CHAPTER

11

Sets, Relations and Functions

Session 1

Definition of Set, Representation of Set, Different Types of Sets, Laws and Theorems, Venn Diagram
(Euler-Venn Diagrams)

Introduction

The concept of set is fundamental in modern Mathematics. Today this concept is being used in different branches of Mathematics and widely used in the foundation of relations and functions. The theory of sets was developed by German Mathematician **Georg Cantor** (1845-1918).

Definition of Set

A set is well-defined collection of distinct objects. Sets are usually denoted by capital letters $A, B, C, X, Y, Z, \dots$.

Examples of sets

(i) The set of all complex numbers.
(ii) The set of vowels in the alphabets of English language.
(iii) The set of all natural numbers.
(iv) The set of all triangles in a plane.
(v) The set of all states in India.
(vi) The set of all months in year which has 30 days.
(vii) The set of all stars in space.

Elements of the Set

The elements of the set are denoted by small letters in the alphabets of English language, i.e. $a, b, c, x, y, z, \dots$. If x is an element of a set A, we write $x \in A$ (read as 'x belongs to A').

If x is not an element of A, then we write $x \notin A$ (read as 'x does not belong to A').

For example,

If $A = \{1, 2, 3, 4, 5\}$, then $3 \in A$, $6 \notin A$.

Representation of a Set

There are two methods for representing a set.

1. Tabulation or Roster or Enumeration Method

Under this method, the elements are enclosed in curly brackets or braces { } after separating them by commas.

Remark

1. The order of writing the elements of a set is immaterial, so $\{a, b, c\}$, $\{b, a, c\}$, $\{c, a, b\}$ all denote the same set.
2. An element of a set is not written more than once, i.e. the set $\{1, 2, 3, 4, 3, 3, 2, 1, 2, 1, 4\}$ is identical with the set $\{1, 2, 3, 4\}$.

For example,

1. If A is the set of prime numbers less than 10, then
$$A = \{2, 3, 5, 7\}$$
2. If A is the set of all even numbers lying between 2 and 20, then
$$A = \{4, 6, 8, 10, 12, 14, 16, 18\}$$

2. Set Builder Method

Under this method, the stating properties which its elements are to satisfy, then we write
$$A = \{x \, P(x)\} \quad \text{or} \quad A = \{x : P(x)\}$$
and read as 'A is the set of elements x, such that x has the property P'.

Remark

1. " : " or " | " means 'such that'.
2. The other names of this method are property method, rule method and symbolic method.

For example,

1. If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then we can write
$$A = \{x \in N : x \le 8\}.$$
2. A is the set of all odd integers lying between 2 and 51, then
$$A = \{x : 2 < x < 51, x \text{ is odd}\}.$$

Some Standard Sets

- N denotes set of all natural numbers $= \{1, 2, 3, \dots\}$.
- Z or I denotes set of all integers
$$= \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$$
- Z_0 or I_0 denotes set of all integers excluding zero
$$= \{\dots, -3, -2, -1, 1, 2, 3, \dots\}.$$
- Z^+ or I^+ denotes set of all positive integers
$$= \{1, 2, 3, \dots\} = N.$$
- E denotes set of all even integers
$$= \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}.$$
- O denotes set of all odd integers
$$= \{\dots, -5, -3, -1, 1, 3, 5, \dots\}.$$
- W denotes set of all whole numbers $= \{0, 1, 2, 3, \dots\}$.
- Q denotes set of all rational numbers $= \{x : x = p/q$, where p and q are integers and $q \neq 0\}$.
- Q_0 denotes set of all non-zero rational numbers $\{x : x = p/q$, where p and q are integers and $p \neq 0$ and $q \neq 0\}$.
- Q^+ denotes set of all positive rational numbers $= \{x : x = p/q$, where p and q are both positive or negative integers$\}$
- R denotes set of all real numbers.
- R_0 denotes set of all non-zero real numbers.
- R^+ denotes set of all positive real numbers.
- $R - Q$ denotes set of all irrational numbers.
- C denotes set of all complex numbers
$$= \{a + ib : a, b \in R \text{ and } i = \sqrt{-1}\}.$$
- C_0 denotes set of all non-zero complex numbers
$$= \{a + ib : a, b \in R_0 \text{ and } i = \sqrt{-1}\}.$$
- N_a denotes set of all natural numbers which are less than or equal to a, where a is positive integer
$$= \{1, 2, 3, \dots, a\}.$$

Different Types of Sets

1. Null Set or Empty Set or Void Set

A set having no element is called a null set or empty set or void set. It is denoted by ϕ or $\{\,\}$.

Remark

1. ϕ is called the null set.
2. ϕ is unique.
3. ϕ is a subset of every set.
4. ϕ is never written within braces i.e., $\{\phi\}$ is not the null set.
5. $\{0\}$ is not an empty set as it contains the element 0 (zero).

For example,

1. $\{x : x \in N, 4 < x < 5\} = \phi$
2. $\{x : x \in R, x^2 + 1 = 0\} = \phi$
3. $\{x : x^2 = 25, x$ is even number$\} = \phi$

2. Singleton or Unit Set

A set having one and only one element is called singleton or unit set.

For example, $\{x : x - 3 = 4\}$ is a singleton set.

Since, $\quad x - 3 = 4 \Rightarrow x = 7$

$\therefore \quad \{x : x - 3 = 4\} = \{7\}$

3. Subset

If every element of a set A is also an element of a set B, then A is called the subset of B, we write $A \subseteq B$ (read as A is subset of B or A is contained in B).

Thus, $\quad A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$

Remark

1. Every set is a subset of itself i.e., $A \subseteq A$.
2. If $A \subseteq B$, $B \subseteq C$, then $A \subseteq C$.

For example,

1. If $A = \{2, 3, 4\}$ and $B = \{5, 4, 2, 3, 1\}$, then $A \subseteq B$.
2. The sets $\{a\}, \{b\}, \{a, b\}, \{b, c\}$ are the subsets of the set $\{a, b, c\}$.

4. Total Number of Subsets

If a set A has n elements, then the number of subsets of $A = 2^n$.

Example 1. Write the letters of the word ALLAHABAD in set form and find the number of subsets in it and write all subsets.

Sol. There are 5 different letters in the word ALLAHABAD i.e., A,L,H,B,D, then set is {A, B, D, H, L}, then number of subsets $= 2^5 = 32$ and all subsets are

ϕ, {A}, {B}, {D}, {H}, {L}, {A, B}, {A, D}, {A, H}, {A, L}, {B, D}, {B, H}, {B, L}, {D, H}, {D, L}, {H, L}, {A, B, D}, {A, B, H}, {A, B, L}, {A, D, H}, {A, D, L}, {A, H, L}, {B, D, H}, {B, D, L}, {B, H, L}, {D, H, L}, {A, B, D, H}, {A, B, D, L}, {B, D, H, L}, {A, D, H, L}, {A, B, H, L}, {A, B, D, H, L}.

5. Equal Sets

Two sets A and B are said to be equal, if every element of A is an element of B, and every element of B is an element of A. If A and B are equal, we write $A = B$.

It is clear that $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$.

For example,

1. The sets {1, 2, 5} and {5, 2, 1} are equal.
2. $\{1, 2, 3\} = \{x : x^3 - 6x^2 + 11x - 6 = 0\}$

6. Power Set

The set of all the subsets of a given set A is said to be the power set A and is denoted by $P(A)$ or 2^A.

Symbolically, $P(A) = \{x : x \subseteq A\}$

Thus, $x \in P(A) \Leftrightarrow x \subseteq A$.

Remark

1. ϕ and A are both elements of $P(A)$.
2. If $A = \phi$, then $P(\phi) = \{\phi\}$, a singleton but ϕ is a null set.
3. If $A = \{a\}$, then $P(A) = \{\phi, \{a\}\}$

   For example, If $A = \{a, b, c\}$, then

   $P(A)$ or $2^A = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

   Also, $n(P(A))$ or $n(2^A) = 2^3 = 8$
4. Since, $P(\phi) = \{\phi\}$

   $\therefore$ $P(P(\phi)) = \{\phi, \{\phi\}\}$

   and $P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$
5. If A has n elements, then $P(A)$ has 2^n elements.

7. Super Set

The statement $A \subseteq B$ can be rewritten as $B \supseteq A$, then B is called the super set of A and is written as $B \supset A$.

8. Proper Subset

A set A is said to be proper subset of a set B, if every element of A is an element of B and B has atleast one element which is not an element of A and is denoted by $A \subset B$ (read as "A is a proper subset of B").

For example,

1. If $A = \{1, 2, 4\}$ and $B = \{5, 1, 2, 4, 3\}$, then $A \subset B$

   Since, $3, 5 \notin A$.
2. If $A = \{a, b, c\}$ and $B = \{c, b, a\}$, then $A \not\subset B$ (since, B does not contain any element which is not in A).
3. $N \subset I \subset Q \subset R \subset C$

9. Finite and Infinite Sets

A set in which the process of counting of elements comes to an end is called a finite set, otherwise it is called an infinite set.

For example,

1. Each one of the following sets is a finite set.
   (i) Set of universities in India.
   (ii) Set of Gold Medalist students in Civil Branch, sec A in A.M.I.E. (India).
   (iii) Set of natural numbers less than 500.
2. Each one of the following is an infinite set.
   (i) Set of all integers.
   (ii) Set of all points in a plane.
   (iii) $\{x : x \in R, 1 < x < 2\}$
   (iv) Set of all concentric circles with centre as origin.

10. Cardinal Number of a Finite Set

The number of distinct elements in a finite set A is called cardinal number and the cardinal number of a set A is denoted by $n(A)$.

For example,

If $A = \{-3, -1, 8, 9, 13, 17\}$, then $n(A) = 6$.

11. Comparability of Sets

Two sets A and B are said to be comparable, if either $A \subset B$ or $B \subset A$ or $A = B$, otherwise A and B are said to be incomparable.

For example,

1. The sets $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 6\}$ are incomparable (since $A \not\subset B$ or $B \not\subset A$ or $A \neq B$)
2. The sets $A = \{1, 2, 4\}$ and $B = \{1, 4\}$ are comparable (since $B \subset A$).

12. Universal Set

All the sets under consideration are likely to be subsets of a set is called the universal set and is denoted by Ω or S or U.

For example,

1. The set of all letters in alphabet of English language $U = \{a, b, c, \ldots, x, y, z\}$ is the universal set of vowels in alphabet of English language.

   i.e., $A = \{a, e, i, o, u\}$.
2. The set of all integers $I = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$ is the universal set of all even integers

   i.e., $\{0, \pm 2, \pm 4, \pm 6, \ldots\}$

13. Union of Sets

The union of two sets A and B is the set of all those elements which are either in A or in B or in both. This set is denoted by $A \cup B$ or $A + B$ (read as 'A union B' or 'A cup B' or 'A join B').

Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

or $A \cup B = \{x : x \in A \vee x \in B\}$

Clearly, $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

For example,

1. If $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6\}$, then $A \cup B = \{1, 2, 3, 4, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$
2. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$, $C = \{7, 8\}$, then $A \cup B \cup C = \{1, 2, 3, 4, 5, 7, 8\}$

Remark

The union of a finite number of sets $A_1, A_2, A_3, \ldots, A_n$ is represented by $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$ or $\bigcup_{i=1}^{n} A_i$.

Symbolically, $\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for atleast one } i\}$

14. Intersection of Sets

The intersection of two sets A and B is the set of all elements which are common in A and B. This set is denoted by $A \cap B$ or AB (read as 'A intersection B' or 'A cap B' or 'A meet B').

Symbolically, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

or $\quad A \cap B = \{x : x \in A \wedge x \in B\}$

Clearly, $\quad x \in A \cap B \Leftrightarrow x \in A \text{ and } x \in B$

For example,

1. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$, then $A \cap B = \{3\}$.
2. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$, then $A \cap B \cap C = \{3\}$

Remark

The intersection of a finite number of sets $A_1, A_2, A_3, \ldots, A_n$ represented by

$$A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n \quad \text{or} \quad \bigcap_{i=1}^{n} A_i.$$

Symbolically, $\bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for all } i\}$

15. Disjoint Sets

If the two sets A and B have no common element.

i.e., $A \cap B = \phi$, then the two sets A and B are called disjoint or mutually exclusive events.

For example, If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$, then $A \cap B = \phi$

Hence, A and B are disjoint sets.

Remark

If $S = \{a_1, a_2, a_3, \ldots, a_n\}$, so

number of ordered pairs of disjoint sets of S is $\frac{3^n + 1}{2}$.

($\because$ each element in either (A) or (B) or neither

$\therefore$ Total ways $= 3^n$ i.e., $A = B$, iff $A = B = \phi$ (1 case) otherwise A and B are interchangeable.

$\therefore$ Number of ordered pairs of disjoint sets of $S = 1 + \frac{3^n - 1}{2} = \frac{3^n + 1}{2}$).

16. Difference of Sets

If A and B be two given sets, then the set of all those elements of A which do not belong to B is called difference of sets A and B. It is written as $A - B$. It is also denoted by $A \sim B$ or $A \backslash B$ or $C_A B$ (complement of B in A).

Symbolically, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Clearly, $x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$.

Remark

1. $A - B \neq B - A$
2. The sets $A - B$, $B - A$ and $A \cap B$ are disjoint sets.
3. $A - B \subseteq A$ and $B - A \subseteq B$
4. $A - \phi = A$ and $A - A = \phi$

For example,

If $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7\}$, then $A - B = \{1, 2, 3\}$.

17. Symmetric Difference of Two Sets

Let A and B be two sets. The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ or $(A \cup B) - (A \cap B)$ and is denoted by $A \,\Delta\, B$ or $A \oplus B$ (A direct sum B).

i.e., $\quad A \oplus B$ or $A \,\Delta\, B = (A - B) \cup (B - A)$

and $\quad A \oplus B$ or $A \,\Delta\, B = (A \cup B) - (A \cap B)$

Remark

1. $A \,\Delta\, B = \{x : x \in A \text{ and } x \notin B\}$
   or $\quad A \,\Delta\, B = \{x : x \in B \text{ and } x \notin A\}$
2. $A \,\Delta\, B = B \,\Delta\, A$ (commutative)

For example,

Let $\quad A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7\}$,

then $\quad A - B = \{2, 4\}$, $B - A = \{7\}$

$\therefore \quad A \,\Delta\, B = (A - B) \cup (B - A) = \{2, 4, 7\}$

18. Complement Set

Let U be the universal set and A be a set, such that $A \subset U$. Then, the complement of A with respect to U is denoted by A' or A^c or $C(A)$ or $U - A$.

Symbolically, A' or A^c or $C(A) = \{x : x \in U \text{ and } x \notin A\}$.

Clearly, $\quad x \in A' \Leftrightarrow x \notin A$.

Remark

1. $U' = \phi$ and $\phi' = U$
2. $A \cup A' = U$ and $A \cap A' = \phi$

For example,

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 3, 5, 7\}$.

Then, $A' = U - A = \{2, 4, 6\}$

Laws and Theorems

1. **Idempotent Laws**

   For **any set A,**

   (i) $\boldsymbol{A \cup A = A}$ (ii) $\boldsymbol{A \cap A = A}$

   Proof

   (i) Let $x \in A \cup A \Leftrightarrow x \in A$ or $x \in A$

   $\Leftrightarrow x \in A$

   Hence, $A \cup A = A$

   (ii) Let $x \in A \cap A \Leftrightarrow x \in A$ and $x \in A$

   $\Leftrightarrow x \in A$

   Hence, $A \cap A = A$

2. **Identity Laws**

   For **any set A,**

   (i) $\boldsymbol{A \cup \phi = A}$ (ii) $\boldsymbol{A \cap \phi = \phi}$

   (iii) $\boldsymbol{A \cup U = U}$ (iv) $\boldsymbol{A \cap U = A}$

   Proof

   (i) Let $x \in A \cup \phi \Leftrightarrow x \in A$ and $x \in \phi$

   $\Leftrightarrow x \in A$

   Hence, $A \cup \phi = A$

   (ii) Let $x \in A \cap \phi \Leftrightarrow x \in A$ and $x \in \phi$

   $\Leftrightarrow x \in \phi$

   Hence, $A \cap \phi = \phi$

   (iii) Let $x \in A \cup U \Leftrightarrow x \in A$ or $x \in U$

   $\Leftrightarrow x \in U$

   Hence, $A \cup U = U$

   (iv) Let $x \in A \cap U \Leftrightarrow x \in A$ and $x \in U$

   $\Leftrightarrow x \in A$

   Hence, $A \cap U = A$

3. **Commutative Laws**

   For any two sets A and B, we have

   (i) $\boldsymbol{A \cup B = B \cup A}$ (ii) $\boldsymbol{A \cap B = B \cap A}$

   Proof

   (i) Let $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

   $\Leftrightarrow x \in B$ or $x \in A$

   $\Leftrightarrow x \in B \cup A$

   $\therefore$ $x \in A \cup B \Leftrightarrow x \in B \cup A$

   Hence, $A \cup B = B \cup A$

   (ii) Let $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$

   $\Leftrightarrow x \in B$ and $x \in A$

   $\Leftrightarrow x \in B \cap A$

   $\therefore$ $x \in A \cap B \Leftrightarrow x \in B \cap A$

   Hence, $A \cap B = B \cap A$.

4. **Associative Laws**

   For any three sets A, B and C, we have

   (i) $\boldsymbol{A \cup (B \cup C) = (A \cup B) \cup C}$

   (ii) $\boldsymbol{A \cap (B \cap C) = (A \cap B) \cap C}$

   Proof

   (i) Let $x \in A \cup (B \cup C) \Leftrightarrow x \in A$ or $x \in B \cup C$

   $\Leftrightarrow x \in A$ or $(x \in B$ or $x \in C)$

   $\Leftrightarrow (x \in A$ or $x \in B)$ or $x \in C$

   $\Leftrightarrow x \in A \cup B$ or $x \in C$

   $\Leftrightarrow x \in (A \cup B) \cup C$

   $\therefore$ $x \in A \cup (B \cup C) \Leftrightarrow x \in (A \cup B) \cup C$

   Hence, $A \cup (B \cup C) = (A \cup B) \cup C$.

   (ii) Let $x \in A \cap (B \cap C) \Leftrightarrow x \in A$ and $x \in B \cap C$

   $\Leftrightarrow x \in A$ and $(x \in B$ and $x \in C)$

   $\Leftrightarrow (x \in A$ and $x \in B)$ and $x \in C$

   $\Leftrightarrow x \in A \cap B$ and $x \in C$

   $\Leftrightarrow x \in (A \cap B) \cap C$

   Hence, $A \cap (B \cap C) = (A \cap B) \cap C$.

5. **Distributive Laws**

   For any three sets A, B and C, we have

   (i) $\boldsymbol{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

   (ii) $\boldsymbol{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

   Proof

   (i) Let $x \in A \cup (B \cap C) \Leftrightarrow x \in A$ or $x \in B \cap C$

   $\Leftrightarrow x \in A$ or $(x \in B$ and $x \in C)$

   $\Leftrightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

   $\Leftrightarrow x \in A \cup B$ and $x \in A \cup C$

   $\Leftrightarrow x \in [(A \cup B) \cap (A \cup C)]$

   $\therefore$ $x \in A \cup (B \cap C) \Leftrightarrow x \in (A \cup B) \cap (A \cup C)$

   Hence, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   (ii) Let $x \in A \cap (B \cup C) \Leftrightarrow x \in A$ and $x \in B \cup C$

   $\Leftrightarrow x \in A$ and $(x \in B$ or $x \in C)$

   $\Leftrightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

   $\Leftrightarrow x \in A \cap B$ or $x \in A \cap C$

   $\Leftrightarrow x \in (A \cap B) \cup (A \cap C)$

   $\therefore$ $x \in A \cap (B \cup C) \Leftrightarrow x \in (A \cap B) \cup (A \cap C)$

   Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. **For any two sets A and B, we have**

   (i) $\boldsymbol{P(A) \cap P(B) = P(A \cap B)}$

   (ii) $\boldsymbol{P(A) \cup P(B) \subseteq P(A \cup B)}$

   where, $P(A)$ is the power set of A.

   Proof

   (i) Let $x \in P(A) \cap P(B) \Leftrightarrow x \in P(A)$ or $x \in P(B)$

   $\Leftrightarrow x \subseteq A$ or $x \subseteq B$

   $\Leftrightarrow x \subseteq A \cap B$

   $\Leftrightarrow x \in P(A \cap B)$

   Hence, $P(A) \cap P(B) = P(A \cap B)$

(ii) Let $x \in P(A) \cup P(B) \Leftrightarrow x \in P(A)$ or $x \in P(B)$

$\Leftrightarrow x \subseteq A$ or $x \subseteq B$

$\Leftrightarrow x \subseteq A \cup B$

$\Leftrightarrow x \in P(A \cup B)$

Hence, $P(A) \cup P(B) \subseteq P(A \cup B)$

7. If A is any set, then $\boldsymbol{(A')' = A}$

Proof Let $x \in (A')' \Leftrightarrow x \notin A' \Leftrightarrow x \in A$

Hence, $(A')' = A$

8. **De-Morgan's Laws**

For any three sets A, B and C, we have

(i) $\boldsymbol{(A \cup B)' = A' \cap B'}$

(ii) $\boldsymbol{(A \cap B)' = A' \cup B'}$

(iii) $\boldsymbol{A - (B \cup C) = (A - B) \cap (A - C)}$

(iv) $\boldsymbol{A - (B \cap C) = (A - B) \cup (A - C)}$

Proof

(i) Let $x \in (A \cup B)' \Leftrightarrow x \notin A \cup B$

$\Leftrightarrow x \notin A$ and $x \notin B$

$\Leftrightarrow x \in A'$ and $x \in B'$

$\Leftrightarrow x \in A' \cap B'$

$\therefore \quad x \in (A \cup B)' \Leftrightarrow x \in A' \cap B'$

Hence, $(A \cup B)' = A' \cap B'$.

(ii) Let $x \in (A \cap B)' \Leftrightarrow x \notin A \cap B$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \in A'$ or $x \in B'$

$\Leftrightarrow x \in A' \cup B'$

$\therefore \quad x \in (A \cap B)' \Leftrightarrow x \in A' \cup B'$

Hence, $(A \cap B)' = A' \cup B'$.

(iii) Let $x \in A - (B \cup C) \Leftrightarrow x \in A$ and $x \notin B \cup C$

$\Leftrightarrow x \in A$ and ($x \notin B$ and $x \notin C$)

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) and ($x \in A$ and $x \notin C$)

$\Leftrightarrow x \in (A - B)$ and $x \in (A - C)$

$\Leftrightarrow x \in (A - B) \cap (A - C)$

Hence, $A - (B \cup C) = (A - B) \cap (A - C)$.

(iv) Let $x \in A - (B \cap C) \Leftrightarrow x \in A$ and $x \notin (B \cap C)$

$\Leftrightarrow x \in A$ and ($x \notin B$ or $x \notin C$)

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \notin C$)

$\Leftrightarrow x \in (A - B)$ or $x \in (A - C)$

$\Leftrightarrow x \in (A - B) \cup (A - C)$

Hence, $A - (B \cap C) = (A - B) \cup (A - C)$.

Aliter

$A - (B \cap C) = A \cap (B \cap C)'$ $\quad [\because A - B = A \cap B']$

$= A \cap (B' \cap C)'$

$= (A \cap B') \cup (A \cap C')$

$= (A - B) \cup (A - C)$

More Results on Operations on Sets

For any two sets A and B, we have

1. $\boldsymbol{A \subseteq A \cup B, B \subseteq A \cup B, A \cap B \subseteq A, A \cap B \subseteq B}$
2. $\boldsymbol{A - B = A \cap B'}$

Proof

Let $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$

$\Leftrightarrow x \in A$ and $x \in B'$

$\Leftrightarrow x \in A \cap B'$

Hence, $A - B = A \cap B'$

3. $\boldsymbol{(A - B) \cup B = A \cup B}$

Proof $(A - B) \cup B = (A \cap B') \cup B$

$= (A \cup B) \cap (B' \cup B)$ [from distributive law]

$= (A \cup B) \cap U$

$= A \cup B$

Hence, $(A - B) \cup B = A \cup B$

4. $\boldsymbol{(A - B) \cap B = \phi}$

Proof $(A - B) \cap B = (A \cap B') \cap B$

$= A \cap (B' \cap B)$ [from associative law]

$= A \cap \phi = \phi$

Hence, $(A - B) \cap B = \phi$

5. $\boldsymbol{A \subseteq B \Leftrightarrow B' \subseteq A'}$

Proof Only if part Let $A \subseteq B$ ...(i)

To prove $B' \subseteq A'$

Let $x \in B' \Rightarrow x \notin B$

$\Rightarrow x \notin A$ $\quad [\because A \subseteq B]$

$\Rightarrow x \in A'$

Thus, $x \in B' \Rightarrow x \in A'$ $\quad [\because B \subseteq A]$

Hence, $B' \subseteq A'$ ...(ii)

If part Let $B' \subseteq A'$ ...(iii)

To prove $A \subseteq B$

Let $x \in A \Rightarrow x \notin A'$

$\Rightarrow x \notin B'$ [from Eq.(iii)]

$\Rightarrow x \in B$

Hence, $A \subseteq B$ ...(iv)

From Eqs. (ii) and (iv), we get $A \subseteq B \Leftrightarrow B' \subseteq A'$

6. $\boldsymbol{A - B = B' - A'}$

Proof $A - B = (A \cap B')$

$= B' \cap A = B' \cap (A')' = B' - A'$

Hence, $A - B = B' - A'$

7. $\boldsymbol{(A \cup B) \cap (A \cup B') = A}$

Proof $(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$

[by distributive law]

$= A \cup \phi = A$

Hence, $(A \cup B) \cap (A \cup B') = A$

8. $\boldsymbol{A \cup B = (A - B) \cup (B - A) \cup (A \cap B)}$

Proof $(A - B) \cup (B - A) \cup (A \cap B)$

$= [(A \cup B) - (A \cap B)] \cup (A \cap B)$

$= [(A \cup B) \cap (A \cap B)'] \cup (A \cap B)$
$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)' \cup (A \cap B)]$
$= (A \cup B) \cap U = A \cup B$

Hence, $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

9. $\boldsymbol{A - (A - B) = A \cap B}$

Proof $A - (A - B) = A - (A \cap B')$
$= A \cap (A \cap B')'$
$= A \cap (A' \cup B)$
$= (A \cap A') \cup (A \cap B)$
$= \phi \cup (A \cap B) = A \cap B$

Hence, $A - (A - B) = A \cap B$

10. $\boldsymbol{A - B = B - A \Leftrightarrow A = B}$

Proof Only if part Let $A - B = B - A$ ...(i)

To prove $A = B$

Let $x \in A \Leftrightarrow (x \in A \text{ and } x \notin B)$ or $(x \in A \text{ and } x \in B)$
$\Leftrightarrow x \in (A - B)$ or $x \in (A \cap B)$
$\Leftrightarrow x \in (B - A)$
or $x \in A \cap B$ [from Eq. (i)]
$\Leftrightarrow (x \in B \text{ and } x \notin A)$ or $(x \in B \text{ and } x \in A)$
$\Leftrightarrow x \in B$

Hence, $A = B$

If part Let $A = B$

To prove $A - B = B - A$

Now, $A - B = A - A = \phi$ $[\because B = A]$
and $B - A = A - A = \phi$ $[\because B = A]$
$\therefore$ $A - B = B - A$
Hence, $A = B \Rightarrow A - B = B - A$

11. $\boldsymbol{A \cup B = A \cap B \Leftrightarrow A = B}$

Proof Only if part Let $A \cup B = A \cap B$

Now, $x \in A \Rightarrow x \in A \cup B$
$\Rightarrow$ $x \in A \cap B$ $[\because A \cup B = A \cap B]$
$\Rightarrow$ $x \in B$
Thus, $A \subseteq B$ ...(i)
Again, $y \in B \Rightarrow y \in A \cup B$
$\Rightarrow$ $y \in A \cap B$ $[\because A \cup B = A \cap B]$
$\Rightarrow$ $y \in A$
Thus, $B \subseteq A$ ...(ii)
From Eqs. (i) and (ii), we have $A = B$
Thus, $A \cup B = A \cap B \Rightarrow A = B$.

If part Let $A = B$ ...(iii)

To prove $A \cup B = A \cap B$

Now, $A \cup B = A \cup A = A$ $[\because B = A]$ ...(iv)
and $A \cap B = A \cap A = A$ $[\because B = A]$ ...(v)
From Eqs. (iv) and (v), we have $A \cup B = A \cap B$
Hence, $A \cup B = A \cap B \Leftrightarrow A = B$

Example 2. ***Let*** A, B ***and*** C ***be three sets such that*** $A \cup B = A \cup C$ ***and*** $A \cap B = A \cap C$***. Show that*** $B = C$***.***

Sol. Given, $A \cup B = A \cup C$ ...(i)
and $A \cap B = A \cap C$ ...(ii)

To prove $B = C$.

From Eq. (i), $(A \cup B) \cap C = (A \cup C) \cap C$
$\Rightarrow$ $(A \cap C) \cup (B \cap C) = (A \cap C) \cup (C \cup C)$
$\Rightarrow$ $(A \cap B) \cup (B \cap C) = (A \cap C) \cup C$
$[\because A \cap C = A \cap B]$
$\Rightarrow$ $(A \cap B) \cup (B \cap C) = C$ $[\because A \cap C \subseteq C]$
Thus, $C = (A \cap B) \cup (B \cap C)$ ...(iii)
Again, from Eq. (i), $(A \cup B) \cap B = (A \cup C) \cap B$
$\Rightarrow$ $(A \cap B) \cup (B \cap B) = (A \cap B) \cup (C \cap B)$
$\Rightarrow$ $(A \cap B) \cup B = (A \cap B) \cup (B \cap C)$
$\Rightarrow$ $B = (A \cap B) \cup (B \cap C)$
$[\because A \cap B \subseteq B]$
Thus, $B = (A \cap B) \cup (B \cap C)$ ...(iv)
From Eqs. (iii) and (iv), we have $B = C$.

Example 3. Let A and B be any two sets. If for some set X, $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$, prove that $A = B$.

Sol. Given, $A \cap X = B \cap X = \phi$ ...(i)
and $A \cup X = B \cup X$ ...(ii)
From Eq. (ii), $A \cap (A \cup X) = A \cap (B \cup X)$
$\Rightarrow$ $A = (A \cap B) \cup (A \cap X)$
$[\because A \subseteq A \cup X \therefore A \cap (A \cup X) = A]$
$\Rightarrow$ $A = (A \cap B) \cup \phi$ $[\because A \cap X = \phi]$
$\Rightarrow$ $A = (A \cap B)$
$\Rightarrow$ $A \subseteq B$ ...(iii)
Again, $A \cup X = B \cup X$
$\Rightarrow$ $B \cap (A \cup X) = B \cap (B \cup X)$
$\Rightarrow$ $(B \cap A) \cup (B \cap X) = B$
$[\because B \subseteq B \cup X \therefore B \cap (B \cup X) = B]$
$\Rightarrow$ $(B \cap A) \cup \phi = B$ $[\because B \cap X = \phi]$
$\Rightarrow$ $B \cap A = B$
$\Rightarrow$ $B \subseteq A$ ...(iv)
From Eqs. (iii) and (iv), we have $A = B$.

Example 4. If A and B are any two sets, prove that $P(A) = P(B) \Rightarrow A = B$.

Sol. Given, $P(A) = P(B)$ ...(i)

To prove $A = B$

Let $x \in A \Rightarrow$ there exists a subset X of A such that $x \in X$.
Now, $X \subseteq A \Rightarrow X \in P(A)$
$\Rightarrow$ $X \in P(B)$ [from Eq. (i)]

$\Rightarrow \quad X \subseteq B$

$\Rightarrow \quad x \in B \quad [\because x \in X]$

Thus, $\quad x \in A \Rightarrow x \in B$

$\therefore \quad A \subseteq B \quad$ ...(ii)

Let $y \in B \Rightarrow$ there exists a subset Y of B such that $y \in Y$.

Now, $\quad Y \subseteq B \Rightarrow Y \in P(B)$

$\Rightarrow \quad Y \in P(A) \quad [\because P(B) = P(A)]$

$\Rightarrow \quad Y \subseteq A$

$\Rightarrow \quad y \in A \quad [\because y \in Y]$

Thus, $\quad y \in B \Rightarrow y \in A$

$\therefore \quad B \subseteq A \quad$ ...(iii)

From Eqs. (ii) and (iii), we have $A = B$

Use of Sets in Logical Problems

$M =$ Set of students which have Mathematics.

$P =$ Set of students which have Physics.

$C =$ Set of students which have Chemistry.

Applying the different operations on the above sets, then we get following important results.

$M' =$ Set of students which have no Mathematics.

$P' =$ Set of students which have no Physics.

$C' =$ Set of students which have no Chemistry.

$M \cup P =$ Set of students which have atleast one subject Mathematics or Physics.

$P \cup C =$ Set of students which have atleast one subject Physics or Chemistry.

$C \cup M =$ Set of students which have atleast one subject Chemistry or Mathematics.

$M \cap P =$ Set of students which have both subjects Mathematics and Physics.

$P \cap C =$ Set of students which have both subjects Physics and Chemistry.

$C \cap M =$ Set of students which have both subjects Chemistry and Mathematics.

$M \cap P' =$ Set of students which have Mathematics but not Physics.

$P \cap C' =$ Set of students which have Physics but not Chemistry.

$C \cap M' =$ Set of students which have Chemistry but not Mathematics.

$(M \cup P)' =$ Set of students which have not both subjects Mathematics and Physics.

$(P \cup C)' =$ Set of students which have not both subjects Physics and Chemistry.

$(C \cup M)' =$ Set of students which have not both subjects Chemistry and Mathematics.

$(M \cap P \cap C) =$ Set of students which have all three subjects Mathematics, Physics and Chemistry.

$(M \cup P \cup C) =$ Set of all students which have three subjects.

Cardinal Number of Some Sets

If A, B and C are finite sets and U be the finite universal set, then

(i) $n(A') = n(U) - n(A)$

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(iii) $n(A \cup B) = n(A) + n(B)$, if A and B are disjoint non-void sets.

(iv) $n(A \cap B') = n(A) - n(A \cap B)$

(v) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

(vi) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(vii) $n(A - B) = n(A) - n(A \cap B)$

(viii) $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$

(ix) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(x) If $A_1, A_2, A_3, \ldots, A_n$ are disjoint sets, then
$n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n)$
$= n(A_1) + n(A_2) + n(A_3) + \ldots + n(A_n)$

Example 5. If A and B be two sets containing 6 and 3 elements respectively, what can be the minimum number of elements in $A \cup B$? Also, find the maximum number of elements in $A \cup B$.

Sol. We have, $\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

$n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum, respectively.

***Case* I** If $n(A \cap B)$ is minimum i.e., $n(A \cap B) = 0$ such that

$$A = \{a, b, c, d, e, f\} \text{ and } B = \{g, h, i\}$$

$\therefore \quad n(A \cup B) = n(A) + n(B) = 6 + 3 = 9$

***Case* II** If $n(A \cap B)$ is maximum i.e., $n(A \cap B) = 3$, such that

$$A = \{a, b, c, d, e, f\} \text{ and } B = \{d, a, c\}$$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = 6 + 3 - 3 = 6$

Example 6. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots, B_n$ are n sets each with three elements.

Let $\quad \bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$

Assume that each element of S belongs to exactly ten of the A_i's and exactly to nine of the B_j's. Find n.

Sol. Given, A's are thirty sets with five elements each, so

$$\sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150 \quad \text{...(i)}$$

If the m distinct elements in S and each element of S belongs to exactly 10 of the A_i's, so we have

$$\sum_{i=1}^{30} n(A_i) = 10m \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get $10m = 150$

$$\therefore \quad m = 15 \quad \text{...(iii)}$$

Similarly, $\sum_{j=1}^{n} n(B_j) = 3n$ and $\sum_{j=1}^{n} n(B_j) = 9m$

$$\therefore \quad 3n = 9m \Rightarrow n = \frac{9m}{3} = 3m$$

$$= 3 \times 15 = 45 \quad \text{[from Eq. (iii)]}$$

Hence, $n = 45$

Example 7. In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali only? How many can speak both Hindi and Bengali?

Sol. Let H and B be the set of those people who can speak Hindi and Bengali respectively, then according to the problem, we have

$$n(H \cup B) = 1000,$$
$$n(H) = 750, n(B) = 400$$

We know that,

$$n(H \cup B) = n(H) + n(B) - n(H \cap B)$$
$$1000 = 750 + 400 - n(H \cap B)$$
$$\therefore \quad n(H \cap B) = 150$$

$\therefore$ Number of people speaking Hindi and Bengali both is 150.

Also, $n(H \cap B') = n(H) - n(H \cap B)$

$$= 750 - 150$$
$$= 600$$

Thus, number of people speaking Hindi only is 600.

Again, $n(B \cap H') = n(B) - n(B \cap H) = 400 - 150 = 250$

Thus, number of people speaking Bengali only is 250.

Example 8. A survey of 500 television watchers produced the following information, 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games? How many watch exactly one of the three games?

Sol. Let F, H and B be the sets of television watchers who watch Football, Hockey and Basketball, respectively.

Then, according to the problem, we have

$$n(U) = 500, n(F) = 285, n(H) = 195,$$
$$n(B) = 115, n(F \cap B) = 45,$$
$$n(F \cap H) = 70, n(H \cap B) = 50$$

and $n(F' \cup H' \cup B') = 50,$

where U is the set of all the television watchers.

Since, $n(F' \cup H' \cup B') = n(U) - n(F \cup H \cup B)$

$$\Rightarrow \quad 50 = 500 - n(F \cup H \cup B)$$
$$\Rightarrow \quad n(F \cup H \cup B) = 450$$

We know that,

$$n(F \cup H \cup B) = n(F) + n(H) + n(B) - n(F \cap H) - n(H \cap B) - n(B \cap F) + n(F \cap H \cap B)$$

$$\Rightarrow \quad 450 = 285 + 195 + 115 - 70 - 50 - 45 + n(F \cap H \cap B)$$

$$\therefore \quad n(F \cap H \cap B) = 20$$

which is the number of those who watch all the three games. Also, number of persons who watch football only

$= n(F \cap H' \cap B')$

$= n(F) - n(F \cap H) - n(F \cap B) + n(F \cap H \cap B)$

$= 285 - 70 - 45 + 20 = 190$

The number of persons who watch hockey only

$= n(H \cap F' \cap B')$

$= n(H) - n(H \cap F) - n(H \cap B) + n(H \cap F \cap B)$

$= 195 - 70 - 50 + 20 = 95$

and the number of persons who watch basketball only

$= n(B \cap H' \cap F')$

$= n(B) - n(B \cap H) - n(B \cap F) + n(H \cap F \cap B)$

$= 115 - 50 - 45 + 20 = 40$

Hence, required number of those who watch exactly one of the three games

$$= 190 + 95 + 40 = 325$$

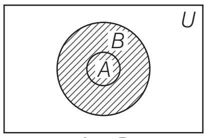
Venn Diagrams

(Euler-Venn Diagrams)

The diagram drawn to represent sets are called Venn diagrams or Euler Venn diagrams. Here, we represent the universal set U by points within rectangle and the subset A of the set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B by two intersecting circles.

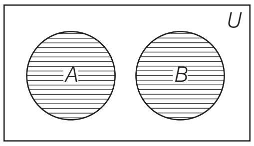
Venn Diagrams in Different Situations

1. Subset

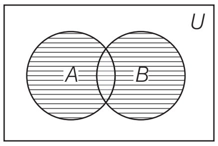


$A \subseteq B$

2. Union of sets

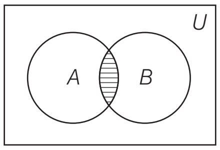


$A \cup B$, when $A \cap B = \phi$

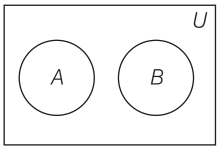


$A \cup B$, when neither $A \subseteq B$ nor $B \subseteq A$

3. Intersection of sets

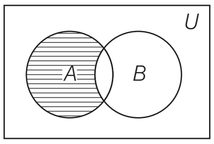


$A \cap B$, when neither $A \subseteq B$ nor $B \subseteq A$

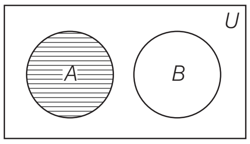


$A \cap B$, when $A \cap B = \phi$ (no shaded one)

4. Difference of sets

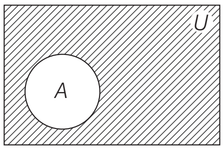


$A - B$, when neither $A \subseteq B$ nor $B \subseteq A$



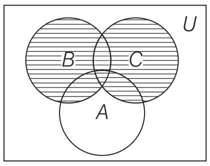
$A - B$, when $A - B = \phi$

5. Complement set

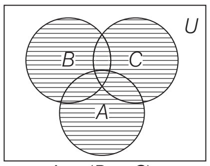


A' = shaded one

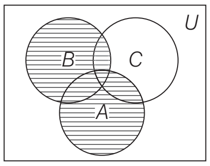
6. $A \cup (B \cup C)$ and $(A \cup B) \cup C$



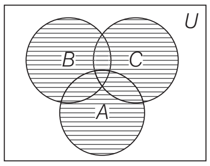
$B \cup C$



$A \cup (B \cup C)$



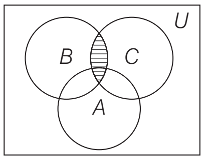
$A \cup B$



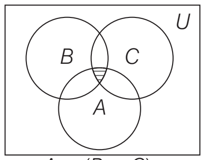
$(A \cup B) \cup C$

Hence, $A \cup (B \cup C) = (A \cup B) \cup C$ which is associative law for union.

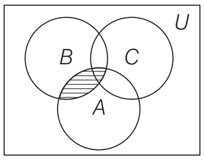
7. $A \cap (B \cap C)$ and $(A \cap B) \cap C$



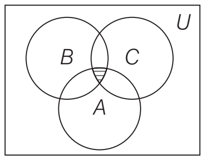
$B \cap C$



$A \cap (B \cap C)$



$A \cap B$



$(A \cap B) \cap C$

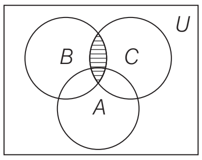
Hence, $A \cap (B \cap C) = (A \cap B) \cap C$ which is associative law for intersection.

8. Distributive law

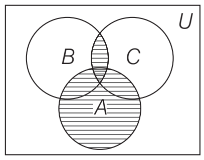
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

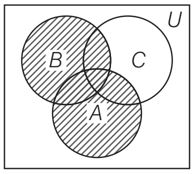
(i)



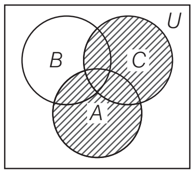
$(B \cap C)$



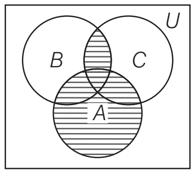
$A \cup (B \cap C)$



$(A \cup B)$



$(A \cup C)$

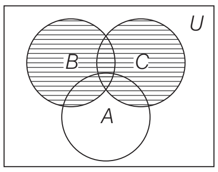


$(A \cup B) \cap (A \cup C)$

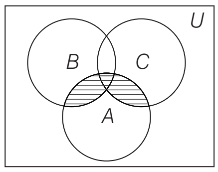
It is clear from diagrams that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

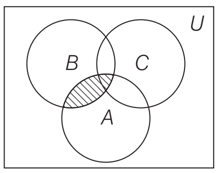
(ii)



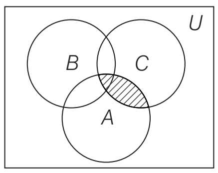
$(B \cup C)$



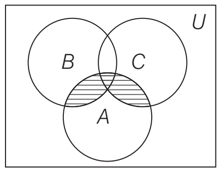
$A \cap (B \cup C)$



$(A \cap B)$



$(A \cap C)$

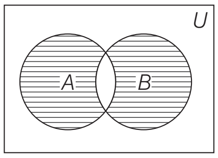


$(A \cap B) \cup (A \cap C)$

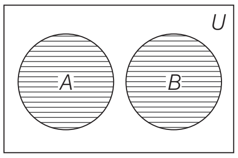
It is clear from diagrams that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

9. Symmetric difference



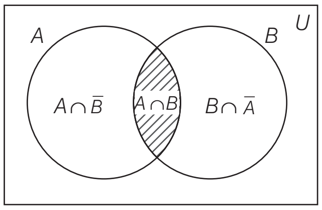
$A \Delta B$, when neither $A \subseteq B$ nor $B \subseteq A$ and $A \cap B \neq \phi$

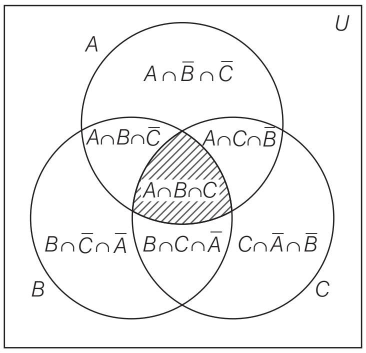


$A \Delta B$, when $A \cap B = \phi$

Remark

Remember with the help of figures.





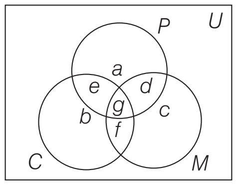
Example 9. A class has 175 students. The following table shows the number of students studying one or more of the following subjects in this case.

Subjects	Number of students
Mathematics	100
Physics	70
Chemistry	46
Mathematics and Physics	30
Mathematics and Chemistry	28
Physics and Chemistry	23
Mathematics, Physics and Chemistry	18

How many students are enrolled in Mathematics alone, Physics alone and Chemistry alone? Are there students who have not offered any one of these subjects?

Sol. Let P, C and M denotes the sets of students studying Physics, Chemistry and Mathematics, respectively.

Let a, b, c, d, e, f, g denote the elements (students) contained in the bounded region as shown in the diagram.



Then,
$$a + d + e + g = 170$$
$$c + d + f + g = 100$$
$$b + e + f + g = 46$$
$$d + g = 30$$
$$e + g = 23$$
$$f + g = 28$$
$$g = 18$$

After solving, we get $g = 18$, $f = 10$, $e = 5$, $d = 12$, $a = 35$, $b = 13$ and $c = 60$

$\therefore \quad a + b + c + d + e + f + g = 153$

So, the number of students who have not offered any of these three subjects = 175 − 153 = 22

Number of students studying Mathematics only, $c = 60$

Number of students studying Physics only, $a = 35$

Number of students studying Chemistry only, $b = 13$

Aliter

Let P, C and M be the sets of students studying Physics, Chemistry and Mathematics, respectively. Then, we are given that

$$n(P) = 70, n(C) = 46, n(M) = 100$$
$$n(M \cap P) = 30, n(M \cap C) = 28$$
$$n(P \cap C) = 23$$

and $\quad n(M \cap P \cap C) = 18$

$\therefore$ The number of students enrolled in Mathematics only

$= n(M \cap P' \cap C') = n(M \cap (P \cup C)')$

[by De-Morgan's law]

$= n(M) - n(M \cap (P \cup C))$

$= n(M) - \{n[(M \cap P) \cup (M \cap C)]\}$

[by distributive law]

$= n(M) - n(M \cap P) - (M \cap C) + n(M \cap P \cap C)$

$= 100 - 30 - 28 + 18 = 60$

Similarly, the number of students enrolled in Physics only,

$n(P \cap M' \cap C')$

$= n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$

$= 70 - 30 - 23 + 18 = 35$

and the number of students enrolled in Chemistry only,

$n(C \cap M' \cap P') = n(C) - n(C \cap M) - n(C \cap P) + n(C \cap M \cap P)$

$= 46 - 28 - 23 + 18 = 13$

and the number of students who have not offered any of the three subjects,

$n(M' \cap P' \cap C') = n(M \cap P \cap C)'$ [by De-Morgan's law]

$= n(U) - n(M \cup P \cup C)$

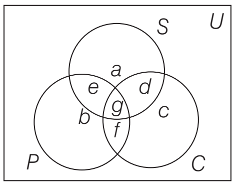
$= n(U) - \{n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(P \cap C \cap M)\}$

$= 175 - \{100 + 70 + 46 - 30 - 28 - 23 + 8\}$

$= 175 - 153 = 22$

Example 10. In a pollution study of 1500 Indian rivers the following data were reported. 520 were polluted by sulphur compounds, 335 were polluted by phosphates, 425 were polluted by crude oil, 100 were polluted by both crude oil and sulphur compounds, 180 were polluted by both sulphur compounds and phosphates, 150 were polluted by both phosphates and crude oil and 28 were polluted by sulphur compounds, phosphates and crude oil. How many of the rivers were polluted by atleast one of the three impurities?
How many of the rivers were polluted by exactly one of the three impurities?

Sol. Let S, P and C denote the sets of rivers polluted by sulphur compounds, by phosphates and by crude oil respectively, and let a, b, c, d, e, f, g denote the elements (impurities) contained in the bounded region as shown in the diagram.



Then, $a + d + e + g = 520$

$c + d + f + g = 425$

$b + e + f + g = 335 \Rightarrow d + g = 100$

$e + g = 180 \Rightarrow f + g = 150$

$g = 28$

After solving, we get

$g = 28, f = 122, e = 152, b = 33, d = 72, c = 203$ and $a = 268$

The number of rivers were polluted by atleast one of the three impurities

$= (a + b + c + d + e + f + g) = 878$

and the number of rivers were polluted by exactly one of the three impurities,

$a + b + c = 268 + 33 + 203 = 504$

Aliter

Let S, P and C denote the sets of rivers polluted by sulphur compounds, by phosphates and by crude oil, respectively.

Then, we are given that

$n(S) = 520, n(P) = 335, n(C) = 425, n(C \cap S) = 100,$
$n(S \cap P) = 180, n(P \cap C) = 150$ and $n(S \cap P \cap C) = 28.$

The number of rivers polluted by atleast one of the three impurities,

$n(S \cup P \cup C)$

$= n(S) + n(P) + n(C) - n(S \cap P) - n(P \cap C) - n(C \cap S) + n(S \cap P \cap C)$

$= 520 + 335 + 425 - 180 - 150 - 100 + 28 = 878$

and the number of rivers polluted by exactly one of the three impurities,

$n\{(S \cap P' \cap C') \cup (P \cap C' \cap S') \cup (C \cap P' \cap S')\}$

$= n\{(S \cap (P \cup C)'\} \cup \{P \cap (C \cup S)'\} \cup \{C \cap P \cup S)'\}$

$= n(S \cap (P \cup C)') + n(P \cap (C \cup S)') + n(C \cap (P \cup S)')$

$= n(S) - n(S \cap P) - n(S \cap C) + n(S \cap P \cap C) + n(P) - n(P \cap C) - n(P \cap S) + n(S \cap P \cap C) + n(C) - n(C \cap P) - n(C \cap S) + n(S \cap P \cap C)$

$= n(S) + n(P) + n(C) - 2n(S \cap P) - 2n(S \cap C) - 2n(P \cap C) + 3n(S \cap P \cap C)$

$= 520 + 335 + 425 - 360 - 200 - 300 + 84 = 504$

Exercise for Session 1

1. If $X = \{4^n - 3n - 1 : n \in N\}$ and $y = \{9(n-1) : n \in N\}$, then $X \cup Y$ equals

(a) X (b) Y (c) N (d) None of these

2. If $N_a = \{an : n \in N\}$, then $N_5 \cap N_7$ equals

(a) N (b) N_5 (c) N_7 (d) N_{35}

3. If A and B are two sets, then $A \cap (A \cup B)'$ equals

(a) A (b) B (c) ϕ (d) None of these

4. If U be the universal set and $A \cup B \cup C = U$, then $[(A - B) \cup (B - C) \cup (C - A)']$ equals

(a) $A \cup B \cup C$ (b) $A \cap B \cap C$ (d) $A \cup (B \cap C)$ (d) $A \cap (B \cup C)$

5. If A and B are two sets, then $(A - B) \cup (B - A) \cup (A \cap B)$ equals

(a) $A \cup B$ (b) $A \cap B$ (c) A (d) B'

6. If $A = \{x : x$ is a multiple of 4$\}$ and $B = \{x : x$ is a multiple of 6$\}$, then $A \subset B$ consists of all multiple of

(a) 4 (b) 8 (c) 12 (d) 16

7. A set contains $2n + 1$ elements. The number of subsets of this set containing more than n elements equals

(a) 2^{n-1} (b) 2^n (c) 2^{n+1} (d) 2^{2n}

8. If $A = \{\phi, \{\phi\}\}$, then the power set of A is

(a) A (b) $\{\phi, \{\phi\}, A\}$

(c) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ (d) None of these

9. Given $n(U) = 20$, $n(A) = 12$, $n(B) = 9$, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n((A \cup B)')$ equals

(a) 3 (b) 9 (c) 11 (d) 17

10. A survey shows that 63% of the Indians like cheese, whereas 76% like apples. If x % of the Indians like both cheese and apples, then x can be

(a) 40 (b) 65 (c) 39 (d) None of these

11. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is

(a) 6 (b) 7 (c) 9 (d) 22

Answers

Exercise for Session 1

1. (b)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (c)
7. (d)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (d)	