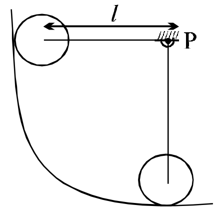
GUIDED REVISION

SECTION-I

Single Correct Answer Type **4 Q. [3 M (–1)]**

1. A sphere of mass M and radius R is attached by a light rod of length ℓ to a point P. The sphere rolls without slipping on a circular track as shown. It is released from the horizontal position. The angular momentum of the system about P when the rod becomes vertical is :



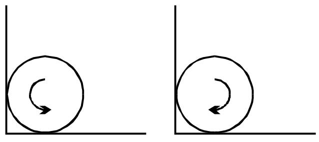
(A) $M\sqrt{\frac{10}{7}gl}\,[l+R]$

(B) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{2}{5}R\right]$

(C) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{7}{5}R\right]$

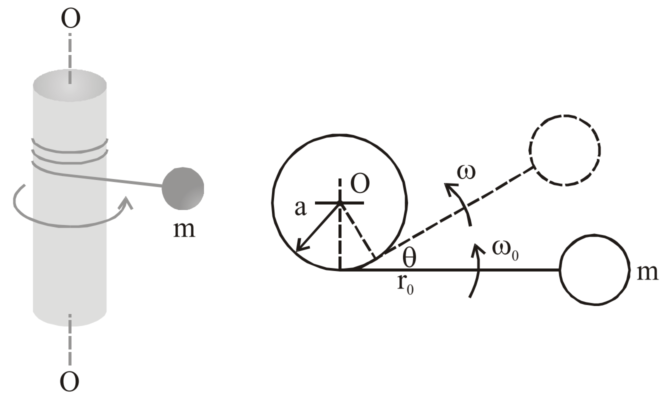
(D) $M\sqrt{\frac{10}{7}gl}\left[l-\frac{2}{5}R\right]$

2. A sphere is placed rotating with its centre initially at rest in a corner as shown in figure (a) & (b). Coefficient of friction between all surfaces and the sphere is 1/3. Find the ratio of the frictional force $\frac{f_a}{f_b}$ by ground in situations (a) & (b).



(A) 1 (B) $\frac{9}{10}$ (C) $\frac{10}{9}$ (D) none

3. The small particle of mass m is given an initial high velocity in the horizontal plane and winds its cord around the fixed vertical shaft of radius 1m. All motion occurs essentially in the horizontal plane. If the angular velocity of the cord is 0.8 rad/s when the distance from the particle to the tangential point is 5m, determine the angular velocity ω (in rad/s) of the cord after it has turned through an angle 1 rad.



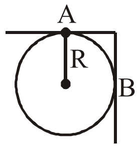
(A) 1 (B) 2 (C) 4 (D) 6

4. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor. At a certain moment, the speed of the end in contact with the horizontal floor is v and the ladder makes an angle α = 30° with the horizontal. Then the speed of the ladder's center must be :-

(A) $2v/\sqrt{3}$ (B) v/2 (C) v (D) None

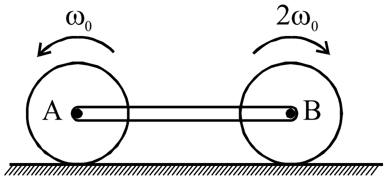
Multiple Correct Answer Type 4 Q. [4 M (–1)]

5. A rod bent at right angle along its centre line, is placed on a rough horizontal fixed cylinder of radius R as shown in figure. Mass of rod is 2 m and rod is in equilibrium. Assume that friction force on rod at A and B are equal in magnitde.



(A) Normal force applied by cylinder on rod at A is 3mg/2.
(B) Normal force applied by cylinder on rod at B must be zero.
(C) Friction force acting on rod at B is upward.
(D) Normal force applied by cylinder on rod at A is mg.

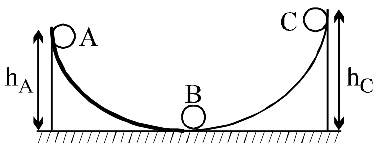
6. Two identical disc having radius R are rotating in opposite sense are joined with light rod having frictionless groove A and B and placed on rough horizontal surface as shown in figure. Initially translation velocity of each disc is zero. Then :-



(A) Final velocity of B is $\frac{R\omega_0}{6}$
(B) Rod's length will increase initially due to stress
(C) Finally rod has tensile stress
(D) Mechanical energy will decrease

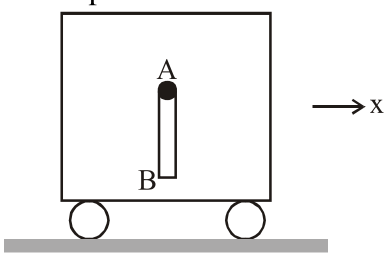
7. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then

[IIT-JEE 2006]



(A) $h_A > h_C$; $K_B > K_C$
(B) $h_A > h_C$; $K_C > K_A$
(C) $h_A = h_C$; $K_B = K_C$
(D) $h_A < h_C$; $K_B > K_C$

8. A uniform rod AB of length ℓ and mass M hangs from point A at which it is freely hinged in a car moving with velocity v_0. The rod can rotate in vertical plane about the axis at A. If the car suddenly stops,



(A) The angular speed ω with which the rod starts rotating is $\frac{3v_0}{2\ell}$

(B) The minimum value of v_0 so that the rod completes the rotation $\sqrt{\frac{8}{3}g\ell}$

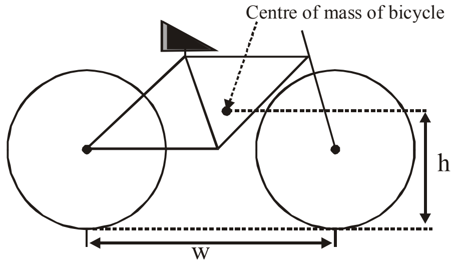
(C) Loss of energy during the process $\frac{1}{8}Mv_0^2$

(D) There is no loss of energy

Linked Comprehension Type **(2 Para × 3Q.) [3 M (-1)]**
(Single Correct Answer Type)

Paragraph for Questions no. 9 to 11

A simplified model of a bicycle of mass M has two tyres that each comes into contact with the ground at a point. The wheelbase of this bicycle (the distance between the points of contact with the ground) is w, and the center of mass of the bicycle is located midway between the tyres and at height h above the ground. Air resistance may be ignored.



Case -1 (Q. 9): Assume that cycle is moving rightwards & has acceleration a. Both tyres roll purely.

9. The direction of friction on the tyres is
(A) forward on both (B) backward on both
(C) forward on front & backward on rear (D) forward on rear & backward on front.

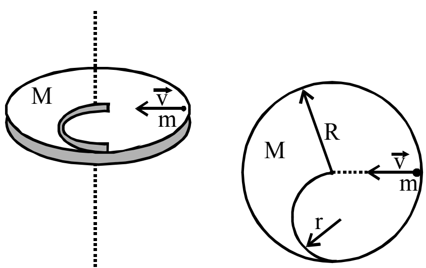
Case-2 (Q. 10 and 11) : The bicycle is moving to the right, but slowing down at a constant rate. The acceleration has a magnitude a. Assume that the coefficient of sliding friction between each tyre and the ground is μ, and that both tyres are skidding (sliding without rotating). Express your answers in terms of w, h, M and g.

10. What is the maximum value of μ so that both tyres remain in contact with the ground?
(A) $\frac{w}{2h}$ (B) $\frac{h}{2w}$ (C) $\frac{2h}{w}$ (D) $\frac{w}{h}$

11. What is the maximum value of a so that both tyres remain in contact with the ground?
(A) $\frac{wg}{h}$ (B) $\frac{wg}{2h}$ (C) $\frac{hg}{2w}$ (D) $\frac{2hg}{w}$

Paragraph for Question Nos. 12 to 14

A smooth disc of mass M and radius R can rotate freely around a vertical axis supported by frictionless bearings. A constraining vertical surface of negligible mass, whose shape is a semicircular arc of radius $r = \frac{R}{2}$, is fixed on the disc as shown in the figure. A small ball of mass m is placed on the stationary disc and is bowled at a speed v in such a way that it reaches the internal side of the constraining surface tangentially.



12. Mark the **CORRECT** statement :-
(A) Momentum of ball & disc system is conserved
(B) Mechanical energy of ball & disc system is not conserved
(C) Angular momentum of ball & disc system is conserved
(D) Mechanical energy of ball & disc system is not conserved due to impulsive reaction from support bearings

13. What is speed of the ball when it leaves the disc :-

(A) $v\sqrt{\frac{M}{M+2m}}$ (B) $v\sqrt{\frac{m}{M+2m}}$ (C) $v\sqrt{\frac{2M}{M+m}}$ (D) $2v\sqrt{\frac{M}{2M+m}}$

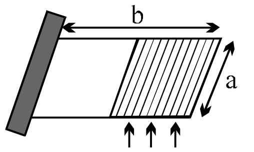
14. What is angular velocity of disc when ball leaves it :-

(A) $2\frac{mv}{MR}\sqrt{\frac{M}{M+2m}}$ (B) $2\frac{mv}{MR}\sqrt{\frac{m}{M+2m}}$ (C) $2\frac{mv}{MR}\sqrt{\frac{2M}{M+m}}$ (D) $2\frac{mv}{MR}\sqrt{\frac{M}{2M+m}}$

SECTION-II

Numerical Answer Type Question (upto second decimal place) 1 Q. [3(0)]

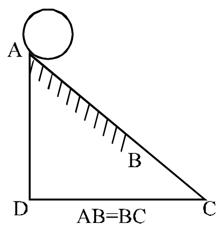
1. There is a rectangular plate of mass M kg of dimensions $(a \times b)$. The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v? It is given n = 100, M = 3 kg, m = 0.01kg; b = 2 m; a = 1m; g = 10 m/s^2. **[JEE 2006]**



SECTION-III

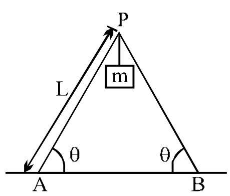
Numerical Grid Type (Ranging from 0 to 9) 1 Q. [4 M (0)]

1. Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. Find the ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C.

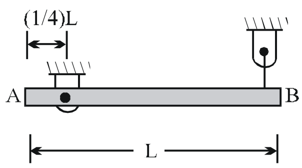


Subjective Type 9 Q. [4 M (0)]

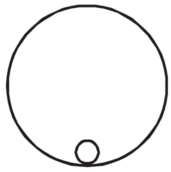
1. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B. **[IIT-JEE 2005]**



2. A uniform beam of length L and mass m is supported as shown. If the cable at B suddenly breaks, determine; (a) the acceleration of end B. (b) the reaction at the pin support.



3. A ring of mass m and radius r is set rolling inside a fixed circular rough loop of radius R. Find minimum speed at A such that it completes the vertical circular motion.



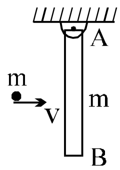
4. Two men, each of mass 75 kg, stand on the rim of a horizontal large disc, diametrically opposite to each other. The disc has a mass 450 kg and is free to rotate about its axis. Each man simultaneously start along the rim clockwise with the same speed and reaches their original starting points on the disc. Find the angle turned by the disc with respect to the ground in this duration.

5. A solid sphere of mass m and radius R is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height h = 4R/5 above the centre line. If I is the impulse of the blow then find

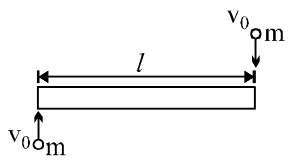
(a) the minimum time after which the highest point B will touch the ground

(b) the displacement of the centre of mass during this internal.

6. A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle comes to rest after the impact, then find the impulse exerted by hinge on rod at A.



7. On a smooth table two particles of mass m each, travelling with a velocity v_0 in opposite directions, strike the ends of a rigid massless rod of length l, kept perpendicular to their velocity. The particles stick to the rod after the collision.Find the tension in rod during subsequent motion.

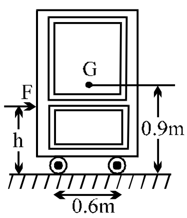


8. Find the MI of a rod about an axis through its centre of mass and perpendicular to the length whose linear density varies as $\lambda = ax$ where a is a constant and x is the position of an element of the rod relative to its left end. The length of the rod is ℓ.

9. A 20 kg cabinet is mounted on small casters that allow it to move freely ($\mu = 0$) on the floor. If a 100 N force is applied as shown, determine

(a) the acceleration of the cabinet,

(b) the range of values of h for which the cabinet will not tip.



ANSWER KEY	GR # ROTATIONAL MOTION-1

SECTION-I

Single Correct Answer Type — **4 Q. [3 M (–1)]**

1. Ans. (D) **2. Ans. (B)** 3. **Ans. (A)** **4. Ans. (C)**

Multiple Correct Answer Type — **4 Q. [4 M (–1)]**

5. Ans. (A,C) **6. Ans. (A, B, D)** **7. Ans. (A,B)** **8. Ans. (A, B, C)**

Linked Comprehension Type — **(2 Para × 3Q.) [3 M (-1)]**

(Single Correct Answer Type)

9. Ans. (D) **10. Ans. (A)** **11. Ans. (B)** **12. Ans. (C)**

13. Ans. (A) **14. Ans. (A)**

SECTION-II

Numerical Answer Type Question — **1 Q. [3(0)]**

(upto second decimal place)

1. Ans. 10 m/s

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) — **1 Q. [4 M (0)]**

1. Ans. 5

Subjective Type — **9 Q. [4 M (0)]**

1. Ans. $f = (M + m)\, g \dfrac{\cot\theta}{2}$ **2. Ans.** (a) $\dfrac{9g}{7}\downarrow$ (b) $\dfrac{4mg}{7}\uparrow$ **3. Ans.** $\sqrt{3g(R-r)}$

4. Ans. $\dfrac{4\pi}{5}$ **5. Ans.** (a) $t = \dfrac{\pi Rm}{2I}$; (b) $s = \dfrac{\pi R}{2}$ **6. Ans.** $\dfrac{mv}{4}$

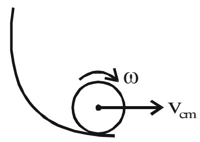
7. Ans. $\dfrac{2mv_0^{\ 2}}{l}$ **8. Ans.** $\dfrac{a\ell^4}{36}$ **9. Ans.** (a) 5 m/s^2 $\rightarrow$, (b) $0.3 < h < 1.5$ m

SOLUTIONS
SECTION-I

Single Correct Answer Type **4 Q. [3 M (–1)]**

1. Ans. (D)

Sol. $L_P = mv_{cm}\ell - \frac{2}{5}mR^2\omega$



$= mv_{cm}\ell - \frac{2}{5}mRv_{cm} \Rightarrow mv_{cm}\left(\ell - \frac{2}{5}R\right)$

$mg\ell = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{5}mR^2\omega^2 \Rightarrow \frac{7}{10}mv_{cm}^2$

$\therefore v_{cm} = \sqrt{\frac{10}{7}g\ell}$

2. Ans. (B)

Sol. $f_2 + N_1 = mg$

$f_a = N_2 = \mu N_1$

$f_a = \mu\,(mg - f_2)$

$f_a = \mu\,[mg - \mu N_2]$

$f_a = \mu\,[mg - \mu f_a]$

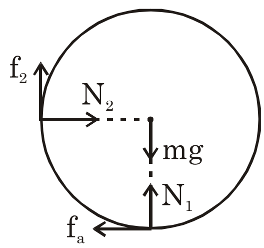
$f_a\,(1 + \mu^2) = \mu\, mg$

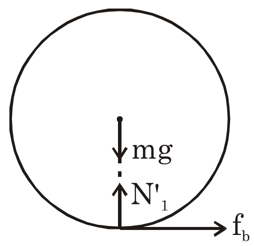
$f_a = \frac{\mu mg}{1+\mu^2} = \frac{3mg}{10}$

$f_b = \mu N'_1 = \mu mg$

$f_b = \frac{mg}{3}$

$\frac{f_a}{f_b} = \frac{9}{10}$





3. Ans. (A)

Sol. Since tension is perpendicular to velocity, therefore work done by tension is zero. Hence speed of particle remains constant.

$v_i = v_f$

$\omega_0 r_0 = \omega r$

$\omega_0 r_0 = \omega(r_0 - a\theta)$

$\omega = \frac{\omega_0 r_0}{r_0 - a\theta}$

$\omega = \frac{0.8r_0}{5 - 1\times 1}$

$\omega = \frac{0.8r_0}{4}$

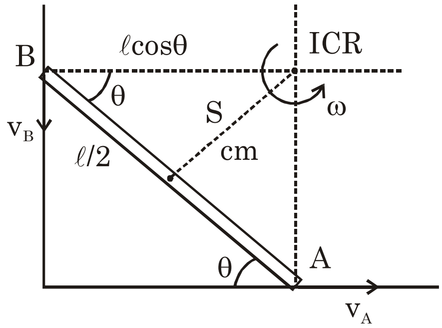
$\omega = (5 \times 0.2)$ rad/sec

$\omega = 1$ rad/sec

4. **Ans. (C)**

Sol. $v_A \cos\theta = v_B \sin\theta$

$v_B = v_A \cot\theta$



$$\omega = \frac{v_A}{\ell \sin\theta} = 2v/\ell$$

$v_{cm} = S\omega$

$$S = \sqrt{\left(\frac{\ell}{2}\right)^2 + (\ell\cos\theta)^2 - 2\frac{\ell}{2}\ell\cos\theta\cos\theta}$$

$$S = \ell\sqrt{\frac{1}{4} + \cos^2\theta - \cos^2\theta}$$

$$S = \frac{\ell}{2}$$

$$v_{cm} = \omega S = \frac{2v}{\ell}\frac{\ell}{2}$$

$v_{cm} = v$

Multiple Correct Answer Type 4 Q. [4 M (–1)]

5. **Ans. (A,C)**

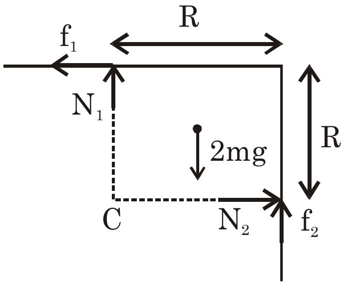
Sol. $f_1 = N_2$

$N_1 + f_2 = 2mg$

$f_1 = f_2 = f$

$N_1 + f = 2\ mg$

$f = N_2$



$$(\vec{\tau}_{net})_C = 0$$

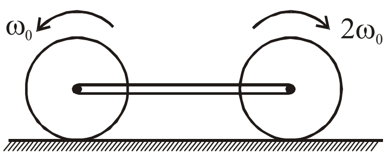
$$2fR - 2mg\frac{R}{2} = 0$$

$$f = \frac{mg}{2}$$

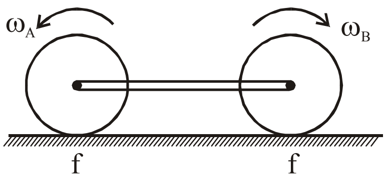
$$N_1 = 2mg - f = \frac{3mg}{2}$$

6. **Ans. (A, B, D)**

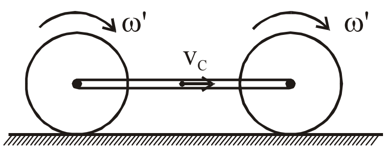
Sol.



Initially



at any instant 't'



Finally pure rolling takes place; $v_C = \omega' R$

Angular momentum conservation gives

$$(2\omega_0)\frac{mR^2}{2} - \frac{mR^2}{2}(\omega_0) = 2mv_C R + \frac{2mR^2}{2}\omega'$$

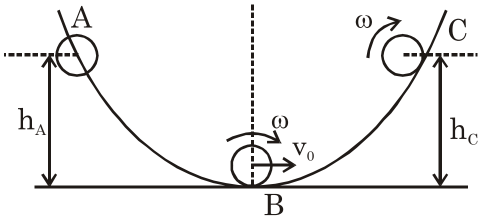
$$\frac{mR^2}{2}\omega_0 = 2mv_C R + mRv_C \text{ (Since } v_C = \omega' R)$$

$$v_C = \frac{R\omega_0}{6}$$

* Second figure shows that rod is under stress & hence its length will increase.
* Finally when the discs roll purely, there is no friction & hence no tensile stress
* Since friction is kinetic therefore mechanical energy will decrease.

7. Ans. (A,B)

Sol.



$mgh_A = K_B = (K_{trans})_B + (K_{rot})_B$

$$mgh_A = \frac{1}{2}mv_0^2 + \frac{1}{2}I_0\omega^2 = K_B \quad \text{... (i)}$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I_0\omega^2 = \frac{1}{2}I_0\omega^2 + mgh_C \text{, where } \frac{1}{2}I_0\omega^2 = K_C$$

$$mgh_C = \frac{1}{2}mv_0^2 \quad \text{... (ii)}$$

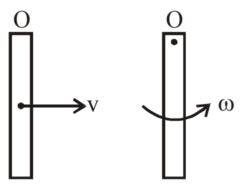
From (i) & (ii)

$h_A > h_C$; $K_B > K_C$

and $h_A > h_C$; $K_C > K_A$

8. Ans. (A, B, C)

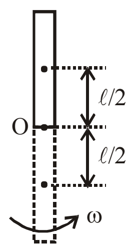
Sol. Angular momentum is conserved about O



$$Mv\frac{\ell}{2} = \frac{m\ell^2}{3}\omega$$

$$\omega = \frac{3v}{2\ell}$$

if rod has to complete the circle,



$$\frac{1}{2}I\omega_{min}^2 = Mg\ell \Rightarrow \omega_{min} = \sqrt{\frac{2Mg\ell \times 3}{M\ell^2}}$$

$$\Delta KE = \frac{1}{2}Mv_0^2 - \frac{1}{2}.\frac{M\ell^2}{3}.\omega^2$$

Linked Comprehension Type (2 Para × 3Q.) [3 M (-1)]

(Single Correct Answer Type)

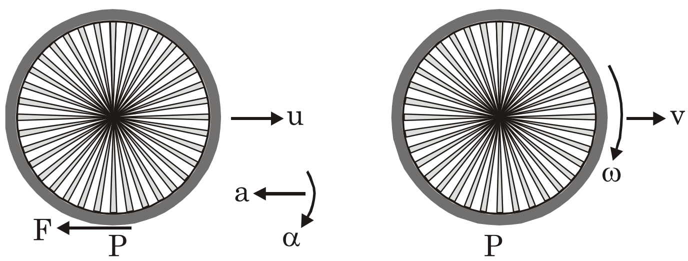
9. Ans. (D)

Sol.

Friction Force on the Wheels during Pedaling

Friction Force on the Front Wheel

The front wheel is connected to the rest of the bicycle by a rod passing through its centre (axle). The torque on the wheel about its centre by the force coming from the rest of the bicycle is zero. Thus, pedaling can give linear velocity to the front wheel but cannot rotate it.



Motion of the Front Wheel on a Rough Horizontal Surface

Let us analyse a simple situation to get the direction of friction force on the front wheel. What will happen if a wheel translating with speed u is gently placed on a rough surface ?

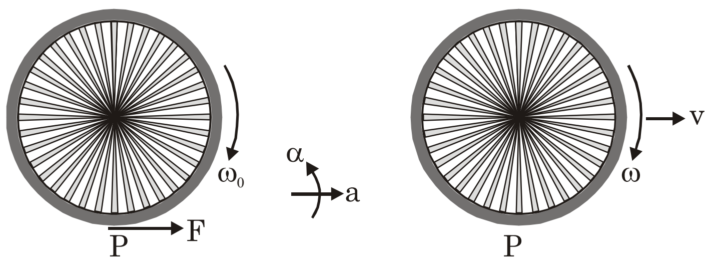
Every point on the wheel, including centre C and contact point P translates with the same speed u.

Thus, P moves towards right relative to the surface. To oppose this, frictional force at P acts towards the left. The frictional force retards the velocity of the centre. The torque about C due to frictional force gives clockwise angular acceleration and the wheel starts rotating in clockwise direction. If the coefficient of friction is sufficiently large then the wheel will start rolling without slipping after sometime. At this instant, velocity of P relative to the surface becomes zero. The frictional force will become zero and the wheel continues to roll without slipping, $\upsilon = \omega r$.

Friction Force on the Rear Wheel

The rear wheel is connected to rest of the bicycle by a rod passing through its centre and a chain connected to the pedals (see figure). Pressing the pedal increases tension in the upper portion of the chain.

This tension gives rise to clockwise torque and wheel starts rotating in clockwise direction.



Motion of Rear Wheel on a Rough horizontal surface during pedaling

Let us analyse a simple situation.

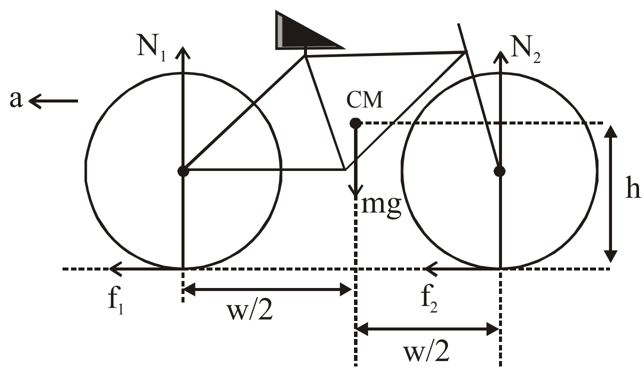
What will happen if a wheel rotating with angular velocity ω_0 is gently placed on a rough surface ? Initial velocity of the contact point P is leftward relative to the surface. To oppose this, frictional force at P acts towards the right. The frictional force increases the velocity of C.

The torque about C due to frictional force is anti-clockwise. This torque gives anti-clockwise angular acceleration.

If the coefficient of friction is sufficiently large then the wheel will start rolling without slipping after some time. At this instant, velocity of P relative to the surface becomes zero. After this, the wheel continues to roll without slipping, $\upsilon = \omega r$.

10. Ans. (A)

Sol.



$f_1 + f_2 = f = ma$ ... (i)

$N = N_1 + N_2 = mg$ ... (ii)

$(\vec{\tau}_{net})_{cm} = 0$

$(f_1 + f_2)h + N_1 w - N_2 w = 0$ ... (iii)

$f_1 + f_2 = f = \mu N_1 + \mu N_2$ ... (iv)

From (ii) & (iii)

$$N_2 = \frac{1}{2}\left[mg - \frac{2fh}{w}\right]$$

$$N_1 = \frac{1}{2}\left[mg + \frac{2fh}{w}\right]$$

For both the tyres to remain in contact with the ground,

$N_2 \geq 0$

$$mg \geq \frac{2h}{w}(mg)\mu \quad \text{[using (iv)]}$$

$\mu \le \frac{w}{2h}$

11. Ans. (B)

Sol. From (i), f = ma

$\mu mg = ma$

$a = \mu g$

$\mu = \frac{a}{g}$

$\mu \le \frac{w}{2h}$

$a \le \frac{wg}{2h}$

12. Ans. (C)

13. Ans. (A)

14. Ans. (A)

Sol. (12 to 14)

Let ball leaves with velocity u & disc rotates with ω clockwise direction

Angular moment conservation about centre of disc

$\vec{L}_i = \vec{L}_f$

$O = muR - \left(\frac{MR^2}{2}\right)\omega$

$\omega = \frac{2mu}{MR}$ ... (i)

Energy conservation

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}I\omega^2$ ... (ii)

$u = v\sqrt{\frac{M}{M+2m}}$, $\omega = 2\frac{mv}{MR}\sqrt{\frac{M}{M+2m}}$

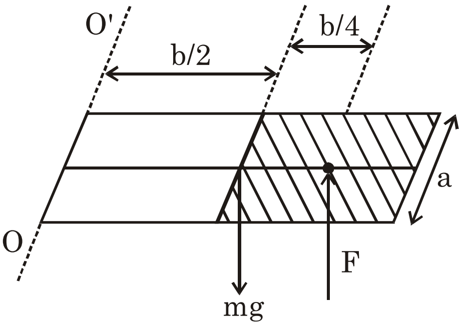
SECTION-II

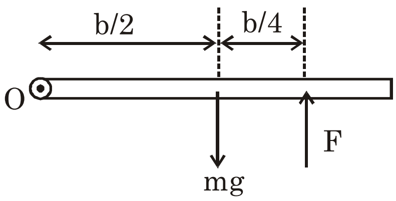
Numerical Answer Type Question **1 Q. [3(0)]**

(upto second decimal place)

1. Ans. 10 m/s

Sol.





Taking torque about axis OO' & putting it equal to zero

$\left(\vec{\tau}_{net}\right)_0 = 0$

$mg\frac{b}{2} - F\frac{3b}{4} = 0$

$F = \frac{2mg}{3}$ .... (i)

$F = n(2mv)\left(\frac{ab}{2}\right)$ .... (ii)

From (i) & (ii), putting the values we get

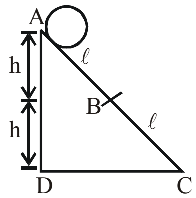
v = 10 m/s

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **1 Q. [4 M (0)]**

1. **Ans. 5**

Sol. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$



$mgh = \frac{3}{4}mv^2$

$2mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$

$2mgh = \frac{1}{2}mv_f^2 + mgh - \frac{1}{2}mv^2$

$mgh = \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv^2$

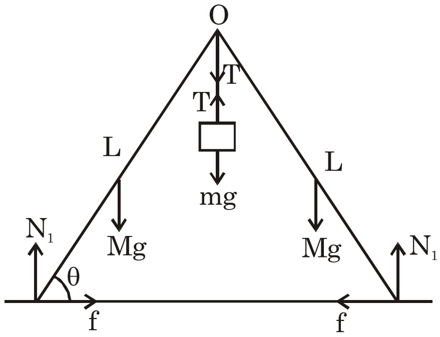
$\frac{1}{2}mv_f{}^2 = \frac{5}{3}mgh$

$\therefore \text{Ratio} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}\frac{mv^2}{2}} = \frac{\frac{5}{3}mgh}{\frac{1}{4}\frac{4}{3}mgh} = 5$

Subjective Type 9 Q. [4 M (0)]

1. **Ans.** $f = (M+m)\, g \dfrac{\cot\theta}{2}$

Sol. For any one ladder



$$\left(\vec{\tau}_{net}\right)_0 = 0$$

$$N_1 L\cos\theta - f\,L\sin\theta - Mg\frac{L}{2}\cos\theta = 0$$

$$\vec{F}_{net} = 0$$

$$2N_1 - 2Mg - mg = 0$$

$$N_1 = \left(M + \frac{M}{2}\right)g$$

$$f = \cot\theta\left[N_1 - \frac{Mg}{2}\right]$$

$$f = \cot\theta\left[M + \frac{m}{2} - \frac{M}{2}\right]g$$

$$f = \frac{(M+m)g(\cot\theta)}{2}$$

2. Ans. (a) $\dfrac{9g}{7}\downarrow$ (b) $\dfrac{4mg}{7}\uparrow$

Sol. about hinge $mg\dfrac{L}{4} = I\alpha$

$$I = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$

$$\therefore \frac{MgL}{4} = \frac{7ML^2\alpha}{48}$$

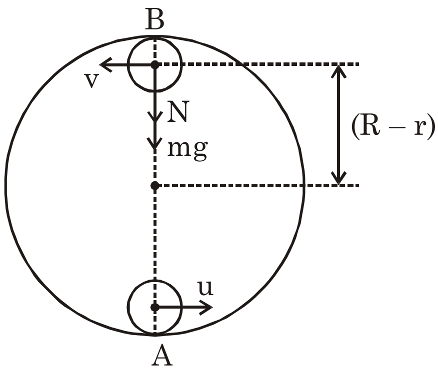
$$\therefore \alpha = \frac{12g}{7L}$$

(a) $\alpha\dfrac{3L}{4} = \dfrac{12g}{7L}\dfrac{3L}{4} = \dfrac{9g}{7}\downarrow$

(b) $a_{cm} = \dfrac{\alpha L}{4} = \dfrac{3g}{7}$ and $mg - F_{hinge} = ma_{cm}$

3. **Ans.** $\sqrt{3g(R-r)}$

Sol. $N + mg = \dfrac{mv^2}{R-r}$



$N = 0,\ v^2_{min} = g(R-r)$

$E_A = E_B$

$$E_A = \frac{1}{2}mu^2\left(1 + \frac{I_C}{mr^2}\right) = mu^2$$

$$E_B = \frac{1}{2}mv^2\left(1+\frac{I_C}{mR^2}\right)+2mg\left(R-r\right)$$

$$E_B = mv^2 + 2mg\ (R - r)$$

$$mu_{min}^2 = mv_{min}^2 + 2mg\left(R-r\right)$$

$$u_{min}^2 = g\left(R-r\right)+2mg\left(R-r\right)$$

$$u_{min} = \sqrt{3g\left(R-r\right)}$$

4. Ans. $\frac{4\pi}{5}$

Sol. From angular momentum conservation,

$$\vec{L}_i = \vec{L}_f$$

$$O = I_m\omega_m - I_D\omega_D$$

Where, $\omega_m = \frac{d\theta_m}{dt}, \omega_D = \frac{d\theta_D}{dt}$

$$I_m\theta_m - I_D\theta_D = 0$$

$$\theta_m + \theta_D = 2\pi$$

$$\theta_D = \left(\frac{I_m}{I_m + I_D}\right)2\pi$$

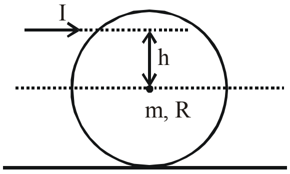
$$\theta_D = \left(\frac{2mR^2}{2mR^2 + \frac{MR^2}{2}}\right)2\pi$$

$$\theta_D = \left[\frac{1}{1+\frac{M}{4m}}\right]2\pi$$

$$\theta_D = \left(\frac{1}{1+\frac{450}{4\times 75}}\right)2\pi = \frac{4\pi}{5}$$

5. Ans. (a) $t = \frac{\pi Rm}{2I}$; (b) $s = \frac{\pi R}{2}$

Sol.



From angular-impulse angular-momentum theorem,

$$Ih = \frac{2}{5}MR^2\omega$$

$$I\frac{4R}{5} = \frac{2}{5}MR^2\omega$$

$$\omega = \frac{2I}{MR}$$

(a) $\theta = \omega t$

$\pi = \omega t$

$$t = t_0 = \frac{\pi}{\omega} = \frac{\pi MR}{2I}$$

(b) From linear-impulse linear-momentum theorem,

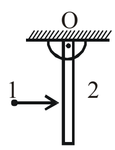
$I = Mv_{cm}$

$$v_{CM} = \frac{I}{M}$$

$$\Delta S_{cm} = v_{cm} t_0 = \frac{\pi R}{2}$$

6. **Ans.** $\frac{mv}{4}$

Sol. Angular momentum conservation about O.



$$mV\frac{\ell}{2} = 0 + \frac{m\ell^2}{3}\omega$$

$$\omega\ell = V\frac{3}{2}$$

Now V_{CM} of rod $= \frac{\omega\ell}{2} = \frac{3V}{4}$

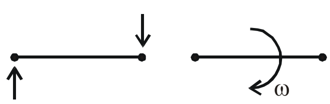
$I_1 = mv$; Impulse on rod by particle

$$I_1 - N_{imp} = m\frac{3v}{4}$$

$$\therefore\ mv - N_{Im} = \frac{3mv}{4}$$

$\therefore\ N_{Im} = \frac{mv}{4}$; Impulse on rod by hinge

7. **Ans.** $\frac{2mv_0{}^2}{l}$

Sol.  (V_{cm} of rod = 0 as $P_i = P_f$)

$L_i = L_f$

$$\therefore\ \omega = \frac{2v_0}{\ell}$$

$$T = m\omega^2\frac{\ell}{2} = \frac{m4v_0^2}{\ell^2}\times\frac{\ell}{2} = \frac{2mv_0^2}{\ell}$$

8. **Ans.** $\frac{a\ell^4}{36}$

Sol. $I_0 = \int dmx^2 = \int_0^{\ell}\lambda dx x^2 = \frac{a\ell^4}{4}$

Now CM = **d =** $\dfrac{\int_r^{\ell}\lambda dx x}{\int_0^{\ell}\lambda dx}=\dfrac{2\ell}{3}$

$\therefore\ I_{CM}=I_0-md^2=\dfrac{a\ell^4}{4}-md^2$

$m=\int\lambda dx=\dfrac{a\ell^2}{2}$

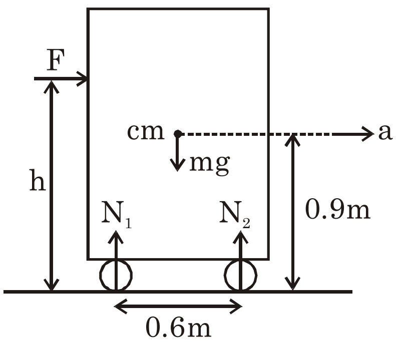
$\therefore\ \dfrac{a\ell^4}{4}-\left(\dfrac{a\ell^2}{2}\right)\dfrac{4}{9}\ell^2=\dfrac{a\ell^4}{36}$

9. **Ans.** (a) 5 m/s² →, (b) 0.3 < h < 1.5 m

Sol. F = ma

$a=5m/s^2$

$N_1+N_2=mg=200$ .... (i)



$(\vec{\tau}_{net})_{cm}=0$

Case-1 : $F(h-0.9)+N_1(0.3)-N_2(0.3)=0$

$\dfrac{100(h-0.9)}{0.3}=-(N_1-N_2)$

$N_2-N_1=\dfrac{1000}{3}(h-0.9)$ .... (ii)

From (i) & (ii)

$2N_1=200-\dfrac{1000}{3}(h-0.9)$

$N_1=100-\dfrac{500}{3}(h-0.9)$

For not tipping, $N_1>0$

$\dfrac{300}{500}>h-0.9$

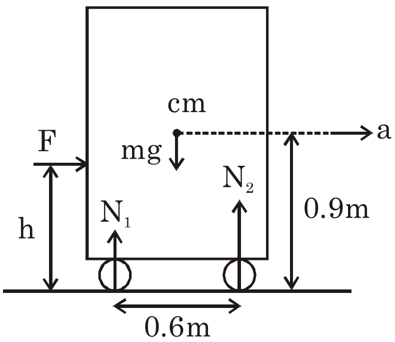
$h<0.9+0.6$

$h<1.5$ m

$(\vec{\tau}_{net})_{cm}=0$

Case-2 : $F(0.9-h)+N_1(0.3)-N_2(0.3)=0$ .... (iii)

From (i) & (iii)



$N_1=100-\dfrac{500}{3}(0.9-h)$

For not tipping, $N_1>0$

$\dfrac{300}{500}>0.9-h$

$h>0.9-0.6$

$h>0.3$ m