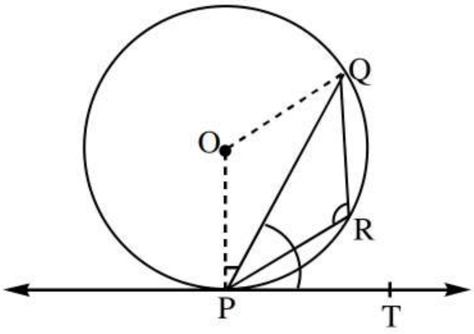
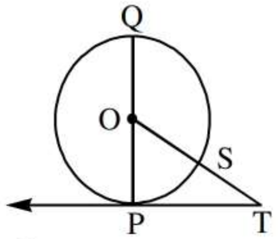
10. CIRCLES

1. In the figure, PQ is a chord of a circle with center O and PT is tangent $\angle QPT = 60^0$, find $\angle PRQ$.



[Ans : $\underline{|PRQ} = 120°$]

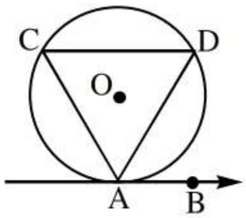
2. TP is a tangent to the circle with center O. If $\angle TOQ = 120^0$, find the diameter of the circle, when OT = 10 cm.



[Ans : diameter of the circle = 10 cm]

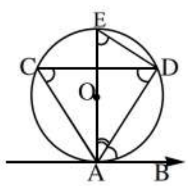
3. **Application of Alternative Segment Theorem :**

Chord CD is parallel to the tangent AB at a point A of the circle with centre O. Prove that ΔACD is isosceles.



[Proof : CD || AB (Given)

So, $\angle CDA = \angle DAB$ (Alternate angles) ....(1)



Now, join OA and produce it to meet the circle at E.

Also, join DE.

$\angle OAB = 90°$ (Angle between radius and tangent)

$\Rightarrow \angle OAD + \angle DAB = 90°$ ....(2)

Also, $\angle ADE = 90°$ (Angle in a semicircle)

So, $\angle E + \angle OAD = 90°$ ....(3)

From (2) and (3),

$\angle E = \angle DAB$ ....(4)

But $\angle E = \angle C$ (Angles in the same segment)

So, $\angle C = \angle DAB$ [From (4)] ....(5)

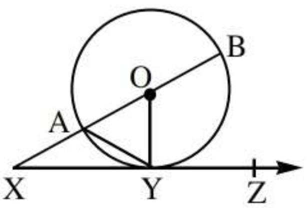
Therefore, from (1) and (5), we get

$\angle C = \angle CDA$

So, AD = AC (Sides opposite the equal angles)

Hence, ΔADC is isosceles.]

4. In the figure, tangent XZ touches the circle with centre O at Y. Diameter BA, when produced meets XZ at X. If $\angle BXY = b$ and $\angle AYX = a$, prove that $b + 2a = 90^0$.



[**Sol. :** From ΔAXY, $\angle OAY = \angle BXY + \angle AYX$ (Exterior angle property)

= b + a.

But, OA = OY (Radii of a circle)

So, $\angle OYA = \angle OAY$

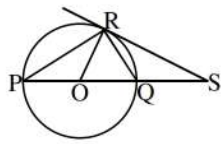
$\Rightarrow \angle OYA = b + a.$

Also, $\angle OYX = 90°$ (Angle between radius and tangent)

$\Rightarrow \angle AYX + \angle OYA = 90°$

$\therefore$ a + a + b = 90° $\Rightarrow$ b + 2a = 90°.]

5. In the figure, PQ is a diameter of a circle with centre O and PR is a chord such that $\angle QPR = 30^0$. If the tangent at R intersects PQ extended at S, then prove that QR = QS.



[**Sol. :** $\angle PRQ = 90°$ (PQ is a diameter) ....(1)

$\angle QPR = 30°$ (Given)

So, $\angle ORP = 30°$ (OP = OR) ....(2)

From (1) and (2),

$\angle ORQ = \angle PRQ - \angle ORP = 90° - 30° = 60°.$

But $\angle ORS = 90°$ (Angle between tangent and radius)

So, $\angle QRS = \angle ORS - \angle ORQ$

$= 90° - 60° = 30°$ ....(3)

Also, $\angle OQR = \angle ORQ = 60°$ (OR = OQ)

Therefore, $\angle QSR = \angle OQR - \angle QRS$ (Exterior angle property)

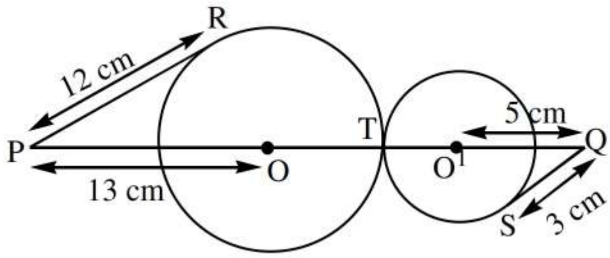
$= 60° - 30° = 30°$ ....(4)

Thus, from (3) and (4), we have

$\angle QRS = \angle QSR$

$\Rightarrow$ $QR = QS$ (Sides opposite the equal angles), proved.]

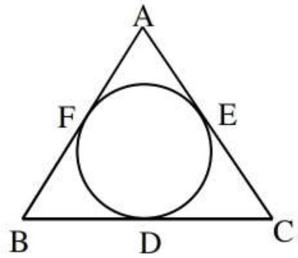
6. Two circles with centres at O and O' touch each other externally at T as shown below :



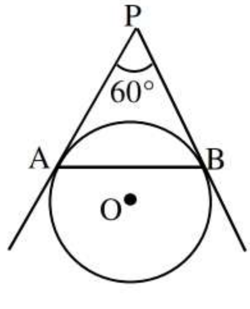
If PR = 12 cm, PO = 13 cm, O^1Q = 5 cm and SQ = 3 cm, find the length of line segment PQ.
[Ans : PQ = 27 cm]

7. In the figure, sides AB, BC and CA of ΔABC touch a circle at F, D and E respectively. Prove that

$AF + BD + CE = \frac{1}{2}$ (Perimeter ΔABC).



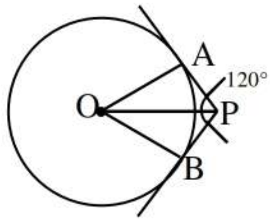
8. In the figure, AP and BP are tangents to a circle with centre O such that AP = 5 cm and $\angle APB = 60^0$. Find the length of chord AB.



[Ans : AB = 5 cm]

9. Two tangents PA and PB are drawn to a circle with centre O such that $\angle APB = 120^0$. Prove that OA = $\sqrt{3}$ AP.

[Sol. : See figure.



Join OA and OB.

Since $\angle APB = 120°$,

therefore $\angle AOB = 180° - 120° = 60°$

Further, $\angle AOP = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60° = 30°.$

Now, from rt. ΔOAP, we get

$$\frac{OA}{AP} = \cot 30° \Rightarrow \frac{OA}{AP} = \sqrt{3}$$

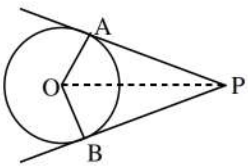
$\Rightarrow$ $OA = AP\sqrt{3}$ or $\sqrt{3}AP$, proved.]

10. Prove that tangents drawn from an external point are equally inclined to line joining the external point and the centre.

(OR)

If two tangents are drawn to a circle from an external point, then prove that they are equally inclined to the line segment, joining the centre to that point.

[Ans : In ΔPAO and ΔPBO,



$\angle PAO = \angle PBO$	(Each 90°)
PO = OP	(Common)
AO = OB	(Radii of the same circle)
$\therefore$ $\Delta PAO \cong \Delta PBO$	(By RHS congruence)
$\Rightarrow$ $\angle APO = \angle BPO$	(By CPCT)

Hence proved.]

11. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle of radius r touches the sides of the triangle. Prove that $r = \frac{a+b-c}{2}$.

[Sol. : Let O be the centre of the circle.

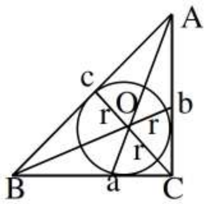
Now, area of $\Delta ABC = \frac{1}{2} BC \times AC = \frac{1}{2} ab$ ......(1)

Also, area of ΔABC = Area of ΔOAB + Area ΔOBC + area of ΔOCA

$$= \frac{1}{2}c \times r + \frac{1}{2}a \times r + \frac{1}{2}b \times r$$

$$= \frac{1}{2}r(a+b+c)$$ ....(2)

From (1) and (2), we get $\frac{1}{2}r(a+b+c)=\frac{1}{2}ab$



$\Rightarrow \qquad r(a+b+c)=ab$

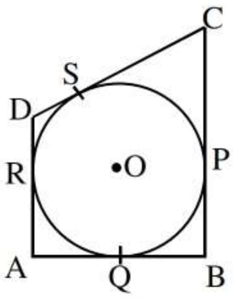
$\Rightarrow \qquad r=\frac{ab}{(a+b+c)}$

$$=\frac{ab\times(a+b-c)}{(a+b+c)(a+b-c)}=\frac{ab(a+b-c)}{(a+b)^2-c^2}$$

$$=\frac{ab(a+b-c)}{(a^2+b^2+2ab)-c^2}=\frac{ab(a+b-c)}{c^2+2ab-c^2}$$

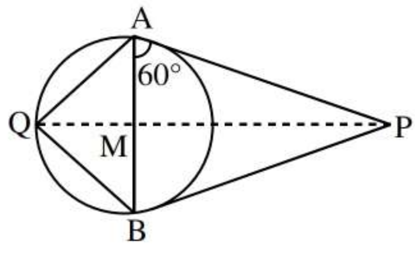
$$=\frac{ab}{2ab}(a+b-c)=\frac{1}{2}(a+b-c),$$ proved.]

12. A circle with centre O is inscribed in a quadrilateral ABCD in which $\angle B=90^0$. If AD = 23cm, AB = 29cm and DS = 5cm, find the radius of the circle.



[Ans : radius = 11cm**]**

13. PA and PB are the tangents to a circle which circumscribes an equilateral ΔABQ. If $\angle PAB=60^0$, as shown in the figure, prove that QP bisects AB at right angles.



[Sol. :

and $\left.\begin{array}{l}\angle QAB=60^\circ\\ \angle QBA=60^\circ\end{array}\right\}(\Delta ABQ$ is equilateral)

So, $\left.\begin{array}{ll}So, & \angle PAQ=\angle PAB+\angle QAB=60^\circ+60^\circ=120^\circ\\ \text{Similarly,} & \angle PBQ=120^\circ\end{array}\right\}$ ....(1)

Now, in ΔPAQ and ΔPBQ,

$PA = PB$ (Tangents from external point)

$AQ = BQ$ (ΔABQ equilateral)

$\angle PAQ = \angle PBQ$ [Each = 120°, shown above)

So, $\Delta PAQ \cong \Delta PBQ$ (By SAS)

Hence, $\angle APQ = \angle BPQ$ (CPCT) ....(2)

Let QP intersect AB at M.

Now, in ΔPAM and ΔPBM,

$\angle APM = \angle BPM$ [From (2)]

$PA = PB$ (Tangents from external point)

$PM = PM$ (Common)

So, $\Delta PAM \cong \Delta PBM$ (SAS)

$\Rightarrow$ $AM = BM$ (CPCT) ....(3)

and $\angle AMP = \angle BMP$ (CPCT)

But $\angle AMP + \angle BMP = 180°$

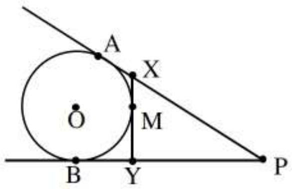
$\Rightarrow$ $\angle AMP + \angle AMP = 180°$

$\Rightarrow$ $2\angle AMP = 180°$

$\Rightarrow$ $\angle AMP = 90°$ ....(4)

From (3) and (4), we get that QP bisects AB at right angles.]

14. In the figure below, ΔPXY is formed using three tangents to a circle centred at O.



Based on the construction, the sum of the tangents PA and PB is _______the perimeter of ΔPXY

a) lesser than

b) greater than

c) equal to

d) (cannot be answered without knowing the tangent lengths)

[Ans : (c) equal to]

15. A circle has a centre O and radii OQ and OR. Two tangents, PQ and PR, are drawn from an external point, P.
In addition to the above information, which of these must also be known to conclude that the quadrilateral PQOR is a square?

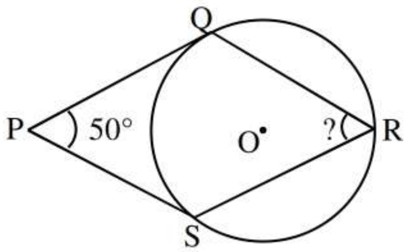
i) OQ and OR are at an angle of 90°.

ii) The tangents meet at an angle of $90°$.

a) only (i) b) only (ii) c) either (i) or (ii) d) both (i) and (ii)

[Ans : (c) either (i) or (ii)**]**

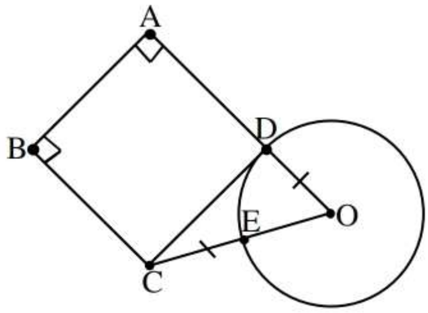
16. In the following figure, O is the centre of the circle and PQ and PS are tangents to the circle at points Q and S respectively.



What is the measure of $\angle QRS$? Show your work.

[Ans : $\angle QRS = 65°$].

17. ABCD is a square. CD is a tangent to the circle with centre O as shown in the figure below.



If OD = CE, what is the ratio of the area of the circle and the area of the square? Show your steps and give valid reasons.

[Sol. : Write that OE = OD (radii of the same circle) and CE = OD (given).

Finds the length of OC as 2OD.

Writes that as CD is tangent to the circle, $OD \perp CD$ and applies Pythagoras' theorem in ΔODC to find the length of CD as:

$OC^2 = CD^2 + OD^2$

$\Rightarrow CD^2 = OC^2 - OD^2 = 4 \times OD^2 - OD^2$

$\Rightarrow CD^2 = 3 \times OD^2$

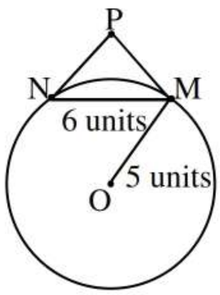
The area of the circle as $\pi \times OD^2$ sq units.

The area of the square as $3 \times OD^2$ sq units, using step 2.

The ratio of the areas as

Area (circle) : Area (square) = $\pi : 3$].

18. Show below is a circle with centre O and radius 5 units. PM and PN are tangents and the length of chord MN is 6 units.

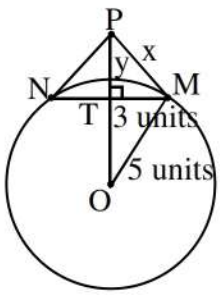


Find the length of (PM + PN). Show your work.

[Sol. : Connects P to O intersecting MN at T. Applies chord properties to conclude that

i) $PO \perp MN$

ii) MT = NT = 3 units.



Applies Pythagoras' theorem in ΔOTM to find OT as 4 units.

Assumes PM = x units and PT = y units and applies Pythagoras' theorem to find two equations in x and y as

From ΔPTM, $x^2 = y^2 + 3^2$ ...... (1)

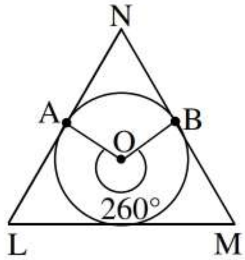
From ΔPMO, $(y + 4)^2 = x^2 + 5^2$ ..... (2)

Solves the pair of equations in the previous step to get y (PT) as $\frac{9}{4}$ or 2.25 units.

Inputs the value of y in one of the equations from step 3 to find the value of x (PM) as $\frac{15}{4}$ or 3.75 units.

Applies tangent property from external point to conclude that PM = PN and finds PM + PN as $\frac{30}{4}$ or 7.5 units].

19. In the figure below, a circle with centre O is inscribed inside ΔLMN. A and B are the points of tangency.

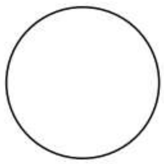


Find $\angle ANB$. Show your steps.

[Sol.: Finds minor $\angle AOB$ as $360° - 260° = 100°$.

Finds $\angle ANB$ as $360° - (90° + 90° + 100°) = 80°$.

20. shown below is a circle whose centre is unknown.



State true or false the statements below and give valid reasons.

i) The centre of the circle can be found using any 2 tangents.

ii) The centre of the circle can be found using any 2 chords.

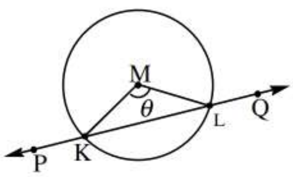
[Sol.: i) True.

For example, drawing 90° angles at the points of contact of the 2 tangents and extending perpendicular lines to meet at a point gives the centre of the circle.

ii) True.

For example, drawing the perpendicular bisectors of the 2 chords and extending them to meet at a point gives the centre of the circle.]

21. Shown below is a circle with centre M. PQ is a secant and $\angle KML = \theta$.



i) Show that, when $\theta = 0°$, PQ becomes a tangent to the circle.

ii) What is the point of contact of the tangent in part i) with the circle?

[Sol.: i) Writes that ΔKLM is an isosceles triangle and finds the measures of $\angle KLM = \angle LKM$

$$= \frac{180° - \theta}{2} = 90° - \frac{\theta}{2}.$$

Writes that angles on a straight line are supplementary and finds the measures of

$$\angle PKM = \angle QLM = 180^\circ - \left(90^\circ - \frac{\theta}{2}\right) = 90^\circ + \frac{\theta}{2}.$$

Writes that, when $\theta = 0^\circ$, KM and LM coincide and $\angle PKM = \angle QLM = 90^\circ$. Hence, concludes that PQ becomes a tangent to the circle.

ii) Writes the point of contact of the tangent in part (i) with the circle as K or L.]