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Dual Nature of Radiation and Matter

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Electrons are regarded as particles because they possess charge, mass and behave according to the laws of particle mechanics. However, it can be considered that a moving electron can be assumed as a wave as it is interpreted as a particle. We considered electromagnetic waves as waves because under suitable circumstances, they exhibit diffraction, interference and polarisation. Similarly, under other circumstances, they behave as a streams of particles. Rather, we can say they have the *dual nature*.

Emission of Electrons

At room temperature, the free electrons move randomly within the conductor, but they cannot leave the surface of the conductor due to attraction of positive charges. Some external energy is required to emit these electrons from a metal surface.

Thus, a sufficient minimum energy which is required to emit these electrons from the surface of the conductor is called the *work function* (denoted by W or ϕ) of the conductor. It is the property of the metallic surface.

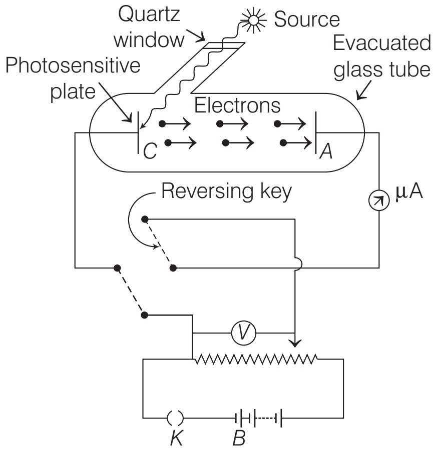
Photoelectric Emission

When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. This is called photoelectric emission. These photo (light) generated electrons are *photoelectrons* and the current, so produced is called *photoelectric current*.

Note ϕ is measured in eV (electron-volt), 1 eV = 1.602×10^{-19} J.

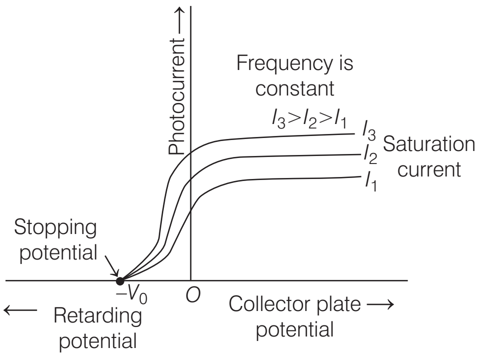
Experimental Study of Photoelectric Effect

On the basis of the experimental arrangement used for studying the photoelectric effect, the variations of photo current with intensity of radiation, frequency of radiation and the potential difference between the plates A and C are as follows.



Effect of Potential on Photoelectric Current

For a fixed intensity and frequency of incident radiation, the variation of photoelectric current with potential difference between cathode and anode is as shown below,



From the above graph, we can observe that,

(i) For a fixed frequency and intensity of incident radiation, the photocurrent increases with increase in the potential applied to the collector. At some stage, the photoelectric current becomes maximum or saturates.

(ii) Maximum value of photoelectric current is called *saturation current*. It corresponds to the case when all the photoelectrons emitted by the emitter plate reaches the collector plate.

(iii) For a particular frequency of incident radiation, the minimum negative (retarding) potential V_0 given to the plate A for which the photocurrent stops or becomes zero is called the *cut-off* or *stopping potential*.

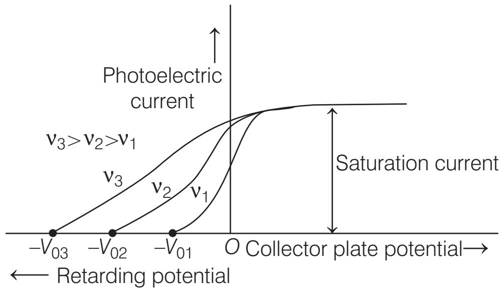
(iv) Photoelectric current is zero when the stopping potential is sufficient to repel even the most energetic photoelectrons with the maximum kinetic energy $K_{\max}$, so that $K_{\max} = eV_0$.

(v) Thus, for a given frequency of the incident radiation, the stopping potential is independent of its intensity.

In other words, the maximum kinetic energy of photoelectrons depends on the light source and the emitter plate material, but it is independent of intensity of incident radiation.

Effect of Frequency of Incident Radiation on Stopping Potential

For a fixed frequency of incident radiation, the variation of photoelectric current against the potential difference between the plates is as shown below.

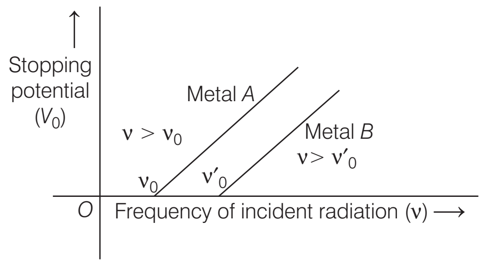


From the above graph, we observe that,

(i) Stopping potentials are in the order of $V_{03} > V_{02} > V_{01}$, if the frequencies are in the order of $\nu_3 > \nu_2 > \nu_1$.

(ii) Thus, greater the frequency of incident light, greater is the maximum kinetic energy of photoelectrons. Consequently, greater retarding potential is required to stop them completely.

The graph between frequency (ν) and stopping potential (V_0) is found to be a straight line which is not passing through the origin.



This graph shows that,

(i) The stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii) There exist a certain minimum cut-off frequency (ν_0) for which the stopping potential is zero.

Note The stopping potential is more negative for higher frequencies of incident radiations.

Implication of the Experimental Study of Photoelectric Effect

(i) Maximum KE of photoelectrons varies linearly with ν of incident radiation, but it is independent of its intensity.

(ii) For a given material, there exists a certain minimum frequency of the incident radiation below which no emissions of photoelectrons take place. This frequency is called *threshold frequency*.

(iii) The photoelectric emission is an instantaneous process. The time lag between the incidence of radiations and emission of photoelectrons is very small, less than 10^{-9}s, even when incident radiation is made exceedingly dim.

Note **Threshold wavelength** (λ_0) is the maximum wavelength of incident radiation above which no photoelectric emission takes place.

Photoelectric Effect and Wave Theory of Light

The wave theory of light could not explain photoelectric effect due to following main reasons.

(i) Wave theory suggests that the kinetic energy of the photoelectrons should increase with the increase in intensity of light. However, $K_{\max} = eV_0$ suggests that it is independent of the intensity of light.

(ii) According to wave theory, the photoelectric effect should occur for any frequency of the light, provided that the light is intense enough. However, $E \geq \phi$ for $\nu \geq \nu_0$ or $\lambda \leq \lambda_0$.

(iii) Wave theory suggests that absorption of energy by electron takes place continuously, so it can take hours or more for a single electron to pick up sufficient energy to overcome the ϕ_0 and come out of the metal. However, photoemission is an instantaneous process.

Einstein's Photoelectric Equation

Albert Einstein explained the implications of photoelectric effect on the basis of Planck's quantum theory. According to this, light radiations consist of packets of energy called quanta and each quanta of light has energy $h\nu$, where h is Planck 's constant and ν is the frequency of incident radiation.

$\therefore$ Einstein's photoelectric equation is given as

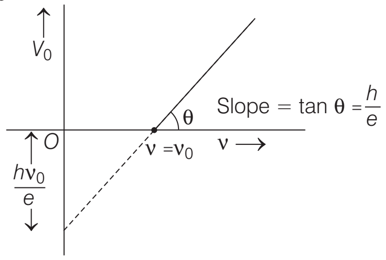
$$K_{\max} = \frac{1}{2} m v_{\max}^2 = h\nu - h\nu_0 = h\nu - \phi_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Here $h = 6.626 \times 10^{-34}$ J-s $= 4.136 \times 10^{-15}$ eV-s

Also, $K_{\max} = eV_0$

$$eV_0 = h\nu - \phi_0 \text{ or } V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$$

The above equation implies that V_0 *versus* ν graph is a straight line with slope $\left(\frac{h}{e}\right)$, which is independent of the nature of the material. Also, the intercept on the V_0-axis is $\frac{-h\nu_0}{e}$.



Example 1. *When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to* **[JEE Main 2020]**

(a) 0.81 eV (b) 1.02 eV (c) 0.52 eV (d) 0.61 eV

Sol. (*d*) Maximum kinetic energy of emitted electrons is given by,

$$K_{\max} = \frac{hc}{\lambda} - \phi_0$$

where, $hc \approx 1240$ eV- nm and $\phi_0 =$ work function.

Here, as per question, $$K = \frac{1240}{500} - \phi_0 \quad \text{...(i)}$$

and $$3K = \frac{1240}{200} - \phi_0 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$3\left(\frac{1240}{500} - \phi_0\right) = \frac{1240}{200} - \phi_0$$

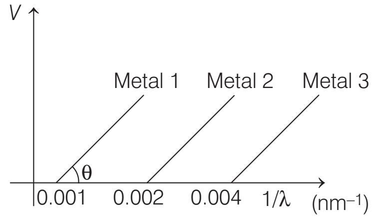
$$\Rightarrow \quad 2\phi_0 = \frac{3 \times 1240}{500} - \frac{1240}{200}$$

$$\Rightarrow \quad 2\phi_0 = \frac{1240}{100}\left(\frac{3}{5} - \frac{1}{2}\right) = \frac{1240}{100}\left(\frac{1}{10}\right)$$

$$\Rightarrow \quad \phi_0 = \frac{1240}{2 \times 1000} = 0.62 \text{ eV}$$

which is close to 0.61 eV.

Example 2. *The graph between $1/\lambda$ and stopping potential (V) of three metals having work-functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure given below. Which of the following statement(s) is/are correct? (Here, λ is the wavelength of the incident ray).*



(a) *Ratio of work-functions* $\phi_1 : \phi_2 : \phi_3 = 1 : 3 : 4$
(b) *Ratio of work-functions* $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(c) $\tan\theta$ *is directly proportional to* hc/e, *where h is Planck constant and c is the speed of light*
(d) *The violet colour light can eject photoelectrons from metals 2 and 3*

Sol. (c) From the relation, $eV = \frac{hc}{\lambda} - \phi$

or $$V = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$$

This is equation of straight line. Slope is $\tan\theta = \frac{hc}{e}$.

Further $V = 0$ at $\phi = \frac{hc}{\lambda}$

$$\therefore \quad \phi_1 : \phi_2 : \phi_3 = \frac{hc}{\lambda_{01}} : \frac{hc}{\lambda_{02}} : \frac{hc}{\lambda_{03}}$$
$$= \frac{1}{\lambda_{01}} : \frac{1}{\lambda_{02}} : \frac{1}{\lambda_{03}}$$
$$= 1 : 2 : 4$$

Violet colour has wavelength 4000 Å.

So, violet colour can eject photoelectrons from metal 1 and metal 2.

Example 3. *Surface of certain metal is first illuminated with light of wavelength* $\lambda_1 = 350$ *nm and then by light of wavelength* $\lambda_2 = 540$ *nm. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to*
(energy of photon $= \frac{1240}{\lambda(\text{in nm})}$ eV*)*

(a) 5.6
(b) 2.5
(c) 1.8
(d) 1.4

Sol. (c) Let velocity of one is twice in factor with second then.
Let $v_1 = 2v$ and $v_2 = v$.
We know that from Einstein's photoelectric equation, energy of incident radiation = work function + KE

or $$\frac{hc}{\lambda} = \phi + \frac{1}{2}mv^2$$

Let when $\lambda_1 = 350$ nm, then $v_1 = 2v$
and when $\lambda_1 = 540$ nm, then $v_2 = v$
$\therefore$ Above equation becomes

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v^2)$$

or $$\frac{hc}{\lambda_1} - \phi = \frac{1}{2}m \times 4v^2 \quad \text{...(i)}$$

and $$\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$$

or $$\frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2 \quad \text{...(ii)}$$

Now, we divide Eq. (i) by (ii) Eq.

$$\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{\frac{1}{2}m \times 4v^2}{\frac{1}{2}mv^2} = 4$$

or $$\frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

or $$\phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$
$$= \frac{1}{3} \times 1240 \left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$

or $$\phi = 1.8 \text{ eV}$$

Example 4. *The magnetic field associated with a light wave is given at the origin, by* $B = B_0$
$[\sin(3.14 \times 10^7)\, ct + \sin(6.28 \times 10^7) ct]$.
If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons?
(Take, $c = 3 \times 10^8$ ms^{-1} *and* $h = 6.6 \times 10^{-34}$ *J-s)*

(a) 7.72 eV
(b) 6.82 eV
(c) 8.52 eV
(d) 12.5 eV

Sol. (a) According to question, the wave equation of the magnetic field which produce photoelectric effect

$$B = B_0[(\sin(3.14 \times 10^7 ct) + \sin(6.28 \times 10^7 ct)]$$

Here, the photoelectric effect produced by the angular frequency

$$\omega = 6.28 \times 10^7 \text{ c}$$
$$\Rightarrow \quad \omega = 6.28 \times 10^7 \times 3 \times 10^8$$
$$\omega = 2\pi \times 10^7 \times 3 \times 10^8 \text{ rad/s} \quad \text{...(i)}$$

Using Eqs. (i)

$$h\nu = \frac{h\omega}{2\pi} = \frac{h \times 2\pi \times 10^7 \times 3 \times 10^8}{2\pi}$$
$$h\nu = 12.4 \text{ eV}$$

Therefore, according to Einstein equation for photoelectric effect

$$E = h\nu = \phi + KE_{max}$$
$$\Rightarrow \quad KE_{max} = E - \phi \quad \text{(where, } \phi = \text{work-function} = 4.7 \text{ eV)}$$
$$KE_{max} = 12.4 - 4.7 = 7.7 \text{ eV}$$
or $$KE_{max} = 7.7 \text{ eV}$$

Particle Nature of Light : Photon

Photoelectric effect gave evidence that light in interaction with matter behaved as if, it was made up of quanta or packets of energy, each having energy $h\nu$. This packet of energy is associated with a particle called *photon*.

Characteristic Properties of Photons

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called **photons**.

(ii) A photon in vacuum travels with a speed of light, c (*i.e.* $3 \times 10^8 \text{ms}^{-1}$) in straight line.

(iii) It has zero rest mass, *i.e.* the photon cannot exist at rest.

(iv) The inertial mass of a photon is given by

$$m = \frac{E}{c^2} = \frac{h}{c\lambda} = \frac{h\nu}{c^2}$$

(v) Irrespective of the intensity of radiation, all the photons of a particular frequency (ν) or wavelength λ have the same energy

$$E\left(= h\nu = \frac{hc}{\lambda}\right) \text{ and momentum } p\left(= \frac{h\nu}{c} = \frac{h}{\lambda}\right).$$

(vi) Energy of a photon depends upon frequency of the photon, so the energy of the photon does not change when photon travels from one medium to another.

(vii) Photons are not deflected by electric and magnetic fields. This shows that photons are electrically neutral.

(viii) Total energy and total momentum are conserved in photon-particle collision. However, the number of photons may not be conserved in collision. As, the photon may be absorbed or a new photon may be created in a collision.

Intensity of Light

As a moving photon carries momentum with it. So, pressure and intensity are also associated with photons.

For area A through which light (photons) of total energy E and intensity I is passing normally,

Total energy of photons, $E = Nh\nu = N\left(\frac{hc}{\lambda}\right)$

$\therefore$ Power of source, $P = \frac{E}{t} = \frac{Nh\nu}{t} = nh\nu = \frac{nhc}{\lambda}$ $\left[\because n = \frac{N}{t}\right]$

$$\Rightarrow \quad n = P/E = \frac{P}{h\nu} = \frac{P\lambda}{hc}$$

$$\Rightarrow \quad \text{Intensity, } I = \frac{P}{A} = \frac{E}{tA} = \frac{nh\nu}{A} = n\frac{hc}{A\lambda}$$

Note At a distance r from a point source of power P, intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$.

For a line source, $I \propto \frac{P}{2\pi r l} \Rightarrow I \propto \frac{1}{r}$.

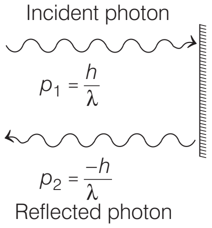
Force and Pressure Exerted by Incident Light

When light is incident on a surface, it exerts a force (or pressure) on the surface.

The expression for force and pressure being exerted on perfectly reflecting and perfectly absorbing surfaces are as follows.

Force and Pressure Exerted on Perfectly Reflecting Surface

For N photons incident on a perfectly reflected surface in time t as shown in the figure.



Force on the surface,

$$F = \frac{\Delta p}{\Delta t} = \frac{2Nh}{\lambda t} = n\left(\frac{2h}{\lambda}\right) \qquad \left[\because n = \frac{N}{t}\right]$$

$$\Rightarrow \quad F = \left(\frac{2h}{\lambda}\right) \times \left(\frac{P\lambda}{hc}\right) = \frac{2P}{c} \qquad \left[\because n = \frac{P\lambda}{hc}\right]$$

$\therefore$ Pressure on the surface $= \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$

where, c is speed of light.

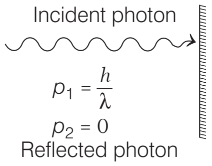
Special case When a beam of light is incident at angle ϕ on perfectly reflecting surface

$$F = \frac{2P}{c}\cos\phi = n\left(\frac{2h}{\lambda}\right)\cos\phi$$

$$\text{Pressure} = \frac{F}{A} = \frac{2P}{cA}\cos\phi = \frac{2I\cos\phi}{c}$$

Force and Pressure on Perfectly Absorbing Surface

For N photons incident on a perfectly absorbing surface, then in time t.



$$\therefore \quad F = \left(\frac{P\lambda}{hc}\right)\left(\frac{h}{\lambda}\right) = \frac{P}{c}$$

Pressure on the surface $= \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$

Photocell

It is a device which converts light energy into electrical energy. It is also called an *electric eye*.

It works on the principle of photoelectric emission.

Example 5. *Two metallic plates A and B each of area $5 \times 10^{-4} m^2$, are placed parallel to each other at a separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$ so that 10^{16} photons fall on it per square metre per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work-function of plate A remains constant at the value 2 eV. Then*

(a) *the number of photoelectrons emitted upto $t = 10$ s is 6×10^7.*

(b) *the magnitude of the electric field between the plates A and B at $t = 10$ s is 2×10^3 NC^{-1}.*

(c) *the kinetic energy of the most energetic photoelectrons emitted at $t = 10$ s when it reaches plate B is 23 eV. Neglect the time taken by the photoelectron to reach plate B.*
(Take, $\varepsilon_0 = 8.85 \times 10^{-12} C^2/N\text{-}m^2$).

(d) *Both (b) and (c)*

Sol. (*d*) Area of plates, $A = 5 \times 10^{-4}$ m^2

Distance between the plates, $d = 1$ cm $= 10^{-2}$ m

(a) Number of photoelectrons emitted upto $t = 10$ s are

$$n = \frac{\text{(number of photons falling on unit area in unit time)} \times \text{(area} \times \text{time)}}{10^6}$$

$$= \frac{1}{10^6}[(10)^{16} \times (5 \times 10^{-4}) \times (10)] = 5.0 \times 10^7$$

(b) At time $t = 10$ s,

Charge on plate A, $q_A = + ne = (5.0 \times 10^7)(1.6 \times 10^{-19})$
$= 8.0 \times 10^{-12}$ C

and charge on plate B,

$$q_B = (33.7 \times 10^{-12} - 8.0 \times 10^{-12})$$
$$= 25.7 \times 10^{-12} \text{ C}$$

$\therefore$ Electric field between the plates, $E = \dfrac{(q_B - q_A)}{2A\varepsilon_0}$

or $$E = \frac{(25.7 - 8.0) \times 10^{-12}}{2 \times (5 \times 10^{-4})(8.85 \times 10^{-12})} = 2 \times 10^3 \text{ N/C}$$

(c) Energy of most energetic photoelectrons at plate A,
$= E - W = (5 - 2)$ eV $= 3$ eV

Increase in energy of photoelectrons
$= (eEd)$ joule $= (Ed)$ eV
$= (2 \times 10^3)(10^{-2})$ eV $= 20$ eV

Energy of photoelectrons at plate $B = (20 + 3)$ eV $= 23$ eV

Example 6. *A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work-function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. The number of photoelectrons liberated in two seconds.*

(a) 1.1×10^{12} (b) 2×10^6
(c) 4×10^8 (d) 9×10^6

Sol. (a) Energy of photon having wavelength 4144 Å,

$$E_1 = \frac{12375}{4144} \text{ eV}$$
$$= 2.99 \text{ eV}$$

Similarly, $$E_2 = \frac{12375}{4972} \text{ eV}$$
$$= 2.49 \text{ eV}$$

and $$E_3 = \frac{12375}{6216} \text{ eV}$$
$$= 1.99 \text{ eV}$$

Since, only E_1 and E_2 are greater than the work-function $W = 2.3$ eV, only first two wavelengths are capable for ejecting photoelectrons. Given intensity is equally distributed in all wavelengths. Therefore, intensity corresponding to each wavelength is

$$\frac{3.6 \times 10^{-3}}{3} = 1.2 \times 10^{-3} \text{ W/m}^2$$

Or energy incident per second in the given area ($A = 1.0$ cm^2 $= 10^{-4}$ m^2) is

$$P = 1.2 \times 10^{-3} \times 10^{-4}$$
$$= 1.2 \times 10^{-7} \text{ J/s}$$

Let n_1 be the number of photons incident per unit time in the given area corresponding to first wavelength, then

$$n_1 = \frac{P}{E_1}$$
$$= \frac{1.2 \times 10^{-7}}{2.99 \times 1.6 \times 10^{-19}}$$
$$= 2.5 \times 10^{11}$$

Similarly, $$n_2 = \frac{P}{E_2}$$
$$= \frac{1.2 \times 10^{-7}}{2.49 \times 1.6 \times 10^{-19}}$$
$$= 3.0 \times 10^{11}$$

Since, each energetically capable photon ejects one electron, total number of photoelectrons liberated in 2 s.

$$= 2(n_1 + n_2)$$
$$= 2(2.5 + 3.0) \times 10^{11}$$
$$= 1.1 \times 10^{12}$$

Wave Nature of Particle

In case of light, some phenomenon like diffraction and interference can be explained on the basis of its wave character. However, certain other phenomenon such as black body radiation and photoelectric effect can be explained only on the basis of its particle nature. Thus, light is said to have a dual character. Such studies on light wave were made by Einstein in 1905.
Louis-de-Broglie, in 1942 extended the idea of photons to material particles such as electron and he proposed that matter also has a dual character as wave and as particle.

Matter Waves : de-Broglie Relation

According to de-Broglie, moving particles of matter should display wave-like properties under suitable conditions. The waves associated with such particles are called *matter waves* or *de-Broglie waves.* These then proposed that the wavelength λ associated with moving particles can expressed as,

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where, m and v are the mass and velocity of the particle, respectively, p is momentum and h is Planck's constant.

de-Broglie wavelength associated with different moving particles are as follows.

(i) For a photon, $\lambda = \frac{h}{mc} = \frac{h}{p}$

where, $p = mc$ is the momentum of a photon.

(ii) For a charged particle, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

where V is the potential through which particle is accelerated and E is kinetic energy.

Specifically for an electron $\lambda = \frac{1.227}{\sqrt{V}}$ nm.

(iii) For a gas molecule, $\lambda = \frac{h}{\sqrt{3mkT}}$

where, T = absolute temperature

and k = Boltzmann's constant = 1.38×10^{-23} J/K

(iv) For neutron, $\lambda = \frac{0.286}{\sqrt{E \text{ (in eV)}}}$ Å.

Example 7. *The de-Broglie wavelength associated with an electron moving with a speed of* 5.4×10^{6} m/s *is* λ_e *and for a ball of mass* 150 g *travelling at* 30 m/s *is* λ_b, *then which of the following relations between* λ_e *and* λ_b *is true?*

(a) $\lambda_e < \lambda_b$ (b) $\lambda_b < \lambda_e$
(c) $\lambda_e = \lambda_b$ (d) $2\lambda_b = \lambda_e$

Sol. (*b*) We know that de-Broglie wavelength is given by

$$\lambda = \frac{h}{p}$$

where, h is Planck's constant and p is momentum.

For the electron,

$$m = 9.11 \times 10^{-31} \text{ kg},$$
$$v = 5.4 \times 10^{6} \text{ m/s}$$
$$p = mv = 9.11 \times 10^{-31} \times 5.4 \times 10^{6}$$
$$= 4.92 \times 10^{-24} \text{ kg-m/s}$$
$$\therefore \quad \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.92 \times 10^{-24}}$$
$$\lambda_e = 0.135 \text{ nm}$$

For the ball, $m' = 0.150$ kg

$$v' = 30 \text{ m/s}$$
$$p' = m'v'$$
$$= 0.150 \times 30$$
$$= 4.50 \text{ kg-m/s}$$
$$\therefore \quad \lambda_b = \frac{h}{p'} = \frac{6.63 \times 10^{-34} \text{ J-s}}{4.50 \text{ kg-m/s}}$$
$$= 1.47 \times 10^{-34} \text{ m}$$
$$\therefore \quad \lambda_b < \lambda_e$$

Example 8. *A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is* 1.878×10^{-4}. *The mass of the particle is close to* ***[JEE Main 2020]***

(a) 4.8×10^{-27} kg (b) 9.1×10^{-31} kg
(c) 1.2×10^{-28} kg (d) 9.7×10^{-28} kg

Sol. (*d*) de-Broglie wavelength of a particle is given by

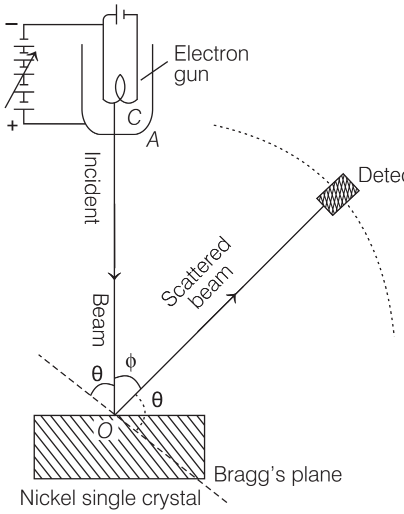
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Given, $\frac{\lambda_{\text{particle}}}{\lambda_{\text{electron}}} = 1.878 \times 10^{-4}$

$$\Rightarrow \quad \frac{m_e v_e}{m_p v_p} = 1.878 \times 10^{-4}$$
$$\Rightarrow \quad \frac{m_e \times v_e}{m_p \times 5v_e} = 1.878 \times 10^{-4} \quad (\because v_p = 5v_e)$$
$$\Rightarrow \quad m_p = \frac{m_e}{5 \times 1.878 \times 10^{-4}}$$
$$= \frac{9.1 \times 10^{-31}}{5 \times 1.878 \times 10^{-4}}$$
$$= 9.7 \times 10^{-28} \text{ kg}$$

Davisson and Germer Experiment

The wave nature of the material particles as predicted by de-Broglie wave confirmed by **Davisson** and **Germer** (1927) in United States and by **GP Thomson** (1928) **Scotland**.



They found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering.

The beam of electron was allowed to fall normally on the surface of nickel crystal. It is observed that below 44 V, the graph of electron intensity is smooth, at 44 V a bump begins to appear and continues to move upwards reaching a pronounced maximum at 54 V. Beyond 54 V, the bump diminishes with increasing potential and almost vanish at around 68 V.

Thus, for angles of incidence 50°, scattering angle relative to the set of Bragg's plane is 65°.

For most of the crystals, the spacing between atomic planes is about 1 Å, therefore the Bragg's equation for maxima in the diffraction pattern becomes

$$2d \sin\theta = \lambda$$

$$\lambda = 2 \times 1 \times \sin 65^\circ = 1.67 \text{ Å}$$

$\therefore$ The de-Broglie wavelength of diffracted electrons is 1.67 Å. Now, for 54 V electron, the de-Broglie wavelength is

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å} = \frac{12.27}{\sqrt{54}} \text{ Å} = 1.67 \text{ Å}$$

Practice Exercise

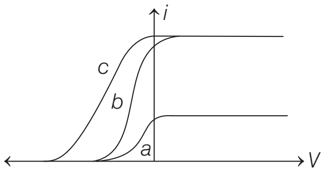
ROUND I Topically Divided Problems

Emission of Electrons and Photoelectric Effect

1. When a point source of light is 1 m away from a photoelectric cell, the photoelectric current is found to be I mA. If the same source is placed at 4 m from the same photoelectric cells, the photoelectric current (in mA) will be

(a) $\frac{I}{16}$ (b) $\frac{I}{4}$ (c) $4I$ (d) $16I$

2. The figures shows the variation of photo-current i with anode potential V for three differential radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively. Then

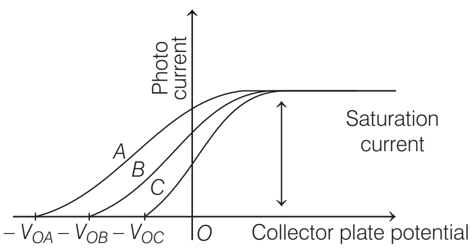


(a) $f_a = f_b$ and $I_a \neq I_b$ (b) $f_a = f_c$ and $I_a = I_c$
(c) $f_a = f_b$ and $I_a = I_b$ (d) $f_b = f_c$ and $I_b = I_c$

3. Which one of the following statements regarding photoemission of electrons is correct?

(a) Kinetic energy of electrons increases with the frequency of incident light.
(b) Electrons are emitted when the wavelength of the incident light is above a certain threshold wavelength.
(c) Photoelectric emission is instantaneous with the incidence of light.
(d) Photo electrons are emitted whenever a gas is irradiated with ultraviolet light.

4. For the graph as shown below



Which of the following statement(s) is/are correct?

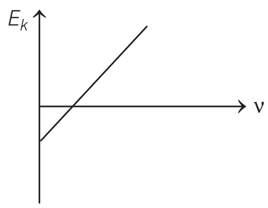
I. Stopping potential are in the order $V_{OA} > V_{OB} > V_{OC}$, if the frequencies are in the order $\nu_A > \nu_B > \nu_C$.
II. Greater the frequency of incident radiation, lesser would be the maximum kinetic energy of photoelectrons.

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

5. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then, **[NCERT Exemplar]**

(a) no electrons will be emitted as only photons can emit electrons
(b) electrons can be emitted but all with an energy, E_0
(c) electons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function)
(d) electrons can be emitted with any energy, with a maximum of E_0

6. The maximum KE (E_k) of the emitted photoelectrons against frequency of the incident radiation is plotted as shown in figure. This graph help in determining the following quantities.



(a) Planck's constant
(b) charge on an electron
(c) threshold frequency
(d) work function of cathode metal

7. The work function of tungsten and sodium are 4.5 eV and 2.3 eV respectively. If the threshold wavelength, λ for sodium is 5460 Å, the value of λ for tungsten is

(a) 2791 Å (b) 3260 Å
(c) 1925 Å (d) 1000 Å

8. Radiations of two photon's energy, twice and ten times the work function of metal are incident on the metal surface successively. The ratio of maximum velocities of photoelectrons emitted in two cases is

(a) 1 : 2 (b) 1 : 3
(c) 1 : 4 (d) 1 : 1

9. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. The work function in eV is **[NCERT Exemplar]**

(a) 1.50 eV (b) 1.02 eV
(c) 1.94 eV (d) 2.76 eV

10. The wavelength of the photoelectric threshold for silver is λ_0. The energy of the electron ejected from the surface of silver by an incident light of wavelength λ ($\lambda < \lambda_0$) will be

(a) $hc(\lambda_0 - \lambda)$ (b) $\frac{hc}{\lambda_0 - \lambda}$
(c) $\frac{h}{c}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ (d) $hc\left(\frac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$

11. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to **[JEE Main 2019]**

(a) 250 nm (b) 2020 nm
(c) 1700 nm (d) 220 nm

12. An isolated lead ball is charged upon continuous irradiation by EM radiation of wavelength, $\lambda = 221$ nm. The maximum potential attained by the lead ball, if its work function is 4.14 eV is (Take, $h = 6.63 \times 10^{-34}$ J-s, $c = 3 \times 10^8$ m/s and $e = 1.6 \times 10^{-19}$ C)

(a) 1.49 V (b) 2.67 V
(c) 3.14V (d) 0.51V

13. A metal surface is illuminated by a light of given intensity and frequency to cause photoemission. If the intensity of illumination is reduced to one-fourth of its original value, then the maximum kinetic energy of the emitted photoelectrons would become

(a) four times the original value
(b) twice the original value
(c) (1/6)th of the original value
(d) unchanged

14. The threshold wavelength for photoelectron emission from a materials is 5200 Å. Photo electrons will be emitted when this material is illuminated with monochromatic radiation from a

(a) 50 W infrared lamp (b) 1 W infrared lamp
(c) 1 W ultraviolet lamp (d) None of these

15. The work function for the surface of Al is 4.2 eV. How much potential difference will be required to just stop the emission of maximum energy electrons emitted by light of 2000 Å ?

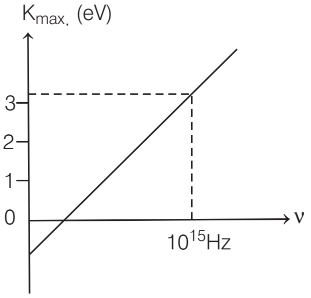
(a) 1.51 V (b) 1.99 V
(c) 2.99 V (d) None of these

16. Light is incident on the cathode of a photocell and the stopping voltages are measured for light of two different wavelengths. From the data given below, determine the work functions of the metal of the cathode in eV.

Wavelength (Å)	Stopping voltage (volt)
4000	0.9
4500	1.3

(a) 1.2 eV (b) 2.3 eV
(c) 4.2 eV (d) 6 eV

17. Following graph represents kinetic energy of most energetic photoelectrons K_{max} (in eV) and frequency ν for a metal used as cathode in photoelectric experiment. The threshold frequency of light for the photoelectric emission from the metal is



(a) 4×10^{14} Hz (b) 3.5×10^{14} Hz
(c) 2.0×10^{14} Hz (d) 2.7×10^{14} Hz

18. Light of wavelength λ strikes a photo sensitive surface and electrons are ejected with kinetic energy E. If the KE is to be increased to $2E$, the wavelength must be changed to λ' where

(a) $\lambda' = \frac{\lambda}{2}$ (b) $\lambda' = 2\lambda$
(c) $\frac{\lambda}{2} < \lambda' < \lambda$ (d) $\lambda' > \lambda$

19. In a photoelectric effect experiment, the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be

Given, $E \text{ (in eV)} = \frac{1237}{\lambda(\text{in nm})}$ **[JEE Main 2019]**

(a) 15.1 eV (b) 3.0 eV
(c) 1.5 eV (d) 4.5 eV

20. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photocurrent is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is **[JEE Main 2019]**

(a) $\frac{4}{3}\nu$ (b) 2ν
(c) $\frac{3\nu}{2}$ (d) $\frac{5\nu}{3}$

21. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 n-m to 400 n-m. The decrease in the stopping potential is close to $\left(\frac{hc}{e} = 1240 \text{ n-mV}\right)$ **[JEE Main 2019]**

(a) 0.5 V (b) 2.0 V
(c) 1.5 V (d) 1.0 V

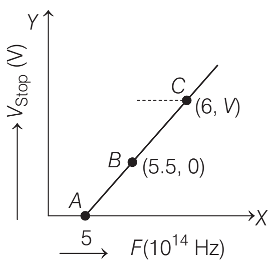
22. Cut off potentials for a metal surface for light of wavelengths λ_1, λ_2 and λ_3 are V_1, V_2 and V_3 respectively. If V_1, V_2 and V_3 are in arithmatic progression, then λ_1, λ_2 and λ_3 are in

(a) arithmetic progression
(b) geometric progression
(c) harmonic progression
(d) None of the above

23. Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

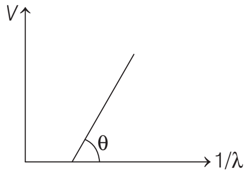
(a) $> v\left(\frac{4}{3}\right)^{1/2}$ (b) $< v\left(\frac{4}{3}\right)^{1/2}$
(c) $= v\left(\frac{4}{3}\right)^{1/2}$ (d) $= v\left(\frac{3}{4}\right)^{1/2}$

24. The following figure shows few data points in a photoelectric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is (Take, Planck's constant, $h = 6.62 \times 10^{-34}$ J-s) **[JEE Main 2020]**



(a) 1.93 eV (b) 2.59 eV
(c) 2.27 eV (d) 2.10 eV

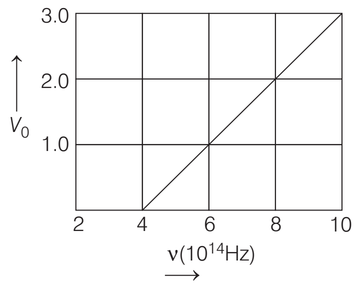
25. In a photoelectric effect experiment, the graph of stopping potential V *versus* reciprocal of wavelength $(1/\lambda)$ obtained is shown in the figure. As the intensity of incident radiation is increased,



[JEE Main 2020]

(a) graph does not change
(b) straight line shifts to left
(c) slope of the straight line get more steep
(d) straight line shifts to right

26. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be
(Take, Planck's constant $(h) = 6.63 \times 10^{-34}$ J-s, electron charge, $e = 1.6 \times 10^{-19}$ C] **[JEE Main 2019]**



(a) 1.82 eV
(b) 1.66 eV
(c) 1.95 eV
(d) 2.12 eV

27. A photon of energy E ejects a photoelectron from a metal surface whose work function is W_0. If this electron enters into a uniform magnetic field of induction B in a direction perpendicular to the field and describes a circular path of radius r, then the radius, r is given by

(a) $\sqrt{\frac{2m(W_0 - E)}{eB}}$ (b) $\sqrt{\frac{2e(E - W_0)}{mB}}$
(c) $\frac{\sqrt{2m(E - W_0)}}{eB}$ (d) $\sqrt{\frac{2mW_0}{eB}}$

28. All electrons ejected from a metallic surface by incident light of wavelength 400 nm travelled 1 m in the direction of uniform electric field of 2 NC^{-1} and came to rest. The work function of the surface is

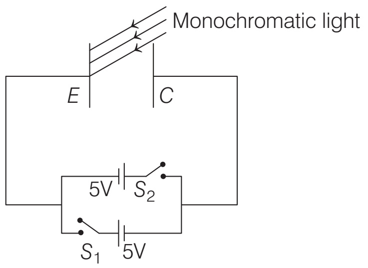
(a) 1.1 eV (b) 2.2 eV
(c) 3.1 eV (d) 5.1 eV

29. The electric field of light wave is given as $\mathbf{E} = 10^{-3} \cos\left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right) \hat{\mathbf{x}}\ \text{NC}^{-1}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photoelectrons is **[JEE Main 2019]**

$$\left(\text{Given, } E \text{ (in eV)} = \frac{12375}{\lambda(\text{in Å})}\right)$$

(a) 0.48 V (b) 0.72 V
(c) 2.0 V (d) 2.48 V

30. In a photoelectric experiment, a monochromatic light is incident on the emitter plate E, as shown in the figure. When switch S_1 is closed and switch S_2 is open, the photoelectrons strike the collector plate C with a maximum kinetic energy of 1 eV. If switch S_1 is open and switch S_2 is closed and the frequency of the incident light is doubled the photoelectrons strike the collector plate with a maximum kinetic energy of 20 eV. The threshold wavelength of the emitter plate is



(a) 5233.3 Å (b) 4133.3 Å
(c) 4166.7 Å (d) 5336.7 Å

31. A small piece of cesium metal ($\phi = 1.9$ eV) is kept at a distance of 17.7 cm from a large metal plate having a charge density of $1.0 \times 10^{-9}\text{C/m}^2$ on the surface facing the cesium piece. A monochromatic light of wavelength 400 nm is incident on the cesium piece. Find the minimum and maximum kinetic energy of the photoelectrons reaching the large metal plate. Neglect any change in electric field due to the small piece of cesium present.
(a) 20 eV, 21.2 eV (b) 21.2 eV, 60 eV
(c) 0.2 eV, 0.31 eV (d) 6 eV, 8 eV

Particle Nature of Light

32. There are two sources of light each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light of wavelength 500 nm. Find the ratio of number of photons of X-rays and photons of visible light of the given wavelength?
(a) 1 : 500 (b) 1 : 250
(c) 1 : 20 (d) 100

33. Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what can inference you about their frequencies ?
(a) The frequency of beam B is twice that of A
(b) The frequency of beam B is half that of A
(c) The frequency of beam A is twice of B
(d) None of the above

34. A parallel beam of light is incident normally on a plane surface absorbing 40% of the light and reflecting the rest. If the incident beam carries 60 W of power, the force exerted by it on the surface is
(a) 3.2×10^{-8} N (b) 3.2×10^{-7} N
(c) 5.12×10^{-7} N (d) 5.12×10^{-8} N

35. Ultraviolet light of wavelength 300 nm and intensity 1.0 Wm^{-2} falls on the surface of a photosensitive material. If one percent of the incident photons produce photoelectrons, then the number of photoelectrons emitted from an area of 1.0 cm^2 of the surface is nearly
(a) $9.61 \times 10^{14}\ \text{s}^{-1}$ (b) $4.12 \times 10^{13}\ \text{s}^{-1}$
(c) $1.51 \times 10^{12}\ \text{s}^{-1}$ (d) $2.13 \times 10^{11}\ \text{s}^{-1}$

36. What will be the number of photons emitted per second by a 10 W sodium vapour lamp assuming that 90% of the consumed energy is converted into light? [Wavelength of sodium light is 590 nm, $h = 6.63 \times 10^{-34}$ J-s]
(a) 0.267×10^{18} (b) 0.267×10^{19}
(c) 0.267×10^{20} (d) 0.267×10^{17}

37. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is (Given, Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^8$ m/s)
(a) 1×10^{16} (b) 5×10^{15} **[JEE Main 2019]**
(c) 1.5×10^{16} (d) 2×10^{16}

38. Photons of wavelength λ emitted by a source of power P incident on a photo cell. If the current produced in the cell is I, then the percentage of incident photons which produce current in the photo cell is. (where, h is Planck's constant and c is the speed of light in vacuum)
(a) $\frac{100\, ePc}{Ih\lambda}$ (b) $\frac{100\, eP\lambda}{Ihc}$ (c) $\frac{100\, Ih\lambda}{ePc}$ (d) $\frac{100 Ihc}{eP\lambda}$

39. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is **[JEE Main 2020]**
(a) 1/250 (b) 500 (c) 250 (d) 1/500

40. A cobalt (Co) plate is placed at a distance of 1 m from a point source of power 1 W. Assume a circular area of the plate of radius, $r = 1$ Å is exposed to the radiation and ejects photoelectrons. The light energy is considered to be spread uniformly and the work function of cobalt is 5 eV. The minimum time the target should be exposed to the light source to eject a photoelectron (Assuming no reflection loses) is

(a) 320 s (b) 450 s
(c) 860 s (d) 100 s

41. A photodiode sensor is used to measure the output of a 300 W lamp kept 10 m away. The sensor has an opening of 2 cm in diameter. How many photons enter the sensor if the wavelength of the light is 660 nm and the exposure time is 100 ms. (Assume that all the energy of the lamp is given off as light and $h = 6.6 \times 10^{-34}$ Js)

(a) 3.6×10^{13} (b) 2.8×10^{13}
(c) 2.5×10^{13} (d) 1.8×10^{13}

42. Photons absorbed in matter are converted to heat. A source emitting n photon/s of frequency ν is used for converting of ice at 0°C to water at 0°C. Then, which amongst the following is incorrect regarding the time, T taken for the conversion?

(a) Decreases with increasing n, with v fixed
(b) Decreases with n fixed, v increasing
(c) Remains constant with n and v changing such that nv = constant
(d) Increases when the product $n\nu$ increases

43. A beam of light consists of five wavelengths 4000 Å, 4800Å, 6000Å, 7000Å and 7800Å. The light beam is falling normally over a metal surface of area 10^{-4} m^2 with a work function of 1.9 eV. Intensity of light beam is 7.5×10^{-3} Wm^{-2}, which is equally divided among the constituent wavelengths. If there is no loss of light energy, number of photoelectrons emitted per second is

(a) 1.12×10^{12} (b) 3.15×10^{12}
(c) 1.77×10^{12} (d) 4.06×10^{12}

44. Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be **[JEE Main 2019]**

(a) 3.5×10^{-6} N-s
(b) 6.3×10^{-4} N-s
(c) 1.4×10^{-6} N-s
(d) 5.0×10^{-3} N-s

45. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively will be (Take, 1 eV = 1.6×10^{-19} J) **[JEE Main 2019]**

(a) 10^{11} and 5 eV (b) 10^{12} and 5 eV
(c) 10^{10} and 5 eV (d) 10^{14} and 10 eV

Wave Nature of Particle

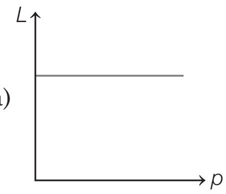
46. A particle of mass 1 mg has the same wavelength as an electron moving with a velocity of 3×10^6 ms^{-1}. The velocity of the particle is

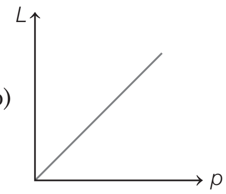
(a) 3×10^{-31} ms^{-1} (b) 2.7×10^{-21} ms^{-1}
(c) 2.7×10^{-18} ms^{-1} (d) 9×10^{-2} ms^{-1}

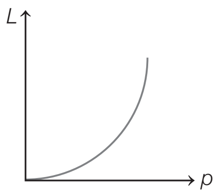
47. An electron and a proton have the same de-Broglie wavelength. Then the kinetic energy of the electron is

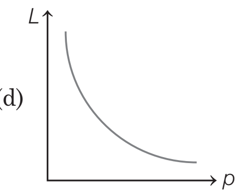
(a) zero
(b) infinity
(c) equal to kinetic energy of the proton
(d) greater than the kinetic energy of proton

48. The de-Broglie wavelength L associated with an elementary particle of linear momentum p is best represented by the graph

(a)


(b)


(c)


(d)


49. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths? ($m_p = 1.00727$ u, $m_e = 0.00055$ u) **[JEE Main 2021]**

(a) 1860 : 1 (b) $(1860)^2$: 1
(b) 41.4 : 1 (d) 43 : 1

50. The energy that should be added to an electron to reduce its de-Broglie wavelength from 10^{-10}m to 0.5×10^{-10} m , will be

(a) four times the initial energy
(b) thrice the initial energy
(c) equal to the initial energy
(d) twice the initial energy

51. A proton, a neutron, an electron and an α-particle have same energy, then their de-Broglie wavelengths compare as **[NCERT Exemplar]**

(a) $\lambda_p = \lambda_n > \lambda_c > \lambda_\alpha$ (b) $\lambda_e > \lambda_p = \lambda_n > \lambda_\alpha$
(c) $\lambda_c = \lambda_p > \lambda_n > \lambda_\alpha$ (d) $\lambda_c = \lambda_p > \lambda_n > \lambda_\alpha$

52. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become $\lambda/2$. Value of ΔE is **[JEE Main 2020]**

(a) $2E$ (b) $4E$ (c) $3E$ (d) E

53. An electron, a doubly ionised helium ion He $^{++}$) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, $\lambda_{He^{++}}$ and λ_p is **[JEE Main 2020]**

(a) $\lambda_e > \lambda_{He^{++}} > \lambda_p$ (b) $\lambda_e < \lambda_{He^{++}} = \lambda_p$
(c) $\lambda_e > \lambda_p > \lambda_{He^{++}}$ (d) $\lambda_e < \lambda_p < \lambda_{He^{++}}$

54. Electrons are accelerated through a potential difference V_0 and protons are accelerated through a potential difference 4 V. The de-Broglie wavelength are λ_e and λ_p for electrons and protons respectively. The ratio of $\dfrac{\lambda_e}{\lambda_p}$ is given by

(Given m_e is mass of electrons and m_p is mass of proton).

(a) $\dfrac{\lambda_e}{\lambda_p} = \sqrt{\dfrac{m_p}{m_e}}$ (b) $\dfrac{\lambda_e}{\lambda_p} = \sqrt{\dfrac{m_e}{m_p}}$

(c) $\dfrac{\lambda_e}{\lambda_p} = \dfrac{1}{2}\sqrt{\dfrac{m_e}{m_p}}$ (d) $\dfrac{\lambda_e}{\lambda_p} = 2\sqrt{\dfrac{m_p}{m_e}}$

55. An electron (of mass m) and a photon have the same energy E in the range of a few electron volt. The ratio of the de Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum) **[JEE Main 2020]**

(a) $\left(\dfrac{E}{2m}\right)^{1/2}$ (b) $\dfrac{1}{c}\left(\dfrac{2E}{m}\right)^{1/2}$

(c) $c\,(2mE)^{1/2}$ (d) $\dfrac{1}{c}\left(\dfrac{E}{2m}\right)^{1/2}$

56. A particle A of mass m and charge q is accelerated by a potential difference of 50 V. Another particle B of mass $4m$ and charge q is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\dfrac{\lambda_A}{\lambda_B}$ is close to **[JEE Main 2019]**

(a) 4.47 (b) 10.00 (c) 0.07 (d) 14.14

57. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV.

If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is **[JEE Main 2020]**

(a) 4 eV (b) 2 eV (c) 1.5 eV (d) 3 eV

58. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to **[NCERT Exemplar]**

(a) H (b) $H^{1/2}$
(c) H^0 (d) $H^{-1/2}$

59. An electron of mass m and magnitude of charge $|e|$ initially at rest gets accelerated by a constant electric field $\mathbf{E}$. The rate of change of de Broglie wavelength of this electron at time t ignoring relativistic effects is **[JEE Main 2020]**

(a) $\dfrac{h}{|e|\,\mathbf{E}\sqrt{t}}$ (b) $-\dfrac{h}{|e|\,\mathbf{E}t}$

(c) $\dfrac{-h}{|e|\,\mathbf{E}t^2}$ (d) $\dfrac{|e|\,\mathbf{E}t}{h}$

60. An electron is moving with an initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ and is in a magnetic field $\mathbf{B} = B_0\,\hat{\mathbf{j}}$. Then it's de-Broglie wavelength **[NCERT Exemplar]**

(a) remains constant
(b) increases with time
(c) decreases with time
(d) increases and decreases periodically

61. A particle A of mass m and initial velocity v collides with a particle B of mass $\dfrac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is **[JEE Main 2017]**

(a) $\dfrac{\lambda_A}{\lambda_B} = 2$ (b) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{2}{3}$

(c) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{2}$ (d) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{3}$

62. Particle A of mass $m_A = \dfrac{m}{2}$ moving along the X-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \dfrac{m}{3}$. If both particles move along the X-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength λ_0 before collision is **[JEE Main 2020]**

(a) $\Delta\lambda = \dfrac{5}{2}\lambda_0$ (b) $\Delta\lambda = 4\lambda_0$

(c) $\Delta\lambda = 2\lambda_0$ (d) $\Delta\lambda = \dfrac{3}{2}\lambda_0$

63. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2, respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ of the final particle, is given by **[JEE Main 2019]**

(a) $\dfrac{1}{\lambda^2} = \dfrac{1}{\lambda_1^2} + \dfrac{1}{\lambda_2^2}$ (b) $\lambda = \sqrt{\lambda_1\lambda_2}$

(c) $\lambda = \dfrac{\lambda_1 + \lambda_2}{2}$ (d) $\dfrac{2}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

64. An electron (mass m) with initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}}$ is in an electric field $\mathbf{E} = -E_0\hat{\mathbf{k}}$. If λ_0 is initial de-Broglie wavelength of electron, then its de Broglie wavelength at time t is given by **[JEE Main 2020]**

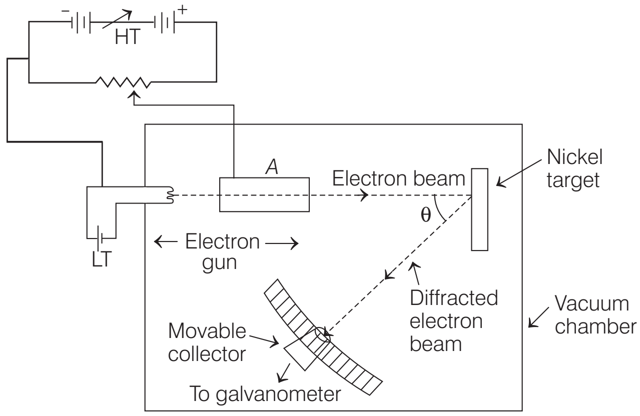
(a) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$ (b) $\dfrac{\lambda_0}{\sqrt{2+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$

(c) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{2m^2v_0^2}}}$ (d) $\dfrac{\lambda_0\sqrt{2}}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$

65. An electron microscope uses electrons accelerated by a voltage of 50 kV. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, then

(a) de-Broglie wavelength associated with e^- is 5.5×10^{-12} m
(b) the resolving power of an electron microscope is about 10^5 times greater than that of an optical microscope which uses yellow light
(c) de-Broglie wavelength associated with e^- is 5.9×10^{-7} m
(d) Both (a) and (b)

66. Consider the figure given below. Suppose the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that **[NCERT Exemplar]**



(a) will be larger than the earlier value
(b) will be the same as the earlier value
(c) will be less than the earlier value
(d) will depend on the target

ROUND II Mixed Bag

Only One Correct Option

1. Two identical metal plates shown photoelectric effect by a light of wavelength λ Å falls on plate A and λ_B on plate $B(\lambda_A = 2\lambda_B)$. The maximum kinetic energy is

(a) $2K_A = K_B$ (b) $K_A < \dfrac{K_B}{2}$
(c) $K_A = 2K_B$ (d) $K_A = \dfrac{K_B}{2}$

2. Given that a photon of light of wavelength 10,000 Å has an energy equal to 1.23 eV. When light of wavelength 5000 Å and intensity I_0 falls on a photoelectric cell, the surface current is 0.40×10^{-6} A and the stopping potential is 1.36 V, then the work function is

(a) 0.43 eV (b) 0.55 eV
(c) 1.10 eV (d) 1.53 eV

3. An electron of mass m when accelerated through a potential difference has de-Broglie wavelength λ. The de-Broglie wavelength associated with a proton of mass M accelerated through the same potential difference will be

(a) $\lambda\dfrac{m}{M}$ (b) $\lambda\sqrt{\dfrac{m}{M}}$
(c) $\lambda\dfrac{M}{m}$ (d) $\lambda\sqrt{\dfrac{M}{m}}$

4. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation: **[JEE Main 2021]**

(a) Phase (b) Intensity
(c) Amplitude (d) Frequency

5. The potential energy of a particle of mass m is given by

$$U(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0\ ; & x > 1 \end{cases}$$

λ_1 and λ_2 are the de-Broglie wavelengths of the particle when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$ the ratio $\dfrac{\lambda_1}{\lambda_2}$ will be

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$

6. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to (Take, $c = 3 \times 10^8$ m/s) **[JEE Main 2019]**

(a) 20×10^{-8} N
(b) 35×10^{-8} N
(c) 15×10^{-8} N
(d) 10×10^{-8} N

7. If the de-Broglie wavelength of an electron is equal to 10^{-3} times, the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to
(Take, speed of light $= 3 \times 10^8$ m/s,
Planck's constant $= 6.63 \times 10^{-34}$ J-s and
mass of electron $= 9.1 \times 10^{-31}$ kg) **[JEE Main 2019]**
(a) 1.45×10^6 m/s
(b) 1.8×10^6 m/s
(c) 1.1×10^6 m/s
(d) 1.7×10^6 m/s

8. A 100 W light bulb is placed at the centre of a spherical chamber of radius 0.10 m. Assume that 66% of the energy supplied to the bulb is converted into light and that the surface of chamber is perfectly absorbing. The pressure exerted by the light on the surface of the chamber is
(a) 0.87×10^{-6} Pa (b) 1.75×10^{-6} Pa
(c) 3.50×10^{-6} Pa (d) None of these

9. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to **[JEE Main 2019]**
(a) 500 keV (b) 1 keV
(c) 100 keV (d) 25 keV

10. Relativistic corrections become neccssary when the expression for the kinetic energy $\frac{1}{2}mv^2$, becomes comparable with mc^2 where m is the mass of the particle. At what de-Broglie wavelength will relativistic corrections become important for an electron?
(a) $\lambda = 10$ nm (b) $\lambda = 10^{-1}$ nm
(c) $\lambda = 10^{-4}$nm (d) $\lambda = 10^{-6}$ nm

11. A photon and electron have same de-Broglie wavelength. Give that v is the speed of electron and c is the velocity of light. E_e, E_p are the kinetic energy of electron and photon respectively. p_e, p_h are the momentum of electron and photon respectively. Then which of the following relation is correct?
(a) $\frac{E_e}{E_p} = \frac{v}{2c}$ (b) $\frac{E_e}{E_p} = \frac{2c}{v}$
(c) $\frac{p_e}{p_h} = \frac{c}{2v}$ (d) $\frac{p_e}{p_h} = \frac{2c}{v}$

12. The de-Broglie wavelength of a photon is twice the de-Broglie wavelength of an electron. The speed of the electron is $v_e = \frac{c}{100}$. Then
(a) $\frac{E_e}{E_p} = 10^{-4}$ (b) $\frac{E_e}{E_p} = 10^{-3}$
(c) $\frac{p_e}{m_e c} = 10^{-2}$ (d) $\frac{p_e}{m_e c} = 10^{-4}$

13. Radiation with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4}T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to **[JEE Main 2020]**
(a) 0.8 eV (b) 1.1 eV
(c) 1.8 eV (d) 1.6 eV

14. A particle p is formed due to a completely inelastic collision of particles x and y having de-Broglie wavelengths λ_x and λ_y, respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of p is **[JEE Main 2019]**
(a) $\lambda_x - \lambda_y$ (b) $\frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y}$
(c) $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$ (d) $\lambda_x + \lambda_y$

Numerical Value Questions

15. At an incident radiation frequency of ν_1, which is greater than the threshold frequency, the stoopping potential for a certain metal is V_1. At frequency $2\nu_1$, the stopping potential is $3V_1$. If the stopping potential at frequency $4\nu_1$ is nV_1, then n is ....

16. 1.5 mW of 400 nm light is directed at a photoelectric cell. If 0.1 % of the incident photons produce photoelectrons, find the current (in μA) in the cell.

17. Light of wavelength 180 nm ejects photoelectrons from a plate of metal whose work-function is 2 eV. If a uniform magnetic field of 5×10^{-5} T be applied parallel to the plate, what would be the radius (in m) of the path followed by electrons ejected normally from the plate with maximum energy?

18. The surface of a metal is illuminated alternately with photons of energies $E_1 = 4$ eV and $E_2 = 2.5$ eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal (in eV) is .... **[JEE Main 2020]**

19. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $V/4$. If the threshold wavelength for the metallic surface is $n\lambda$, then value of n will be .... **[JEE Main 2020]**

20. The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. The photons (nearly) per square metre are incident on the earth per second is 3.838×10^n, where the value of n is ....

Assume that, the photons in the sunlight have an average wavelength of 550 nm. **[NCERT]**

21. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength $\lambda = 310$ nm. It falls normally on a metal (work function $\phi = 2$ eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. ($hc = 1240$ eVnm, 1 eV $= 1.6 \times 10^{-19}$ J), then x is .... **[JEE Main 2020]**

22. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60^\circ$, then E (in eV) is ....
(Given, Planck's constant, $h = 6.64 \times 10^{-34}$ Js,
1 eV $= 1.6 \times 10^{-19}$ J and electron mass,
$m = 9.1 \times 10^{-31}$ kg. **[JEE Main 2020]**

23. A photon with energy of 4.9 eV ejects photoelectrons from tungsten. When the ejected electron enters a constant magnetic field of strength $B = 2.5$ mT at an angle of 60° with the field direction, the maximum pitch of the helix described by the electron is found to be 2.7 mm. Find the work-function of the metal in electron-volt. Given that, specific charge of electron is 1.76×10^{11} C/ kg.

24. The maximum kinetic energy (in eV) of a photoelectron liberated from the surface of lithium with work function 2.35 eV by electromagnetic radiation whose electric component varies with time as : $E = a[1 + \cos(2\pi f_1 t)]\cos 2\pi f_2 t$ (where a is a constant) is ...... ($f_1 = 3.6 \times 10^{15}$ Hz, $f_2 = 1.2 \times 10^{15}$ Hz and Planck's constant $h = 6.6 \times 10^{-34}$ Js) **[JEE Main 2021]**

Answers

Round I

1. (a)	2. (a)	3. (a)	4. (a)	5. (d)	6. (a)	7. (a)	8. (b)	9. (b)	10. (d)
11. (a)	12. (a)	13. (d)	14. (c)	15. (b)	16. (b)	17. (d)	18. (c)	19. (c)	20. (c)
21. (d)	22. (c)	23. (a)	24. (c)	25. (a)	26. (b)	27. (c)	28. (a)	29. (a)	30. (b)
31. (a)	32. (a)	33. (a)	34. (b)	35. (c)	36. (c)	37. (b)	38. (d)	39. (b)	40. (a)
41. (c)	42. (d)	43. (a)	44. (d)	45. (a)	46. (c)	47. (d)	48. (d)	49. (d)	50. (b)
51. (b)	52. (c)	53. (c)	54. (d)	55. (d)	56. (d)	57. (a)	58. (d)	59. (c)	60. (a)
61. (a)	62. (b)	63. (a)	64. (c)	65. (d)	66. (c)				

Round II

1. (b)	2. (c)	3. (b)	4. (d)	5. (c)	6. (a)	7. (a)	8. (b)	9. (d)	10. (d)
11. (a)	12. (c)	13. (b)	14. (b)	15. 7	16. 0.48	17. 0.148	18. 2	19. 9	20. 21
21. 11	22. 50	23. 4.5	24. 17.45						

Solutions

Round I

1. Photoelectric current $(I) \propto$ Intensity of incident light

and intensity $\propto \dfrac{1}{(\text{distance})^2}$

So, $\quad I \propto \dfrac{1}{(\text{distance})^2}$

Hence $\quad I' = I\left(\dfrac{1}{4}\right)^2 = \dfrac{I}{16}$

2. Stopping potentials for b and a are same

$\therefore \quad f_a = f_b$

and saturation current for b and a are different

$\therefore \quad I_b \neq I_a$

3. KE of photoelectrons increases with increase in frequency of the incident light and is independent of the intensity of incident light.

4. The statement I is correct but rest is incorrect and it can be corrected as,

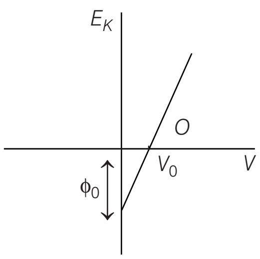
From the given graph, we can say that we have obtained different values of stopping potential but the same value of the saturation current for incident radiation of different frequencies. Since, the energy of emitted electron depends on the frequency of the incident radiations. This means, stopping potential is more negative for higher frequencies of incident radiation. Thus, we can conclude that,

I. stopping potential are in the order $V_{OA} > V_{OB} > V_{OC}$, if the frequencies are in the order $\nu_A > \nu_B > \nu_C$.

II. This further implies that greater the frequency of incident radiation, greater is the maximum kinetic energy of the photoelectrons. Consequently, we need greater retarding potential to stop them completely.

5. When a beam of electrons of energy E_0 is incident on a metal surface kept in an evacuated chamber, electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0, when part of incident energy of electron is used in liberating the electrons from the surface of metal.

6. According to graph between frequency (ν) of incident photon and maximum kinetic energy (E_K) of emitted electrons is given as,



$$E_K = h\nu - \phi_0$$

Comparing above equation with $y = mx + C$, we get

Slope $m = h$ (Planck's constant)

7. As, $\quad W_0 = \dfrac{hc}{\lambda_0}; \dfrac{(W_0)_T}{(W_0)_{Na}} = \dfrac{\lambda_{Na}}{\lambda_T}$

or $\quad \lambda_T = \dfrac{\lambda_{Na} \times (W_0)_{Na}}{(W_0)_T} = \dfrac{5460 \times 2.3}{4.5} = 2791 \text{ Å}$

8. We have, $\dfrac{1}{2} mv_1^2 = 2\phi_0 - \phi_0 = \phi_0$

and $\quad \dfrac{1}{2} mv_2^2 = 10\phi_0 - \phi_0 = 9\phi_0$

$\therefore \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{\phi_0}{9\phi_0}} = \dfrac{1}{3}$

9. The maximum energy $= h\nu - \phi$

$$K_1 = \frac{K_2}{2}$$

$$\left(\frac{hc}{\lambda_1} - \phi\right) = \frac{1}{2}\left(\frac{hc}{\lambda_2} - \phi\right)$$

$$\Rightarrow \quad \left(\frac{1240}{600} - \phi\right) = \frac{1}{2}\left(\frac{1240}{400} - \phi\right)$$

$(\because hc = 1240 \text{ eV-nm})$

$$\Rightarrow \quad \phi = \frac{1240}{1200} = 1.03 \approx 1.02 \text{ eV}$$

10. As, $E_k = \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0} = hc\left(\dfrac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$

11. Minimum wavelength occurs when mercury atom de-excites from highest energy level.

$\therefore$ Maximum possible energy absorbed by mercury atom $= \Delta E = 5.6 - 0.7 = 4.9$ eV

Wavelength of photon emitted in deexcitation is

$$\lambda = \frac{hc}{E} \approx \frac{1240 \text{ eV-nm}}{4.9 \text{ eV}} \approx 250 \text{ nm}$$

12. Given, $\lambda = 2.21 \times 10^{-7}$ m

So, energy of imparted by radiation in the lead ball,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2.21 \times 10^{-7}} = 9 \times 10^{-19} \text{J}$$

or $\quad E = \dfrac{9 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 5.63 \text{ eV}$

Work function of lead ball $= 4 \cdot 14$ eV

So, maximum energy attained by ball

$= 5 \cdot 63 - 4 \cdot 14 = 1 \cdot 49$ eV

Equivalent maximum potential is 1.49 V.

13. The maximum KE of the emitted photoelectrons is independent of the intensity of the incident light but depends upon the frequency of the incident light. Therefore, when the intensity of illumination is reduced to one-fourth of its original value, then the maximum kinetic energy of photoelectrons would be unchanged.

14. Here, $\lambda_{th} = 5200$ Å,

Thus, wavelength greater than 5200 Å cannot produce the photoelectric effect. The wavelength of UV-rays is 100-4000Å, so photoelectrons will be emitted by using 1W UV lamp.

15. As, $E_k = h\nu - W_0 = \frac{hc}{\lambda} - W_0$

$$W_0 = 4.2 \text{ eV} = 4.2 \times 1.6 \times 10^{-19} = 6.72 \times 10^{-19} \text{ J}$$

So, $E_k = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}} - 6.72 \times 10^{-19}$

$\Rightarrow E_k = 9.9 \times 10^{-19} - 6.72 \times 10^{-19}$

$\Rightarrow E_k = 3.18 \times 10^{-19}$ J

Hence, stopping potential

$$V_0 = \frac{E_k}{e} = \frac{3.18 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.99 \text{ V}$$

16. As, $K_{max} = hf - W_0$

or $V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$

$$\therefore \Delta V_0 = (V_0)_2 - (V_0)_1 = \left[\frac{hc}{e\lambda_2} - \frac{W_0}{e}\right] - \left[\frac{hc}{e\lambda_1} - \frac{W_0}{e}\right] = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$\Delta V_0 = \frac{hc}{e}\left[\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right]$$

$$\Rightarrow \frac{hc}{e} = \frac{\Delta V_0 \cdot \lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Substituting given values, we get

$$= \frac{(1.3 - 0.9) \times 4000 \times 10^{-10} \times 4500 \times 10^{-10}}{500 \times 10^{-10}}$$

$$= 1.44 \times 10^{-6} \text{ Vm}$$

Also, $V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$

$$\frac{W_0}{e} = \frac{hc}{e\lambda} - V_0 = \frac{1.44 \times 10^{-6}}{4000 \times 10^{-10}} - 1.3 = 2.3 \text{ V}$$

$\Rightarrow W_0 = 2.3$ eV

17. From graph, $\nu = 10^{15}$ Hz

$$K_{max} = 3\text{eV} = 3 \times 1.6 \times 10^{-19} \text{ J}$$

As, $K_{max} = h\nu - h\nu_0$

or $\nu_0 = \nu - \frac{K_{max}}{h}$

$$= 10^{15} - \frac{3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = (10 - 7.3) \times 10^{14} = 2.7 \times 10^{14} \text{ Hz}$$

18. As, $E = \frac{hc}{\lambda} - W_0$ and $2E = \frac{hc}{\lambda'} - W_0$

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{E + W_0}{2E + W_0}$$

$$\Rightarrow \lambda' = \lambda \frac{\left(1 + \frac{W_0}{E}\right)}{\left(2 + \frac{W_0}{E}\right)}$$

Since, $\frac{\left(1 + \frac{W_0}{E}\right)}{\left(2 + \frac{W_0}{E}\right)} > \frac{1}{2}$

So, $\lambda' > \frac{\lambda}{2}$

Hence, $\frac{\lambda}{2} < \lambda' < \lambda$.

19. Given, threshold wavelength, $\lambda_0 = 380$ nm

Wavelength of incident light, $\lambda = 260$ nm

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{max} = eV_0 = h\nu - h\nu_0 \quad \text{...(i)}$$

But $h\nu = E = \frac{1237}{\lambda(\text{nm})}$ eV (Given)

$$\therefore E_0 = \frac{1237}{\lambda_0(\text{nm})} \text{ eV} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{max} = E - E_0 \left(\frac{1237}{\lambda} - \frac{1237}{\lambda_0}\right) \text{eV}$$

$$= 1237\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \text{eV } (\lambda \text{ in nm}) \quad \text{... (iii)}$$

By putting values of λ and λ_0 in Eq. (iii), we get

$$(\text{KE})_{max} = 1237\left(\frac{1}{260} - \frac{1}{380}\right) \text{eV} = 1237 \times \left[\frac{380 - 260}{380 \times 260}\right] \text{eV}$$

$\Rightarrow (\text{KE})_{max} = 1.5$ eV

20. Relation between stopping potential and incident light's frequency is $eV_0 = hf - \phi_0$, where V_0 is the stopping potential and ϕ_0 is the the work function of the photosensitive surface.

So, from given data, we have

$$-e\frac{V_0}{2} = h\nu - \phi_0 \quad \text{...(i)}$$

and $-eV_0 = \frac{h\nu}{2} - \phi_0 \quad$ ...(ii)

Subtracting Eqs. (i) from (ii), we have

$$-eV_0-\left(-\frac{eV_0}{2}\right)=\frac{h\nu}{2}-h\nu$$

$$\Rightarrow \quad -\frac{eV_0}{2}=-\frac{h\nu}{2}$$

$$\Rightarrow \quad eV_0=h\nu$$

Substituting this in Eq. (i), we get

$$-\frac{eV_0}{2}=eV_0-\phi_0$$

$$\Rightarrow \quad -\left(\frac{3}{2}eV_0\right)=-\phi_0 \text{ or } \frac{3}{2}h\nu=\phi_0$$

If threshold frequency is ν_0, then

$$h\nu_0=\frac{3}{2}h\nu \Rightarrow \nu_0=\frac{3}{2}\nu$$

21. Given, $\lambda_1=300$ nm; $\lambda_2=400$ nm

$$\frac{hc}{e}=1240\text{ nm}$$

Using Einstein equation for photoelectric effect,

$$E=h\nu=\phi+eV_0 \quad \text{...(i)}$$

(here, ϕ is work function of the metal and V_0 is stopping potential)

For λ_1 wavelength's wave,

$$E_1=h\nu_1=\phi+eV_{01}$$

or

$$\frac{hc}{\lambda_1}=\phi+eV_{01} \quad \text{...(ii)}$$

Similarly,

$$\frac{hc}{\lambda_2}=\phi+eV_{02} \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$hc\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right]=e(V_{01}-V_{02}) \text{ or } \frac{hc}{e}\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right]=\Delta V$$

By using given values,

$$\Delta V=1240\left[\frac{1}{300}-\frac{1}{400}\right]\frac{\text{nm-V}}{\text{nm}}$$

$$=1240\times\frac{1}{1200}\text{ V}$$

$$\Rightarrow \quad \Delta V=1.03.\text{ V}\approx 1\text{V}$$

22. $eV_1=\frac{hc}{\lambda_1}-\phi_0,\ eV_2=\frac{hc}{\lambda_2}-\phi_0,\ eV_3=\frac{hc}{\lambda_3}-\phi_0$

$$\Rightarrow \quad V_1=\frac{hc}{e\lambda_1}-\frac{\phi_0}{e}$$

$$V_2=\frac{hc}{e\lambda_2}-\frac{\phi_0}{e}$$

$$V_3=\frac{hc}{e\lambda_3}-\frac{\phi_0}{e}$$

$\because$ V_1, V_2 and V_3 are in AP.

$$\Rightarrow \quad 2V_2=V_1+V_3$$

$$\Rightarrow \quad \frac{2hc}{e\lambda_2}-\frac{2\phi_0}{e}=\frac{hc}{e\lambda_1}+\frac{hc}{e\lambda_3}-\frac{2\phi_0}{e}$$

$$\Rightarrow \quad \frac{2}{\lambda_2}=\frac{1}{\lambda_1}+\frac{1}{\lambda_3}$$

$\Rightarrow$ λ_1, λ_2 and λ_3 are in harmonic progression.

23. According to the law of conservation of energy, *i.e.* Energy of a photon ($h\nu$) = Work function (ϕ) + Kinetic energy of the photoelectron $\left(\frac{1}{2}mv^2_{\text{max}}\right)$

According to Einstein's photoelectric emission of light

$$\Rightarrow \quad (\text{KE})_{\text{max}}=\frac{hc}{\lambda}-\phi$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{hc}{\lambda}-\phi \quad \text{(i)}$$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$$\frac{1}{2}mv_2^2=\frac{hc}{3\lambda/4}-\phi=\frac{4hc}{3\lambda}-\phi$$

$$\Rightarrow \quad \frac{1}{2}mv_2^2=\frac{4hc}{3\lambda}-\phi \quad \text{(ii)}$$

Clearly, from Eqs. (i) and (ii), we get

$$\frac{1}{2}mv_2^2=\frac{4}{3}\left(\frac{1}{2}mv^2+\phi\right)-\phi$$

$$\frac{1}{2}mv_2^2=\frac{2}{3}mv^2+\frac{\phi}{3}$$

$$\Rightarrow \quad \frac{1}{2}mv_2^2>\frac{2}{3}mv^2$$

$$v_2^2>\frac{4}{3}v^2$$

$$\Rightarrow \quad v_2>v\left(\frac{4}{3}\right)^{1/2}$$

i.e. $$v'>v\left(\frac{4}{3}\right)^{1/2}$$

24. From Einstein's photoelectric equation,

$\phi=\phi_0+eV_0$ [here, V_0 = stopping potential]

$$\Rightarrow hf=\phi_0+eV_0 \quad \text{...(i)}$$

From graph, take point B $(5.5\times10^{14}$ Hz, 0 V)

On satisfying Eq. (i) with point B, we get

$$6.62\times10^{-34}\times5.5\times10^{14}=\phi_0+1.6\times10^{-19}\times0$$

$$36.41\times10^{-20}=\phi_0+0$$

$$\phi_0=36.41\times10^{-20}\text{ J}$$

$$=\frac{36.41\times10^{-20}}{1.6\times10^{-19}}\text{ eV}$$

$$=22.75\times10^{-1}\text{ eV}$$

$$=2.275\text{ eV}$$

$$\simeq 2.27\text{eV}$$

25. From photoelectric equation,

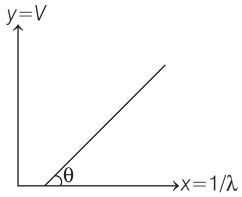
$$\phi=\phi_0+eV$$

$$\Rightarrow \quad \frac{hc}{\lambda}=\phi_0+eV$$

$$eV=\frac{hc}{\lambda}-\phi_0$$

$$\Rightarrow \quad V=\frac{hc}{\lambda e}-\frac{\phi_0}{e}$$

$$V = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{\phi_0}{e}$$



On comparing the above equation with equation of straight line, *i.e.* $y = mx + c$, we get

Slope, $\quad m = \dfrac{hc}{e}$

y-intercept $c = -\dfrac{\phi_0}{e}$

Now, if we change intensity of incident radiation, then h, c, e and ϕ_0 will not change. Therefore, neither slope of the graph, nor the y-intercept cut by graph will change on changing the intensity of incident radiation.

So, graph will not change.

26. Given, Planck's constant,

$$h = 6.63 \times 10^{-34} \text{ J-s}, \; e = 1.6 \times 10^{-19} \text{ C}$$

and there is a graph between stopping potential and frequency.

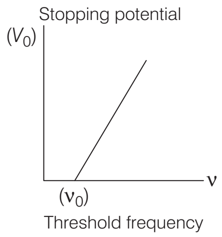
We need to determine work function W.

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{\max} = eV_0 = h\nu - h\nu_0 = h\nu - W \qquad [\because W = h\nu_0]$$

or $\quad V_0 = \dfrac{h}{e}\nu - \dfrac{W}{e}$

From graph at $V_0 = 0$ and $\nu = 4 \times 10^{14}$ Hz



$\therefore \quad 0 = \dfrac{6.63 \times 10^{-34}}{e} \times 4 \times 10^{14} - \dfrac{W}{e}$

$\Rightarrow \quad \dfrac{W}{e} = \dfrac{6.63 \times 10^{-34} \times 4 \times 10^{14}}{e}$ J

or $\quad W = 6.63 \times 4 \times 10^{-20}$ J

or $\quad W = \dfrac{6.63 \times 4 \times 10^{-20}}{1.6 \times 10^{-19}}$ eV $= 1.657$ eV

$\therefore \quad W = 1.66$ eV

27. We know that, according to Einstein equation,

$$E = W_0 + \frac{1}{2}mv^2$$

or $\quad \sqrt{\dfrac{2(E - W_0)}{m}} = v$

When emitted electron enters into magnetic field B in $\perp$ direction, it moves on circular path and radius of circular path is given as,

$\Rightarrow \quad r = \dfrac{mv}{eB} = \dfrac{\sqrt{2m(E - W_0)}}{eB}$

28. Given, wavelength of incident radiation,

$$\lambda = 400 \text{ nm} = 4 \times 10^{-7} \text{ m}$$

electric field, $E = 2$ N/C and distance, $s = 1$ m

$\therefore$ Energy of the incident light,

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}}$$
$$= 4.95 \times 10^{-19} \text{ J}$$
$$= \frac{4.95 \times 10^{19}}{1.6 \times 10^{-19}} \text{ J}$$
$$= 3.09 \simeq 3.1 \text{ eV}$$

If a be the retardation of emitted electrons in the electric field,

then, $\quad a = \dfrac{qE}{m}$

initial speed u of emitted electron is calculated as

$$v^2 = u^2 - 2as$$

$\Rightarrow \quad 0 = u^2 - 2a \times 1$

$\Rightarrow \quad u^2 = 2a = \dfrac{2qE}{m}$

$\therefore$ Maximum kinetic energy of the electron,

$$K_{\max} = \frac{1}{2}mu^2$$
$$= \frac{1}{2}m \cdot \frac{2qE}{m} = qE$$
$$= 1.6 \times 10^{-19} \times 2$$
$$K_{\max} = 3.2 \times 10^{-19} \text{ J} = 2 \text{ eV}$$

$\therefore$ Work function of the surface,

$$W_0 = E - K_{\max} = 3.1 - 2 = 1.1 \text{ eV}$$

29. Given, $\mathbf{E} = 10^{-3} \cos\left(\dfrac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right) \hat{\mathbf{x}} \text{NC}^{-1}$

By comparing it with the general equation of electric field of light, *i.e.*

$$E = E_0 \cos(kx - \omega t)\,\hat{x}, \text{ we get}$$

$$k = \frac{2\pi}{5 \times 10^{-7}} = 2\pi/\lambda \qquad (\text{from definition, } k = 2\pi/\lambda)$$

$\Rightarrow \quad \lambda = 5 \times 10^{-7}$ m $= 5000$ Å $\quad$ ...(i)

The value of λ can also be calculated as, after comparing the given equation of $\mathbf{E}$ with standard equation, we get

$$\omega = 6 \times 10^{14} \times 2\pi$$

$\Rightarrow \quad \nu = 6 \times 10^{14} \qquad [\because 2\pi\nu = \omega]$

As, $\quad c = \nu\lambda$

$\Rightarrow \quad \lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}}{6 \times 10^{14}} = 5 \times 10^{-7}$ m $= 5000$ Å

According to Einstein's equation for photoelectric effect, *i.e.*

$$\frac{hc}{\lambda} - \phi = (\text{KE})_{\max} = eV_0 \qquad \ldots\text{(ii)}$$

For photon, substituting the given values,

$$E = \frac{hc}{\lambda} = \frac{12375 \text{ eV}}{\lambda} \qquad \text{[given]}$$

or

$$\frac{hc}{\lambda} = \frac{12375}{5000} \text{ eV} \qquad \text{[using Eq. (i)]} \ldots\text{(iii)}$$

Now, substituting the values from Eq. (iii) in Eq. (ii), we get

$$\frac{12375}{5000} \text{ eV} - 2\text{eV} = eV_0$$

$$\Rightarrow \quad 2.475 \text{ eV} - 2 \text{ eV} = eV_0$$

or $\quad V_0 = 2.475 \text{ V} - 2 \text{ V} = 0.475 \text{ V}$

$\Rightarrow \quad V_0 \approx 0.48 \text{ V}$

30. Let threshold frequency of emitter plate be ν_0

Energy of photon in first case is E.

When switch S_1 is closed and switch S_2 is open,

So, $\quad E = h\nu_0 + (5 + 1) \text{ eV} \qquad \ldots\text{(i)}$

For second case, when switch S_1 is open and switch S_2 is closed and frequency of incident light is doubled.

then, $\quad 2E = h\nu_0 + (20 - 5) \text{ eV} \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad 2(h\nu_0 + 6\text{eV}) = h\nu_0 + 15\text{eV}$$

$$\Rightarrow \quad 2h\nu_0 + 12\text{eV} = h\nu_0 + 15\text{eV}$$

$$\Rightarrow \quad h\nu_0 = 3 \text{ eV}$$

$$\Rightarrow \quad \nu_0 = \frac{3 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 7.25 \times 10^{14} \text{ Hz}$$

$$\because \quad \lambda_0 = \frac{c}{\nu_0} = \frac{3 \times 10^8}{7.25 \times 10^{14}}$$

$$\lambda_0 = 4133.3 \text{ Å}$$

Hence, the threshold wavelength of the emitter plate is 4133.3 Å.

31. Electric field between the plate, $E = \dfrac{\sigma}{\varepsilon_0}$

The potential difference between the plates,

$$V = E \cdot d = \frac{\sigma}{\varepsilon_0} \cdot d$$

$$\Rightarrow \quad V = \frac{1.0 \times 10^{-9} \times 17.7}{8.85 \times 10^{-12} \times 100} = 20 \text{ V}$$

From Einstein photoelectric equation,

$$K_{\max} = \frac{hc}{\lambda} - W_0 = \frac{12400}{4000} - 1.9 = 1.2 \text{ eV}$$

But $\quad K_{\max} = eV_0$

$\Rightarrow \quad 1.2 \text{ eV} = eV_0$

$\Rightarrow \quad V_0 = 1.2 \text{ V}$

As V_0 is less than the potential difference between the plates (20 V). Hence, the minimum energy required to reach the charged plate should be $K_{\min} = 20$ eV.

For maximum kinetic energy, the potential difference between the plates should be an accelerating one. Hence maximum kinetic energy of electron should be

$$K_{\max} = 20 + 1.2 = 21.2 \text{ eV}$$

32. Total E is constant, let n_1 and n_2 be the number of photons of X-rays and visible region respectively.

Then, $\quad n_1 E_1 = n_2 E_2$

$$\Rightarrow \quad n_1 \frac{hc}{\lambda_1} = n_2 \frac{hc}{\lambda_2}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{1}{500} = 1:500$$

33. As, $\quad E = hn_A\nu_n = hn_B\nu_B$

$$\Rightarrow \quad \frac{n_A}{n_B} = 2 = \frac{\nu_B}{\nu_A}$$

The frequency of beam B is twice as that of A.

34. Momentum of incident light per second

$$p_1 = \frac{E}{c} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7}$$

Momentum of reflected light per second

$$p_2 = \frac{60}{100} \times \frac{E}{c} = \frac{60}{100} \times \frac{60}{3 \times 10^8} = 1.2 \times 10^{-7}$$

$\therefore$ Force on the surface = Change in momentum per second

$$= p_2 - (-p_1) = p_2 + p_1$$

$$= (2 + 1.2) \times 10^{-7} = 3.2 \times 10^{-7} \text{ N}$$

35. Energy incident over 1 cm^2 = 1.0×10^{-4} J ;

Energy required to produce photoelectrons

$$= 1.0 \times 10^{-4} \times 10^{-2} = 10^{-6} \text{ J}$$

As, number of photoelectrons ejected = number of photons

which can produce photoelectrons = energy required for producing electron/energy of photon.

$$= \frac{10^{-6}}{hc/\lambda} = \frac{10^{-6} \times 300 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 1.51 \times 10^{12} \text{ s}^{-1}$$

36. Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{590 \times 10^{-9}} = \frac{6.63 \times 3}{59} \times 10^{-18}$$

Light energy produced per second $= \dfrac{90}{100} \times 10 = 9$ W

$\therefore$ Number of photons emitted per sec

$$= \frac{9 \times 59}{6.63 \times 3 \times 10^{-18}} = 2.67 \times 10^{19} \approx 0.267 \times 10^{20}$$

37. Power of laser is given as $P = \dfrac{\text{Energy}}{\text{Time}}$

$$= \frac{\text{Number of photons emitted} \times \text{Energy of one photon}}{\text{Time}}$$

$\Rightarrow \quad P = \frac{NE}{t} = \left(\frac{N}{t}\right) \cdot E$

So, number of photons emitted per second

$$= \frac{N}{t} = \frac{P}{E}$$

$$= \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \qquad \left[\because E = h\nu = \frac{hc}{\lambda}\right]$$

Here, $h = 6.6 \times 10^{-34}$ J-s, $\lambda = 500$ nm

$= 500 \times 10^{-9}$ m; $c = 3 \times 10^8$ ms^{-1}

$P = 2$ mW $= 2 \times 10^{-3}$ W

$$\therefore \quad \frac{N}{t} = \frac{2 \times 10^{-3} \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$= 5.56 \times 10^{15}$

$\approx 5 \times 10^{15}$ photons per second

38. In photoemission, only one electron can produce one photon.

So, if only $x\%$ of incident photons can produce electrons, then

$$\text{then, } x\% = \frac{\text{Energy of incident photons which emit electrons}}{\text{Total energy of source}} \times 100$$

$$= \frac{hc/\lambda}{P \times t} \times 100 = \frac{hc \times 100}{P\lambda t} = \frac{100hc}{P\lambda\left(\frac{e}{I}\right)} \qquad [\because e = It]$$

$x\% = 100hcI/P\lambda e$

39. Power of source $= \frac{\text{Energy emitted}}{\text{Time}}$

$= \frac{\text{Number of photons} \times \text{Energy of 1 photon}}{\text{Time}}$

= Number density of photons × Energy of 1 photon

$= n \cdot \frac{hc}{\lambda}$

Now, $\quad P_{\text{X-rays}} = P_{\text{visible light}}$

$$\Rightarrow \quad n_X\left(\frac{hc}{\lambda_X}\right) = n_V\left(\frac{hc}{\lambda_V}\right)$$

$$\Rightarrow \quad \frac{n_V}{n_X} = \frac{\lambda_V}{\lambda_X} = \frac{500}{1} = 500$$

40. Given,

source power, $P = 1$W

Intensity of light source at a distance of 1 m is given by

$$I = \frac{P}{4\pi r^2} = \frac{1}{4\pi . 1^2} = \frac{1}{4\pi} \text{ W/m}^2$$

Power absorbed by the circular plates of radius 1 Å is

$P = I \times$ Area of plate,

$$= \frac{1}{4\pi} \times \pi r^2 \qquad [\because r = 1 \text{ Å} = 10^{-10}\text{m}]$$

$$P = \frac{1}{4}(10^{-10})^2 = \frac{10^{-20}}{4}$$

Work function of cobalt

$W = 5$ eV

$= 5 \times 1.6 \times 10^{-19}$ J

$P = \frac{W}{t}$

$t = \frac{W}{P}$

$$\therefore \quad t = \frac{5 \times 1.6 \times 10^{-19}}{\frac{10^{-20}}{4}} = 320 \text{ s}$$

41. Given that, power of lamp, $P = 300$ W

distance of lamp from sensor, $d = 10$ m,

radius of sensor opening, $r = 1$ cm, wavelength of light emitted from photodiode, $\lambda = 660$ nm and exposure time, $t = 100$ ms $= 100 \times 10^{-3}$ s.

Engergy of photons, $E = \frac{hc}{\lambda}$

Putting the given values, we get

$$\Rightarrow \quad E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \text{ nm}} = 3 \times 10^{-19}$$

The area of of exposure of lamp at a radius of 10 m

$$A_0 = 4\pi r^2 = 4\pi \times 10^2 \text{m}^2$$

Similarly, The area of sensor

$$A's = \pi r^2 = \pi \times (10^{-2})^2 = \pi \times 10^{-4} \text{m}^2$$

Energy emitted by lamp in exposure time,

E_0 = Power of lamp (p) × Exposure time (t)

$$E_0 = 300 \times 100 \times 10^{-3} = 30 \text{ J}$$

So, the number of photons entering to sensor is given by

$$N = \frac{E_0}{E} \times \frac{A'_s}{A_0} = \frac{30}{3 \times 10^{-19}} \times \frac{\pi \times 10^{-4}}{4\pi \times 10^2}$$

$\Rightarrow \quad N = 2.5 \times 10^{13}$

42. Energy spent in conversion $= mL = 80000$ cal

Energy used by photon $= nT \times E = nT \times h\nu$

$\Rightarrow \quad nTh\nu = mL$

or $\quad T = mL/nh\nu$

$\therefore \; T \propto \frac{1}{n}$, when ν is fixed, $T \propto \frac{1}{\nu}$ when n is fixed.

43. $E_1 = \frac{12400}{4000} = 3.1$ eV

$E_2 = \frac{12400}{4800} = 2.58$ eV

$E_3 = \frac{12400}{6000} = 2.06$ eV

$E_4 = \frac{12400}{7000} = 1.77$ eV

$E_5 = \frac{12400}{7800} = 1.58$ eV

Clearly 4th and 5th wavelengths are not emitting any electron because energy of photons corresponding to these wavelengths is less than work function.

Now, number of photoelectrons emitted per second

= Number of photons incident per second

$$= \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3}$$

$$= IA\left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{1}{E_3}\right) \quad (\because I_1A_1 = I_2A_2 = I_3A_3 = IA)$$

$$= \frac{7.5\times 10^{-3}}{5} \times 10^{-4} \times \frac{1}{1.6\times 10^{-19}} \times \left(\frac{1}{3.1} + \frac{1}{2.58} + \frac{1}{2.06}\right)$$

$= 1.12\times 10^{12}$ electrons per second

44. Radiation pressure over an absorbing surface is,

$$p = \frac{I}{c}$$

where, I = intensity or energy flux

and c = speed of light.

If A = area of surface, then force due to radiation on the surface is

$$F = p \times A = \frac{IA}{c}$$

If force F acts for a duration of Δt seconds, then momentum transferred to the surface is

$$\Delta p = F \times \Delta t = \frac{IA}{c} \times \Delta t$$

Here, $I = 25\ \text{W cm}^{-2}$, $A = 25\ \text{cm}^2$,

$c = 3 \times 10^8\ \text{ms}^{-1}$,

$\Delta t = 40\ \text{min} = 2400\ \text{s}$

So, momentum transferred to the surface,

$$\Delta p = \frac{25 \times 25 \times 2400}{3 \times 10^8} = 5 \times 10^{-3}\ \text{N-s}$$

45. We know that, intensity of a radiation I with energy 'E' incident on a plate per second per unit area is given as

$$\Rightarrow \quad I = \frac{dE}{dA \times dt}$$

$$\Rightarrow \quad \frac{dE}{dt} = IdA \text{ or } IA$$

i.e. energy incident per unit time = IA

Substituting the given values, we get

$$\frac{dE}{dt} = 16\times 10^{-3} \times 1\times 10^{-4}$$

$$\frac{dE}{dt} = 16\times 10^{-7}\ \text{W} \quad \ldots\text{(i)}$$

Using Einstein's photoelectric equation, we can find kinetic energy of the incident radiation as

$$E = \frac{1}{2}mv^2 + \phi$$

(Here, ϕ is work function of metal)

or $\quad E = \text{KE} + \phi$

$\text{KE} = E - \phi = 10\ \text{eV} - 5\ \text{eV}$

$\Rightarrow \quad \text{KE} = 5\ \text{eV} \quad \ldots\text{(ii)}$

Now, energy per unit time for incident photons will be

$\because \quad E = Nh\nu$

$$\therefore \quad \frac{dE}{dt} = h\nu\frac{dN}{dt} \text{ or } h\nu\dot{N} \quad \ldots\text{(iii)}$$

From Eqs. (i) and (iii), we get

$$h\nu\dot{N} = 16 \times 10^{-7} \text{ or } E\dot{N} = 16 \times 10^{-7}$$

But $\quad E = 10$ eV, so

$$\dot{N}(10\times 1.6\times 10^{-19}) = 16\times 10^{-7} \Rightarrow \dot{N} = 10^{12}$$

$\because$ Only 10% of incident photons emit electrons.

So, emitted electrons per second are

$$\frac{10}{100} \times 10^{12} = 10^{11}$$

46. de-Broglie wavelength, $\lambda = \dfrac{h}{mv}$

As both particle and electrons have same wavelength, therefore their momentum will be equal to

$\Rightarrow \quad m_pv_p = m_ev_e$

$$\Rightarrow \quad v_p = \frac{m_ev_e}{m_p} = \frac{9.1\times 10^{-31} \times 3\times 10^6}{10^{-6}}$$

$\Rightarrow \quad v_p = 2.7\times 10^{-18}$ m/s

47. As, $E_K = \dfrac{1}{2}mv^2$ or $mv = \sqrt{2mE_K}$

As per question, $\quad m_pv_p = m_ev_e$

or $\quad \sqrt{2m_pE_{K_p}} = \sqrt{2m_eE_{K_e}}$

or $\quad \dfrac{E_{K_e}}{E_{K_p}} = \dfrac{m_p}{m_e} > 1$

or $\quad E_{K_e} > E_{K_p}$

48. As, $\lambda = \dfrac{h}{p}$ or $L = \dfrac{h}{p}$, *i.e.* $L \propto \dfrac{1}{p}\cdot$ The curve (d) is correct.

49. $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1831.4} = 42.79 \simeq 43$$

50. As, $\quad \lambda = \dfrac{h}{\sqrt{2mE}}$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{E}}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \quad \frac{10^{-10}}{0.5\times 10^{-10}} = \sqrt{\frac{E_2}{E_1}}$$

$\Rightarrow \quad E_2 = 4E_1$

Therefore, added energy = $E_2 - E_1 = 3E_1$

51. Kinetic energy of particle $K = \dfrac{1}{2}mv^2$

or $\quad mv = \sqrt{2mK}$

de-Broglie wavelength $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mK}}$

For the given value of $K, \lambda \propto 1/\sqrt{m}$.

$\therefore \quad \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha = \dfrac{1}{\sqrt{m_p}} : \dfrac{1}{\sqrt{m_n}} : \dfrac{1}{\sqrt{m_e}} : \dfrac{1}{\sqrt{m_\alpha}}$

Since $m_p = m_n$, hence $\lambda_p = \lambda_n$

As $m_\alpha > m_p$, therefore $\lambda_\alpha < \lambda_p$,

As $m_e < m_n$, therefore $\lambda_e > \lambda_n$,

Hence, $\quad \lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$

or $\quad \lambda_e > \lambda_p = \lambda_n > \lambda_\alpha$

52. de-Broglie wavelength of a moving particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

where, K = kinetic energy of the particle.

Now, in given condition,

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{...(i)}$$

and $$\frac{\lambda}{2} = \frac{h}{\sqrt{2m(E + \Delta E)}} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$2 = \frac{\sqrt{E + \Delta E}}{\sqrt{E}}$$

$\Rightarrow \quad \Delta E = 3E$

53. de-Broglie wavelength of a particle having kinetic energy, is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\,(\text{KE})}}$$

For same kinetic energy, $\lambda \propto \dfrac{1}{\sqrt{m}}$

$\because \quad m_{\text{He}^{++}} > m_p > m_e$

$\Rightarrow \quad \lambda_{\text{He}^{++}} < \lambda_p < \lambda_e$

54. We have, $E = qV$, we know that $E = \dfrac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\dfrac{2E}{m}}$

As, $\quad \lambda = \dfrac{h}{mv} = \dfrac{h}{m\sqrt{\dfrac{2E}{m}}}$

$\Rightarrow \quad \lambda = \dfrac{h}{\sqrt{2mqV}} \quad \text{...(i)}$

For electron, $\quad \lambda_e = \dfrac{h}{\sqrt{2m_e qV}} \quad \text{...(ii)}$

For protron, $\quad \lambda_p = \dfrac{h}{\sqrt{2m_p qV}}$

$\therefore \quad \lambda_p = \dfrac{h}{\sqrt{2m_p q \cdot 4V}} \quad (\because V = 4V) \text{ ...(iii)}$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$$

55. Here, energy of electron = E

$\Rightarrow \quad \dfrac{1}{2}mv^2 = E$

$\Rightarrow \quad m^2v^2 = 2mE \Rightarrow p^2 = 2mE$

or momentum of electron, $p = \sqrt{2mE}$

So, de Broglie wavelength associated with electron,

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Now, photon energy = E

$\Rightarrow \quad hf = E \Rightarrow \dfrac{hc}{\lambda_p} = E$

or wavelength of photon, $\lambda_p = \dfrac{hc}{E}$

Hence, ratio $\dfrac{\lambda_e}{\lambda_p} = \dfrac{\left(\dfrac{h}{\sqrt{2mE}}\right)}{\left(\dfrac{hc}{E}\right)} = \dfrac{1}{c}\left(\dfrac{E}{2m}\right)^{1/2}$

56. de-Broglie wavelength associated with a moving charged particle of charge q is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

where, V = accelerating potential

Ratio of de-Broglie wavelength for particle A and B is,

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{m_B q_B V_B}}{\sqrt{m_A q_A V_A}} = \sqrt{\frac{m_B}{m_A}} \cdot \sqrt{\frac{q_B}{q_A}} \cdot \sqrt{\frac{V_B}{V_A}}$$

Substituting the given values, we get

$$= \sqrt{\frac{4m}{m}} \cdot \sqrt{\frac{q}{q}} \cdot \sqrt{\frac{2500}{50}} = 2 \times 1 \times 5 \times 1.414 = 14.14$$

57. Kinetic energy T of electron and its de Broglie wavelength λ are related as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{m^2v^2}} = \frac{h}{\sqrt{2m \times \frac{1}{2}mv^2}} = \frac{h}{\sqrt{2mT}}$$

and given, $\lambda_B = 2\lambda_A$

$\Rightarrow \quad \dfrac{h}{\sqrt{2mT_B}} = 2 \cdot \dfrac{h}{\sqrt{2mT_A}} \Rightarrow T_A = 4T_B$

Also given,

$$T_B = T_A - 1.5 \Rightarrow T_B = 4T_B - 1.5$$

$\Rightarrow \quad 3T_B = 1.5 \Rightarrow T_B = 0.5 \text{ eV}$

Now, for metal B,

incident energy = kinetic energy of electron + work function

$\Rightarrow \quad 4.5 = 0.5 + \phi_0 \Rightarrow \phi_0 = 4 \text{ eV}$

58. Velocity acquired by a particle while falling from a height H is, $v = \sqrt{2gH}$

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}} \text{ or } \lambda \propto H^{-\frac{1}{2}}$$

59. Initial acceleration of electron,

$$a = \frac{F}{m} = \frac{|e|\mathbf{E}}{m}$$

Velocity of electron after time t,

$$v = u + at = \frac{|e|\mathbf{E}}{m}\cdot t$$

Momentum of electron, $p = mv = |e|\mathbf{E}\cdot t$

de Broglie wavelength λ associated with electron

$$\lambda = \frac{h}{p} = \frac{h}{|e|\mathbf{E}\cdot t}$$

Rate of change of de Broglie wavelength with time,

$$\frac{d\lambda}{dt} = \frac{-h}{|e|\mathbf{E}t^2}$$

Here, negative sign shows wavelength decreases with time.

60. Here, $\mathbf{v} = v_0\hat{\mathbf{i}}, \mathbf{B} = B_0\hat{\mathbf{j}}$

Force on moving electron due to magnetic field is

$$\mathbf{F} = -e(\mathbf{v}\times\mathbf{B}) = -e[v_0\hat{\mathbf{i}}\times B_0\hat{\mathbf{j}}] = -eV_0 B_0\hat{\mathbf{k}}$$

As this force is perpendicular to $\mathbf{v}$ and $\mathbf{B}$, so the magnitude of $\mathbf{v}$ will not change, *i.e.* momentum $(= mv)$ will remain constant in magnitude. Hence, de-Broglie wavelength $\lambda = h/mv$ remains constant.

61. For elastic collision,

$$p_{\text{before collision}} = p_{\text{after collision}}.$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \quad \cdots \text{(i)}$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collision $e = 1$

$\therefore \quad 1 = \dfrac{v_B - v_A}{v}$

$\Rightarrow \quad v = v_B - v_A \quad \ldots\text{(ii)}$

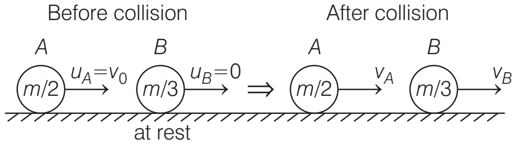
From Eqs. (i) and (ii), we get

$$v_A = \frac{v}{3}$$

and $\quad v_B = \dfrac{4v}{3}$

Hence, $\dfrac{\lambda_A}{\lambda_B} = \dfrac{\left(\dfrac{h}{mv_A}\right)}{\dfrac{h}{\dfrac{m}{2}\cdot v_B}} = \dfrac{v_B}{2v_A} = \dfrac{4/3}{2/3} = 2$

62.



Using law of conservation of linear momentum, we have

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\frac{m}{2}\times v_0 + \frac{m}{3}\times 0 = \frac{m}{2}\times v_A + \frac{m}{3}\times v_B$$

$$\frac{v_0}{2} = \frac{v_A}{2} + \frac{v_B}{3}$$

$$\Rightarrow \quad \frac{v_0}{2} = \frac{3v_A + 2v_B}{6}$$

$$3v_A + 2v_B = 3v_0 \quad \ldots\text{(i)}$$

By the definition of coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - u_B} \Rightarrow 1 = \frac{v_B - v_A}{v_0 - 0}$$

($\because$ for elastic collision, $e = 1$)

$\Rightarrow \quad -v_A + v_B = v_0$

$\Rightarrow \quad -3v_A + 3v_B = 3v_0 \quad \ldots\text{(ii)}$

On adding Eqs. (i) and (ii), we get

$\Rightarrow \quad 5v_B = 6v_0$

$\Rightarrow \quad v_B = \dfrac{6v_0}{5}$

Substituting this value in eq. (i), we get

$$3v_A + 2\left(\frac{6v_0}{5}\right) = 3v_0 \Rightarrow 3v_A = 3v_0 - \frac{12}{5}v_0$$

$$3v_A = \frac{3}{5}v_0 \Rightarrow v_A = \frac{v_0}{5}$$

Now, de-Broglie wavelength of particle A before collision,

$$(\lambda_A)_i = \frac{h}{(p_A)_i}$$

$$\lambda_0 = \frac{h}{m_A u_A} \quad [\because (\lambda_A)_i = \lambda_0 \text{ (given)}]$$

$$= \frac{h}{\frac{m}{2}\times v_0} = \frac{2h}{mv_0} \quad \ldots\text{(iii)}$$

and de-Broglie wavelength of particle A after collision,

$$(\lambda_A)_f = \frac{h}{(p_A)_f} = \frac{h}{m_A v_A}$$

$$= \frac{h}{\frac{m}{2}\times\frac{v_0}{5}} = \frac{10h}{mv_0}$$

$$= 5\times\frac{2h}{mv_0} = 5\lambda_0$$

So, change in de-Broglie wavelength of particle A is

$$(\Delta\lambda)_A = (\lambda_A)_f - (\lambda_A)_i = 5\lambda_0 - \lambda_0 = 4\lambda_0$$

63. Given, de-Broglie wavelengths for particles are λ_1 and λ_2.

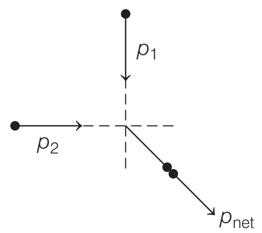
So, $\quad \lambda_1 = \dfrac{h}{p_1}$ and $\lambda_2 = \dfrac{h}{p_2}$

and momentum of particles are

$$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

Given that, particles are moving perpendicular to each other and collide inelastically.

So, they move as a single particle.



So, by conservation of momentum and vector addition law, net momentum after collision,

$$p_{net} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 90^\circ}$$

$$= \sqrt{p_1^2 + p_2^2}$$

Since, $p_1 = \dfrac{h}{\lambda_1}$ and $p_2 = \dfrac{h}{\lambda_2}$

So, $p_{net} = \sqrt{\dfrac{h^2}{\lambda_1^2} + \dfrac{h^2}{\lambda_2^2}}$ ...(i)

Let the de-Broglie wavelength after the collision is λ_{net}, then

$$p_{net} = \frac{h}{\lambda_{net}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{h}{\lambda_{net}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$$

$$\Rightarrow \quad \frac{1}{\lambda_{net}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

$$\Rightarrow \quad \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} \quad (\because \lambda_{net} = \lambda)$$

64. Initial velocity of electron, $\mathbf{v} = v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}}$

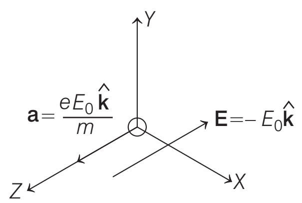
$$\Rightarrow \quad |\mathbf{u}| = \sqrt{v_0^2 + v_0^2} = v_0\sqrt{2}$$

Initial de Broglie wavelength associated with electron,

$$\lambda_0 = \frac{h}{p} = \frac{h}{mv_0\sqrt{2}} \quad \text{...(i)}$$

After t seconds velocity of electron,

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$



or $\mathbf{v} = (v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}}) + \dfrac{eE_0}{m}\cdot t\cdot\hat{\mathbf{k}}$

Note that electron is accelerated opposite to the field.

Magnitude of velocity after time t,

$$|\mathbf{v}| = \sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0}{m}t\right)^2}$$

$$= \sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}}$$

$$= v_0\sqrt{2 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$$

$\therefore$ de-Broglie wavelength of electron at time t,

$$\lambda = \frac{h}{p} = \frac{h}{m|\mathbf{v}|}$$

$$\Rightarrow \quad \lambda = \frac{h}{mv_0\sqrt{2 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$$

$$= \frac{\lambda_0\sqrt{2}}{\sqrt{2 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}} = \frac{\lambda_0}{\sqrt{1 + \dfrac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

65. Here, $V = 50\text{ kV} = 5\times10^4\text{ V}$, $m_e = 9.11\times10^{-31}$ kg

KE of an electron,

$$K = 50\text{ eV} = 1.6\times10^{-19}\times5\times10^4\text{ J} = 8\times10^{-15}\text{ J}$$

$\therefore$ de-Broglie wavelength of electrons is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times8\times10^{-15}}}\text{ m}$$

$$= \frac{6.63\times10^{-11}}{12.07}\text{ m} = 5.5\times10^{-12}\text{ m}$$

Wavelength of yellow light, $\lambda_y = 5.9\times10^{-7}$ m

Resolving power of a microscope $\propto 1/\lambda$

$$\therefore \quad \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} = \frac{\lambda_y}{\lambda}$$

$$= \frac{5.9\times10^{-7}}{5.5\times10^{-12}} \equiv 10^5$$

66. In Davisson-Germer experiment, the de-Broglie wavelength associated with electron is

$$\lambda = \frac{12.27}{\sqrt{V}}\text{ Å} \quad \text{...(i)}$$

where, V is the applied voltage.

If there is a maxima of the diffracted electrons at an angle θ, then

$$2d\sin\theta = \lambda \quad \text{...(ii)}$$

From Eq. (i), we note that if V is inversely proportional to the wavelength λ.

i.e. V will increase with the decrease in the λ.

From Eq. (ii), we note that wavelength λ is directly proportional to $\sin\theta$ and hence, θ.

So, with the decrease in λ, θ will also decrease.

Thus, when the voltage applied to A is increased, the diffracted beam will have the maxima at a value of θ that will be less than the earlier value.

Round II

1. We know, $K_A = \dfrac{hc}{\lambda_A} - \phi_0$

and $K_B = \dfrac{hc}{\lambda_B} - \phi_0$

$\therefore$ $\dfrac{K_A}{K_B} = \dfrac{\dfrac{hc}{2\lambda_B}}{\dfrac{hc}{\lambda_B}} < \dfrac{1}{2}$

or $K_A < K_B/2$

2. As, $E = \dfrac{hc}{\lambda}$ or $E \propto \dfrac{1}{\lambda}$

$\therefore$ $\dfrac{E_2}{E_1} = \dfrac{\lambda_1}{\lambda_2}$

or $E_2 = E_1 \times \dfrac{\lambda_1}{\lambda_2} = 1.23 \times \dfrac{10000}{5000} = 2.46 \text{ eV}$

Now, $h\nu - \phi_0 = \dfrac{1}{2} mv_{\max}^2 = eV_s$

$\phi_0 = h\nu_2 - eV_s = E_s - eV_s$

or $= 2.46 - 1.36 = 1.10 \text{ eV}$

3. As, $\lambda = \dfrac{h}{\sqrt{2m_0E}} = \dfrac{h}{\sqrt{2m_0qV}} \Rightarrow \lambda \propto \dfrac{1}{\sqrt{m_0}}$

where, m_0 = mass of the charge

$\Rightarrow$ $\dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{M}{m}}$

$\Rightarrow$ $\lambda_2 = \lambda_1 \sqrt{\dfrac{m}{M}}$

or $\lambda_2 = \lambda \sqrt{\dfrac{m}{M}}$ $(\because \lambda_1 = \lambda)$

4. Stopping potential changes linearly with frequency of incident radiation.

5. From Einstein's photoelectric equation,

$\text{KE} = 2E_0 - E_0 = E_0$ (for $0 \le x \le 1$)

$\Rightarrow$ $\lambda_1 = \dfrac{h}{\sqrt{2mE}}$ ...(i)

Similarly, $\text{KE} = 2E_0$ (for $x > 1$)

$\Rightarrow$ $\lambda_2 = \dfrac{h}{\sqrt{4mE_0}}$

From Eqs. (i) and (ii), we get

$\dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$

6. Radiation pressure or momentum imparted per second per unit area when light falls is

$$p = \begin{cases} \dfrac{2I}{c} & ; \text{ for reflection of radiation} \\ \dfrac{I}{c} & ; \text{ for absorption of radiation} \end{cases}$$

where, I is the intensity of the light.

In given case, there is 25% reflection and 75% absorption, so radiation pressure = force per unit area

$= \dfrac{25}{100} \times \dfrac{2I}{c} + \dfrac{75}{100} \times \dfrac{I}{c}$

$= \dfrac{1}{2} \times \dfrac{I}{c} + \dfrac{3}{4} \times \dfrac{I}{c}$

$= \dfrac{5}{4} \times \dfrac{I}{c} = \dfrac{5}{4} \times \dfrac{50}{3 \times 10^8}$

$= 20.83 \times 10^{-8} \text{N}$

$\approx 20 \times 10^{-8} \text{ N}$

7. Wavelength of the given photon is given as,

$\lambda_p = \dfrac{c}{\nu_p} = \dfrac{3 \times 10^8}{6 \times 10^{14}} \text{ m} = 5 \times 10^{-7} \text{m}$ ...(i)

As, it is given that, de-Broglie wavelength of the electron is

$\lambda_e = 10^{-3} \times \lambda_p$ [$\because$ using Eq. (i)]

$= 5 \times 10^{-10} \text{m}$

Also, the de-Broglie wavelength of an electron is given as,

$\lambda_e = \dfrac{h}{p} = \dfrac{h}{mv_e} \Rightarrow v_e = \dfrac{h}{\lambda_e m_e}$

Substituting the given values, we get

$= \dfrac{6.63 \times 10^{-34}}{5 \times 10^{-10} \times 9.1 \times 10^{-31}} \text{ m/s}$

$= 1.45 \times 10^6 \text{ m/s}$

8. Light falling per second on the surface of sphere

$E = \dfrac{66}{100} \times 100 = 66 \text{ W}$

Momentum of the light falling per second on the surface of sphere $= \dfrac{E}{c}$

Momentum of the reflected light = 0; as the light is completely absorbed.

Force exerted by light, $F = \dfrac{E}{c} - 0 = \dfrac{E}{c}$

Pressure on surface, $p = \dfrac{F}{4\pi r^2} = \dfrac{E/c}{4\pi r^2}$

$= \dfrac{66/(3 \times 10^8)}{4 \times (22/7) \times (0.10)^2}$

$= 1.75 \times 10^{-6} \text{ Pa}$

9. Given, resolution achieved in electron microscope is of the order of wavelength.

So, to resolve 7.5×10^{-12}m seperation wavelength associated with electrons is

$\lambda = 7.5 \times 10^{-12} \text{m}$

$\therefore$ Momentum of electrons required is

$p = \dfrac{h}{\lambda}$

or kinetic energy of electron must be

$$\text{KE} = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$$

Substituting the given values, we get

$$= \frac{\left(\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right)^2}{2 \times 9.1 \times 10^{-31}} \text{ J}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (7.5 \times 10^{-12})^2 \times (1.6 \times 10^{-19})} \text{ eV}$$

$$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

$$= 26593.4 \simeq 26.6 \times 10^3 \text{ eV} \simeq 26 \text{ keV}$$

which is nearest to 25 keV.

10. Velocity of electron, $v = h/(m\lambda)$

Let $h = 6.6 \times 10^{-9}$ Js and $m = 9 \times 10^{-31}$ kg.

(a) When $\lambda_1 = 10 \text{ nm} = 10 \times 10^{-9} \text{ m} = 10^{-8}$ kg

$$v_1 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-8}} = \frac{2.2}{3} \times 10^5 = 10^5 \text{ m/s}$$

(b) When $\lambda_2 = 10^{-1} \text{ nm} = 10^{-1} \times 10^{-9} \text{ m} = 10^{-10}$ kg.

$$v_2 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-10}} \approx 10^7 \text{ m/s}$$

(c) When $\lambda_3 = 10^{-4} \text{ nm} = 10^{-4} \times 10^{-9} \text{ m} = 10^{-13}$ m

$$v_3 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-13}} \approx 10^{10} \text{ m/s}$$

(d) When $\lambda_4 = 10^{-4} \text{ nm} = 10^{-4} \times 10^{-9} \text{ m} = 10^{-13}$ m

$$v_4 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-15}} \approx 10^{12} \text{ m/s}$$

As v_3 and v_4 are greater than velocity of light ($= 3 \times 10^8$ m/s), hence relativistic correction is needed for $\lambda = 10^{-4}$ nm and $\lambda = 10^{-6}$ nm.

11. As, $E_e = \frac{1}{2} mv^2 = \frac{1}{2}(mv)v = \frac{1}{2}\left(\frac{h}{\lambda}\right) v$

and $\quad E_p = \frac{hc}{\lambda}$;

$$\therefore \quad \frac{E_e}{E_p} = \frac{v}{2c}$$

Now, $\quad p_e = mv = \frac{h}{\lambda}$ and $p_h = \frac{h}{\lambda}$

$$\therefore \quad \frac{p_e}{p_h} = 1$$

12. For electron, $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{m_e(c/100)} = \frac{100h}{m_e c}$ $\quad$ ...(i)

Kinetic energy, $E_e = \frac{1}{2} m_e v_e^2$ or $m_e v_e = \sqrt{2E_e m_e}$

$$\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}} \text{ or } E_e = \frac{h_2}{\lambda_e^2 m_e} \quad \text{...(ii)}$$

For photon of wavelength λ_p, energy $= E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e}$

$$(\because \lambda_p = 2\lambda_e)$$

$$\therefore \quad \frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2\lambda_e^2 m_e}{h_2} = \frac{\lambda_e m_e c}{h} = \frac{100h}{m_e c} \times \frac{m_e c}{h} = 100$$

$$\therefore \quad \frac{E_e}{E_p} = \frac{1}{100} = 10^{-2}$$

For electron, $p_e = m_e v_e = m_e \times c/100$

$$\therefore \quad \frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2}$$

13. Radius of largest circular path of electron in perpendicular magnetic field,

$$r = \frac{mv_{\text{max}}}{Be}$$

$\because$ Velocity of most energetic electron emitted from metal surface,

$$v_{\text{max}} = \frac{Ber}{m}$$

Kinetic energy with which electron is emitted,

$$K_{\text{max}} = \frac{1}{2} mv_{\text{max}}^2 = \frac{1}{2} m \cdot \frac{B^2e^2r^2}{m^2}$$

$$\Rightarrow \quad K_{\text{max}} = \frac{1}{2}\left(\frac{B^2e^2r^2}{m}\right) \text{ joule}$$

Kinetic energy of emitted electron (in electron volts),

$$K_{\text{max}} \text{ (eV)} = \frac{K_{\text{max}} \text{ (in joule)}}{e}$$

$$\therefore \quad K_{\text{max}} \text{ (eV)} = \frac{1}{2}\left(\frac{B^2er^2}{m}\right)$$

Here given, $B = 3 \times 10^{-4}$ T, $e = 1.6 \times 10^{-19}$ C,

$$r = 10 \text{ mm} = 10^{-2} \text{ m}$$

and $\quad m = 9.1 \times 10^{-31}$ kg

So, $\quad K_{\text{max}} = \frac{(3 \times 10^{-4})^2 \times (1.6 \times 10^{-19}) \times (10^{-2})^2}{2 \times (9.1 \times 10^{-31})}$

$$= \frac{9 \times 1.6}{2 \times 9.1} \times 10^{31-31} \approx 0.8 \text{ eV}$$

$\therefore$ Energy of incident photon $= hf = \frac{hc}{\lambda}$

$$= \frac{12400 \text{ eV-Å}}{6561 \text{ Å}} \approx 1.9 \text{ eV}$$

(**Note** $hc = 1240$ eV- nm $= 12400$ eV-Å)

So, work function of metal,

ϕ_0 = incident energy – kinetic energy of emitted electron

$$= 1.9 - 0.8 = 1.1 \text{ eV}$$

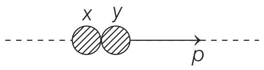
14. Initially,

We have, de-Broglie wavelengths associated with particles are

$$\lambda_x = \frac{h}{p_x} \text{ and } \lambda_y = \frac{h}{p_y}$$

$$\Rightarrow \quad p_x = \frac{h}{\lambda_x} \text{ and } p_y = \frac{h}{\lambda_y}$$

Finally, particles collided to form a single particle.



As we know that, linear momentum is conserved in collision.

So, $\mathbf{p}_p = |\mathbf{p}_x - \mathbf{p}_y| \Rightarrow \mathbf{p}_p = \left|\frac{h}{\lambda_x} - \frac{h}{\lambda_y}\right|$

So, de-Broglie wavelength of combined particle is

$$\lambda_p = \frac{h}{|\mathbf{p}_p|} = \frac{h}{\left|\frac{h}{\lambda_x} - \frac{h}{\lambda_y}\right|} = \frac{h}{\left|\frac{h\lambda_y - h\lambda_x}{\lambda_x\lambda_y}\right|}$$

$$= \frac{\lambda_x\lambda_y}{|\lambda_x - \lambda_y|}$$

15. According to Einstein's equation, $h\nu = h\nu_0 + eV$

$\Rightarrow \quad V = \frac{h}{e}(\nu - \nu_0)$

Given, $\quad V_1 = \frac{h}{e}(\nu_1 - \nu_0)$ and $3V_1 = \frac{h}{e}(2\nu_1 - \nu_0)$

$\Rightarrow \quad \frac{3V_1}{V_1} = \frac{2\nu_1 - \nu_0}{\nu_1 - \nu_0} \Rightarrow \nu_1 = 2\nu_0$

$\therefore \quad V_1 = \frac{h}{e}(2\nu_0 - \nu_0) = \frac{h}{e}\nu_0$

At frequency $4\nu_1$, $nV_1 = \frac{h}{e}(4\nu_1 - \nu_0)$

$$= \frac{h}{e}(8\nu_0 - \nu_0) = 7 \times \frac{h}{e}\nu_0$$

$\Rightarrow \quad nV_1 = 7V_1 \Rightarrow n = 7$

16. Number of photons incident per second

$$= \frac{\text{Power}}{\text{Energy of one photon}}$$

$$= \frac{P}{(hc/\lambda)} = \frac{P\lambda}{hc}$$

Number of electrons emitted per second = 0.1% of

$$\frac{P\lambda}{hc} = \frac{P\lambda}{1000\,hc}$$

$\therefore$ Current = Charge (on photoelectrons per second)

$$= \frac{P\lambda e}{1000\,hc}$$

$$= \frac{(1.5\times 10^{-3})\,(400\times 10^{-9})\,(1.6\times 10^{-19})}{(1000)\,(6.63\times 10^{-34})\,(3\times 10^{8})}$$

$$= 0.48\times 10^{-6}\ \text{A} = 0.48\ \mu\text{A}$$

17. $E = \frac{12375}{1800} = 6.875$ eV

$K_{\max}$ or $K = E - W = 4.875$ eV

$$r = \frac{\sqrt{2Km}}{Bq}$$

$$= \frac{\sqrt{2\times 4.875\times 1.6\times 10^{-19}\times 9.1\times 10^{-31}}}{5\times 10^{-5}\times 1.6\times 10^{-19}} = 0.148\ \text{m}$$

18. Let the work function of the metal be ϕ.

$$\text{KE} = \frac{hc}{\lambda} - \phi$$

For case I, $\frac{1}{2}mv_1^2 = 4 - \phi$ ...(i)

For case II, $\frac{1}{2}mv_2^2 = 2.5 - \phi$ ...(ii)

Given, $\quad \frac{v_1}{v_2} = 2$

From Eqs. (i) and (ii), we get

$$\frac{v_1^2}{v_2^2} = \frac{4-\phi}{2.5-\phi} \Rightarrow (2)^2 = \frac{4-\phi}{2.5-\phi}$$

$\Rightarrow \quad 4(2.5 - \phi) = 4 - \phi \Rightarrow 10 - 4\phi = 4 - \phi$

$\Rightarrow \quad 3\phi = 6 \Rightarrow \phi = \frac{6}{3} = 2$ eV

So, the work function of the metal is 2 eV.

19. With radiation wavelength λ and work function ϕ_0, stopping potential is V.

$\Rightarrow \quad \frac{hc}{\lambda} - \phi_0 = eV$ ...(i)

When wavelength is 3λ, stopping potential is $\frac{V}{4}$.

$\Rightarrow \quad \frac{hc}{3\lambda} - \phi_0 = \frac{eV}{4}$ ...(ii)

Now, multiply Eq. (ii) by 3 and subtracting Eq (i) from it, we get

$\Rightarrow \left(\frac{hc}{\lambda} - 3\phi_0\right) - \left(\frac{hc}{\lambda} - \phi_0\right) = \frac{3\,\text{eV}}{4} - \text{eV}$

$\Rightarrow \quad 2\phi_0 = \frac{1}{4}eV \Rightarrow eV = 8\phi_0$

Putting this value in Eq. (i), we get

$$\frac{hc}{\lambda} - \phi_0 = 8\phi_0 \Rightarrow \frac{hc}{\lambda} = 9\phi_0 \Rightarrow \frac{hc}{\lambda} = 9\frac{hc}{\lambda_0}$$

{As $\phi_0 = \frac{hc}{\lambda_0}$, where λ_0 = threshold wavelength}

$\Rightarrow \quad \lambda_0 = 9\lambda \quad \therefore \quad n = 9$

20. Given, energy per unit area per second,

$$P = 1.388\times 10^3\ \text{W/m}^2$$

Let n be the number of photons incident on the earth per square metre. Wavelength of each photon

$$= 550\ \text{nm} = 550\times 10^{-9}\ \text{m}.$$

Energy of each photon, $E = \frac{hc}{\lambda}$

(where h is the Planck's constant)

$$= \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{550\times 10^{-9}} = 3.616\times 10^{-19}\ \text{J}$$

Number of photons incident on the earth's surface

$$N = \frac{P}{E} = \frac{1.388\times 10^3}{3.616\times 10^{-19}} = 3.838\times 10^{21}$$

$$= 3.838\times 10^{21}\ \text{photon/m}^2\text{-s}$$

$\Rightarrow \quad n = 21$

21. Given, intensity of falling radiation,

$$I = 6.4 \times 10^{-5}\ \text{Wcm}^{-2}$$

Wavelength of radiation, $\lambda = 310$ nm

Work function of metal, $\phi = 2$ eV

Area of metal surface, $A = 1\ \text{cm}^2$

Energy of one photon of radiation,

$$E_1 = \frac{hc}{\lambda} = \frac{1240\ \text{eVnm}}{\lambda\ (\text{nm})}$$

or $$E_1 = \frac{1240}{310}\ \text{eV}$$

$$= \frac{1240}{310} \times 1.6 \times 10^{-19}\ \text{J}$$

$$= 6.4 \times 10^{-19}\ \text{J}$$

So, photon flux of radiation falling over surface = number of photons falling over surface per second

$$= \frac{\text{Total incident energy}}{\text{Energy of one photon}} = \frac{I \times A}{E_1}$$

$$= \frac{6.4 \times 10^{-5} \times 1}{6.4 \times 10^{-19}}$$

$$= 10^{14}\ \text{photons/second}$$

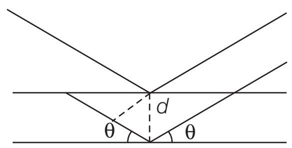
As only one in 10^3 photons are able to emit an electron, number of ejected electrons in 1 second

$$= \frac{10^{14}}{10^3} = 10^{11}$$

$\Rightarrow \quad x = 11$

22. From Bragg's law,

$$2d \sin\theta = n\lambda$$



For first maxima, $n = 1$

$$\Rightarrow \quad 2d \sin\theta = \lambda$$

$$\Rightarrow \quad 2d \sin\theta = \frac{h}{\sqrt{2Em}} \qquad \left\{\begin{array}{l} \because \frac{1}{2}mv^2 = E \\ p = \sqrt{2Em} \\ \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \end{array}\right\}$$

$$\Rightarrow \quad 2Em = \frac{h^2}{4d^2 \sin^2\theta}$$

$$\Rightarrow \quad E = \frac{h^2}{8md^2 \sin^2\theta}$$

Given, $d = 1\ \text{Å} = 10^{-10}$ m, $\theta = 60°$

$$h = 6.64 \times 10^{-34}\ \text{Js},\ m = 9.1 \times 10^{-31}\ \text{kg}$$

Substituting these values in above equation, we get

$$E = \frac{(6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2 \sin^2 60°}$$

$$= \frac{44.0896 \times 10^{-68}}{8 \times 9.1 \times 10^{-51} \times \left(\frac{3}{4}\right)}$$

$$= 0.8075 \times 10^{-17}\ \text{J}$$

$$= \frac{0.8075 \times 10^{-17}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$= 50.47\ \text{eV} \approx 50\ \text{eV}$$

23. Pitch of helical path, $p = (v\cos\theta)\,T = \frac{vT}{2}$. (as $\theta = 60°$)

$$T = \frac{2\pi m}{Bq} = \frac{2\pi}{B\alpha} \qquad \left(\alpha = \frac{q}{m}\right)$$

$\therefore$ $$p = \frac{\pi v}{B\alpha}$$

or $$v = \frac{B\alpha p}{\pi} \qquad \ldots\text{(i)}$$

$$\text{KE} = \frac{1}{2}mv^2 = E - W$$

$\therefore$ $$W = E - \frac{1}{2}mv^2 \qquad \ldots\text{(ii)}$$

Substituting value of v from Eq. (i) in Eq. (ii), we get

$$W = 4.9 - \frac{1}{2}\,9.1 \times 10^{-31} \times (2.5 \times 10^{-3})^2\,(1.76 \times 10^{11})^2 \times \frac{(2.7 \times 10^{-3})^2}{\pi^2 \times 1.6 \times 10^{-19}}$$

$$= (4.9 - 0.4)\ \text{eV} = 4.5\ \text{eV}$$

24. Given, work function, $W_0 = 2.35$ eV and the electric component of electromagnetic radiation

$$E = a[1 + \cos(2\pi f_1 t)]\cos(2\pi f_2 t)$$

$$\Rightarrow E = [a\cos(2\pi f_2 t) + a\cos(2\pi f_1 t)\cos(2\pi f_2 t)]$$

$$\left(\because \cos A \cos B = \frac{1}{2}[\cos(A + B) - \cos(A - B)]\right)$$

$$\Rightarrow E = a\cos(2\pi f_2 t) + \frac{a}{2}\cos 2\pi(f_1 + f_2)t - \frac{a}{2}\cos 2\pi(f_1 - f_2)t$$

So, the electric component has 3 sub-components with frequencies are,

$$f_2,\ (f_1 + f_2)\ \text{and}\ (f_1 - f_2)$$

So, for maximum kinetic energy of photoelectron, we take photon of maximum frequency. Hence,

$$E_{\text{max}} = \frac{h\nu_{\text{max}}}{e} = \frac{6.6 \times 10^{-34} \times (3.6 \times 10^{15} + 1.2 \times 10^{15})}{1.6 \times 10^{-19}}$$

$$= 19.8\ \text{eV}$$

Hence, the maximum kinetic energy,

$$\text{KE}_{\text{max}} = E_{\text{max}} - W_0 = 19.8 - 2.35 = 17.45\ \text{eV}$$