MOCK TEST

General Instructions :

(*i*) **All** *questions are compulsory.*

(*ii*) *This question paper contains* **29** *questions.*

(*iii*) *Questions* **1 – 4** *in* **Section–A** *are very short-answer type questions carrying* **1** *mark each.*

(*iv*) *Questions* **5 – 12** *in* **Section–B** *are short-answer type questions carrying* **2** *marks each.*

(*v*) *Questions* **13 – 23** *in* **Section–C** *are long-answer-I type questions carrying* **4** *marks each.*

(*vi*) *Questions* **24–29** *in* **Section–D** *are long-answer-II type questions carrying* **6** *marks each.*

CLASS—XII
MATHEMATICS

Time Allowed : 3 Hours] **[Maximum Marks : 100**

SECTION–A

(Question numbers 1 to 4 carry 1 mark each)

1. Let A = {1, 2, 3,........, 9}. Let R be the equivalence relation on A × A defined by (a, b) R (c, d) iff $a + d = b + c$. Find the equivalence class [(2, 5)].

2. If A_{ij} is the co-factor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$.

3. Find $|\vec{x}|$, if for unit vector $\vec{a}$, $(\vec{x}-\vec{a}) \cdot (\vec{x}+\vec{a}) = 12$.

4. Find the identity element in **Z** with respect to the operation 'x' defined by $a * b = a + b + 1$ $\forall\, a, b \in \mathbf{Z}$.

SECTION–B

(Question numbers 5 to 12 carry 2 marks each)

5. If $\sin^{-1} x + 4 \cos^{-1} x = \pi$, find the value of x.

6. Consider the matrix A = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$.

   Prove that $A^2 - 7A - 2I = O$ and hence find A^{-1}.

7. Prove that $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right) = 2\sin^{-1} x$, where $\dfrac{-1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}}$.

8. If $y = x^4 - 10$ and if x changes from 2 to 1.99, what is the approximate change in y ?

9. Evaluate: $\displaystyle\int \frac{2+\sin x}{1+\cos x} e^{x/2}\, dx$.

10. Show that $y = ax^3 + bx^2 + c$ is a solution of the differential equation :

    $$\frac{d^3y}{dx^3} - 6a = 0.$$

11. Find the projection (vector) of $7\hat{i}+\hat{j}-\hat{k}$ on $2\hat{i}+6\hat{j}+3\hat{k}$.

12. If P(A) = 0.6, P(B) = 0.7 and P(A ∪ B) = 0.9 , then find P(A/B) and P(B/A).

SECTION–C

(Question numbers 13 to 23 carry 4 marks each)

13. Using properties of determinants, prove that :

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$

14. For what values of 'a' and 'b' the function :

$$f(x) = \begin{cases} x^2, & x \le 0 \\ ax+b, & x > 0 \end{cases}$$ is differentiable at $x = 0$.

Or

If $$f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \dfrac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases}$$

is continuous at $x = 0$, then find the values of a and b.

15. If $y = \sin(m \tan^{-1} x)$, prove that :

$(1 + x^2)^2 y_2 + 2x(1 + x^2) y_1 + m^2 y = 0$.

16. Find the equations of the tangents to the curve :

$y = x^3 + 2x - 4$,

which are perpendicular to the line $x + 14y + 3 = 0$

Or

Prove that $\dfrac{x}{1+x} < \log(1+x) < x$ for $x > 0$.

17. If performance of the students 'y' depends on the number of hours 'x' given by the relation :

$$y = 4x - \frac{x^2}{2}.$$

Find the number of hours, the students work to have the best performance.

'Hours to hard work are necessary for success', Justify.

18. Evaluate: $\displaystyle\int \frac{dx}{\sin x(3+2\cos x)}$.

19. Find the particular solution of the differential equation :

$$\frac{dy}{dx} = -\frac{x + y\cos x}{1+\sin x}, \text{ given } y(0) = 1.$$

Or

Show that $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$ is homogeneous and solve it.

20. Prove that $\left\{(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})\right\}.(\vec{a}+\vec{b}) = 2[\vec{a}\ \vec{b}\ \vec{c}]$.

21. Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines :

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

and $$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

22. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black.

23. Two bad eggs are mixed accidently with 10 good ones. Find the probability distribution of the number of bad eggs in 3 eggs drawn at random in succession, without replacement from the lot. Find the mean number of bad eggs drawn.

SECTION–D

(Question numbers 24 to 29 carry 6 marks each)

24. Let $f : \mathbf{N} \to \mathbf{N}$ be a function defined by:

$f(x) = 9x^2 + 6x - 5$.

Show that $f : \mathbf{N} \to S$, where S is the range of 'f', is invertible. Find the inverse of 'f' and hence, find $f^{-1}(43)$ and $f^{-1}(163)$.

Or

Let $A = \mathbf{R} \times \mathbf{R}$ and '*' be a binary operation on A defined by :

$(a, b) * (c, d) = (a + c, b + d)$.

Show that '*' is commutative and associative. Find the identity element for '*' on A. Also, find the inverse of every element $(a, b) \in A$.

25. Find A^{-1} : $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$.

Hence, solve the following system of linear equations :

$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11$.

Or

Find the inverse of the matrix A by elementary row transformations and verify that $A^{-1} A = I$ when :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

26. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

27. Evaluate = $\int_{\pi/3}^{\pi/2} \sqrt{\frac{1+\cos x}{(1-\cos x)^{5/2}}}\,dx$.

Or

Evaluate = $\int_0^1 (2-3x+x^2)dx$ as the limit of a sum.

28. Find the distance of the point (1, – 2, 3) from the plane $x - y + z = 5$, measured parallel to the line : $\frac{x}{2} = \frac{y}{3} = \frac{z}{-0}$.

29. A company manufactures two types of a novalty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is ₹ 5 each for type A and ₹ 6 each for type B souvenirs. How many souvenirs of type should the company manufacture in order to maximise the profit ?

Answers

1. [(2, 5)] = [(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)]

2. 110.

3. $\sqrt{13}$

4. –1

5. $\frac{\sqrt{3}}{2}$

6. $\begin{bmatrix} -5/2 & 3/2 \\ 2 & -1 \end{bmatrix}$

8. Decrease of 0.32

9. $2\tan\frac{x}{2}e^{x/2}+c$

11. $\frac{17}{49}(2\hat{i}+6\hat{j}+3\hat{k})$

12. $\frac{4}{7}, \frac{2}{3}$

14. $a = 2c$, $b = -c^2$ **Or** $a = -1$, $b = 4$

16. $14x - y - 20 = 0$, $14x - y + 12 = 0$

17. 4 hours per day

By doing work, we can create skill in using the things, learnt by us ?

Thus Don't make mistake in the competition when things are asked.

18. $\frac{1}{5}\log|\cos x-1|-\frac{1}{2}\log|\cos x+1|+\frac{2}{5}\log|3+2\cos x|+c$.

19. $y(1+\sin x)=\frac{-x^2}{2}+1$ **Or** $\log|x|-2\log|x-y|-\frac{y}{x}+c=0$

21. $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

22. $\frac{4}{17}$

23. $\frac{1}{2}$

24. $f(x)=\frac{\sqrt{x+6}-1}{3}; f^{-1}(43)=2, f^{-1}(163)=4$ **Or** (0, 0), $(-a, -b)$

25. $x = 3$, $y = -2$, $z = 1$

26. $\frac{1}{3}$

27. $\frac{3}{2}$ **Or** $\frac{5}{6}$

28. 1 unit

29. Type A : 8; Type B : 20; Max. Profit = ₹ 160.