UNIT TEST 1

Time Allowed : 1½ Hours **Max. Marks : 50**

Notes : **1.** *All question are compulsory.*
2. *Marks have been indicated against each question.*

1. A relation R on A = {1, 2, 3} defined by :
R = {(1, 1), (1, 2), (3, 3)} is not symmetric. Why ? **(1)**

2. Let '*' be a binary operation on **N** given by
$a * b$ = H.C.F.(a, b), $a, b \in \mathbf{N}$. Write the value of 22*4. **(1)**

3. Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. **(2)**

4. Evaluate $\cot(\tan^{-1} a + \cot^{-1} a)$. **(2)**

5. Show that the relation defined by :
(a, b) R $(c, d) = a + d = b + c$
in the set **N** is an equivalace relation. **(2)**

6. Find *fog* and *gof* if
$f(x) = |x|$ and $g(x) = |5x - 2|$. **(4)**

7. Consider $f : \{1, 2, 3\} \to \{a, b, c\}$, given by :
$f(1) = a, f(2) = b$ and $f(3) = c$.
Find f^{-1} and show that $(f^{-1})^{-1} = f$. **(4)**

8. Let '*' be a binary operation on **Q** defined by $a * b = \frac{3ab}{5}$.
Show that '*' is commutative as well as associative.
Also, find its identity element, if it exists. **(4)**

9. Prove that :
$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$ **(4)**

10. Solve for x :
$$3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}.$$ **(4)**

11. Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$. **(4)**

12. Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by :
R = {(a, b) : both a and b are either odd or even}.
Show that R is an equivalence relation.
Further, slow that all the elements of the subset {1, 3, 5, 7} are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}. **(6)**

13. A binary operation '*' on the set {0, 1, 2, 3, 4, 5} defined as :
$$a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & \text{if } a+b \geq 6. \end{cases}$$
Show that zero is the identity for this operation and each element 'a' of the set is invertible with $6 - a$, being the inverse of 'a'. **(6)**

14. Prove that :
$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}; x \in \left(0, \frac{\pi}{4}\right).$$ **(6)**

Answers

1. $(1, 2) \in$ R whereas $(2, 1) \notin$ R. **2.** 2. **3.** $\frac{\pi}{3}$.

4. 0. **6.** $|5x - 2|, |5|x| - 2|$. **8.** $\frac{5}{3}$. **10.** $x = \frac{1}{\sqrt{3}}$.