GUIDED REVISION

SECTION-I

Single Correct Answer Type **15 Q. [3 M (–1)]**

1. The shape of a wave propagating in the positive x or negative x- direction is given $y = \frac{1}{\sqrt{1+x^2}}$ at $t = 0$ and $y = \frac{1}{\sqrt{2-2x+x^2}}$ at t = 1s where x and y are in meters. The shape the wave disturbance does not change during propagation. Find the velocity of the wave.

(A) 1 m/s in positive x direction (B) 1 m/s in negative x direction

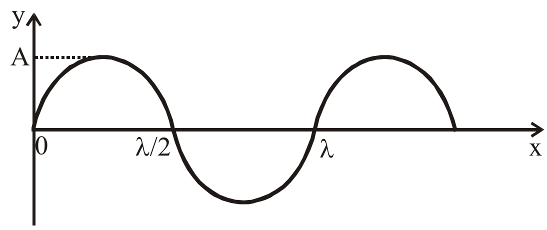
(C) $\frac{1}{2}$ m/s in positive x direction (D) $\frac{1}{2}$ m/s in negative x direction

2. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment t = T/6 (T is the time period). Assume that the source was at mean position at t = 0. The wavelength of the running wave is

(A) 0.96 m (B) 0.48 m (C) 0.24 m (D) 0.12 m

3. Here given snap shot of a progressive wave at t = 0 with time period = T. Then the equation of the wave if wave is going in +ve x-direction and if wave is going in –ve x-direction will be respectively.

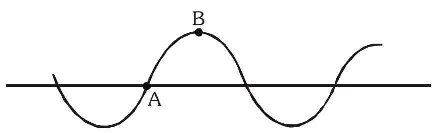
$\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$



(A) $y = A \sin(kx + \omega t)$, $y = A \sin(kx - \omega t)$ (B) $y = A \cos(kx + \omega t)$, $y = A \cos(kx - \omega t)$

(C) $y = A \sin(\omega t - kx)$, $y = A \sin(\omega t + kx)$ (D) $y = A \sin(kx - \omega t)$, $y = A \sin(kx + \omega t)$

4. A progressive wave is travelling in a string as shown. Then which of the following statement about KE and potential energy of the elements A and B is true?



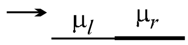
(A) For point A : kinetic energy is maximum and potential energy is min.

(B) For point B : kinetic energy is minimum and potential energy is min.

(C) For point A : kinetic energy is minimum and potential energy is max.

(D) For point B : kinetic energy is minimum and potential energy is max.

5. A string consists of two parts attached at $x = 0$. The right part of the string $(x > 0)$ has mass μ_r per unit length and the left part of the string $(x < 0)$ has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string ?



(A) 1 (B) $\frac{2}{1+\sqrt{\mu_l/\mu_r}}$ (C) $\frac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$ (D) $\frac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

6. The equation of a wave on a string having linear mass density 0.07 kgm^{-1} is given as

$$y = 0.03(m)\cos\left[2\pi\left(\frac{t}{0.08(s)} - \frac{x}{0.8(m)}\right)\right]$$

the tension in string is :

(A) 14N (B) 3.5N (C) 7 N (D) 70N

7. Five waveforms moving with equal speeds on the x-axis

$y_1 = 8\sin(\omega t + kx)$; $y_2 = 6\sin(\omega t + \frac{\pi}{2} + kx)$; $y_3 = 4\sin(\omega t + \pi + kx)$; $y_4 = 2\sin(\omega t + \frac{3\pi}{2} + kx)$;

$y_5 = 4\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$ are superimposed on each other. The resulting wave is :

(A) $8\sqrt{2}\cos kx \sin(\omega t + \frac{\pi}{4})$ (B) $8\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$

(C) $8\sqrt{2}\sin kx \cos(\omega t + \frac{\pi}{4})$ (D) $8\sin(\omega t + kx)$

8. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin\omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then **[JEE 2001 (Scr)]**

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

9. A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is :- **[AIEEE-2011]**

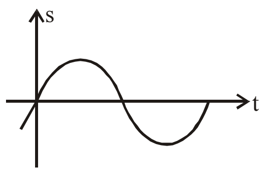
(A) A standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2$

(B) A wave travelling along + x direction

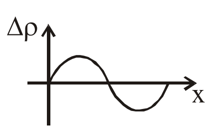
(C) A wave travelling along –x direction

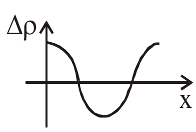
(D) A standing wave having nodes at $x = \frac{n\lambda}{2}$; n = 0, 1, 2

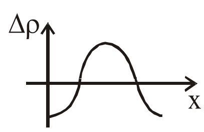
10. A sound waves is travelling towards right and its s-t graph is as shown for x = 0.

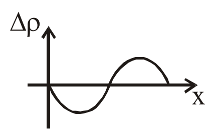


What will be the variation in density vs x graph at t = T/4 :-

(A)


(B)


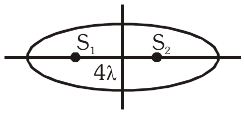
(C)


(D)


11 n identical waves each of intensity I_0 interfere with each other. The ratio of maximum intensities if the interference is (i) coherent and (ii) incoherent is :-

(A) n^2 (B) $\frac{1}{n}$ (C) n (D) $\frac{1}{n^2}$

12. S_1, S_2 are two coherent sources of sound located along x – axis separated by 4λ where λ is wavelength of sound emitted by them. Number of maxima located on the elliptical boundary around it will be :



(A) 16 (B) 12 (C) 8 (D) 4

13. A source when at rest in a medium produces waves with a velocity v and a wavelength of λ. If the source is set in motion with a velocity v_s what would be the wavelengths produced directly in front of the source?

(A) $\lambda\left(1-\frac{v_s}{v}\right)$ (B) $\lambda\left(1+\frac{v_s}{v}\right)$ (C) $\lambda\left(1+\frac{v}{v_s}\right)$ (D) $\frac{\lambda v}{v+v_s}$

14. A police van moving with velocity 22 m/s and emitting sound of frequency 176 Hz, follows a motor cycle in turn is moving towards a stationary car and away from the police van. The stationary car is emitting frequency 165 Hz. If motorcyclist does not hear any beats then his velocity is (v_s = 330 m/s) **[JEE 2003 (Scr)]**

(A) 22 m/s (B) 24 m/s (C) 20 m/s (D) 18 m/s

15. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1}) **[JEE Main - 2017]**

(A) 17.3 GHz (B) 15.3 GHz (C) 10.1 GHz (D) 12.1 GHz

Multiple Correct Answer Type 8 Q. [4 M (–1)]

16. Choose the incorrect one

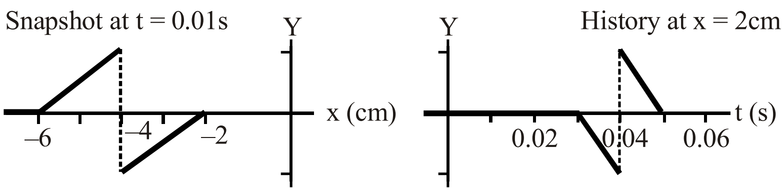
(A) When an ultrasonic wave travels from air into water, it bends towards the normal to the air water interface

(B) Any function of the form y(x, t) = f(vt + x) represents a travelling wave

(C) The velocity, wavelength and frequency of wave undergo change when it is reflected from a surface

(D) None of the above

17. Figure shows a snapshot graph and a history graph for a wave pulse on a stretched string. They describe the same wave from two perspectives.



(A) the wave is travelling in positive x-direction

(B) the wave is travelling in negative x-direction

(C) the speed of the wave is 2 m/s.

(D) the peak is located at x = –6 cm at t = 0.

18. In a travelling one dimensional mechanical sinusoidal wave

(A) potential energy and kinetic energy of an element become maximum simultaneously.

(B) all particles oscillate with the same frequency and the same amplitude

(C) all particles may come to rest simultaneously

(D) we can find two particles, in a length equal to half of a wavelength, which have the same non zero acceleration simultaneously.

19. One end of a taut string of length 3m along the x–axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) :-

[JEE-Advance-2014]

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

20. Which of the following statements is/are **INCORRECT :-**

(A) in wave motion, particle speed depends on tension and linear mass density of the string.

(B) in wave motion particle speed can be equal to the wave speed

(C) The speed of sound in air at 20ºC is twice that at 5ºC

(D) A 60 dB sound has twice the intensity of a 30dB sound.

21. A car moves towards a hill with speed v_c. It blows a horn of frequency f which is heard by an observer following the car with speed v_0. The speed of sound in air is v.

(A) the wavelength of sound reaching the hill is $\frac{v}{f}$

(B) the wavelength of sound reaching the hill is $\frac{v - v_c}{f}$

(C) the beat frequency observed by the observer is $\left(\frac{v + v_o}{v - v_c}\right)f$

(D) the beat frequency observed by the observer is $\frac{2v_c(v + v_o)f}{v^2 - v_c^2}$

22. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe. **[JEE 2012]**

(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

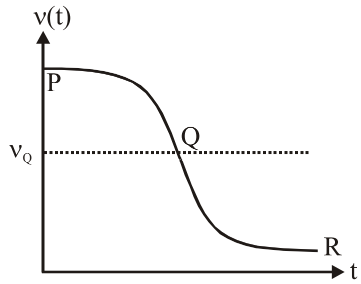
(B) a low -pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(C) a low pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

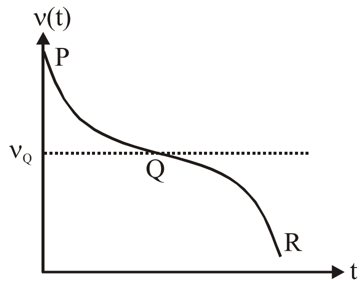
(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

23. Two loudspeakers M and N are located 20m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let ν(t) represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person ? **[JEE Advanced 2016]**

(A) The plot below represents schematically the variation of beat frequency with time



(B) The plot below represents schematically the variations of beat frequency with time



(C) The rate of change in beat frequency is maximum when the car passes through Q

(D) $\nu_P + \nu_R = 2\nu_Q$

Linked Comprehension Type (3 Para × 3Q. & 1 Para × 4 Q.) [3 M (-1)]
(Single Correct Answer Type)

Paragraph for Question Nos. 24 to 26

A wave represented by equation $y = 2(mm)\sin[4\pi(sec^{-1})t - 2\pi(m^{-1})x]$ is superimposed with another wave $y = 2\,(mm)\sin[4\pi(sec^{-1})t + 2\pi(m^{-1})x + \pi/3]$ on a tight string.

24. Phase difference between two particles which are located at $x_1 = 1/7$ and $x_2 = 5/12$ is :-

(A) 0 (B) $\frac{5\pi}{6}$ (C) π (D) $\frac{5\pi}{3}$

25. Which of the following is not a location of antinode?

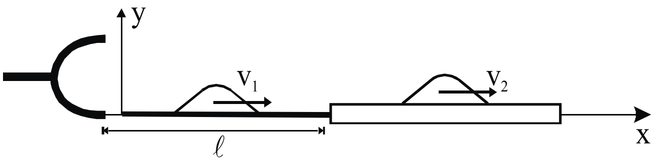
(A) $\frac{2}{3}$ (B) $\frac{11}{12}$ (C) $\frac{5}{12}$ (D) $\frac{17}{12}$

26. The location having maximum potential energy is

(A) 1/7 (B) 1/6 (C) 5/12 (D) 23/12

Paragraph for Question Nos. 27 to 29

A harmonic oscillator at x = 0, oscillates with a frequency $\frac{\omega}{2\pi}$ and amplitude a. It is generating waves at end of a thin string in which velocity of wave is v_1 and which is connected to another heavier string in which velocity of wave is v_2 as shown, length of first string is ℓ.



27. If harmonic oscillator oscillates by an equation y = a sinωt. The equation of incident wave in first string is

(A) $y = a\sin\omega\left(t - \frac{x}{v_1}\right)$ (B) $y = a\sin\omega\left(t + \frac{x}{v_1}\right)$

(C) $y = a\sin\left[\omega\left(t - \frac{x}{v_1}\right) + \pi\right]$ (D) $y = a\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$

28. Equation of transmitted wave in second string if its amplitude is a_t is

(A) $y = a_t \sin\omega\left(t - \frac{x}{v_2}\right)$ (B) $y = a_t \sin\omega\left(t - \frac{\ell}{v_1}\right)$

(C) $y = a_t \sin\omega\left(t - \frac{\ell}{v_1} - \frac{x-\ell}{v_2}\right)$ (D) $y = a_t \sin\omega\left(t - \frac{x}{v_2}\right)$

29. Equation of reflected wave, if it is reflecting at the joint and amplitude of reflected wave is a_R

(A) $y = a_R \sin\omega\left(t - \frac{x}{v_2}\right)$ (B) $y = a_R\sin\left[\omega\left(t - \frac{\ell}{v_1} - \frac{\ell - x}{v_1}\right) + \pi\right]$

(C) $y = a_R\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$ (D) $y = a_R\sin\left[\omega\left(t + \frac{2\ell + x}{v_1}\right) + \pi\right]$

Paragraph for Question Nos. 30 to 32

A 2m string has tension 1N is fixed at both end and its is vibrating in its third harmonic with antinode amplitude 3 cm and frequency 100 Hz, then

30. Possible stationary wave equation for the vibration of the string will be (assume origin at left end of the string and x is measured in meters towards right and t is measured in seconds)

(A) $y = (0.06\text{ m})\sin\left(\frac{3\pi}{2}x\right)\cos(200\pi t)$ (B) $y = (0.03\text{ m})\sin\left(\frac{3\pi}{4}x\right)\cos(200\pi t)$

(C) $y = (0.06\text{ m})\sin\left(\frac{3\pi}{4}x\right)\cos(200\pi t)$ (D) $y = (0.03\text{ m})\sin\left(\frac{3\pi}{2}x\right)\cos(200\pi t)$

31. Total wave energy on the string will be nearly equal to

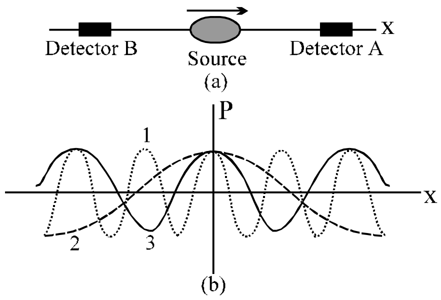
(A) 40 mJ (B) 10 mJ (C) 30 mJ (D) 20 mJ

32. At what time from the start (by the answer of first question in this paragraph) string will have maximum kinetic energy first time (second)

(A) $\frac{1}{200}$ (B) $\frac{1}{100}$ (C) $\frac{1}{400}$ (D) $\frac{1}{800}$

Paragraph for Question No. 33 to 36

A source emitting a sound wave at a certain frequency moves with constant speed along an x-axis figure (a). The source moves directly towards a stationary detector A and directly away from another stationary detector B. The superimposed three plots of figure (b) indicate the pressure function P(x) of the sound wave as measured by detector A, by detector B, and by someone (c) in the rest frame of the source.



33. Which of the following plot corresponds to the measurement done by detector A?

(A) 1 (B) 2 (C) 3 (D) These plots are not possible

34. The plot corresponding to the measurement done by detector B is

(A) 1 (B) 2 (C) 3 (D) These plots are not possible

35. The plot corresponding to the measurement done by the detector C is

(A) 1 (B) 2 (C) 3 (D) These plots are not possible

36. Now the source stops and begins to move along y-axis with same speed, the plot which corresponds to the measurement of B now is

(A) 1 (B) 2 (C) 3 (D) none of these

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

37. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be, **[JEE-Advance-2019]**

	List-I		**List-II**
(I)	String-1(μ)	(P)	1
(II)	String-2 (2μ)	(Q)	1/2
(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$
(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

(1) I→P, II→R, III→S, IV→Q (2) I→P, II→Q, III→T, IV→S

(3) I→Q, II→S, III→R, IV→P (4) I→Q, II→P, III→R, IV→T

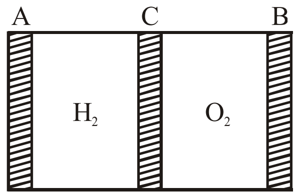
SECTION-II

Numerical Answer Type Question **4 Q. [3(0)]**

(upto second decimal place)

1. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement?

2. AB is a cylinder of length 1m fitted a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibration for which diaphragm C is a node ? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **[IIT-1978]**



3. A steel wire 8×10^{-4} m in diameter is fixed to a support at one end and is wrapped round a cylindrical tuning peg 5 mm in diameter at the other end. The length of the wire between the peg and the support is 0.06 m. The wire is initially kept taut but without any tension. What will be the fundamental frequency of vibration of the wire if it is tightened by giving the peg a quarter of a turn?
Density of steel = 7800 kg/m^3, Y of steel = 20×10^{10} N/m^2.

4. A steel of wire of length 25 cm is fixed at its ends to rigid walls. Young's modulus of steel = 200 GPa, coefficient of linear thermal expansion = 10^{-5} / °C. Initially, the wire is just taut at 20°C temperature. The density of steel = 8.0 g/cc. A tuning fork of frequency 200 Hz is touched to the wire, to execute oscillations. Simultaneously, the temperature is slowly lowered. At what temperature will resonance occur corresponding to the third overtone?

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **2 Q. [4 M (0)]**

1. A super car is moving with velocity 150 m/sec towards a fixed wall producing a sound of frequency 75Hz. Speed of sound is 300 m/sec then find the wavelength (in m) of reflected sound.

2. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is _____

[JEE Advanced 2018]

SECTION-IV

Matrix Match Type (4 × 5) **3 Q. [8 M (for each entry +2(0)]**

1. In a string a standing wave is set up whose equation is given as $y = 2A \sin kx \cos \omega t$. The mass per unit length of the string is μ.

	Column-I		**Column-II**
(A)	at $t = 0$	(P)	Total energy per unit length at $x = 0$ is $2\mu A^2\omega^2$.
(B)	at $t = \frac{T}{8}$	(Q)	Total energy per unit length at $x = \lambda/4$ is $2\mu A^2\omega^2$.
(C)	at $t = \frac{T}{4}$	(R)	Total energy per unit length at $x = \lambda$ is $2\mu A^2\omega^2$.
(D)	at $t = \frac{T}{2}$	(S)	power transmitted through a point at $x = \lambda$ is 0.
		(T)	power transmitted through a point at $x = \lambda/4$ is 0.

2.

	Column I		**Column II**
(A)	Beats	(P)	Redistribution of energy
(B)	Standing waves	(Q)	Nearly equal frequencies
(C)	Interference	(R)	Varying amplitude
(D)	Echo	(S)	Reflection from a rigid support
		(T)	Multiple reflection

3. **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves. **[IIT-JEE 2011]**

Column I / **Column II**

(A) Pipe closed at one end

(P) Longitudinal waves

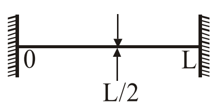
(B) Pipe open at both ends

(Q) Transverse Waves

(C) Stretched wire clamped at both ends

(R) $\lambda_f = L$

(D) Stretched wire clamped at both ends and at mid-point



(S) $\lambda_f = 2L$

(T) $\lambda_f = 4L$

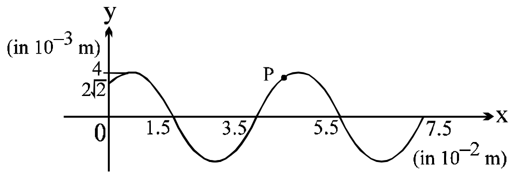
Subjective Type **14 Q. [4 M (0)]**

1. The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with x-axis at P is 6°.

(a) Find the direction in which the wave is moving

(b) the equation of the wave

(c) the total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm/m.



2. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. Ignore gravity. Find the time required for a transverse wave to travel from one end of the rope to the other.

3. A plane wave given by equation $y = 0.04 \sin(0.5\pi x - 100\pi t)$, where x and y are in meter and t in sec is incident normally on a boundary between two media beyond which wave speed becomes doubled. State boundary condition and find the equation of the reflected and transmitted waves. Take x = 0 as the boundary between two media.

4. A string between x = 0 and $x = l$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[IIT-JEE 2003(Scr)]**

5. A long string under tension of 100 N has one end at x = 0. A sinusoidal wave is generated at x = 0 whose equation is given by $y = (0.01\text{ cm}) \sin\left[\left(\frac{\pi x}{10}\text{ m}\right) - 50\pi t\,(\text{sec})\right]$

(i) Sketch the shape of the string at $t = \frac{1}{50}$ sec.

(ii) Find the average power transmitted by the wave.

(iii) Draw velocity time graph of particle at x = 5 m.

6. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **[JEE 2002]**

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

7. A, B and C are three tuning forks. Frequency of A is 350Hz. Beats produced by A and B are 5 per second and by B and C are 4 per second. When a wax is put on A beat frequency between A and B is 2Hz and between A and C is 6Hz. Then, find the frequency of B and C respectively.

8. A plane sound wave of frequency f_0 and wavelength λ_0 travels horizontally toward the right. It strikes and is reflected from a large, rigid, vertical plane surface, perpendicular to the direction of propagation of the wave and moving towards the left with a speed v.

(a) How many positive wave crests strike the surface in a time interval t ?

(b) At the end of this time interval, how far to the left of the surface is the wave that was reflected at the beginning of the time interval ?

(c) What is the wavelength of the reflected waves, in terms of λ_0 ?

(d) What is the frequency, in terms of f_0 ?

(e) A listener is at rest at the left of the moving surface. Describe the sensation of sound that he hears as a result of the combined effect of the incident and reflected wave trains.

9. A boat is travelling in a river with a speed of 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible. **[JEE 2001]**

(a) What will be the frequency detected by a receiver kept inside the river downstream ?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/sec in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C ; Density of river water = 10^3 Kg/m^3; Bulk modulus of the water = 2.088×10^9 Pa; Gas constant R = 8.31 J/mol-K; Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4)

Note: Boat velocity is with respect to ground & receiver is stationary w.r.t. ground

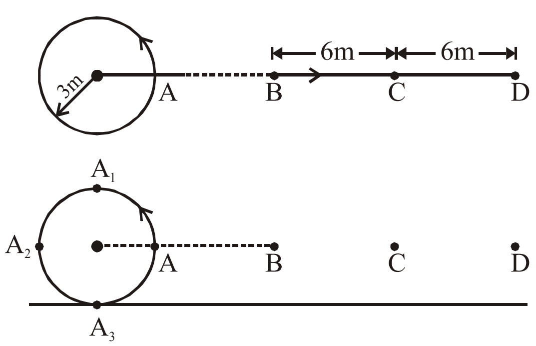
10. A train approaching a hill at speed of 40 km/hr sound a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find

(i) the frequency of the whistle as heard by an observer on the hill.

(ii) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.

(Velocity of sound in air = 1,200 km/hr) **[IIT-1988]**

11. A source of sound is moving along a circular orbit of radius 3 meteres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC = CD = 6 metres. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point A when the detector is at the point B. If the source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. **[IIT-1990]**



12. The displacement of the medium in a sound wave is given by the equation $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave. **[IIT-1991]**

(a) What are the wavelength and frequency of incident wave?

(b) Write the equation for the reflected wave.

(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

(d) Express the resultant wave as a superpositions of standing wave and a travelling wave. What are the positions of the antinodes of the standing wave? What is the directions of propagation of travelling wave?

13. The air colomn in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.

(a) Find the length L of the air column.

(b) What is the amplitude of pressure variation at the middle of the column?

(c) What are the maximum and minimum pressure at the open end of the pipe?

(d) What are the maximum and minimum pressure at the closed end of the pipe? **[IIT-1998]**

14. Sound of wavelength λ passes through a Q'uincke's tube, which is adjusted to give a maximum intensity I_0. Find the distance through the sliding tube should be moved to give an intensity $I_0/2$.

SECTION-I

Single Correct Answer Type **15 Q. [3 M (–1)]**

1. Ans. (A) **2. Ans. (B)** **3. Ans. (D)** **4. Ans. (B)**
5. Ans. (C) **6. Ans. (C)** **7. Ans. (A)** **8. Ans. (C)**
9. Ans. (A) **10. Ans. (A)** **11 Ans. (C)** **12. Ans. (A)**
13. Ans. (A) **14. Ans. (A)** **15. Ans. (A)**

Multiple Correct Answer Type **8 Q. [4 M (–1)]**

16. Ans. (A,C) **17. Ans. (A,C,D)** **18. Ans. (A,B,D)** **19. Ans. (A,C,D)**
20. Ans. (A,C,D) **21. Ans. (B,D)** **22. Ans. (B,D)** **23. Ans. (A, C, D)**

Linked Comprehension Type **(3 Para × 3Q. & 1 Para × 4 Q.) [3 M (-1)]**
(Single Correct Answer Type)

24. Ans. (C) **25. Ans. (A)** **26. Ans. (B)** **27. Ans. (A)**
28. Ans. (C) **29. Ans. (B)** **30. Ans. (D)** **31. Ans. (B)**
32. Ans. (C) **33. Ans. (A)** **34. Ans. (B)** **35. Ans. (C)**
36. Ans. (D)

Matching List Type (4 × 4) **1 Q. [3 M (–1)]**

37. Ans. (1)

SECTION-II

Numerical Answer Type Question **4 Q. [3(0)]**
(upto second decimal place)

1. Ans. 300 Hz **2. Ans.** 1650 Hz **3. Ans.** 10800 Hz **4. Ans.** 17.5°

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **2 Q. [4 M (0)]**

1. Ans. 2 **2. Ans. 5 [4.99, 5.01]**

SECTION-IV

Matrix Match Type (4 × 5) **3 Q. [8 M (for each entry +2(0)]**

1. Ans. (A)–(P,R,S,T) ; (B)–(S,T) ; (C)–(Q,S,T) ; (D)–(P,R,S,T)
2. Ans. (A) → (Q,R) ; (B) → (R,S) ; (C) → (P) ; (D) → (T)
3. Ans. (A) → P,T; (B) → P,S; (C) → Q,S; (D) → Q,R

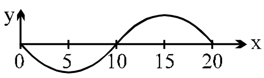
Subjective Type **14 Q. [4 M (0)]**

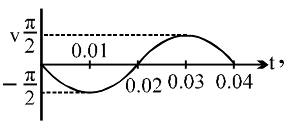
1. Ans. (a) negative x ; (b) $y = 4\times10^{-3} \sin 100\pi\left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2 \times 10^{-5}$ J

2. Ans. $\frac{\pi}{\sqrt{2}\omega}$

3. Ans. $A_t = \frac{4}{3}A_i$, $A_r = \frac{1}{3}A_i$, $y_r = -\frac{0.04}{3}\sin(0.5\,\pi x + 100\,\pi t)$; $y_t = +\frac{0.16}{3}\sin(0.25\pi x - 100\,\pi t)$

4. Ans. $E = \frac{A^2\pi^2 T}{4l}$

5. Ans. (i) 

(ii) 25×10^{-6} W (iii) 

, solved

6. Ans. (a) 2.116, (b) $\frac{3}{4}$ **7. Ans.** 345, 341 or 349 Hz

8. Ans. (a) $\left(\frac{v+\lambda_0 f_0}{\lambda_0}\right)t$ (b) $(\lambda_0 f_0 - v)t$ (c) $\lambda_0\left(\frac{\lambda_0 f_0 - v}{\lambda_0 f_0 + v}\right)$ (d) $f_0\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0 - v}\right)$ (e) $\frac{2vf_0}{\lambda_0 f_0 - v}$

9. Ans. (a) 100696 Hz (b) 103038 Hz

10. Ans. (i) 599 Hz, (ii) 0.935 km, 620 Hz

11. Ans. 438.7 Hz, 257.3 Hz

12. Ans. (a) $\frac{2\pi}{a}, \frac{b}{2\pi}$, (b) y= –0.8 Acos(ax – bt) **OR** y = 0.8 Acos(ax – bt), (c) 1.8 Ab, 0

(d) $y = -1.6\ A \sin ax \sin bt + 0.2\ A \cos(ax + bt)$, $\left[n + \frac{(-1)^2}{2}\right]\frac{\pi}{a}$, –X direction **OR**

$y = 0.2\ A \cos(ax + bt) + 1.6\ A \cos ax \cos bt$, $x = \frac{n\pi}{a}$, –X direction

13. Ans. (a) $\frac{15}{16}$m (b) $\frac{\Delta P_0}{\sqrt{2}}$ (c) equal to mean pressure (d) $P_0 + \Delta P_0$, $P_0 - \Delta P_0$

14. Ans. $\lambda/8$

GUIDED REVISION

SOLUTIONS
SECTION-I

Single Correct Answer Type **15 Q. [3 M (–1)]**

1. Ans. (A)

Sol. $y = \frac{1}{\sqrt{1+x^2}}$ at t = 0

$y = \frac{1}{\sqrt{x^2 - 2x + 2}}$ at t = 1

$= \frac{1}{\sqrt{(x^2-1)^2 + 1}}$

$\because y = \frac{1}{\sqrt{1+(x-t)^2}}$

v = 1 m/s towards the x-axis.

2. Ans. (B)

Sol. $y = A\sin(\omega t - kx)$

At x = 4cm, t = T/6, y = A/2

$\frac{A}{2} = A\sin\left[\frac{2\pi}{T}\left(\frac{T}{6}\right) - K\left(\frac{4}{100}\right)\right]$

$\sin\left[\frac{\pi}{3} - K\left(\frac{4}{100}\right)\right] = \frac{1}{2}$

$\frac{\pi}{3} - \frac{4k}{100} = \frac{\pi}{6}$

$\frac{4k}{100} = \frac{\pi}{6}$

$k = \frac{2\pi}{\lambda} = \frac{100\pi}{24}$

$\lambda = \frac{48}{100}\text{ m} = 48\text{ cm}$

3. Ans. (D)

Sol. Wave moving in +ve x direction

At t = 0

At t = 0, x = 0, y = 0

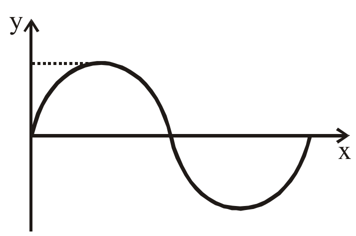
$\phi = 0$

$y = A\sin(kx - \omega t)$

Wave moving in –ve x direction

$y = A\sin(\omega t + kx + \phi)$

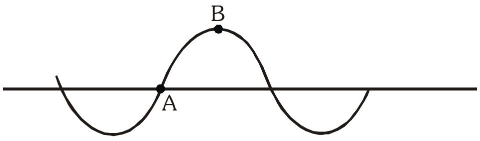
at t = 0, x = 0, y = 0



$\phi = 0$
$y = 0\ A \sin(\omega t + kx)$

4. **Ans. (B)**

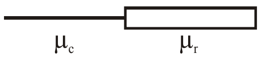
Sol.



Both KE and PE are maximum at y = 0 and minimum at y = A.
V = 0 at point B so KE = 0.
Slope zero at B so PE = 0.

5. **Ans. (C)**

Sol.



A = 1
A_t – Amplitue of transmitted wave.

$$A_t = \left[\frac{2\sqrt{\mu_\ell}}{\sqrt{\mu_\ell} + \sqrt{\mu_r}}\right]$$

6. **Ans. (C)**

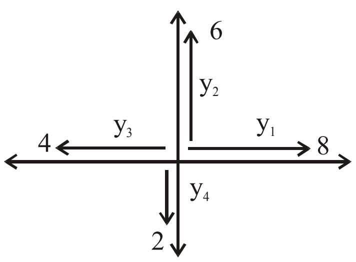
Sol. Compare with

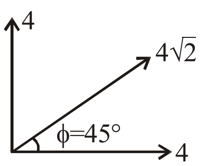
$$y = A\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$\lambda = vt$ & $v = \sqrt{\frac{F}{\mu}}$

7. **Ans. (A)**

Sol. $y_1 = 8\sin(\omega t + kx)$
$y_2 = 6\sin(\omega t + kx + \pi/2)$
$y_3 = 4\sin(\omega t + kx + \pi)$
$y_4 = 2\sin(\omega t + kx + 3\pi/2)$



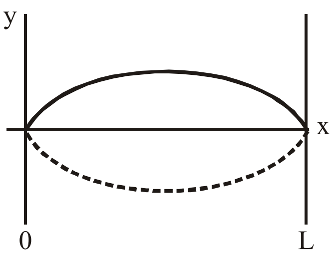


$y_5 = 4\sqrt{2}\sin(\omega t - kx + \pi/4)$
$y_1 + y_2 + y_3 + y_4$
$y' = 4\sqrt{2}\sin(\omega t + kx + \pi/4)$
$y_{net} = y' + y_5$
$= 4\sqrt{2}\ (\sin(\omega t - kx + \pi/4) + \sin(\omega t + kx + \pi/4)$
$= 4\sqrt{2}\ [2\sin(\omega t + \pi/4)\cos kx]$

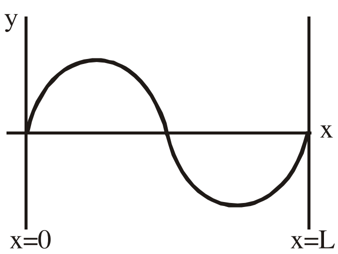
$$= 8\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)\cos kx$$

8. **Ans. (C)**

Sol. $y_1 = A\sin\left[\frac{\pi x}{L}\right]\sin\omega t$ $y_2 = A\sin\left[\frac{2\pi x}{L}\right]\sin(2\omega t)$



$\lambda = 2L$



$\lambda = L$

Energy per unit loop = $\frac{1}{8}\mu(\omega^2 A^2)\lambda$

$$E_1 = 1\times\left[\frac{1}{8}\mu\omega^2 A^2(2L)\right]$$

$$E_2 = 2\times\left[\frac{1}{8}\mu(2\omega)^2 A^2(L)\right]$$

$E_2 = 4E_1$

9. **Ans. (A)**

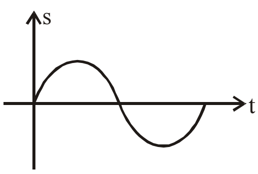
Sol. $y_1 = A\sin(\omega t - kx)$

$y_2 = A\sin(\omega t + kx)$

$y_R = y_1 + y_2 = A(\sin(\omega t - kx) + \sin(\omega t + kx))$

$y_R = 2A\cos(kx)\sin(\omega t)$

10. **Ans. (A)**

Sol. 

$$B = -\frac{dp}{\frac{dv}{v}}$$

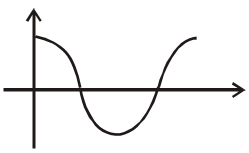
$$dp = -B\frac{dv}{v}$$

$s = A\sin(\omega t - kx)$

$$\frac{ds}{dx} = A(-k)\cos(\omega t - kx)$$

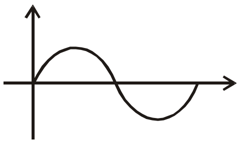
$dp = d\rho = AkB\cos(\omega t - kx)$

at $t = 0$



$t = \frac{T}{4}$

$t = 0$

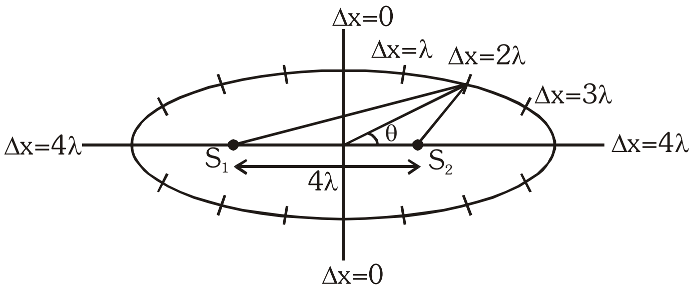


11 Ans. (C)

Sol. $\frac{(I_{coh})_{max}}{(I_{incoh})_{min}} = \frac{n^2 I_0}{nI_0} = n$

12. Ans. (A)

Sol.



$d\cos\theta = n\lambda$

$4\lambda\cos\theta = n\lambda$

$\cos\theta = \frac{n}{4}$

13. Ans. (A)

Sol. $f' = f\left(\frac{v}{v - v_s}\right)$

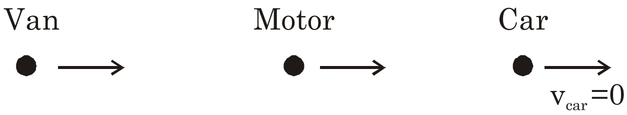
$\frac{v}{\lambda'} = \frac{v}{\lambda}\left(\frac{v}{v - v_s}\right)$

$\lambda\left(\frac{v - v_s}{v}\right) = \lambda'$

$\lambda' = \lambda\left(1 - \frac{v_s}{v}\right)$

14. Ans. (A)

Sol.



$176\left(\frac{330 - v_m}{330 - 22}\right) = 165\left(\frac{330 + v_m}{330}\right)$

$v_m = 22$

15. Ans. (A)

Sol. Doppler effect in light (speed of observer is not very small compare to speed of light)

$$f^1 = \sqrt{\frac{1+V/C}{1-V/C}}\, f_{source} = \sqrt{\frac{1+1/2}{1-1/2}}(10\text{ GHz}) = 17.3\text{ GHz}$$

$$f' = f_{max}\sqrt{\frac{1+V/C}{1-V/C}}$$

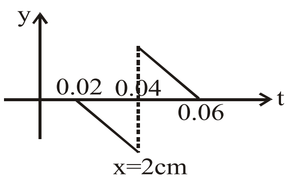
Multiple Correct Answer Type 8 Q. [4 M (–1)]

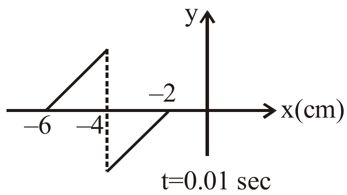
16. Ans. (A,C)

Sol. For sound, air is denser medium than water.

17. Ans. (A,C,D)

Sol.



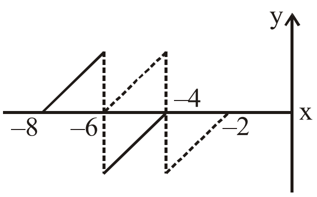


Disturbance at x = –4 at t = 0.01sec and disturbance reaches x = 2 at t = 0.04 sec hence wave is travelling in positive x-direction.

$$v_w = \left(\frac{6\text{cm}}{0.03\,\text{sec}}\right)$$

= 200 cm/sec = 2m/sec

At t = 0 sec



18. Ans. (A,B,D)

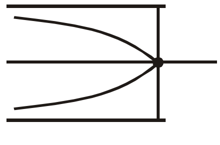
Sol. PE and KE both maximum at y = 0. [Simultaneously for two particles in phase]

Amplitude and frequency same for all particles.

There will be phase diffrence in motion of all particles.

19. Ans. (A,C,D)

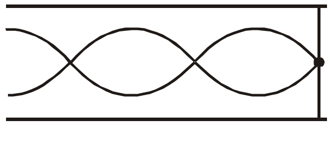
Sol. (A)



$$\frac{\lambda}{4} = 3$$

$\lambda = 12$

(C)



$$\frac{5\lambda}{4} = 3$$

$$\lambda = \frac{12}{5}$$

20. Ans. (A,C,D)

Sol. $V \propto \sqrt{T}$

$$60dB = 10 \log \frac{I_1}{I_0} \text{ and } 30 \text{ dB} = 10 \log \frac{I_1}{I_0}$$

$$V_P = -V_W \frac{dy}{dx}$$

$$V_{sound} = \sqrt{\frac{\gamma RT}{M}}$$

21. Ans. (B,D)

Sol. v_C it blows a horn of frequency f.

$$v - v_C = f\lambda' \qquad \lambda' = \frac{v - v_C}{f}$$

$$f_1 = f\left(\frac{v + v_0}{v}\right) \qquad f_2 = f_1\left(\frac{v}{v - v_0}\right)$$

$$f_2 = f_1\left(\frac{v + v_0}{v - v_0}\right)$$

$$\Delta f = f_2 - f_1 = f_0\left(\frac{v + v_0}{v - v_0}\right) - f$$

$$= f_0\left(\frac{v + v_0 - v + v_0}{v - v_0}\right)$$

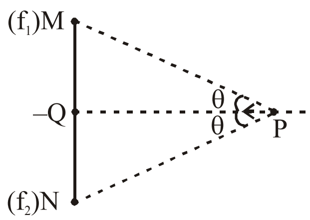
$$\Delta f = f_0\left(\frac{2v_0}{v - v_0}\right)\left(\frac{v + v_0}{v + v_0}\right) = f_0\left(\frac{2v_0(v + v_0)}{v^2 - v_0^2}\right)$$

22. Ans. (B,D)

Sol. From open end compression will be reflected as refraction and from closed end these is no phase change so compression will be reflected back as compression.

23. Ans. (A, C, D)

Sol. $f_M = \frac{C + V\cos\theta}{C} f_1$



$$f_N = \frac{C + V\cos\theta}{C} f_2$$

$$\Delta f = f_N - f_M$$

$$= \frac{C + V\cos\theta}{C}(f_2 - f_1)$$

$$\frac{d(\Delta f)}{dt} = -\frac{V}{C}(f_2 - f_1)\sin\theta\frac{d\theta}{dt}$$

$\therefore$ & $\frac{d(\Delta f)}{dt}$ is maximum when $\theta = 90°$

[$\therefore$ C is correct]

$\nu_P = \left(1+\frac{V}{C}\cos\theta\right)\Delta f$

$\nu_Q = \Delta f$

$\nu_R = \left(1-\frac{V}{C}\cos\theta\right)\Delta f$

$\therefore$ $\nu_P + \nu_r = 2\nu_Q$

Linked Comprehension Type (3 Para × 3Q. & 1 Para × 4 Q.) [3 M (-1)]

(Single Correct Answer Type)

24. Ans. (C)

25. Ans. (A)

26. Ans. (B)

Sol. (Q. No. 24 to 26)

$y_1 = A \sin[\omega t - kx]$

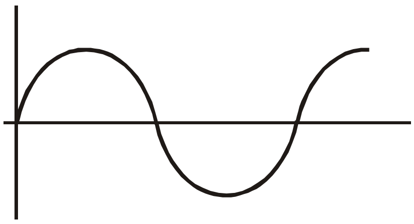
$y_1 = A \sin(\omega t + kx + \pi/3]$

$A = 2mm$, $w = 4\pi$, $k = 2\pi$

$y_{Res.} = 2A\sin\left(\omega t+\frac{\pi}{6}\right)\cos\left(kx+\frac{\pi}{6}\right)$

$= 2A\cos\left(kx+\frac{\pi}{6}\right)\sin\left(\omega t+\frac{\pi}{6}\right)$

$x_1 = \frac{1}{7}$, $x_2 = \frac{5}{12}$



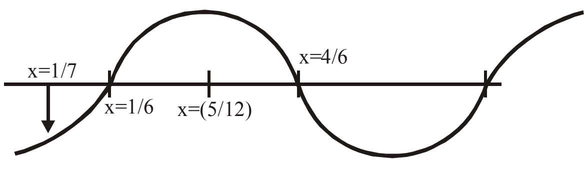
Nodes : y = 0

$\cos\left(kx+\frac{\pi}{6}\right) = 0$

$kx+\frac{\pi}{6} = (2n+1)\frac{\pi}{2}$

$x_0 = \frac{\pi}{3k} = \frac{\pi}{3\times 2\pi} = \frac{1}{6}$

$x_1 = \frac{4\pi}{3k} = \frac{4k}{3\times 2\pi} = \frac{4}{6}$



$\Delta\phi = \pi$

Antinodes

$$\cos\left(kx+\frac{\pi}{6}\right)=1$$

$$kx=\frac{\pi}{6}=n\pi$$

$$x=\left(n-\frac{1}{6}\right)\frac{\pi}{k}$$

$$x=\left(n-\frac{1}{6}\right)\frac{\pi}{k}=\frac{1}{2}\left(n-\frac{1}{6}\right)$$

$$x=\frac{5}{12},\frac{11}{12},\frac{17}{12},\frac{23}{12}.....$$

Location of max. potetnial energy, are nodes

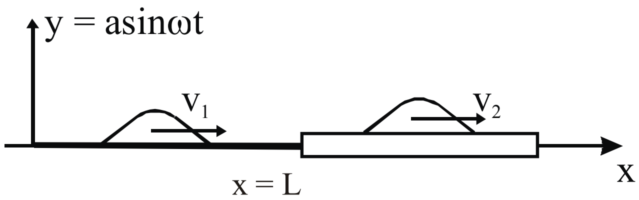
$$x=\frac{1}{6}$$

27. Ans. (A)

28. Ans. (C)

29. Ans. (B)

Sol. (Q. No. 27 to 29)



$$Y_i=a\sin\omega\left(t-\frac{x}{v_1}\right)$$

Transmitted at x = L

$$Y_{(x=L)}=a\sin\omega\left(t-\frac{L}{v_1}\right)$$

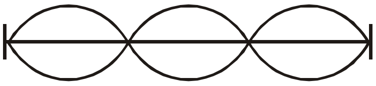
$$Y_t=a_t\sin\omega\left[t-\frac{L}{v_1}-\left(\frac{x-\ell}{v_2}\right)\right]$$

Reflected at x = L

$$y_{x=2L}=a\sin\omega\left(t-\frac{\ell}{v_1}\right)$$

$$y_{Reff}=a_r\sin\omega\left[t-\frac{\ell}{v_1}+\left(\frac{x-\ell}{v_1}\right)+\pi\right]$$

30. Ans. (D)

Sol. 

A = 3 cm
f = 100 Hz

$$2m = \frac{3\lambda}{2}$$

$$\lambda = \frac{4}{3}m$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \times 3}{4} = \frac{3\pi}{2}$$

31. Ans. (B)

Sol. $E = (\text{No of loop's}) \times \left(\frac{1}{8}\omega^2 A_S^2 \lambda\right)$

32. Ans. (C)

Sol. $V = f \times \lambda$ $\qquad \lambda = \frac{4}{3}$

$$\lambda = 100 \times \frac{4}{3} = \frac{400}{3}$$

$$\text{time} = \frac{\lambda}{4V} = \frac{4}{4 \times 3} \times \frac{3}{400} = \frac{1}{400}$$

33. Ans. (A)

Sol. $f_A = f\left(\frac{v}{v - v_s}\right)$

34. Ans. (B)

Sol. $f_B = f\left(\frac{v}{v + v_s}\right)$

35. Ans. (C)

Sol. $f_C = f$

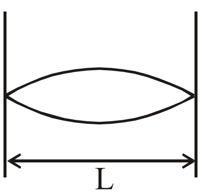
36. Ans. (D)

Sol. $f_D = f\left(\frac{v}{v + v_s \cos\theta}\right)$

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

37. Ans. (1)

Sol. For fundamental mode



$$\frac{\lambda}{2} = L \quad \Rightarrow \lambda = 2L$$

$$f = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (1)

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \Rightarrow (P)$$

For string (2)

$$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_0}{\sqrt{2}} \Rightarrow (R)$$

For string (3)

$$f = \frac{1}{2L}\sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \Rightarrow (S)$$

For string (4)

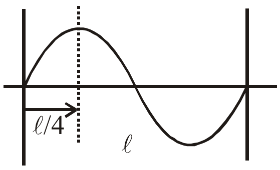
$$f = \frac{1}{2L}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \Rightarrow (Q)$$

SECTION-II

Numerical Answer Type Question **4 Q. [3(0)]**
(upto second decimal place)

1. **Ans.** 300 Hz

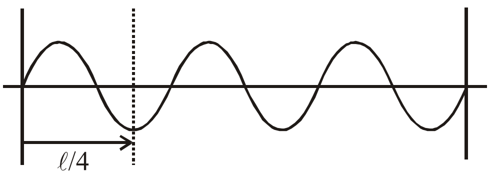
Sol. $\frac{\ell}{4} = \frac{\lambda}{4}$



$\lambda = \ell$

f = 100

$100 = \frac{v}{\lambda}$



$f' = \frac{v}{\lambda'}$ $\qquad \frac{\ell}{4} = \frac{3\lambda'}{4}$

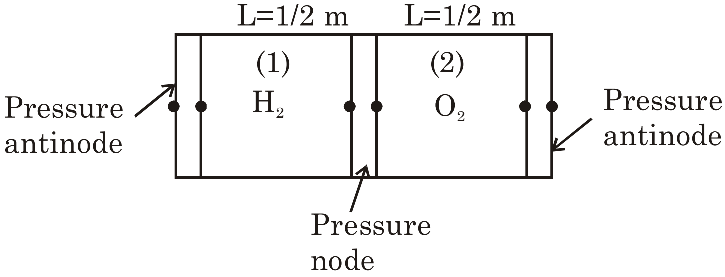
$f' = \frac{v}{(\lambda/3)}$ $\qquad \lambda' = \frac{\ell}{3}$

$f' = 3f$ $\qquad \lambda' = \frac{\lambda}{3}$

f' = 300 Hz

2. **Ans. 1650 Hz**

Sol.



$$n_1 \times \frac{v_1}{4L} = n_2 \times \frac{v_2}{4L}$$

$$\frac{n_1}{n_2} = \frac{300}{1100} = \frac{3}{11}$$

$$f_1 = \frac{3 \times 1200}{4 \times \frac{1}{2}} = 1650\text{Hz}$$

3. **Ans.** 10800 Hz

Sol. $\ell = 2\pi r = \pi d$

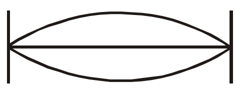
$\ell = \pi d = 3.14 \times 8 \times 10^{-4}$ m $= 24.42 \times 10^{-4}$ m

$$\Delta\ell = \frac{24 \cdot 42 \times 10^{-4}\,\text{m}}{4} = 6 \times 10^{-4}\,\text{m}$$

$$\Delta\ell = \frac{T\ell}{AY} \rightarrow T = \left(AY\frac{\Delta\ell}{\ell}\right)$$

$V = f \times \lambda$

$$V = \sqrt{\frac{T}{\mu}} = f \times \lambda$$



$$\frac{AY}{\mu}\frac{\Delta\ell}{\ell} = f \times \lambda$$

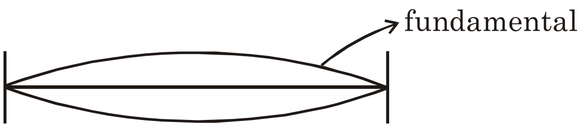
$$\frac{\lambda}{2} = L$$

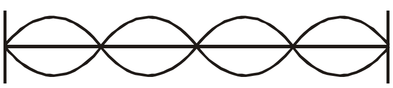
$$\frac{AY\Delta\ell}{A\rho\ell} = f \times \lambda$$

$\lambda = 2L$

4. **Ans.** 17.5°

Sol.





$$f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$f^2 = \frac{4}{\ell^2} \times \frac{Y\alpha\Delta\theta A}{\rho A}$$

$$f^2 = \frac{4}{\ell^2} \times \frac{Y\alpha\Delta\theta}{\rho}$$

$$\Delta\theta = \frac{f^2\ell^2\rho}{4Y\alpha}$$

$\Delta\theta = 2.5°C$

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 2 Q. [4 M (0)]

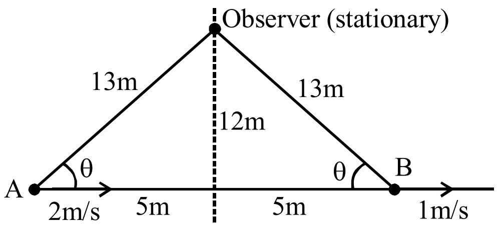
1. Ans. 2

Sol. $f_{reflected} = \frac{(300-150)}{150} \times 75Hz$

$\therefore \lambda_{reflected} = 2$ m

2. Ans. 5 [4.99, 5.01]

Sol.



$\cos\theta = \frac{5}{13}$

$$f_A = 1430\left[\frac{330}{330-2\cos\theta}\right] = 1430\left[\frac{1}{1-\frac{2\cos\theta}{330}}\right] = 1430\left[1+\frac{2\cos\theta}{330}\right]$$ (By bionomial expansion)

$$f_B = 1430\left[\frac{330}{330+1\cos\theta}\right] = 1430\left[1-\frac{\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

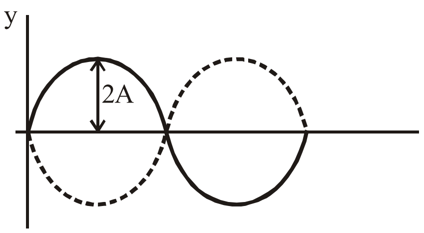
$$= 13\left(\frac{5}{13}\right) = 5.00Hz$$

SECTION-IV

Matrix Match Type (4 × 5) 3 Q. [8 M (for each entry +2(0)]

1. Ans. (A)–(P,R,S,T) ; (B)–(S,T) ; (C)–(Q,S,T) ; (D)–(P,R,S,T)

Sol.



$y = 2A \sin kx \cos \omega t$

(A) At $t = 0$

$y = 2A \sin kx$

Energy per unit length = $\frac{1}{2}\mu((2A)\omega)^2$

$= 2\mu A^2\omega^2$

Power transmission is zero at nodes and antinodes because energy is trapped between nodes and antinodes n standing waves

So, at $x = \frac{\lambda}{4}$ $P_t = 0$ (Antinode)

$x = \lambda$ $P_t = 0$ (Node)

(B) at $t = \frac{T}{8}$

$y = 2A \text{ ssin } kx \cos\left(\frac{\pi}{4}\right)$

$= \left(\sqrt{2}A\right)\sin kx$

$E = \frac{1}{2}\mu\left(\sqrt{2}A\omega\right)^2$

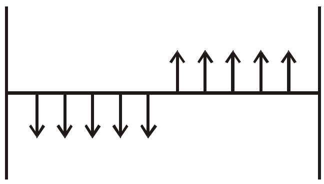
$= u\ A^2\omega^2$

At $x = \lambda$(Node), $\lambda/4$ (Antinode) Power transmitted = 0.

(C) At t = T/4

$y = 2A \sin kx \cos(\pi/2)$

$y = 0$



$dk = \frac{1}{2}(udx)v_p^2$

$\frac{dk}{dx} = \frac{1}{2}u\left(\frac{dy}{dt}\right)^2 = \frac{1}{2}u(2A\omega)^2 (\sin kx \sin \omega t)^2$

$= 2\ uA^2\omega^2\ (\sin^2 kx)\sin^2\omega t$

At x = 0 $\frac{dk}{dx} = 0$

At x = 0 $2\mu A^2\omega^2$

(D) Same as (A)

2. Ans. (A) → (Q,R) ; (B) → (R,S) ; (C) → (P) ; (D) → (T)

Sol. (A) (1) Nearly same frequency

(2) Varying amplitude

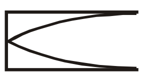
(B) Varying amplitude/reflection from a liquid support

(C) Interference Redistribution of energy

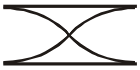
(D) Echo (sound after reflection from fixed support)

3. Ans. (A) → P,T; (B) → P,S; (C) → Q,S; (D) → Q,R

Sol. For (A) : Sound wave is longitudinal wave

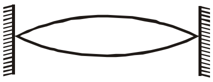
 $\frac{\lambda_F}{4} = L \Rightarrow \lambda_F = 4L$

For (B) : Sound wave is longitudinal wave

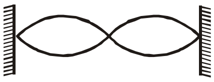


$\frac{\lambda_F}{2} = L \Rightarrow \lambda_F = 2L$

For (C) : String wave is transverse



$\frac{\lambda_F}{2} = L \Rightarrow \lambda_F = 2L$

For (D) :  $\lambda_F = L \Rightarrow$ so (q & r)

Subjective Type — 14 Q. [4 M (0)]

1. **Ans.** (a) negative x ; (b) $y = 4\times10^{-3} \sin 100\pi\left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2 \times 10^{-5}$ J

Sol. (a) $v_P = -v_\omega \times$ slope

at P, $v_P \rightarrow +ve$

slope $\rightarrow +ve$

so, $v_\omega = -ve$ [wave travelling towards left]

(b) $y = 4 \sin (kx + \frac{\pi}{4})$

$\lambda = 4 \times 10^{-2}$ m $\qquad k = \frac{2\pi}{4} \times 100 = 50\pi$

$v_P = -v_\omega \times$ slope

$\left(\frac{20\pi}{100}\right)$ m/s $= -v_\omega \tan 6°$

$v_\omega = \frac{20\pi}{100} \times \frac{180}{6\pi} = 6$ m/s.

$y = 4 \times 10^{-3} \sin (50x + 300\pi t + \frac{\pi}{4})$

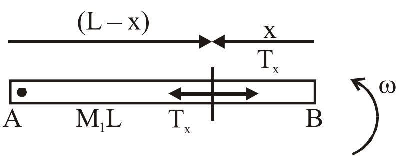
$P = \frac{1}{2}\mu A^2\omega^2 v$

$= \frac{1}{2} 50 \times 10^{-3} \, (4 \times 10^{-3})^2 (300\,\pi)^2 \times 6$

$= 144\pi^2 \times 10^{-5}$

2. **Ans.** $\frac{\pi}{\sqrt{2}\omega}$

Sol.



$T_x = \left(\frac{M}{L}\right) x \, \omega^2 \left[L - x + \frac{x}{2}\right]$

$$T_x = \left(\frac{M}{L}\omega^2\right)x\left[L - \frac{x}{2}\right]$$

$$v_x = \sqrt{\frac{T_x}{(M/L)}} = \omega\sqrt{x\left(L - \frac{x}{2}\right)}$$

$$v^2 = \omega^2 x\left(L - \frac{x}{2}\right)$$

$$2v\frac{dv}{dx} = \omega^2 L - \omega^2 x$$

$$a = \frac{1}{2}\omega^2 (L - x) \qquad a \alpha - x$$

Angular frequency $\Rightarrow \sqrt{\frac{\omega^2}{2}}$

$$T = \frac{2\pi}{\omega}\left(\sqrt{2}\right) \Rightarrow \frac{2\sqrt{2}\pi}{\omega}$$

Time from B $\rightarrow$ A = $\frac{T}{4} = \frac{\pi}{\sqrt{2}\omega}$

3. **Ans.** $A_t = \frac{4}{3}A_i$, $A_r = \frac{1}{3}A_i$, $y_r = -\frac{0.04}{3}$ sin(0.5 πx + 100 πt) ; $y_t = +\frac{0.16}{3}$ sin(0.25πx – 100 πt)

Sol. y = 0.04 sin (0.5 πx – 100πt)

v → double

μ → one-fourth (T → same)

$A_i = 0.04$

$$A_r = \left(\frac{\sqrt{\mu_1} - \sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}}\right)A_i = \frac{0.04}{3}$$

$$A_t = \left(\frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}}\right)A_i = 4\left(\frac{0.04}{3}\right)$$

Transmittet wave → same phase

Reflected wave → (Direction charge)

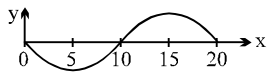
π phase difference

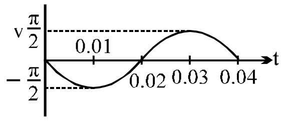
$$Y_R = \left(\frac{0.04}{3}\right) \sin(0.5\,\pi x + 100\pi t + \pi)$$

$$Y_t = \frac{4 \times 0.04}{3} \sin(0.5\,\pi x - 100\pi t)$$

4. **Ans.** $E = \dfrac{A^2\pi^2 T}{4l}$

Sol. $E = \left(\text{no of loop's}\left(\dfrac{1}{2}\omega^2 A s^2\right)\dfrac{\lambda}{4}\right)$

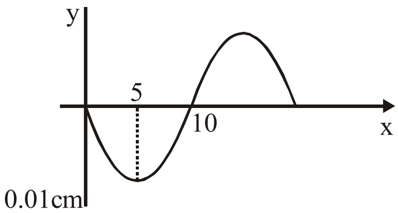
5. **Ans.** (i) 

(ii) 25×10^{-6} W (iii) 

, solved

Sol. At $t = \dfrac{1}{50}$ sec

$y = (0.01 \text{ cm}) \sin\left[\dfrac{\pi x}{10} - \pi\right] = -0.01 \sin\left[\dfrac{\pi x}{10}\right]$



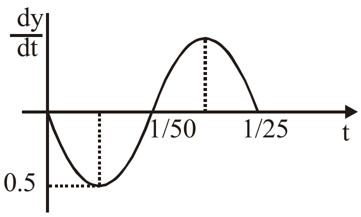
$\langle P \rangle = \dfrac{1}{2}\mu v A^2\omega^2 = \dfrac{1}{2}\mu v A^2 v^2 k^2$

$= 25 \times 10^{-6}\ \omega$

y at (x = 5) $\qquad y = 0.01 \sin\left(\dfrac{\pi}{2} - 50\pi t\right)$

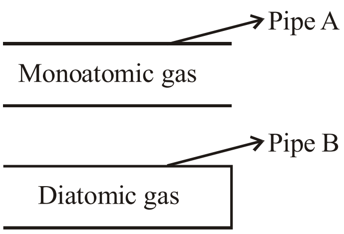
$= 0.01 \cos(50\pi t)$

$\dfrac{dy}{dt} = -0.5\ \pi \sin(50\ \pi t)$



6. **Ans.** (a) 2.116, (b) $\dfrac{3}{4}$

Sol.



(a) $F_1 = \frac{2}{2L} \times \sqrt{\frac{5/3RT}{M_A}}$

$F_2 = \frac{3}{4L}\sqrt{\frac{7/5RT}{M_B}}$

$F_1 = F_2$ (given)

$\frac{1}{L} \times \sqrt{\frac{5RT}{3M_A}} = \frac{3}{4L}\sqrt{\frac{7RT}{5M_B}}$

$\frac{5}{3M_A} = \frac{9}{16} \times \frac{7}{5M_B}$

$\frac{M_A}{M_B} = \frac{5 \times 5 \times 16}{3 \times 9 \times 7} = \frac{400}{189} = 2.116$

(b) If pipe B is closed

$F_B = \frac{1}{2L}\sqrt{\frac{7RT}{5M_B}}$

$F_A = \frac{1}{2L}\sqrt{\frac{5RT}{3M_A}}$

$\frac{F_A}{F_B} = \sqrt{\frac{5}{3} \times \frac{5}{7} \times \frac{M_B}{M_A}}$

$= \sqrt{\frac{25}{27} \times \frac{189}{400}}$

$= \frac{3}{4}$

7. **Ans.** 345, 341 or 349 Hz

Sol. $|f_A - f_B| = \pm 5$ | $|f_B - f_C| = \pm 4$

$350 - f_B = \pm 5$ | $f_B - f_C = \pm 4$

$f_B = 350 + 5 = 355$ | $345 - f_C = \pm 4$

$= 350 - 5 = 345$ | $f_C = 349$

$= 341$

8. **Ans.** (a) $\left(\frac{v + \lambda_0 f_0}{\lambda_0}\right)t$ (b) $(\lambda_0 f_0 - v)t$ (c) $\lambda_0\left(\frac{\lambda_0 f_0 - v}{\lambda_0 f_0 + v}\right)$ (d) $f_0\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0 - v}\right)$ (e) $\frac{2vf_0}{\lambda_0 f_0 - v}$

Sol. (a) $\frac{(\lambda_0 f_0 + v)t}{\lambda_0}$ = No of crest's = $\frac{\text{Length of wave travelled}}{\text{wave length}}$

(b) $x = \lambda_0 f_0 t - vt$

$x = (\lambda_0 f_0 - v)t$

(c) $\lambda_{\text{reflected}} = \lambda_0\left(\frac{\lambda_0 f_0 - v}{\lambda_0 f_0 + v}\right)$

(d) $f' = f_0\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0}\right)$

$$f'' = f'\left(\frac{\lambda_0 f_0}{\lambda_0 f_0 - v}\right)$$

$$f'' = f_0\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0 - v}\right)$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0 - v}\right)$$

(e) Beats $\Delta f = \frac{2vf_0}{\lambda_0 f_0 - v}$

9. Ans. (a) 100696 Hz (b) 103038 Hz

Sol. Velocity of sound in water is

(1) $v_{water} = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.088\times10^9}{10^3}}$ = 1445 m/s

Frequency of sound in water will be $f_0 = \frac{v_{water}}{\lambda_{water}} = \frac{1445}{1445\times10^{-3}}$

$f_0 = 10^5$ Hz

(2) $v_{Boat} = 10$m/s

(a) $f_1 = f_0\left(\frac{v_{water} + v_{river}}{v_{water} + v_{river} - v_{boat}}\right)$

$$= \left(10^5\right)\left(\frac{1445+2}{1445+2-10}\right)$$

$= 1.0069 \times 10^5$ Hz

(b) $v_A = \sqrt{\frac{\gamma RT}{M}}$

$v_A = \sqrt{\frac{1.4\times8.31\times(293)}{28.8\times10^{-3}}}$ = 344 m/s

$$f = 10^5\left(\frac{344-5}{344-5-10}\right)$$

$f = 1.0304 \times 10^5$ Hz

10. Ans. (i) 599 Hz, (ii) 0.935 km, 620 Hz

Sol. V_{Train} = 40 km/hr

f = 580 Hz

$$f_0 = f\left(\frac{v + v_{medium} + v_0}{v + v_{medium}}\right)$$

(i) $f_0 = f\left(\frac{1200+40+40}{1200+40}\right)$

$= 580\left(\frac{1280}{1240}\right) = 580\left(\frac{128}{124}\right)$

$f_0 = (580)\left(\frac{32}{31}\right)$

(ii) $\frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$

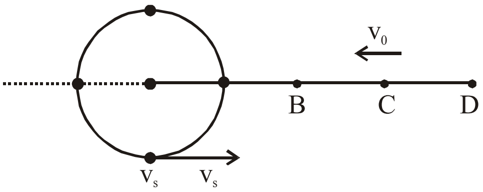
x = 0.935 km

$f'' = 580\left(\frac{1200-40+40}{1200-40-40}\right)$

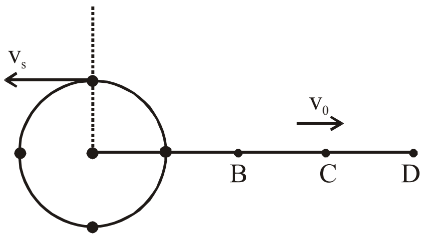
f" = 621.43 Hz

11. **Ans.** 438.7 Hz, 257.3 Hz

Sol.



$f_{max} = f\left(\frac{v+v_0}{v-v_s}\right) = 340\left(\frac{340+60}{340-30}\right) = 4387\,\text{Hz}$



$f_{min} = 340\left(\frac{340-60}{340+30}\right) = 257.3\,\text{Hz}$

12. **Ans.** (a) $\frac{2\pi}{a}, \frac{b}{2\pi}$, (b) y= –0.8 Acos(ax – bt) **OR** y = 0.8 Acos(ax – bt), (c) 1.8 Ab, 0

(d) y = –1.6 A sin ax sin bt + 0.2 A cos(ax + bt), $\left[n + \frac{(-1)^2}{2}\right]\frac{\pi}{a}$, –X direction **OR**

y = 0.2 A cos (ax + bt) + 1.6 A cos ax cos bt, $x = \frac{n\pi}{a}$, –X direction

Sol. y_1 = Acos (ax + bt)

$k = a = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{a}$

$$\omega = \frac{2\pi}{T} = b \Rightarrow T = \frac{2\pi}{b}$$

$I \propto A^2$

$$\frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 \Rightarrow A_r = 0.8A_i$$

x = 0 is a fixed end.

Hence there will phase difference of π

y_1 = A cos (bt) equation of particle

y_r = 0.8 A cos (bt + π)

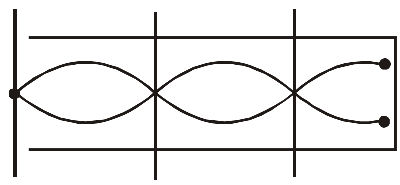
equation of reflected wave

$$= 0.8\ A\ \cos\left[b\left(t - \frac{x}{v}\right) + \pi\right]$$

= 0.8 A cos [bt – ax + π]

13. **Ans.** (a) $\frac{15}{16}$m (b) $\frac{\Delta P_0}{\sqrt{2}}$ (c) equal to mean pressure (d) $P_0 + \Delta P_0, P_0 - \Delta P_0$

Sol. Part (a) $L = \frac{5\lambda}{4} = \frac{5}{4}\left(\frac{330}{440}\right) = \frac{5}{4} \times \frac{3}{4} = \frac{15}{16}$m



Part (b) $\Delta P = \Delta P_0 \cos(-kx)$

$$\Delta P = \Delta P_0 \cos\left(\frac{2\pi}{3} \times 4 \times \frac{15}{32}\right) = \Delta P_0 \cos\left(\frac{5\pi}{4}\right)$$

$$\Delta P = \frac{\Delta P_0}{\sqrt{2}}$$

14. **Ans.** $\lambda/8$

Sol. $I_R = I_0 \cos^2\left(\frac{\phi}{2}\right)$

$$\cos^2\left(\frac{\Delta\phi}{2}\right) = \frac{1}{2}$$

$$\frac{\Delta\phi}{2} = \frac{\pi}{4}$$

$$\Delta\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda}(\Delta\lambda)$$

$\frac{\lambda}{4} = (\Delta x)$ Path of difference

$$(\Delta x) = 2x = \frac{\lambda}{4}$$

$x = \frac{\lambda}{8}$ = movemoent of pipe