Chapter : *Eight*

Introduction to Trigonometry

Competency Based Questions

✦ Multiple Choice Questions

1. The value of cos 0° . cos 1° . cos 2° . cos 3° ... cos 89° cos 90° is

(*a*) 1 (*b*) – 1 (*c*) 0 (*d*) $\frac{1}{\sqrt{2}}$

Ans. (*c*) 0

2. If x tan 45° sin 30° = cos 30° tan 30°, then x is equal to

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{\sqrt{2}}$ (*d*) 1

Ans. (*d*) 1

3. If x and y are complementary angles, then

(*a*) sin x = sin y (*b*) tan x = tan y
(*c*) cos x = cos y (*d*) sec x = cosec y

Ans. (*d*) sec x = cosec y

4. sin 2B = 2 sin B is true when B is equal to

(*a*) 90° (*b*) 60° (*c*) 30° (*d*) 0°

Ans. (*d*) 0°

5. If A, B and C are interior angles of a ΔABC then cos $\left(\frac{B+C}{2}\right)$ is equal to

(*a*) sin $\frac{A}{2}$ (*b*) – sin $\frac{A}{2}$
(*c*) cos $\frac{A}{2}$ (*d*) – cos $\frac{A}{2}$

Ans. (*a*) sin $\frac{A}{2}$

6. If A and (2A – 45°) are acute angles such that sin A = cos (2A – 45°), then tan A is equal to

(*a*) 0 (*b*) $\frac{1}{\sqrt{3}}$ (*c*) 1 (*d*) $\sqrt{3}$

Ans. (*c*) 1

7. If y sin 45° cos 45° = tan² 45° – cos² 30°, then y = ...

(*a*) $-\frac{1}{2}$ (*b*) $\frac{1}{2}$ (*c*) – 2 (*d*) 2

Ans. (*b*) $\frac{1}{2}$

8. If sin θ + sin² θ = 1, then cos² θ + cos⁴ θ = ...

(*a*) – 1 (*b*) 0 (*c*) 1 (*d*) 2

Ans. (*c*) 1

9. 5 tan² A – 5 sec² A + 1 is equal to

(*a*) 6 (*b*) – 5 (*c*) 1 (*d*) – 4

Ans. (*d*) – 4

10. If sec A + tan A = x, then sec A =

(*a*) $\frac{x^2-1}{x}$ (*b*) $\frac{x^2-1}{2x}$ (*c*) $\frac{x^2+1}{x}$ (*d*) $\frac{x^2+1}{2x}$

Ans. (*d*) $\frac{x^2+1}{2x}$

11. If sec A + tan A = x, then tan A =

(*a*) $\frac{x^2-1}{x}$ (*b*) $\frac{x^2-1}{2x}$ (*c*) $\frac{x^2+1}{x}$ (*d*) $\frac{x^2+1}{2x}$

Ans. (*b*) $\frac{x^2-1}{2x}$

12. $\frac{1-\cos A}{\sin A}$ is equal to

(*a*) $\frac{\sin A}{1-\cos A}$ (*b*) $\frac{\sin A}{1+\cos A}$ (*c*) $\frac{\cos A}{1-\cos A}$ (*d*) $\frac{\cos A}{1+\cos A}$

Ans. (*b*) $\frac{\sin A}{1+\cos A}$

13. If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2$ =

(*a*) ab (*b*) $b^2 + a^2$ (*c*) a^2b^2 (*d*) a^4b^4

Ans. (*c*) a^2b^2

14. What is the maximum value of $\frac{1}{\text{cosec A}}$?

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) 2

Ans. (*b*) 1

COMPETENCY BASED

15. What is the minimum value of sin A, $0 \le A \le 90°$

(a) –1 (b) 0 (c) 1 (d) $\frac{1}{2}$

Ans. (b) 0

16. What is the minimum value of cos θ, $0 \le \theta \le 90°$

(a) –1 (b) 0 (c) 1 (d) $\frac{1}{2}$

Ans. (b) 0

17. Given that $\sin\theta = \frac{a}{b}$, then $\tan\theta$ =

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{\sqrt{b^2-a^2}}{b}$

(c) $\frac{a}{\sqrt{b^2-a^2}}$ (d) $\frac{\sqrt{b^2-a^2}}{a}$

Ans. (c) $\frac{a}{\sqrt{b^2-a^2}}$

18. If $\cos 9A = \sin A$ and $9A < 90°$, then the value of tan 5A is

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

Ans. (b) 1

19. If in ΔABC, $\angle C = 90°$, then sin (A + B) =

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

Ans. (d) 1

20. If sin A – cos A = 0, then the value of $\sin^4 A + \cos^4 A$ is

(a) 2 (b) 1 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

Ans. (d) $\frac{1}{2}$

✦ Assertion-Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assetion (A) is false but Reason (R) is true.

1. Assertion: The value of $\sin\theta = \frac{4}{3}$ is not possible.

Reason: Hypotenuse is the largest side in any right angled triangle.

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Explanation: As we know, $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{4}{3}$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

2. Assertion: $\sin^2 67° + \cos^2 67° = 1$

Reason: For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Explanation: As we know, $\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2 67° + \cos^2 67° = 1$

3. Assertion: $\sin 47° = \cos 43°$.

Reason: $\sin\theta = \cos(90° + \theta)$, where θ is an acute angle.

Ans. (c) Assertion (A) is true but Reason (R) is false.

Explanation: As we know, $\sin\theta = \cos(90° - \theta)$

$\sin 47° = \cos(90° - 47°) =$ **cos 43°**

4. Assertion: If $\cos A + \cos A = 1$ then $\sin^2 A + \sin^4 A = 2$.

Reason: $1 - \sin^2 A = \cos^2 A$, for any value of A.

Ans. (d) Assertion (A) is false but Reason (R) is true.

Explanation: Here, $\cos A + \cos^2 A = 1$ ...(i)

$\cos A = 1 - \cos^2 A = \sin^2 A$

Now, $\sin^2 A + \sin^4 A = \cos A + \cos^2 A = 1$ **...[From (i)**

$\therefore \sin^2 A + \sin^4 A =$ **1**

5. Assertion: The value of $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta)$ equals to 1.

Reason: $\tan\theta = \sec(90° - \theta)$

Ans. (c) Assertion (A) is true but Reason (R) is false.

Explanation: $\sin\theta \cdot \cos\theta(90° - \theta) + \cos\theta\sin(90° - \theta)$

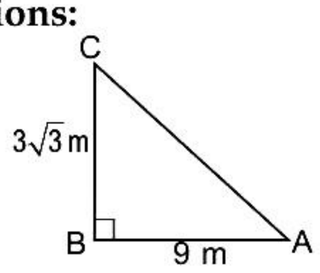
$= \sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta$

$= \sin^2\theta + \cos^2\theta = 1$

Also, $\tan\theta = \cot(90° - \theta)$

✦ Case Based Questions

I. Three friends—Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal has to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure that forms a right angled triangle such that AB = 9 m, BC = $3\sqrt{3}$ m and $\angle B = 90°$, then answer the following questions:



(i) The measure of ∠A is

(a) 30° (b) 45°

(c) 60° (d) None of these

Ans. (a) 30°

Explanation: We have AB = 9 m and BC = $3\sqrt{3}$ m

In ΔABC, we have

$\Rightarrow \tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan A = \tan 30° \quad \therefore \quad \angle A = 30°$

(ii) The measure of ∠C is

(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) None of these

Ans. (*c*) 60°

Explanation: Similarly, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan C = \tan 60^\circ \Rightarrow \angle C = 60^\circ$

(*iii*) The length of AC is

(*a*) $2\sqrt{3}$ m (*b*) $\sqrt{3}$ m
(*c*) $4\sqrt{3}$ m (*d*) $6\sqrt{3}$ m

Ans. (*d*) $6\sqrt{3}$ m

Explanation: Since, $\sin A = \frac{BC}{AC}$

$\Rightarrow \sin 30^\circ = \frac{BC}{AC}$ ...[Using point (*i*)

$\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow$ **AC = $6\sqrt{3}$ m**

(*iv*) cos 2A =

(*a*) 0 (*b*) $\frac{1}{2}$
(*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{\sqrt{3}}{2}$

Ans. (*b*) $\frac{1}{2}$

Explanation: From point (*i*), $\angle A = 30^\circ$

$\therefore \cos 2A = \cos(2 \times 30^\circ) = \cos 60^\circ = \mathbf{\frac{1}{2}}$

(*v*) $\sin\left(\frac{C}{2}\right)$ =

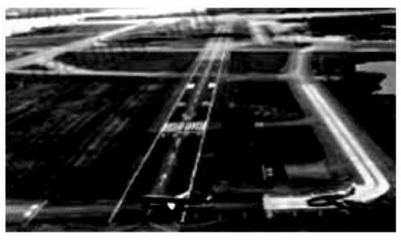
(*a*) 0 (*b*) $\frac{1}{2}$
(*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{\sqrt{3}}{2}$

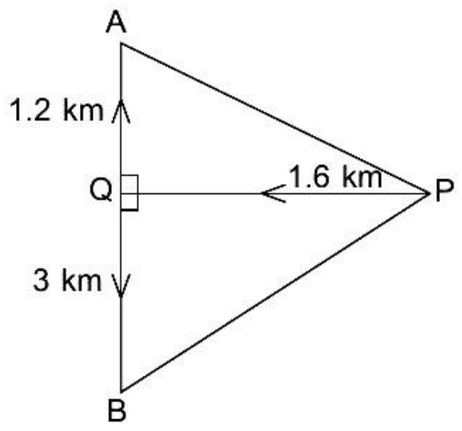
Ans. (*b*) $\frac{1}{2}$

Explanation: From point (*ii*), $\angle C = 60^\circ$

$\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^\circ}{2}\right) = \sin 30^\circ = \mathbf{\frac{1}{2}}$

II. Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400 km/hr and 1000 km/hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions:





(*i*) Find tan θ; if ∠APQ = θ.

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{\sqrt{2}}$
(*c*) $\frac{\sqrt{3}}{2}$ (*d*) $\frac{3}{4}$

Ans. (*d*) $\frac{3}{4}$

Explanation: In ΔAPQ, $\tan\theta = \frac{AQ}{PQ} = \frac{1.2}{1.6} = \mathbf{\frac{3}{4}}$

(*ii*) Find cot B.

(*a*) $\frac{3}{4}$ (*b*) $\frac{15}{4}$
(*c*) $\frac{3}{8}$ (*d*) $\frac{15}{8}$

Ans. (*d*) $\frac{15}{8}$

Explanation: In ΔPBQ, $\cot B = \frac{QB}{PQ} = \frac{3}{1.6} = \mathbf{\frac{15}{8}}$ ...(*M*)

(*iii*) Find tan A

(*a*) 2 (*b*) $\sqrt{2}$
(*c*) $\frac{4}{3}$ (*d*) $\frac{2}{\sqrt{3}}$

Ans. (*c*) $\frac{4}{3}$

Explanation: In ΔAPQ, $\tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \mathbf{\frac{4}{3}}$ ...(*N*)

(*iv*) Find sec A.

(*a*) 1 (*b*) $\frac{2}{3}$
(*c*) $\frac{4}{3}$ (*d*) $\frac{5}{3}$

Ans. (*d*) $\frac{5}{3}$

Explanation: We know that, $\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \sec A = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \mathbf{\frac{5}{3}}$ ...[From (*N*)

(*v*) Find cosec B.

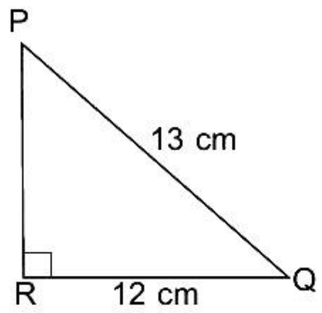
(*a*) $\frac{17}{8}$ (*b*) $\frac{12}{5}$
(*c*) $\frac{5}{12}$ (*d*) $\frac{8}{17}$

Ans. (*a*) $\frac{17}{8}$

Explanation: We know that, $\cot^2 B + 1 = \text{cosec}^2 B$

$= \text{cosec } B = \sqrt{\left(\frac{15}{8}\right)^2 + 1} = \sqrt{\frac{289}{64}} = \mathbf{\frac{17}{8}}$ ...[From (*M*)

III. Anita a student of class 10th, has to make a project on 'Introduction to Trigonometry.' She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as shown in the figure. Considering the front side of bird house as right angled triangled PQR, right-angled at R, answer the following questions:



(i) **If $\angle PQR = \theta$, then $\cos\theta =$**

(*a*) $\frac{12}{5}$ (*b*) $\frac{5}{12}$

(*c*) $\frac{12}{13}$ (*d*) $\frac{13}{12}$

Ans. (*c*) $\frac{12}{13}$

Explanation: In rt. ΔPQR,

$PR^2 + RQ^2 = PQ^2$ ...[Pythagoras' theorem

$\Rightarrow PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5$ cm

$\therefore \cos\theta = \frac{QR}{PQ} = \frac{12}{13}$

(ii) **The value of $\sec\theta =$**

(*a*) $\frac{5}{12}$ (*b*) $\frac{12}{5}$

(*c*) $\frac{13}{12}$ (*d*) $\frac{12}{13}$

Ans. (*c*) $\frac{13}{12}$

Explanation: $\sec\theta = \frac{1}{\cos\theta} = \mathbf{\frac{13}{12}}$

(iii) **The value of $\frac{\tan\theta}{1+\tan^2\theta} =$**

(*a*) $\frac{5}{12}$ (*b*) $\frac{12}{5}$

(*c*) $\frac{60}{169}$ (*d*) $\frac{169}{60}$

Ans. (*c*) $\frac{60}{169}$

Explanation: $\tan\theta = \frac{PR}{RQ} = \frac{5}{12}$

$\therefore \frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{5}{12}}{1+\frac{25}{144}} = \frac{\frac{5}{12}}{\frac{169}{144}} = \mathbf{\frac{60}{169}}$

(iv) **The value of $\cot^2\theta - \operatorname{cosec}^2\theta =$**

(*a*) -1 (*b*) 0

(*c*) 1 (*d*) 2

Ans. (*a*) -1

Explanation: As we know, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow -(\cot^2\theta - \operatorname{cosec}^2\theta) = 1$

$\therefore \cot^2\theta - \operatorname{cosec}^2\theta = \mathbf{-1}$

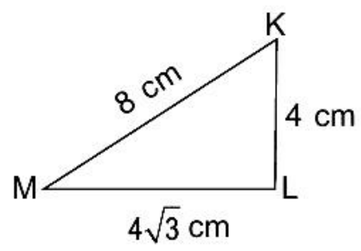
(v) **The value of $\sin^2\theta + \cos^2\theta =$**

(*a*) 0 (*b*) 1

(*c*) -1 (*d*) 2

Ans. (*b*) 1

IV. Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right-angled triangle with sides 4 cm, $4\sqrt{3}$ cm and 8 cm.



On the basis of the above information, answer the following questions:

(i) **The value of $\angle M =$**

(*a*) 30° (*b*) 60°

(*c*) 45° (*d*) None of these

Ans. (*a*) 30°

Explanation: $\tan M = \frac{KL}{LM} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan M = \tan 30^\circ \Rightarrow \mathbf{\angle M = 30^\circ}$

(ii) **The value of $\angle K =$**

(*a*) 45° (*b*) 30°

(*c*) 60° (*d*) None of these

Ans. (*c*) 60°

Explanation: $\tan K = \frac{ML}{KL} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60^\circ$

$\Rightarrow \mathbf{\angle K = 60^\circ}$

(iii) **Find the value of cot M.**

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}$

(*c*) 1 (*d*) None of these

Ans. (*a*) $\sqrt{3}$

Explanation: We have, $\angle M = 30^\circ$ ...[From part (*i*)

$\therefore \cot M = \cot 30^\circ = \mathbf{\sqrt{3}}$

(iv) **$\sec^2 M - 1 =$**

(*a*) $\tan M$ (*b*) $\tan 2M$

(*c*) $\tan^2 M$ (*d*) None of these

Ans. (*c*) $\tan^2 M$

(v) **The value of $\frac{\tan^2 45^\circ - 1}{\tan^2 45^\circ + 1}$ is**

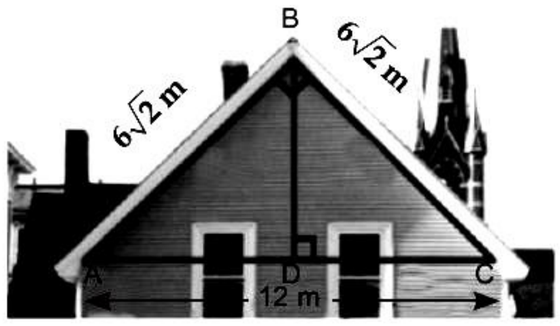
(*a*) 0 (*b*) 1

(*c*) 2 (*d*) -1

Ans. (*a*) 0

Explanation: $\frac{\tan^2 45^\circ - 1}{\tan^2 45^\circ + 1} = \frac{(1)^2 - 1}{(1)^2 + 1} = \frac{0}{2} = \mathbf{0}$ ...[$\because \tan 45^\circ = 1$

V. Aanya and her father go to meet her friend Juhi for a party. When they reached Juhi's place, Aanya saw the roof of the house which is triangular in shape. If she imagined the dimensions of the roof as given in the figure then answer the following questions:



(*i*) If D is the mid point of AC, then BD =

(*a*) 2 m **(*b*)** 3 m
(*c*) 4 m **(*d*)** 6 m

Ans. (*d*) 6 m

Explanation: We have, AB = BC = $6\sqrt{2}$ m & AC = 12 m
$\because$ D is mid-point of AC.
$\therefore$ AD = DC = 6 m
Now, $AB^2 = BD^2 + AD^2$ ($\therefore$ ΔABD is a right triangle)
$\Rightarrow$ $BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$
$\Rightarrow$ **BD = 6 m** ...(1)

(*ii*) Measure of $\angle A$ =

(*a*) 30° **(*b*)** 60°
(*c*) 45° **(*d*)** None of these

Ans. (*c*) 45°

Explanation: In ΔABD, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [Using (1)

$\Rightarrow$ $\sin A = \sin 45^\circ$ $\therefore$ $\angle$**A = 45°**

(*iii*) Measure of $\angle C$ =

(*a*) 30°
(*b*) 60°
(*c*) 45°
(*d*) None of these

Ans. (*c*) 45°

Explanation: In ΔBDC, $\tan C = \frac{BD}{DC} = \frac{6}{6}$ [Using (1)

$\Rightarrow$ $\tan C = 1 = \tan 45^\circ$ $\therefore$ $\angle$**C = 45°**

(*iv*) Find the value of sin A + cos C.

(*a*) 0 **(*b*)** 1
(*c*) $\frac{1}{\sqrt{2}}$ **(*d*)** $\sqrt{2}$

Ans. (*d*) $\sqrt{2}$

Explanation: $\sin A = \frac{1}{\sqrt{2}}$ and $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore$ $\sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(*v*) Find the value of $\tan^2 C + \tan^2 A$.

(*a*) 0 **(*b*)** 1
(*c*) 2 **(*d*)** $\frac{1}{2}$

Ans. (*c*) 2

Explanation: $\tan C = 1$, $\tan A = \tan 45^\circ = 1$
$\therefore$ $\tan^2 C + \tan^2 A = 1 + 1 =$ **2**