energy

$$K' = K + \frac{300}{100}K = 4K \quad \begin{pmatrix}\text{initial KE} = K \\ \text{final KE} = K'\end{pmatrix}$$

Therefore, momentum is given by

$$p' = \sqrt{2m \times 4K} \quad \begin{pmatrix}\text{initial momentum} = p \\ \text{Final momentum} = p'\end{pmatrix}$$

$$= 2\sqrt{2mK} = 2p$$

Hence, percentage change (increase) in momentum

$$\frac{\Delta p}{p} \times 100 = \frac{p' - p}{p} \times 100$$
$$= \left(\frac{p'}{p} - 1\right) \times 100$$
$$= \left(\frac{2p}{p} - 1\right) \times 100$$
$$= 100\%$$

24 A stone is thrown at an angle of 45° to the horizontal with kinetic energy K. The kinetic energy at the highest point is **[CBSE AIPMT 2001]**

(a) $\frac{K}{2}$ (b) $\frac{K}{\sqrt{2}}$
(c) K (d) zero

Ans. *(a)*

At the highest point

$$v_x = u \cos\theta$$
$$v_y = 0$$
$$K_H = \frac{1}{2} m v_x^2$$

or $$K_H = \frac{1}{2} mu^2 \cos^2\theta \quad \text{...(i)}$$

Initial kinetic energy is

$$K = \frac{1}{2} mu^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$K_H = K \cos^2\theta$$
$$= K \cos^2 45°$$
$$= K \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{K}{2}$$

25 A child is swinging a swing. Minimum and maximum heights of swing from the earth's surface are 0.75 m and 2 m respectively. The maximum velocity of this swing is **[CBSE AIPMT 2001]**

(a) 5 m/s (b) 10 m/s
(c) 15 m/s (d) 20 m/s

Ans. *(a)*

From energy conservation

$$\frac{1}{2} m v_{max}^2 = mg(H_2 - H_1)$$

Here, H_1 = minimum height of swing from the earth's surface

= 0.75 m

H_2 = maximum height of swing from earth's surface

= 2 m

$$\therefore \quad \frac{1}{2} m v_{max}^2 = mg(2 - 0.75)$$

or $$v_{max} = \sqrt{2 \times 10 \times 1.25}$$
$$= \sqrt{25} = 5 \text{ m/s}$$

26 Two bodies with kinetic energies in the ratio 4 : 1 are moving with equal linear momentum. The ratio of their masses is **[CBSE AIPMT 1999]**

(a) 1 : 2 (b) 1 : 1 (c) 4 : 1 (d) 1 : 4

Ans. *(d)*

As we know that, relation between kinetic energy and momentum is given by

$$\text{KE} = \frac{p^2}{2m}$$

If $p_1 = p_2$ for two bodies

So, $$\text{KE}_1 \propto \frac{1}{m_1}$$

and $$\text{KE}_2 \propto \frac{1}{m_2}$$

Therefore, ratio of two masses is given by

$$\frac{m_1}{m_2} = \frac{\text{KE}_2}{\text{KE}_1} = \frac{1}{4} \qquad \left[\because \frac{\text{KE}_1}{\text{KE}_2} = \frac{4}{1}\right]$$

27 A force acts on a 3.0 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metre and t in second. The work done during the first 4 s is **[CBSE AIPMT 1998]**

(a) 570 mJ (b) 450 mJ
(c) 490 mJ (d) 528 mJ

Ans. *(d)*

Given, $x = 3t - 4t^2 + t^3$

So, velocity

$$v = \frac{dx}{dt} = 3 - 8t + 3t^2$$

At $t = 0$ s, $v_1 = 3 - 0 + 0 = 3$ m/s

At $t = 4$ s, $v_2 = 3 - 8 \times 4 + 3 \times 4^2$
$= 3 - 32 + 48 = 19$ m/s

Now, work done during $t = 0$ to $t = 4$ s

= gain in kinetic energy

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_2^2 - v_1^2)$$
$$= \frac{1}{2} \times 3 \times 10^{-3} [(19)^2 - (3)^2]$$
$$[\text{Using, } a^2 - b^2 = (a+b)(a-b)]$$
$$= 1.5 \times 10^{-3} \times [(19 + 3)(19 - 3)]$$
$$= 1.5 \times 10^{-3} \times 22 \times 16$$
$$= 528 \times 10^{-3} \text{ J} = 528 \text{ mJ}$$

28 A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of **[CBSE AIPMT 1998]**

(a) 16/25 (b) 2/5
(c) 3/5 (d) 9/25

Ans. *(b)*

Potential energy = Kinetic energy

i.e. $mgh = \frac{1}{2} mv^2 \Rightarrow v = \sqrt{2gh}$

If h_1 and h_2 are the initial and final heights, then

$$v_1 = \sqrt{2gh_1}, v_2 = \sqrt{2gh_2}$$

Loss in velocity

$$\Delta v = v_1 - v_2 = \sqrt{2gh_1} - \sqrt{2gh_2}$$

∴ Fractional loss in velocity

$$= \frac{\Delta v}{v_1} = \frac{\sqrt{2gh_1} - \sqrt{2gh_2}}{\sqrt{2gh_1}} = 1 - \sqrt{\frac{h_2}{h_1}}$$

Substituting the values, we have

$$\therefore \quad \frac{\Delta v}{v_1} = 1 - \sqrt{\frac{1.8}{5}} = 1 - \sqrt{0.36} = 1 - 0.6$$
$$= 0.4 = \frac{2}{5}$$

29 If the momentum of a body is increased by 50%, then the percentage increase in its kinetic energy is **[CBSE AIPMT 1995]**

(a) 50% (b) 100% (c) 125% (d) 200%

Ans. *(c)*

Let p_1 be the initial momentum and p_2 be the inversed momentum

So, $$p_2 = \frac{150}{100} p_1$$

i.e. $$m v_2 = \frac{15}{10} m v_1 \qquad \begin{pmatrix} p_1 = m v_1 \\ p_2 = m v_2 \end{pmatrix}$$

or $$v_2 = \frac{15}{10} v_1$$

Now, $$\frac{E_2}{E_1} = \frac{\frac{1}{2} m v_2^2}{\frac{1}{2} m v_1^2} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{15}{10}\right)^2 = \frac{225}{100}$$

Clearly, $E_2 > E_1$

So, percentage increase in KE

$$= \frac{(E_2 - E_1)}{E_1} \times 100$$
$$= \left(\frac{225}{100} - 1\right) \times 100 = 125\%$$

30 The KE acquired by a mass m in travelling a certain distance d, starting from rest, under the action of a constant force is directly proportional to **[CBSE AIPMT 1994]**

(a) m
(b) $\sqrt{m}$
(c) $\frac{1}{\sqrt{m}}$
(d) Independent of m

Ans. *(d)*

Kinetic energy acquired by the body

$$= \frac{1}{2} mv^2.$$

If body starts from rest, then final velocity achieved by body in displacement d is

$$v^2 = 0 + 2ad = 2ad$$

but $a = \frac{F}{m}$

$\therefore$ $v^2 = 2\left(\frac{F}{m}\right)d$

Hence, $\text{KE} = \frac{1}{2} m \times 2\left(\frac{F}{m}\right)d = Fd$

or KE acquired = Work done

$$= F \times d = \text{constant}$$

So, KE acquired is independent of mass m.

31 Two masses 1 g and 9 g are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momenta is **[CBSE AIPMT 1993]**

(a) 1 : 9 (b) 9 : 1
(c) 1 : 3 (d) 3 : 1

Ans. *(c)*

Given, $\text{KE}_1 = \text{KE}_2$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$$

or $\frac{v_2^2}{v_1^2} = \frac{m_1}{m_2}$ or $\frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}}$

As, $p_2 = m_2 v_2$ and $p_1 = m_1 v_1$

$\therefore$ $\frac{p_2}{p_1} = \frac{m_2 v_2}{m_1 v_1} = \frac{m_2}{m_1}\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m_2^2 m_1}{m_1^2 m_2}}$

$$\frac{p_2}{p_1} = \sqrt{\frac{m_2}{m_1}}$$

or $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

Here, $m_1 = 1\,\text{g}, m_2 = 9\,\text{g}$

$\therefore$ $\frac{p_1}{p_2} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

Alternative

The relation between KE and p is given by

$$\text{KE} = \frac{p^2}{2m}$$

$\Rightarrow$ $p^2 = 2\,m\text{KE}$

$\Rightarrow$ $p = \sqrt{2\,m\text{KE}}$

If KE of two bodies are equal.

So, $p_1 \propto \sqrt{m_1}$

and $p_2 \propto \sqrt{m_2}$

$\Rightarrow$ $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

32 A position dependent force $F = (7 - 2x + 3x^2)\,\text{N}$, acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. Work done in joule is **[CBSE AIPMT 1992]**

(a) 35 (b) 70
(c) 135 (d) 270

Ans. *(c)*

Work done by a variable force F in displacement from $x = x_1$ to $x = x_2$ is given by

$$W = \int_{x_1}^{x_2} F(dx)$$

Given, $x_1 = 0, x_2 = 5$

$F = (7 - 2x + 3x^2)\,\text{N}$

$\therefore$ $W = \int_0^5 (7 - 2x + 3x^2)\,dx$

$$= [7x - x^2 + x^3]_0^5$$
$$= [7 \times 5 - (5)^2 + (5)^3]$$
$$= [35 - 25 + 125] = 135\,\text{J}$$

33 A bullet of mass 10 g leaves a rifle at an initial velocity of 1000 m/s and strikes the earth at the same level with a velocity of 500 m/s. The work done in joule to overcome the resistance of air will be **[CBSE AIPMT 1989]**

(a) 375 (b) 3750 (c) 5000 (d) 500

Ans. *(b)*

According to work-energy theorem, work done by a force in displacing a body measures the change in kinetic energy of the body or work done by a force is equal to change in KE of the body.

or $W = \Delta E$

$= \text{final KE} - \text{initial KE}$

$= \frac{1}{2} mv^2 - \frac{1}{2} mu^2$

Given, $v = 1000\,\text{m/s}$, $m = 10\,\text{g}$

$u = 500\,\text{m/s}, = 0.01\,\text{kg}$

Putting the values of m, u, v

$\therefore$ $W = \frac{1}{2} \times 0.01\,[(1000)^2 - (500)^2]$

$= 3750\,\text{J}$

TOPIC 2
Work Energy Theorem, Power and Verticle Circle

34 Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine? ($g = 10\,\text{m/s}^2$) **[NEET 2021]**

(a) 10.2 kW (b) 8.1 kW
(c) 12.3 kW (d) 7.0 kW

Ans. *(b)*

Given, the flow rate of the water,

$$\frac{m}{t} = 15\,\text{kg/s}$$

The height of the water fall, $h = 60$ m

Loss due to frictional force = 10%

The power used in the turbine

= (100 – 10) % = 90%

The acceleration due to gravity,

$g = 10\,\text{m/s}^2$

We know that, power generated by the turbine

= change in potential energy = $0.90\,\frac{mgh}{t}$

= 0.90 × 15 × 10 × 60 = 8100 W = 8.1 kW

35 A point mass m is moved in a vertical circle of radius r with the help of a string. The velocity of the mass is $\sqrt{7gr}$ at the lowest point. The tension in the string at the lowest point is **[NEET (Oct.) 2020]**

(a) 6 mg (b) 7 mg (c) 8 mg (d) 1 mg

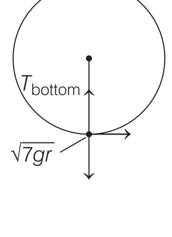
Ans. *(c)*

Velocity of point mass in vertical circle at lowest point, $V_l = \sqrt{7gr}$

$\therefore$ $V_l = \sqrt{7gr} > \sqrt{5gr}$

Hence, point mass will have completed the vertical circular path.

We know that,

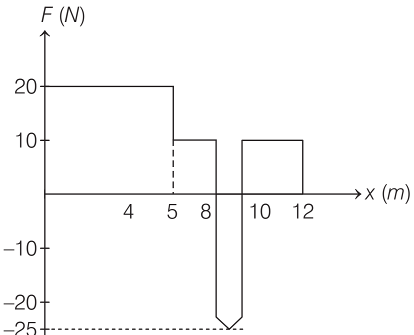


$$T_{bottom} - mg = \frac{mv^2}{r} = \frac{m}{r}(\sqrt{7gr})^2$$

$$\Rightarrow \quad T_{bottom} - mg = 7mg$$

$$\Rightarrow \quad T_{bottom} = 8mg$$

36 An object of mass 500 g, initially at rest acted upon by a variable force whose X component varies with X in the manner shown. The velocities of the object a point $X = 8$ m and $X = 12$ m, would be the respective values of (nearly) **[NEET (Odisha) 2019]**



(a) 18 m/s and 24.4 m/s
(b) 23 m/s and 24.4 m/s
(c) 23 m/s and 20.6 m/s
(d) 18 m/s and 20.6 m/s

Ans. ***(c)***

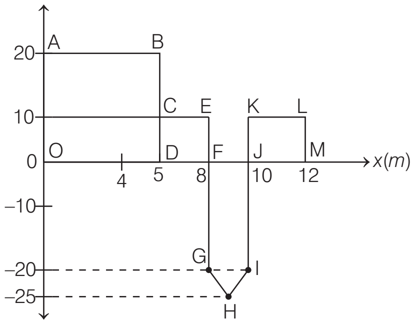
The area under the force displacement curve give the amount of work done.

From work-energy theorem,

$$W = \Delta KE \quad \text{...(i)}$$

$\therefore$ At $x = 8$ m,

$$W = \text{Area } ABDO + \text{Area } CEFD$$
$$= 20 \times 5 + 10 \times 3 = 130 \text{ J}$$



Using Eq. (i)

$$\Rightarrow \quad 130 = \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{500}{1000}v^2$$

$$\Rightarrow \quad v = 2\sqrt{130} = 22.8 \text{ ms}^{-1} \approx 23 \text{ ms}^{-1}$$

At $\quad x = 12$ m

$$W = \text{Area } ABDO + \text{Area } CEFD + \text{Area } FGHIJ + \text{Area } KLMJ$$

$$W = 20 \times 5 + 10 \times 3 + (-20 \times 2) + \left(\frac{1}{2} \times -5 \times 2\right) + 10 \times 2$$

[$\because$ Area FGHIJ = Area FGIJ + Area GHI]

$$= 100 + 30 - 40 - 5 + 20 = 105 \text{ J}$$

Using Eq. (i)

$$\therefore \quad 105 = \frac{1}{2} \times \frac{1}{2} \times v^2$$

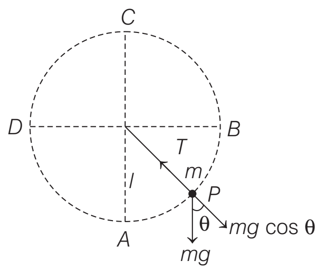
$$\Rightarrow \quad v = 2\sqrt{105} \simeq 20.6 \text{ ms}^{-1}$$

37 A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when: **[NEET (National) 2019]**

(a) the wire is horizontal
(b) the mass is at the lowest point
(c) inclined at an angle of 60° from vertical
(d) the mass is at the highest point

Ans. ***(b)***

Let the mass m which is attached to a thin wire and is whirled in a vertical circle is shown in the figure below.



The tension in the string at any point P be T.

According to Newton's law of motion, In equilibrium, net force towards the centre = centripetal force

$$\Rightarrow \quad T - mg\cos\theta = \frac{mv^2}{l}$$

Here, l = length of wire and v = linear velocity of the particle whirling in a circle.

$$\Rightarrow \quad T = mg\cos\theta + \frac{mv^2}{l}$$

At $A, \theta = 0° \Rightarrow T_A = mg + \frac{mv^2}{l}$

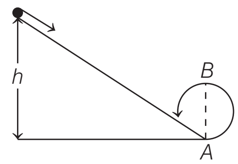
At $B, \theta = 90° \Rightarrow T_B = \frac{mv^2}{l}$

At $C, \theta = 180° \Rightarrow T_C = -mg + \frac{mv^2}{l}$

At $D, \theta = 90° \Rightarrow T_D = T_B = \frac{mv^2}{l}$

So, from the above analysis, it can be concluded that the tension is maximum at A i.e. the lowest point of circle, So chance of breaking is maximum.

38 A body initially at rest and sliding along a frictionless track from a height h (as shown in the figure) just completes a vertical circle of diameter $AB = D$. The height h is equal to **[NEET 2018]**



(a) $\frac{7}{5}D$ (b) D

(c) $\frac{3}{2}D$ (d) $\frac{5}{4}D$

Ans. ***(d)***

If a body is moving on a frictionless surface, then its total mechanical energy remains conserved.

According to the conservation of mechanical energy,

$$(\text{TE})_{\text{initial}} = (\text{TE})_{\text{final}}$$

$$\Rightarrow \quad (\text{KE})_i + (\text{PE})_i = (\text{KE})_f + (\text{PE})_f$$

$$0 + mgh = \frac{1}{2}mv_A^2 + 0$$

$$\Rightarrow \quad gh = \frac{v_A^2}{2} \text{ or } h = \frac{v_A^2}{2g} \quad \text{...(i)}$$

In order to complete the vertical circle, the velocity of the body at point A should be

$$v_A = v_{min} = \sqrt{5gR}$$

where, R is the radius of the body.

Here, $\quad R = \frac{AB}{2} = \frac{D}{2}$

$$\Rightarrow \quad v_{min} = v_A = \sqrt{\frac{5}{2}gD}$$

Substituting the value of v_A in Eq. (i), we get

$$h = \frac{\left(\sqrt{\left(\frac{5}{2}gD\right)}\right)^2}{2g}$$

$$= \frac{5gD}{2 \times 2g} = \frac{5}{4}D$$

39 Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take g constant with a value of 10 m/s^2. The work done by the (i) gravitational force and the (ii) resistive force of air is **[NEET 2017]**

(a) (i) –10 J, (ii) – 8.25 J
(b) (i) 1.25 J, (ii) – 8.25 J
(c) (i) 100 J, (ii) 8.75 J
(d) (i) 10 J, (ii) – 8.75 J

Ans. *(d)*

By work-KE theorem, we have change in KE = work done by all of the forces.

Work done by gravitational force,

$$W_g = mgh = 10^{-3} \times 10 \times 1 \times 10^3 = 10 \text{ J}$$

Now, from work-KE theorem, we have

$$\Delta K = W_{\text{gravity}} + W_{\text{air resistance}}$$

$$\Rightarrow \quad \frac{1}{2} \times mv^2 = mgh + W_{\text{air resistance}}$$

$$\Rightarrow W_{\text{air resistance}} = \frac{1}{2} mv^2 - mgh$$

$$= 10^{-3} \left(\frac{1}{2} \times 50 \times 50 - 10 \times 10^3 \right)$$

$$= -8.75 \text{ J}$$

40 A body of mass 1 kg begins to move under the action of a time dependent force $\mathbf{F} = (2t\,\hat{\mathbf{i}} + 3t^2\,\hat{\mathbf{j}})$ N, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along X and Y axes. What power will be developed by the force at the time (t)? **[NEET 2016]**

(a) $(2t^2 + 4t^4)$ W (b) $(2t^3 + 3t^4)$ W
(c) $(2t^3 + 3t^5)$ W (d) $(2t + 3t^3)$ W

Ans. *(c)*

According to question, a body of mass 1 kg begins to move under the action of time dependent force,

$$\mathbf{F} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\, N$$

where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along X and Y-axes.

$$\because \quad \mathbf{F} = m\mathbf{a} \quad \Rightarrow \quad \mathbf{a} = \frac{\mathbf{F}}{m}$$

$$\Rightarrow \quad \mathbf{a} = \frac{(2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})}{1} \qquad (\because m = 1 \text{ kg})$$

$$\Rightarrow \quad \mathbf{a} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}}) \text{ m/s}^2$$

$$\because \text{ acceleration, } a = \frac{dv}{dt}$$

$$\Rightarrow \qquad dv = a\, dt \qquad \text{...(i)}$$

Integrating both sides, we get

$$\int dv = \int a\, dt = \int (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\, dt$$

$$\mathbf{v} = t^2\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}$$

$\because$ Power developed by the force at the time t will be given as

$$P = \mathbf{F} \cdot \mathbf{v} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}}) \cdot (t^2\hat{\mathbf{i}} + t^3\hat{\mathbf{j}})$$

$$= (2t.t^2 + 3t^2.t^3)$$

$$P = (2t^3 + 3t^5) \text{ W}$$

41 What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? **[NEET 2016]**

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$ (c) $\sqrt{5gR}$ (d) $\sqrt{gR}$

Ans. *(c)*

According to question, we have

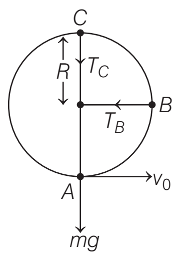
Let the tension at point A be T_A. So, from Newton's second law

$$T_A - mg = \frac{mv_c^2}{R}$$

$$\text{Energy at point } A = \frac{1}{2} mv_0^2 \qquad \text{...(i)}$$

Energy at point C is

$$\frac{1}{2} mv_c^2 + mg \times 2R \qquad \text{...(ii)}$$



Applying Newton's 2nd law at point C

$$T_c + mg = \frac{mv_c^2}{R}$$

To complete the loop $T_c \geq 0$

$$\text{So,} \quad mg = \frac{mv_c^2}{R}$$

$$\Rightarrow \quad v_c = \sqrt{gR} \qquad \text{...(iii)}$$

From Eqs. (i) and (ii) by conservation of energy

$$\frac{1}{2} mv_0^2 = \frac{1}{2} mv_c^2 + 2mgR$$

$$\Rightarrow \quad \frac{1}{2} mv_0^2 = \frac{1}{2} mgR + 2mgR \times 2$$

$$(\because v_c = \sqrt{gR})$$

$$\Rightarrow \quad v_0^2 = gR + 4gR$$

$$\Rightarrow \quad v_0 = \sqrt{5gR}$$

42 A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest, the force on the particle at time t is **[CBSE AIPMT 2015]**

(a) $\sqrt{\frac{mk}{2}}\, t^{-1/2}$ (b) $\sqrt{mk}\; t^{-1/2}$
(c) $\sqrt{2mk}\; t^{-1/2}$ (d) $\frac{1}{2}\sqrt{mk}\; t^{-1/2}$

Ans. *(a)*

As the machine delivers a constant power

So $F.v$ = constant = k (watts)

$$\Rightarrow \quad m\frac{dv}{dt} \cdot v = k \quad \Rightarrow \quad \int v dv = \frac{k}{m} \int dt$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{m} t \Rightarrow v = \sqrt{\frac{2k}{m} t}$$

Now, force on the particle is given by

$$F = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{2kt}{m}\right)^{\frac{1}{2}}$$

$$= \sqrt{2km} \cdot \left(\frac{1}{2} t^{-\frac{1}{2}}\right) = \sqrt{\frac{mk}{2}} \cdot t^{-\frac{1}{2}}$$

43 The heart of a man pumps 5 L of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6×10^3 kg / m^3 and $g = 10$ m / s^2, then the power of heart in watt is **[CBSE AIPMT 2015]**

(a) 1.70 (b) 2.35 (c) 3.0 (d) 1.50

Ans. *(a)*

Given, pressure = 150 mm of Hg

Pumping rate of heart of a man

$$= \frac{dV}{dt} = \frac{5 \times 10^{-3}}{60} \text{ m}^3 \text{ / s}$$

$$\text{Power of heart} = P \cdot \frac{dV}{dt} = \rho gh \cdot \frac{dV}{dt}$$

$$[p = \rho gh]$$

$$\frac{(13.6 \times 10^3 \text{ kg/m}^3)(10 \times 0.15 \times 5 \times 10^{-3})}{60}$$

$$= 1.70 \text{ W}$$

44 An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m s^{-1}. The mass per unit length of water in the pipe is 100 kg m^{-1}. What is the power of the engine? **[CBSE AIPMT 2010]**

(a) 400 W (b) 200 W
(c) 100 W (d) 800 W

Ans. *(d)*

Given, Velocity of water $v = 2\ \text{m/s}$

Mass per unit length of water in the pipe

$= 100\ \text{kg/m}$

So, power = (mass per unit length of water in pipe) $\times v^3$

$$= \frac{m}{l} \times v^3$$

$$= 100 \times 2 \times 2 \times 2 = 800\ \text{W}$$

45 An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water? **[CBSE AIPMT 2009]**

(a) $\frac{1}{2}mv^3$ (b) mv^3

(c) $\frac{1}{2}mv^2$ (d) $\frac{1}{2}m^2v^2$

Ans. *(a)*

As m is the mass per unit length, then

rate of mass per second $= \frac{mx}{t} = mv$

$\therefore$ Rate of KE $= \frac{1}{2}(mv)v^2 = \frac{1}{2}mv^3$

46 Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine? (Take $g = 10\ \text{m/s}^2$) **[CBSE AIPMT 2008]**

(a) 8.1 kW (b) 10.2 kW

(c) 12.3 kW (d) 7.0 kW

Ans. *(a)*

$$P_{\text{generated}} = P_{\text{input}} \times \frac{90}{100} = \frac{mgh}{t} \times \frac{90}{100}$$

$$= \frac{15 \times 10 \times 60}{1} \times \frac{90}{100} = 8.1\ \text{kW}$$

47 A stone is tied to a string of length l and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is **[CBSE AIPMT 2004]**

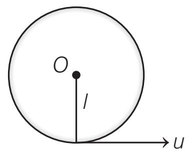
(a) $\sqrt{2(u^2 - gl)}$ (b) $\sqrt{u^2 - gl}$

(c) $u - \sqrt{u^2 - 2gl}$ (d) $\sqrt{2gl}$

Ans. *(a)*

When stone is at its lowest position, it has only kinetic energy, given by

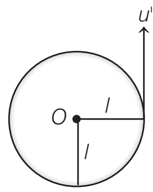
$$K = \frac{1}{2}mu^2$$



At the horizontal position, it has energy

$$E = U + K = \frac{1}{2}mu'^2 + mgl$$

According to conservation of mechanical energy, $K = E$



$\therefore \quad \frac{1}{2}mu^2 = \frac{1}{2}mu'^2 + mgl$

or $\quad \frac{1}{2}mu'^2 = \frac{1}{2}mu^2 - mgl$

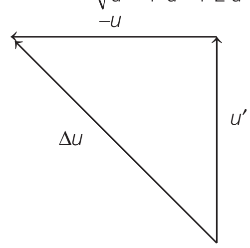
or $\quad u'^2 = u^2 - 2gl$

or $\quad u' = \sqrt{u^2 - 2gl}$ ...(i)

So, the magnitude of change in velocity

$$|\Delta \mathbf{u}| = |\mathbf{u}' - \mathbf{u}|$$

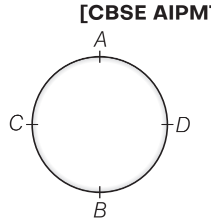
$$= \sqrt{u'^2 + u^2 + 2u'u\cos 90°}$$



$$|\Delta \mathbf{u}| = \sqrt{u'^2 + u^2} = \sqrt{2(u^2 - gl)}$$

[from Eq. (i)]

48 A stone is attached to one end of a string and rotated in a vertical circle. If string breaks at the position of maximum tension, it will break at **[CBSE AIPMT 2000]**

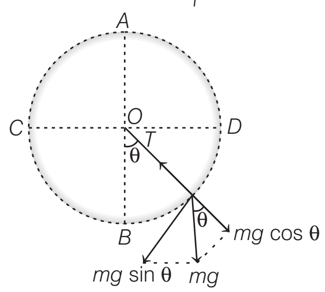


(a) A (b) B (c) C (d) D

Ans. *(b)*

When string makes an angle θ with the vertical in a vertical circle, then balancing the force we get

$$T - mg\cos\theta = \frac{mv^2}{l}$$



or $\quad T = mg\cos\theta + \frac{mv^2}{l}$

Tension is maximum when $\cos\theta = +1$

i.e. $\quad \theta = 0$

Thus, θ is zero at lowest point B. At this point tension is maximum. So, string will break at point B.

49 How much water a pump of 2 kW can raise in one minute to a height of 10 m? (Take $g = 10\ \text{m/s}^2$) **[CBSE AIPMT 1990]**

(a) 1000 L (b) 1200 L (c) 100 L (d) 2000 L

Ans. *(b)*

Power of a body is defined as the rate at which the body can do the work, i.e.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{W}{t}$$

Given, power, $P = 2\ \text{kW} = 2000\ \text{W}$

$W = Mgh = M \times 10 \times 10 \quad [\because g = 10\ \text{m/s}^2]$

$= 100\,M$

Time, $t = 60$ s

$\therefore \quad 2000 = \frac{100M}{60}$

$\therefore \quad M = 1200\ \text{kg}$ and $V = 1200$ L

TOPIC 3
Collision

50 An object flying in air with velocity $(20\hat{\mathbf{i}} + 25\hat{\mathbf{j}} - 12\hat{\mathbf{k}})$ suddenly breaks in two pieces whose masses are in the ratio 1 : 5. The smaller mass flies off with a velocity $(100\hat{\mathbf{i}} + 35\hat{\mathbf{j}} + 8\hat{\mathbf{k}})$. The velocity of the larger piece will be **[NEET (Odisha) 2019]**

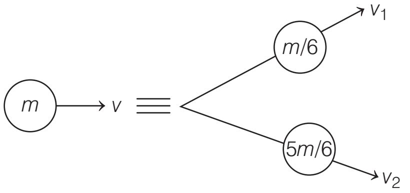
(a) $4\hat{\mathbf{i}} + 23\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$ (b) $-100\hat{\mathbf{i}} - 35\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

(c) $20\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 80\hat{\mathbf{k}}$ (d) $-20\hat{\mathbf{i}} - 15\hat{\mathbf{j}} - 80\hat{\mathbf{k}}$

Ans. *(a)*

Let m be the mass of an object flying with velocity v in air. When it split into two pieces of masses in ratio 1 : 5, the mass of smaller piece is $m/6$ and of bigger piece is $\frac{5m}{6}$.

This situation can be interpreted diagrammatically as below



As the object breaks in two pieces, so the momentum of the system will remains conserved i.e. the total momentum (before breaking) = total momentum (after breaking)

$$m\mathbf{v} = \frac{m}{6}\mathbf{v_1} + \frac{5m}{6}\mathbf{v_2}$$

$$\Rightarrow \quad \mathbf{v} = \frac{\mathbf{v}_1}{6} + \frac{5\mathbf{v}_2}{6}$$

Here, $\mathbf{v} = 20\hat{\mathbf{i}} + 25\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$

$$\mathbf{v}_1 = 100\hat{\mathbf{i}} + 35\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$\Rightarrow 20\hat{\mathbf{i}} + 25\hat{\mathbf{j}} - 12\hat{\mathbf{k}} = \frac{(100\hat{\mathbf{i}} + 35\hat{\mathbf{j}} + 8\hat{\mathbf{k}})}{6} + \frac{5\mathbf{v}_2}{6}$$

$$\Rightarrow (120\hat{\mathbf{i}} + 150\hat{\mathbf{j}} - 72\hat{\mathbf{k}}) = (100\hat{\mathbf{i}} + 35\hat{\mathbf{j}} + 8\hat{\mathbf{k}}) + 5\mathbf{v}_2$$

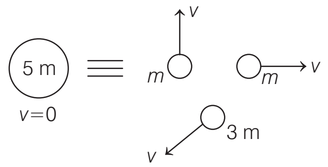
$$\Rightarrow \quad \mathbf{v}_2 = \frac{1}{5}(20\hat{\mathbf{i}} + 115\hat{\mathbf{j}} - 80\hat{\mathbf{k}}) = 4\hat{\mathbf{i}} + 23\hat{\mathbf{j}} - 16\hat{\mathbf{k}}$$

51 A particle of mass 5 m at rest suddenly breaks on its own into three fragments. Two fragments of mass m each move along mutually perpendicular direction with each speed v. The energy released during the process is **[NEET (Odisha) 2019]**

(a) $\frac{3}{5}mv^2$ (b) $\frac{5}{3}mv^2$
(c) $\frac{3}{2}mv^2$ (d) $\frac{4}{3}mv^2$

Ans. *(d)*

The particle of mass 5m breaks in three fragments of mass m, m and 3m respectively. Two fragments of mass m each, move in perpendicular direction with velocity v and the left fragment will move in a direction with velocity $\mathbf{v}'$ such that the total momentum of the system must remain conserved.



By law of conservation of momentum,

$$5m \times 0 = mv\hat{\mathbf{i}} + mv\hat{\mathbf{j}} + 3m\mathbf{v}'$$

$$\Rightarrow \quad \mathbf{v}' = -\frac{v}{3}\hat{\mathbf{i}} - \frac{v}{3}\hat{\mathbf{j}}$$

$$\therefore \quad |\mathbf{v}'| = \sqrt{\left(-\frac{v}{3}\right)^2 + \left(-\frac{v}{3}\right)^2} = \frac{v\sqrt{2}}{3}$$

$\therefore$ Energy released

$$E = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}\times 3m\left(\frac{v\sqrt{2}}{3}\right)^2$$

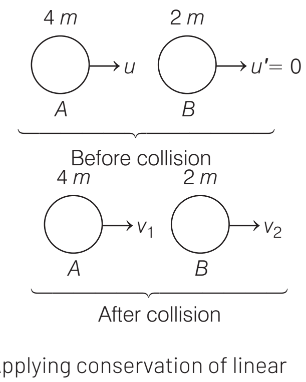
$$= mv^2 + \frac{mv^2}{3} = \frac{4}{3}mv^2$$

52 Body A of mass $4m$ moving with speed u collides with another body B of mass $2m$, at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is **[NEET (National) 2019]**

(a) $\frac{8}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{1}{9}$

Ans. *(a)*

In head-on elastic collision, momentum and kinetic energy before and after the collision is conserved. The given situation of collision can be drawn as



Applying conservation of linear momentum,

Initial momentum of system = Final momentum of system

$$\Rightarrow \quad (4m)u + (2m)u' = (4m)v_1 + (2m)v_2$$

$$4mu = 4mv_1 + 2mv_2$$

or $\quad 2u = 2v_1 + v_2 \quad \ldots(i)$

The kinetic energy of A before collision is

$$KE_A = \frac{1}{2}(4m)u^2 = 2mu^2$$

Kinetic energy of B before collision,

$$KE_B = 0$$

The kinetic energy of A after collision is

$$KE'_A = \frac{1}{2}(4m)v_1^2 = 2mv_1^2$$

Kinetic energy of B after collision,

$$KE'_B = \frac{1}{2}(2m)v_2^2 = mv_2^2$$

As, Initial kinetic energy of the system = Final kinetic energy of the system

$$\Rightarrow KE'_A + KE'_B = KE'_A + KE'_B$$

$$2mu^2 + 0 = 2mv_1^2 + mv_2^2$$

$$2mu^2 = 2mv_1^2 + mv_2^2 \text{ or } 2u^2 = 2v_1^2 + v_2^2 \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$v_1 = \frac{1}{3}u \text{ and } v_2 = \frac{4}{3}u$$

or the final velocity of A can be directly calculated by using the formula.

The velocity after collision is given by

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$= \left(\frac{4m - 2m}{4m + 2m}\right)u + \frac{2(2m)\times 0}{(4m + 2m)} \quad [\because u_2 = u' = 0]$$

$$= \frac{2m}{6m}u = \frac{1}{3}u$$

$\therefore$ Net decreases in kinetic energy of A

$$\Delta KE = KE_A - KE_A$$

$$= 2mu^2 - 2mv_1^2 = 2m(u^2 - v_1^2)$$

Substituting the value of v_1, we get

$$\Delta KE = 2m\left(u^2 - \frac{u^2}{9}\right) = \frac{16mu^2}{9}$$

$\therefore$ The fractional decreases in kinetic energy is

$$\frac{\Delta KE}{KE_A} = \frac{16mu^2}{9} \times \frac{1}{2mu^2} = \frac{8}{9}$$

53 A moving block having mass m, collides with another stationary block having mass $4m$. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be **[NEET 2018]**

(a) 0.8 (b) 0.25
(c) 0.5 (d) 0.4

Ans. *(b)*

Since, the collision mentioned is an elastic head-on collision. Thus, according to the law of conservation of linear momentum, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

where, m_1 and m_2 are the masses of the two blocks, respectively and u_1 and u_2 are their initial velocities and v_1 and v_2 are their final velocities, respectively.

Here, $m_1 = m, m_2 = 4m$

$$u_1 = v, u_2 = 0 \text{ and } v_1 = 0$$

$$mv + 4m \times 0 = 0 + 4mv_2$$

$$\Rightarrow \quad mv = 4mv_2 \quad \text{or} \quad v_2 = \frac{v}{4} \quad \ldots(i)$$

As, the coefficient of restitution is given as,

$$e = \frac{\text{relative velocity of separation after collision}}{\text{relative velocity of approach}}$$

$$= \frac{v_2 - v_1}{u_2 - u_1} = \frac{\frac{v}{u} - 0}{0 - v} \quad \text{[from Eq. (i)]}$$

$$= \frac{1}{4}$$

$\therefore \; e = 0.25$

54 Two identical balls A and B having velocities of 0.5 m/s and –0.3 m/s respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be **[NEET 2016]**

(a) –0.5 m/s and 0.3 m/s
(b) 0.5 m/s and –0.3 m/s
(c) –0.3 m/s and 0.5 m/s
(d) 0.3 m/s and 0.5 m/s

Ans. *(c)*

In elastic collision, kinetic energy of the system remains unchanged and momentum is also conserved.

It is given that mass of balls are same and collision is perfectly elastic ($e = 1$) so their velocities will be interchanged.

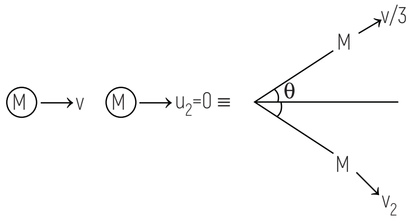
Thus, $v'_A = v_B = -0.3$ m/s,

$v'_B = v_A = 0.5$ m/s

55 On a frictionless surface, a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $v/3$. The second block's speed after the collision is **[CBSE AIPMT 2015]**

(a) $\frac{2\sqrt{2}}{3}v$ (b) $\frac{3}{4}v$
(c) $\frac{3}{\sqrt{2}}v$ (d) $\frac{\sqrt{3}}{2}v$

Ans. *(a)*



According to law of conservation of kinetic energy, we have

$$\frac{1}{2}Mv^2 + 0 = \frac{1}{2}M\left(\frac{v}{3}\right)^2 + \frac{1}{2}Mv_2^2$$

$$\Rightarrow \quad v^2 = \frac{v^2}{9} + v_2^2$$

$$\Rightarrow \quad v^2 - \frac{v^2}{9} = v_2^2 \;\Rightarrow\; \frac{8v^2}{9}$$

Velocity of second block after collision

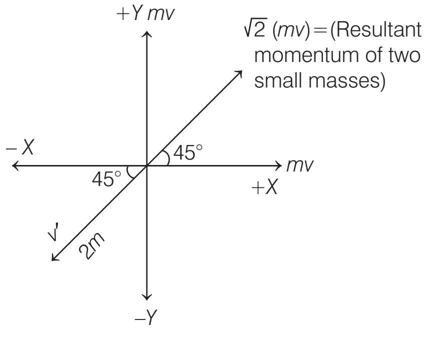
$$v_2 = \frac{2\sqrt{2}}{3}v$$

56 A body of mass $(4m)$ is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is **[CBSE AIPMT 2014]**

(a) mv^2 (b) $\frac{3}{2}mv^2$
(c) $2mv^2$ (d) $4\,mv^2$

Ans. *(b)*

Problem Solving Strategy Conserve the momentum of third mass with the resultant momentum of 1st and 2nd masses. After getting velocity of third mass, calculate total kinetic energy.



According to question, the third part of mass $2m$ will move as shown in the figure, because the total momentum of the system after explosion must remain zero. Let the velocity of third part be v'.

From the conservation of momentum

$$\sqrt{2}\,(mv) = (2m) \times v'$$

$$\Rightarrow \quad v' = \frac{v}{\sqrt{2}}$$

So, total kinetic energy generated by the explosion

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\,v'^2$$

$$= mv^2 + m \times \left(\frac{v}{\sqrt{2}}\right)^2$$

$$= mv^2 + \frac{mv^2}{2}$$

$$= \frac{3}{2}mv^2$$

57 A ball moving with velocity 2 ms^{-1} collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in ms^{-1}) after collision will be **[CBSE AIPMT 2010]**

(a) 0, 1 (b) 1, 1
(c) 1, 0.5 (d) 0, 2

Ans. *(a)*

If two bodies collide head on with coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \ldots(i)$$

From, the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad v_1 = \left[\frac{m_1 - em_2}{m_1 + m_2}\right]u_1 + \left[\frac{(1+e)\,m_2}{m_1 + m_2}\right]u_2$$

Substituting $u_1 = 2\,ms^{-1}$, $u_2 = 0$, $m_1 = m$

and $m_2 = 2m$, $e = 0.5$

we get, $v_1 = \left[\frac{m - m}{m + 2m}\right] \times 2$

$$\Rightarrow \quad v_1 = 0$$

Similarly,

$$v_2 = \left[\frac{(1+e)\,m_1}{m_1 + m_2}\right]u_1 + \left[\frac{m_2 - em_1}{m_1 + m_2}\right]u_2$$

$$= \left[\frac{1.5 \times m}{3m}\right] \times 2 = 1\,ms^{-1}$$

58 An explosion blows a rock into three parts. Two parts go off at right angles to each other. These two are, 1 kg first part moving with a velocity of $12\,ms^{-1}$ and 2 kg second part moving with a velocity of $8\,ms^{-1}$. If the third part flies off with a velocity of $4\,ms^{-1}$, its mass would be **[CBSE AIPMT 2009]**

(a) 5 kg (b) 7 kg (c) 17 kg (d) 3 kg

Ans. *(a)*

Momentum of first part $= 1\times 12 = 12$ kg ms^{-1}

Momentum of the second part

$$= 2\times 8 = 16\ \text{kg ms}^{-1}$$

∴ Resultant momentum

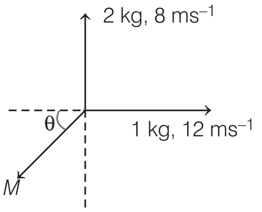
$$= \sqrt{(12)^2 + (16)^2} = 20\ \text{kg ms}^{-1}$$

The third part should also have the same momentum.

Let the mass of the third part be M, then

$$4\times M = 20$$
$$M = 5\ \text{kg}$$

Alternative



$$Mv\cos\theta = 12 \quad \text{...(i)}$$
$$Mv\sin\theta = 16 \quad \text{...(ii)}$$

Dividing Eqs. (ii) and (i), we get

$$\tan\theta = \frac{16}{12} = \frac{4}{3}$$
$$M = \frac{12\times 5}{4\times 3} = \frac{60}{12} = 5\ \text{kg}$$

59 A shell of mass 200 g is ejected from a gun of mass 4 kg by an explosion that generates 1.05 kJ of energy. The initial velocity of the shell is **[CBSE AIPMT 2008]**

(a) $100\ ms^{-1}$ (b) $80\ ms^{-1}$
(c) $40\ ms^{-1}$ (d) $120\ ms^{-1}$

Ans. *(a)*

Problem Solving Strategy Make two equations, one from conservation of momentum and other from convservation of energy and solve it.

Let the velocity of shell be v and that of gun be V. Then, according to conservation of linear momentum.

$$4V + 0.2v = 0 \quad \text{...(i)}$$

Using conservation of energy

$$\frac{1}{2}\times 4\times V^2 + \frac{1}{2}\times 0.2\times v^2 = 1050 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v = 100\ \text{m/s}$$

60 A stationary particle explodes into two particles of masses m_1 and m_2, which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is **[CBSE AIPMT 2003]**

(a) 1 (b) $\frac{m_1v_2}{m_2v_1}$
(c) $\frac{m_2}{m_1}$ (d) $\frac{m_1}{m_2}$

Ans. *(c)*

From conservation of linear momentum,

Initial momentum p_{initial} = Find momentum p_{final}

$$0 = m_1v_1 - m_2v_2$$

or $m_1v_1 = m_2v_2$

or $\frac{v_1}{v_2} = \frac{m_2}{m_1}$ ...(i)

Thus, ratio of kinetic energies,

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{m_1}{m_2}\times\left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$$

61 Two equal masses m_1 and m_2 moving along the same straight line with velocities +3 m/s and – 5 m/s respectively collide elastically. Their velocities after the collision will be respectively **[CBSE AIPMT 1998]**

(a) + 4 m/s for both
(b) – 3 m/s and + 5 m/s
(c) – 4 m/s and + 4 m/s
(d) – 5 m/s and + 3 m/s

Ans. *(d)*

Given,

$u_1 = 3\ \text{m/s}, u_2 = -5\ \text{m/s}, m_1 = m_2 = m$ According to principle of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m\times 3 - m\times 5 = mv_1 + mv_2$$

or $v_1 + v_2 = -2$ ...(i)

In an elastic collision,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$\Rightarrow \quad v_2 - v_1 = e(u_1 - u_2)$

$\Rightarrow \quad v_2 - v_1 = (1)(3+5)$ $\quad (\because e = 1)$

$\Rightarrow \quad v_1 - v_2 = -8$ ...(ii)

Adding Eqs. (i) and (ii), we obtain

$$2v_1 = -10$$

$\Rightarrow \quad v_1 = -5\ \text{m/s}$

From Eq. (i),

$$v_2 = -2 - v_1 = -2 + 5 = 3\ \text{m/s}$$

Thus, $v_1 = -5\ \text{m/s}, v_2 = +3\ \text{m/s}$

If two bodies collide elastically, then their velocities are interchanged. Since, it is an elastic collision hence, velocities after collision will be – 5 m/s and 3 m/s.

62 A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is **[CBSE AIPMT 1997]**

(a) 140 J (b) 100 J (c) 60 J (d) 40 J

Ans. *(c)*

Initial momentum = Final momentum

∴ $m_1v_1 + m_2v_2 = (m_1 + m_2)v$

Given, $v_1 = 36\ \text{km/h}$

$$= 36\times\frac{5}{18} = 10\ \text{m/s},$$

$$v_2 = 0$$
$$m_1 = 2\ \text{kg}, m_2 = 3\ \text{kg}$$

∴ $v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$$= \frac{2\times 10 + 3\times 0}{2+3}$$

or $v = \frac{20}{5} = 4\ \text{m/s}$

Loss in kinetic energy

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1+m_2)v^2$$
$$= \frac{1}{2}\times 2\times(10)^2 + 0 - \frac{1}{2}(2+3)\times(4)^2$$
$$= 100 - 40 = 60\ \text{J}$$

63 A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is **[CBSE AIPMT 1997]**

(a) 140 J (b) 100 J
(c) 60 J (d) 40 J

Ans. *(c)*

Initial momentum = Final momentum

$\therefore \quad m_1v_1 + m_2v_2 = (m_1 + m_2)\,v$

Given, $v_1 = 36\text{ km/h} = 36 \times \frac{5}{18} = 10\text{ m/s}$,

$v_2 = 0$

$m_1 = 2\text{ kg},\ m_2 = 3\text{ kg}$

$\therefore \quad v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$= \frac{2\times 10 + 3\times 0}{2+3}$

or $\quad v = \frac{20}{5} = 4\text{ m/s}$

Loss in kinetic energy

$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1 + m_2)\,v^2$

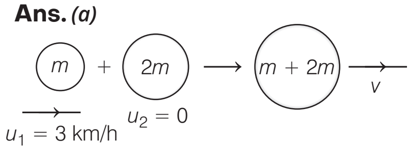
$= \frac{1}{2}\times 2\times(10)^2 + 0 - \frac{1}{2}(2+3)\times(4)^2$

$= 100 - 40 = 60\text{ J}$

64 A body of mass *m* moving with velocity 3 km / h collides with a body of mass 2 *m* at rest. Now, the coalesced mass starts to move with a velocity **[CBSE AIPMT 1996]**

(a) 1 km/h (b) 2 km/h
(c) 3 km/h (d) 4 km/h

Ans. *(a)*



Let v be velocity of combined mass after collision.
Applying law of conservation of linear momentum

Initial momentum = Final momentum

$m\times 3 + 2m\times 0 = (m + 2m)\,v$

or $\quad 3m = 3mv$ or $v = 1\text{ km / h}$

65 Two identical balls *A* and *B* moving with velocities + 0.5 m/s and – 0.3 m/s respectively, collide head on elastically. The velocity of the balls *A* and *B* after collision will be respectively **[CBSE AIPMT 1991]**

(a) + 0.5 m/s and + 0.3 m/s
(b) – 0.3 m/s and + 0.5 m/s
(c) + 0.3 m/s and 0.5 m/s
(d) – 0.5 m/s and + 0.3 m/s

Ans. *(b)*

When two bodies of equal masses undergo head on elastic collision in one dimension their velocities are just interchanged.

66. The coefficient of restitution *e* for a perfectly elastic collision is **[CBSE AIPMT 1988]**

(a) 1 (b) zero
(c) infinite (d) –1

Ans. *(a)*

The degree of elasticity of a collision is determined by a quantity called coefficient of restitution or coefficient of resilience of the collision. It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is represented by *e*.

$$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of approach (before collision)}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

where, u_1, u_2 are the velocities of two bodies before collision and v_1, v_2 are their respective velocities after collision.

For a perfectly elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision

$\therefore \quad e = 1$