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Waves

Waves occur when a system is disturbed from its equilibrium position and this disturbance propagates from one region to other.

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Wave Motion

Wave motion involves transfer of disturbance (energy) from one point to another without actual transport of matter between the two points.

In this motion, the disturbance travels through the medium due to the repeated periodic oscillations of the particles of the medium about their mean positions.

Types of Waves

Waves can be classified into two broad categories

(i) **Mechanical waves** These are the waves which requires a material medium for their propagation, *e.g.* sound waves, water waves, etc.

For propagation of mechanical waves, medium must possess inertia, elasticity and minimum frictional force.

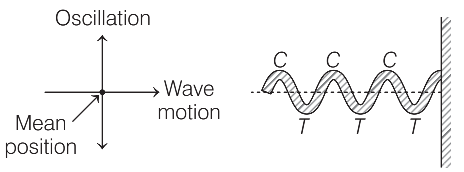
(ii) **Non-mechanical waves** These are the waves which can travel with or without vacuum, *e.g.* electromagnetic waves.

Note All electromagnetic waves are transverse in nature.

Mechanical waves are further classified into two categories

(i) Transverse Wave

In this type of wave, the particles of the medium oscillate perpendicular to the direction in which the wave travels. *e.g.* Travelling waves on a tight rope. In this, disturbance travels along the rope in the form of crests (upward peaks) and troughs (downward peaks).



(ii) Longitudinal Wave

In this type of wave, the particles of the medium oscillate about their mean or equilibrium position along the direction of propagation of the wave motion itself. *e.g.* Waves on springs or sound waves in air.

This wave travels in the form of compressions and rarefactions.

Some important terms related to wave motion are as follows

(i) **Wavelength** It is the distance travelled by the disturbance in one time period, *i.e.* the time in which particle of the medium completes one oscillation.

In case of a transverse wave, wavelength (λ) is equal to the distance between two consecutive crests or troughs.

However, in case of a longitudinal wave, wavelength (λ) is equal to the distance between two consecutive compressions or rarefactions.

(ii) **Frequency** (ν) It is the number of waves produced per unit time in the given medium.

If T is period of oscillation of the particle in the medium, then

$$\nu = \frac{1}{T}$$

(iii) **Velocity** Since, in one time period (T), wave travels a distance which is equal to its wavelength (λ).

Hence, velocity of wave is given by

$$v = \frac{\lambda}{T} = \left(\frac{1}{T}\right)\lambda \quad \text{or} \quad v = \nu\lambda$$

Progressive Waves

A wave which travels from one point of the medium to another continuously in the same direction without any change in its amplitude is called a progressive wave or a *travelling wave*. It may be transverse or longitudinal.

General form of a Progressive Wave

For a wave travelling from left to right, *i.e.* along positive direction of X-axis with velocity v, its displacement relation is given by $y(x, t) = f(x - vt)$

Here, x is the distance of wave pulse from origin.

Similarly, for a wave pulse travelling from right to left, *i.e.* along negative direction of X-axis, the wave function will be

$$y(x, t) = f(x + vt)$$

Displacement Relation for Plane Progressive Harmonic Wave

During the propagation of a wave through a medium, if the particles of the medium oscillate simple harmonically about their mean position, then the wave is said to be *plane progressive harmonic wave.*

For such a wave travelling in + x-direction, displacement relation is given as

$$y = A\sin\left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \pm \phi\right\}$$

In terms of velocity of wave (v), displacement relation is

$$y = A\sin\left\{\frac{2\pi}{\lambda}(vt - x) \pm \phi\right\}$$

Different forms of displacement relation for progressive waves are as follows

$$\begin{aligned} y &= A\sin\{(\omega t \pm kx) \pm \phi\} \\ &= A\sin\{k(vt \mp x) \pm \phi\} \\ &= A\sin\left\{\omega\left(t \mp \frac{x}{v}\right) \pm \phi\right\} \end{aligned}$$

Description of the various terms in the displacement relation of plane progressive waves

- ω is the *angular frequency* of the particle oscillating in SHM.

  $\omega = 2\pi\nu = \frac{2\pi}{T}$, where ν is the natural frequency and T is the time period.
- t is the time elapsed from when the wave begin its motion, *i.e.* time elapsed, since origin of the wave.
- k is the *propagation constant* or wave number.

  $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$, where v is wave velocity.
- The coefficient of sin or cos function, *i.e.* A gives the amplitude of the wave while its argument ($\omega t \pm kx$) denotes its phase.
- ϕ represents *phase constant* or initial phase.

Important points related to plane progressive wave

- The following equations

  $y = A\sin(kx - \omega t)$ and $y = A\sin(\omega t - kx)$ represent a wave travelling in positive x-direction with speed $v = \frac{\omega}{k}$.

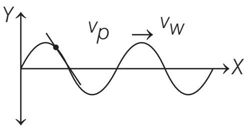
  However, the difference between them is that they are out of phase, *i.e.* phase difference between them is π.
- For a particle with displacement, $y = A\sin(\omega t - kx)$

  Particle's velocity, $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

  $\therefore$ Particle's acceleration, $a_p = \frac{dv_p}{dt} = -\omega^2 y$.
- For a wave travelling along positive X-axis as Shown,

  $$v_p = -\text{ wave velocity} \times \text{slope}$$

i.e. Particle's velocity at a given position and time is equal to negative of the product of wave velocity with the slope of the wave at that point at that instant.



- **Phase difference and path difference** At any instant t, if ϕ_1 and ϕ_2 are the phases of two particles whose distances from the origin are x_1 and x_2 respectively, then

$$\phi_2 - \phi_1 = k(x_2 - x_1)$$

or Phase difference, $\Delta\phi = \frac{2\pi}{\lambda}$ (Path difference Δx)

- **Phase difference and time difference** If the phases of a particle at distance x from the origin is ϕ_1 at time t_1 and ϕ_2 at time t_2, then $\phi_1 - \phi_2 = \omega(t_1 - t_2)$

or Phase difference $(\Delta\phi) = \frac{2\pi}{T}$(Time difference Δt)

Example 1. *A wave travelling along a string is described by*

$$y(x, t) = 0.005 \sin(80x - 3t)$$

in which numerical constants are in SI units (0.005 m, 80 rad m^{-1}, 3 rad s^{-1}). The wavelength λ is then given by

(a) 2.85 *cm* (b) 7.85 *cm* (c) 10.83 *cm* (d) 18.2 *cm*

Sol. (*b*) Comparing the given displacement equation with standard equation,

$$y(x, t) = a\sin(kx - \omega t)$$

We have, amplitude of the wave is 0.005 m = 5 mm, angular wave number k and angular frequency ω are $k = 80\ \text{m}^{-1}$ and $\omega = 3\ \text{s}^{-1}$.

Then, from equation

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{80\ \text{m}^{-1}} = 7.85\ \text{cm}$$

Example 2. *Equation of a transverse wave travelling in a rope is given by $y = 5\sin(4.0t - 0.02x)$, where y and x are expressed in cm and time in seconds. Then, the velocity of the wave and acceleration of the particle of rope respectively is*

(a) 200 cms^{-1}, 40 cms^{-2} (b) 200 cms^{-1}, 80 cms^{-2}
(c) 250 cms^{-1}, 20 cms^{-2} (d) *None of these*

Sol. (*b*) Given, $y = 5\sin(4.0t - 0.02x)$.

Comparing this with the standard equation of wave motion,

$$y = A\sin\left(2\pi ft - \frac{2\pi}{\lambda}x\right)$$

where, A, f and λ are amplitude, frequency and wavelength, respectively. Thus, amplitude, $A = 5$ cm, $2\pi f = 4$

Frequency, $f = \frac{4}{2\pi} = 0.673$ cycle s^{-1}

Again, $\frac{2\pi}{\lambda} = 0.02$

or wavelength, $\lambda = \frac{2\pi}{0.02} = 100\pi$ cm

Velocity of the wave, $v = f\lambda = \frac{4}{2\pi} \times \frac{2\pi}{0.02} = 200\ \text{cms}^{-1}$

Transverse velocity of the particle,

$$u = \frac{\partial y}{\partial t} = 5 \times 4\cos(4.0t - 0.02x)$$

$$= 20\cos(4.0t - 0.02x)$$

Maximum velocity of the particle $= 20\ \text{cms}^{-1}$

Particle acceleration, $a = \frac{\partial y}{\partial t} = 20 \times 4\cos(4.0t - 0.02x)$

Maximum particle acceleration $= 80\ \text{cms}^{-2}$

Energy in Wave Motion

Related to the energy associated with the wave motion. There are three terms namely : energy density (u), power (P) and intensity (I).

Energy density (u) It is defined as the total mechanical energy (kinetic + potential) per unit volume of the medium through which the wave is travelling.

$$\therefore \quad u = \frac{E}{V} = \frac{\frac{1}{2}m\omega^2A^2}{V} = \frac{1}{2}\rho\omega^2A^2 \qquad \left(\because \rho = \frac{m}{V}\right)$$

Power (P) If we consider a transverse wave on a string, then the instantaneous rate at which energy is transferred along the string is called power. Thus,

$$\text{Power}\ (P) = \frac{1}{2}\rho\omega^2A^2\Delta v$$

Intensity (I) It is the flow of energy per unit area of cross-section of the string in unit time. Thus,

$$I = \frac{\text{Power}}{\text{Area of cross-section}} = \frac{P}{S}$$

or $$I = \frac{1}{2}\rho\omega^2A^2v$$

Note The intensity of waves emitting in all directions due to a point source varies inversely as the square of the distance (r).

i.e. $I \propto \frac{1}{r^2}$

The intensity of waves from a linear source varies inversely as the distance (r) perpendicular to the source, *i.e.* $I \propto \frac{1}{r}$.

Speed of Transverse Waves

The expression for speed of transverse waves in a stretched string is given as $v = \sqrt{\frac{T}{m}}$

where, T is tension and m is linear mass density of the string.

Example 3. *A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire, so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C? (Take, speed of sound in dry air at 20° C = 343 m/s)*

(a) 2.06×10^3 *N* (b) 2.06×10^4 *N*
(c) 3.8×10^5 *N* (d) 3.8×10^6 *N*

Sol. (*b*) Here, speed of sound in air, $v = 343\ \text{ms}^{-1}$;

Length of the wire, $l = 12.0$ m;

Total mass of the wire, $M = 2.10$ kg

Therefore, mass per unit length of the wire,

$$m = \frac{M}{l} = \frac{2.10}{12.0}$$
$$= 0.175\ \text{kgm}^{-1}$$

Now, $$v = \sqrt{\frac{T}{m}}$$

or $$T = v^2 m = (343)^2 \times 0.175$$
$$= 20588.6\ \text{N}$$
$$= 2.06 \times 10^4\ \text{N}$$

Example 4. *Equation of travelling wave on a stretched string of linear mass density 5 g/m is $y = 0.03\ \sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is* **[JEE Main 2019]**

(a) 5 *N* (b) 12.5 *N*
(c) 7.5 *N* (d) 10 *N*

Sol. (*b*) Given equation can be rewritten as

$$y = 0.03\ \sin 450\left(t - \frac{9x}{450}\right) \quad \ldots\text{(i)}$$

We know that, the general equation of a travelling wave is given as

$$y = A \sin \omega (t - x/v) \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

Velocity, $v = \frac{450}{9} = 50$ m/s

and angular velocity, $\omega = 450$ rad/s

As, the velocity of wave on stretched string with tension (T) is given as $v = \sqrt{T/\mu}$

where, μ is linear density.

$$\therefore \quad T = \mu v^2$$
$$= 5 \times 10^{-3} \times 50 \times 50$$
$$= 12.5\ \text{N} \quad (\because \text{given}, \mu = 5\ \text{g/m} = 5 \times 10^{-3}\ \text{kg/m})$$

Example 5. *A stretched string is forced to transmit transverse waves by means of an oscillator coupled to one end. The string has a diameter of 4 mm. The amplitude of the oscillation is 10^{-4} m and the frequency is 10 Hz. Tension in the string is 100 N and mass density of wire is $4.2 \times 10^3\ \text{kgm}^{-3}$. Then,*

(a) *the equation of the waves along the string is $(10^{-4})\ m \sin [(1.44\ m^{-1}) \times - (62.83\ rads^{-1})]\ t$*

(b) *the energy per unit volume of the wave $8.29 \times 10^{-2}\ Jm^{-3}$.*

(c) *the average energy flow per unit time across any section of the string $4.53 \times 10^{-5}\ Js^{-1}$.*

(d) *All of the above*

Sol. (*d*) Speed of transverse wave on the string, $v = \sqrt{\frac{T}{\rho S}}$

Substituting the values, we have

$$v = \sqrt{\frac{100}{(4.2 \times 10^3)\left(\frac{\pi}{4}\right)(4.0 \times 10^{-3})^2}}$$
$$= 43.53\ \text{ms}^{-1}$$
$$\omega = 2\pi f = 20\ \pi\ \text{rad/s}$$
$$= 62.83\ \text{rad/s}$$
$$k = \frac{\omega}{v} = 1.44\ \text{m}^{-1}$$

$\therefore$ Equation of the waves along the string,

$$y(x, t) = A \sin (kx - \omega t)$$
$$= (10^{-4}\ \text{m}) \sin [(1.44\ \text{m}^{-1})\, x - (62.83\ \text{rad s}^{-1})\, t]$$

Energy per unit volume of the string,

$$u = \text{energy density} = \frac{1}{2}\rho\omega^2 A^2$$

Substituting the values, we have

$$u = \left(\frac{1}{2}\right)(4.2 \times 10^3)\,(62.83)^2\,(10^{-4})^2$$
$$= 8.29 \times 10^{-2}\ \text{Jm}^{-3}$$

Average energy flow per unit time,

$$= \left(\frac{1}{2}\rho\omega^2 A^2\right)(Sv) = (u)\,(Sv)$$

Substituting the values, we have

$$P = (8.29 \times 10^{-2})\left(\frac{\pi}{4}\right)(4.0 \times 10^{-3})^2\,(43.53)$$
$$= 4.53 \times 10^{-5}\ \text{Js}^{-1}$$

Sound Waves

These waves are longitudinal in nature. They are classified into three groups according to their range of frequencies, namely, *infrasonic waves* (below 20Hz), *audible waves* (20Hz-20 kHz) and *ultrasonic waves* (above 20 kHz).

Speed of Sound Wave

The speed of longitudinal wave (sound wave) in a medium depends on the elastic and inertial properties of medium.

- Speed of sound wave in a medium is given as

$$v = \sqrt{\frac{B}{\rho}}$$

where, B is bulk modulus and ρ is the density of the medium.

- Speed of longitudinal wave (sound wave) in a solid bar,

$$v = \sqrt{\frac{Y}{\rho}}$$

where, Y is the Young's modulus of the medium.

- Speed of sound wave through a solid of bulk modulus B and modulus of rigidity η is given as

$$v = \sqrt{\frac{B + (4/3)\eta}{\rho}}$$

- **Newton's formula** Speed of longitudinal waves in gases,

$$v = \sqrt{\frac{p}{\rho}}$$

where, p is the pressure exerted by the gas,

ρ is the density of the gas at given temperature.

By this formula, speed of sound at STP is approx. 250 m/s, which is smaller as compared to the experimental value of 331 ms^{-1}.

- **Laplace's correction** After modifying Newton's formula, speed of sound in gases is given as

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

where, γ = ratio of two specific heat capacities (C_p/C_V) of the gas.

By this formula, speed of sound comes out to be 331.3 ms^{-1}.

Factors Affecting Speed of Sound in a Gas

- **Effect of temperature** $v \propto \sqrt{T}$

i.e. with the increase in temperature (in kelvin), velocity of sound in a gas also increases.

If velocity of sound at 0°C is 332 m/s, then its value at t°C will be

$$v_t = 332 + 0.61\,t$$

Thus, velocity of sound in air increases roughly by 0.61 m/s per degree centigrade rise in temperature.

- **Effect of pressure** If the temperature of the gas remains constant, then there is no effect of the pressure change on the speed of sound.
- **Effect of humidity** Speed of sound increases with the increase in humidity. Thus, speed of sound in moist air is slightly greater than that in dry air.

Note (i) Speed of sound is greater in solids, and liquids than in gases even though they are denser than gases.

(ii) Speed of sound in air is independent of its frequency.

Example 6. *The density of air at NTP is 1.29 kgm^{-3}. Assume air to be diatomic with $\gamma = 1.4$. The velocity of sound at 127°C is*

(a) 382.8 ms^{-1} (b) 350 ms^{-1}
(c) 350.6 ms^{-1} (d) 348.6 ms^{-1}

Sol. (a) Velocity of sound in air at NTP

$$= \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{1.4 \times 1.013 \times 10^5\ \text{Nm}^{-2}}{1.29\ \text{kgm}^{-3}}} = 331.6\ \text{ms}^{-1}$$

The velocity of sound is proportional to the square root of absolute temperature

$$\Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\Rightarrow \quad v_2 = v_1\sqrt{\frac{T_2}{T_1}} = 331.6\sqrt{\frac{273+127}{273+27}} = 382.8\ \text{ms}^{-1}$$

Displacement or Pressure Waves

A sound wave can be considered as either a displacement wave,

$$y = A\sin(\omega t - kx)$$

or a pressure wave,

$$p = p_0\cos(\omega t - kx)$$

i.e. Pressure wave is 90° out of phase with respect to displacement wave.

Also, amplitude of pressure wave,

$$p_m = ABk = Ak\rho v^2 = \rho v A\omega$$

where, B is bulk modulus.

Example 7. *The pressure wave $p = 0.01\sin[1000\,t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0° C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is* **[JEE Main 2019]**

(a) 15° C (b) 11° C (c) 12° C (d) 4° C

Sol. (d) Given, $p = 0.01\sin(1000\,t - 3x)$ N/m^2

Comparing with the general equation of pressure wave of sound, *i.e.* $p_0\sin(\omega t - kx)$, we get

$$\omega = 1000 \text{ and } k = 3$$

Also, $$k = \frac{\omega}{v}$$

$$\Rightarrow \quad v = \omega / k$$

$$\therefore \text{ Velocity of sound, } |v_1| = \frac{1000}{3}$$

Or

Speed of sound wave can also be calculated as

$$v = -\frac{(\text{coefficient of } t)}{(\text{coefficient of } x)} = -\frac{1000}{(-3)} = \frac{1000}{3}\ \text{m/s}$$

Now, relation between velocity of sound and temperature is

$$v = \sqrt{\frac{\gamma RT}{m}}$$

$$\Rightarrow \quad v \propto \sqrt{T}$$

or $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow T_2 = \frac{v_2^2}{v_1^2}\cdot T_1$$

Here, $v_2 = 336$ m/s, $v_1 = 1000/3$ m/s,

$$T_1 = 0°\text{C} = 273\ \text{K}$$

$$\therefore \quad T_2 = \frac{(336)^2}{(1000/3)^2} \times 273 = 277.38\ \text{K}$$

$$\therefore \quad T_2 = 4.38°\text{C} \simeq 4°\text{C}$$

Intensity of Sound Waves

Intensity of sound waves in terms of pressure is given as

$$I = \frac{v(\Delta p)_m^2}{2B}$$

where, Δp_m is the amplitude of pressure variation.

If the intensity of sound waves in watt per square metre is I, then the intensity level β in decibels (dB) is given by

$$\beta = 10\log_{10}\frac{I}{I_0}$$

where, $I_0 \cong 10^{-12}\ \text{W/m}^2$

For comparison of two different sounds in dB, we can write

$$\beta_2 - \beta_1 = 10\log_{10}\frac{I_2}{I_1}$$

Note Threshold of hearing is 0 dB and threshold of pain is 120 dB.

Example 8. *Assume that the displacement(s) of air is proportional to the pressure difference Δp created by a sound wave. Displacement further depends on the speed of sound v, density of air ρ and the frequency f. If $\Delta p = 10$Pa, v = 300 m/s, $\rho = 1$ kg/m^3 and $f = 1000$ Hz, then displacement(s) will be the order of (Take, the multiplicative constant to be 1)* **[JEE Main 2020]**

(a) 10 mm (b) 1 mm
(c) $\frac{1}{10}$ mm (d) $\frac{3}{100}$ mm

Sol. (d) The pressure difference is given by

$$\Delta p = \frac{B\omega}{v}\times s$$

$\therefore$ Displacement of air, $s = \frac{\Delta p \times v}{B\omega} = \frac{\Delta p v}{\rho v^2 \omega}$

$$= \frac{\Delta p}{\rho v \omega} = \frac{10}{1\times 300\times 1000}$$

$$\approx \frac{3}{100}\text{ mm}$$

Example 9. *A window whose area is 2 m^2 opens on a street where the street noise results in an intensity level at the window of 60 dB. Now, if a sound absorber is fitted at the window, how much energy from the street will it collect in a day?*

(a) 0.73 J (b) 0.173 J (c) 2.73 J (d) 1.73 J

Sol. (b) By definition, sound level $= 10\log\frac{I}{I_0} = 60$

or $\frac{I}{I_0} = 10^6$

$I = 10^{-12}\times 10^6 = 1\mu\ \text{Wm}^{-2}$ $[\because I_0 = 10^{-12}\ \text{Wm}^{-2}]$

Power entering the room $= 1\times 10^{-6}\times 2 = 2\ \mu$W

Energy collected in a day $= 2\times 10^{-6}\times 86400 = 0.173$ J

Example 10. *A small speaker delivers 2 W of audio output. At what distance from the speaker will an observer detect 120 dB intensity sound? (Take, reference intensity of sound as 10^{-12} W/m^2)* **[JEE Main 2019]**

(a) 40 cm (b) 20 cm (c) 10 cm (d) 30 cm

Sol. (a) Loudness of sound in decibel is given by $\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$

where, I = intensity of sound in W/m^2,

I_0 = reference intensity $(= 10^{-12}\ \text{W/m}^2)$, chosen because it is near the lower limit of the human hearing range.

Here, $\beta = 120$ dB

So, we have $120 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$

$\Rightarrow \quad 12 = \log_{10}\left(\frac{I}{10^{-12}}\right)$

Taking antilog, we have

$\Rightarrow \quad 10^{12} = \frac{I}{10^{-12}}$

$\Rightarrow \quad I = 1\ \text{W/m}^2$

This is the intensity of sound reaching the observer.

Now, intensity, $I = \frac{P}{4\pi r^2}$

where, r = distance from source,
P = power of output source.

Here, $P = 2$ W, we have

$$1 = \frac{2}{4\pi r^2}$$

$\Rightarrow \quad r^2 = \frac{1}{2\pi}$

$\Rightarrow \quad r = \sqrt{\frac{1}{2\pi}}\ \text{m} = 0.398\ \text{m} \approx 40\ \text{cm}$

Principle of Superposition of Waves

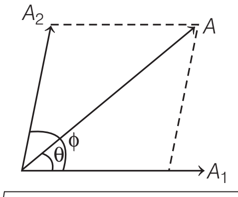
Two or more waves can travel independently in a medium without affecting the motion of one another. Thus, the resultant displacement of each particle of the medium at any instant is equal to the vector sum of displacements produced by the two waves separately. This principle is called principle of superposition of waves.

At a given point for two waves arriving with a phase difference ϕ and with equations

$$y_1 = A_1\sin(\omega t - kx)$$
$$y_2 = A_2\sin(\omega t - kx + \phi)$$

By principle of superposition, resultant wave is given by

$$\mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2 = A\sin(\omega t - kx + \phi)$$



where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Interference of Waves

When two coherent waves of same frequency propagate in same direction and superimpose on each other, then the intensity of the resultant wave becomes maximum at some points and minimum at some points. This phenomenon of intensity variation is called interference of waves.

If I_1 and I_2 are intensities of the interfering waves and ϕ is the phase difference, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

If $I_1 = I_2 = I_0$, then $I = 4I_0 \cos^2 \frac{\phi}{2}$.

Constructive interference These are those points, where the resultant amplitude/intensity is maximum.

For maximum amplitude,

$$\cos\phi = +1$$

or $\phi = 0, 2\pi, \ldots, 2n\pi$

$\Rightarrow$ $A = A_{\max} = A_1 + A_2$

$$I = I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

Destructive interference These are those points, where resultant amplitude/ intensity is minimum.

For minimum amplitude,

$$\cos\phi = -1$$

or $\phi = \pi, 3\pi, \ldots, (2n-1)\pi$

$\Rightarrow$ $A = A_{\min} = A_1 - A_2$

$$I = I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

In interference,

$$\frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

Example 11. *Two coherent sound sources are at distances $x_1 = 0.20$ m and $x_2 = 0.48$ m from a point. The intensity of the resultant wave at that point, if the frequency of each wave is $f = 400$ Hz and velocity of wave in the medium is $v = 448\ ms^{-1}$ is (Take, the intensity of each wave is $I_0 = 60\ Wm^{-2}$)*

(a) 120 Wm^{-2} (b) 125 Wm^{-2}
(c) 130 Wm^{-2} (d) 135 Wm^{-2}

Sol. (a) Path difference, $\Delta x = x_2 - x_1 = 0.48 - 0.20 = 0.28$ m

Phase difference, $\phi = \frac{2\pi}{\lambda}\Delta x = \left(\frac{2\pi f}{v}\right)\Delta x$

$$= \frac{2\pi(400)(0.28)}{448} = \frac{\pi}{2}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

or $I = I_0 + I_0 + 2I_0\cos(\pi/2)$

$$= 2I_0 = 2(60) = 120\ \text{Wm}^{-2}$$

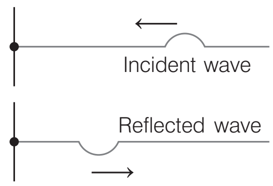
Reflection and Transmission of Waves

When sound waves are incident on a boundary separating two media, a part of it is reflected back into the initial medium while the remaining is partly absorbed and partly transmitted into the second medium.

Following are the characteristics of reflection and transmission of waves

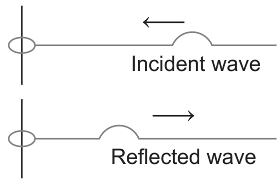
(i) In case of reflection from a denser medium or rigid support or fixed end, there is an inversion of the reflected displacement wave, *i.e.* if the incident wave is $y = A_i \sin(\omega t - kx)$, then reflected wave will be

$$y = -A_r \sin(\omega t + kx)$$
$$= A_r \sin(\omega t + kx + \pi)$$



Reflection from fixed end

i.e. In case of reflection from a denser medium, displacement wave change its phase by π. While in case of reflection from a rarer medium or free end no inversion of wave or phase change occurs.



Reflection from free end

(ii) In case of pressure wave, there is no phase change when the wave is reflected from a denser medium or fixed end.

Wave property	Reflection	Transmission (Refraction)
v	does not change	changes
f, T, ω	do not change	do not change
λ, k	do not change	change
A, I	change	change
ϕ	$\Delta\phi = 0$, from a rarer medium $\Delta\phi = \pi$, from a denser medium	$\Delta\phi = 0$

Note (i) The concept of rarer or denser medium for a wave is through speed (and not density of medium). For example, water is rarer for sound and denser for light than air, as for sound $v_w > v_a$, while for light $v_w < v_a$.

(ii) Multiple reflection of sound is called an *echo*. If the distance of reflector from the source is, d and t is the time of echo, then

$$d = \frac{vt}{2}.$$

Standing or Stationary Waves

When two harmonic waves of equal frequency and amplitude travelling through a medium in the opposite directions along the same line superimpose, then we get stationary waves.

For two such waves having equations,

$$y_1 = A\sin(kx - \omega t) \text{ and } y_2 = A\sin(kx + \omega t)$$

The resultant equation after interference will be

$$y = 2A\sin(kx)\cos(\omega t) = 2A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$$

This is the equation of standing wave.

Following are some important points related to standing waves

(i) **Nodes** are the points at which the amplitude is always zero, *i.e.* destructive interference points.

Position of node is, $x = \frac{n\lambda}{2}$ (where, $n = 0, 1, 2 \ldots$)

At nodes, there is no motion.

(ii) **Antinodes** are the points at which amplitude is maximum, *i.e.* constructive interference points.

Position of antinode is,

$$x = (2n + 1)\frac{\lambda}{4} \quad \text{(where, } n = 0, 1, 2, \ldots)$$

(iii) The distance between two consecutive nodes or two consecutive antinodes is $\lambda/2$. However, the distance between a node and the next antinode is $\lambda/4$.

(iv) All the particle at any particular point x executes simple harmonic motion except those at nodes.

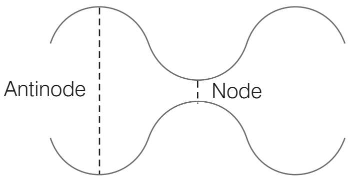
(v) The amplitude is not the same for different particles. It varies gradually from zero at nodes to maximum at antinodes.

(vi) Energy is not transported along the string to the right or to the left, because energy cannot flow past the nodes points in the string which are permanently at rest.

(vii) Due to persistance of vision, these waves appear in the form of loops. All the particles in a loop are in the same phase. But the particles in adjacent loops differ in phase by π.

(viii) Stationary waves may be transverse or longitudinal.

(ix) Two identical waves moving in opposite directions along the string will still produce standing waves even, if their amplitudes are unequal (as shown in figure).



The standing wave ratio (SWR) is defined as

$$= \frac{A_{\text{max}}}{A_{\text{min}}} = \frac{A_i + A_r}{A_i - A_r}$$

For 100% reflection SWR = ∞ and for no reflection SWR = 1

(x) Energy in standing waves in given loop between two nodes oscillates between elastic PE and KE of the particles of the medium. When the particles are at their mean position, KE is maximum while elastic PE is minimum.

When particles are at their extreme positions KE is minimum while elastic PE is maximum.

Vibrations of Air Columns and Strings

Few terms related to standing waves in air columns and strings are given below

(i) When a sound source produces sound waves, it consists of mixture of many frequencies. This mixture of sound is called *note*.

(ii) If produced sound constains only one frequency, then it is called *tone*.

(iii) Tone of minimum frequency is called *fundamental tone*.

(iv) Those tones which are having frequencies greater than fundamental tones are called *overtones*.

(v) Sound produced from a source consists of frequencies which contains fundamental frequency as well as frequencies which are multiple of fundamental frequency, called *harmonics*.

The ratio between the frequencies of two notes is called the *musical interval*.

Following are the names of some musical intervals

(a) Unison $\frac{n_2}{n_1} = 1$

(b) Octave $\frac{n_2}{n_1} = 2$

(c) Major tone $\frac{n_2}{n_1} = \frac{9}{8}$

(d) Minor tone $\frac{n_2}{n_1} = \frac{10}{9}$

(e) Semi tone $\frac{n_2}{n_1} = \frac{16}{15}$

(f) Fifth tone $\frac{n_2}{n_1} = \frac{3}{2}$

Stationary Waves in Strings

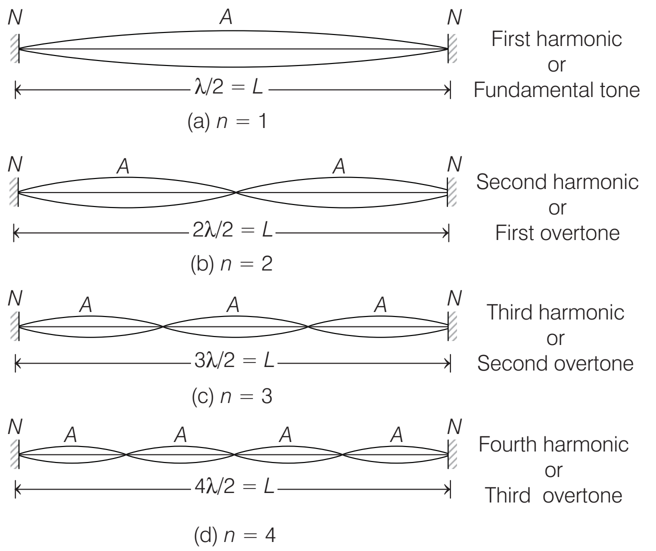
Stationary waves in the strings can be produced by two ways, *i.e.*

(i) **When both the ends of the string are fixed**

A string of length L is stretched between two fixed points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end.

The incident and the reflected waves interfere to produce a stationary transverse wave in which the ends are always nodes.

Various modes of vibrations of a stretched string are shown below



Possible wavelength of stationary wave are

$$\lambda = \frac{2L}{n};\ n = 1, 2, 3, \ldots$$

$\therefore$ Natural frequencies of oscillations are

$$f = n\left(\frac{v}{2L}\right) = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

So, fundamental frequency or first harmonic is

$$f_1 = \frac{v}{2L}$$

Similarly, $f_2 = \dfrac{2v}{2L} = 2f_1$

(first overtone or second hermonic)

$$f_3 = \frac{3v}{2L} = 3f_1$$

(second overtone or third harmonic)

(ii) **When one end of the string is fixed and other end is free to move**

Fundamental frequency of vibration or first harmonic is

f_0 or $n_0 = \dfrac{v}{4l}$

Frequency of third harmonic, $n_1 = \dfrac{3v}{4l} = 3n_0$

Frequency of fifth harmonic, $n_2 = \dfrac{5v}{4l} = 5n_0$

$$n_0 : n_1 : n_2 \ldots\ldots = 1:3:5 : \ldots..$$

Laws of Vibration of Stretched String

- **Law of length** For a given wire under a given tension, the frequency of wire varies inversely as its vibrating length, *i.e.* $f \propto \dfrac{1}{L}$

  or $f_1 L_1 = f_2 L_2$

  where, T and m are constants.
- **Law of tension** For a uniform wire of given length and material, the frequency of the wire varies directly as the square root of tension, *i.e.*

  $$f \propto \sqrt{T}$$

  or $$\frac{f_1}{f_2} = \sqrt{\left(\frac{T_1}{T_2}\right)}$$

  where, L and m are constants.
- **Law of mass** When L and T are constants, the frequency of vibration of the wire varies inversely as the square root of mass per unit length of the wire, *i.e.*

  $$f \propto \frac{1}{\sqrt{m}}$$

  where, L and T are constants.

  So, $$f \propto \frac{1}{L} \propto \frac{1}{\sqrt{m}}$$

  Hence, a graph between L and $\sqrt{m}$ is a straight line.

Example 12. *A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is* ***[JEE Main 2019]***

(a) 180 m/s, 80 Hz
(b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz
(d) 180 m/s, 120 Hz

Sol. (*b*) Frequency of vibration of a string in nth harmonic is given by

$$f_n = n \cdot \frac{v}{2L} \qquad \ldots \text{(i)}$$

where, v = speed of sound and L = length of string.

Here, $f_3 = 240$ Hz, $L = 2$ m and $n = 3$

Substituting these values in Eq. (i), we get

$$240 = 3 \times \frac{v}{2 \times 2}$$

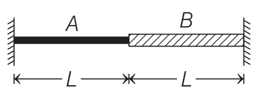
$$\Rightarrow \quad v = \frac{4 \times 240}{3} = 320 \text{ ms}^{-1}$$

Also, fundamental frequency,

$$f = \frac{f_n}{n} = \frac{f_3}{3}$$

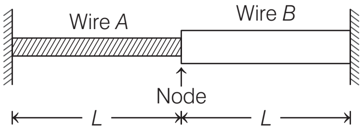
$$= \frac{240}{3} = 80 \text{ Hz}$$

Example 13. *A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q, then the ratio p : q is* ***[JEE Main 2019]***



(a) 3 : 5
(b) 4 : 9
(c) 1 : 2
(d) 1 : 4

Sol. (c)



Let mass per unit length of wires are μ_A and μ_B, respectively.

$\because$ For same material, density is also same.

So, $$\mu_A = \frac{\rho \pi r^2 L}{L} = \mu$$

and $$\mu_B = \frac{\rho 4 \pi r^2 L}{L} = 4\,\mu$$

Tension (T) in both connected wires are same.

So, speed of wave in wires are

$$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{T}{\mu}} \qquad (\because \mu_A = \mu \text{ and } \mu_B = 4\,\mu)$$

and $$v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{T}{4\,\mu}}$$

So, nth harmonic in such wires system is

$$f_{\text{nth}} = \frac{pv}{2L}$$

$$\Rightarrow \quad f_A = \frac{pv_A}{2L} = \frac{p}{2L}\sqrt{\frac{T}{\mu}} \qquad \text{(for } p \text{ antinodes)}$$

Similarly, $$f_B = \frac{qv_B}{2L} = \frac{q}{2L}\sqrt{\frac{T}{4\,\mu}} = \frac{1}{2}\left(\frac{q}{2L}\sqrt{\frac{T}{\mu}}\right) \qquad \text{(for } q \text{ antinodes)}$$

As frequencies f_A and f_B are given equal.

So, $$f_A = f_B$$

$$\Rightarrow \quad \frac{p}{2L}\sqrt{\frac{T}{\mu}} = \frac{q}{2}\left[\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right]$$

$$\frac{p}{q} = \frac{1}{2}$$

$$\Rightarrow \quad p : q = 1 : 2$$

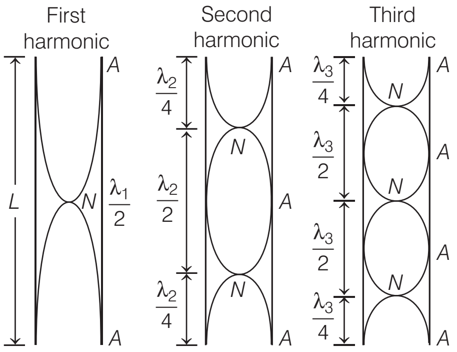
Stationary Waves in Air Column

When longitudinal waves propagate in a fluid in a pipe with finite length, the waves are reflected from the ends. Thus, the superposition of the waves travelling in opposite directions forms a longitudinal standing wave.

- Longitudinal standing waves are described either in terms of the displacement or the pressure variation in the fluid.
- Displacement node and antinode refers to the points, where the particles of the fluid have zero and maximum displacement, respectively.
- Pressure node and antinode refers to the points, where the pressure and density variation is zero and maximum, respectively.

Open Organ Pipe

If both ends of a pipe of length L are open and a system of air is directed against an edge, standing longitudinal waves can be set up in the tube. The open end has a displacement antinode.



(a) (b) (c)

Possible wavelengths of standing waves are

$$L = \frac{n\lambda}{2};\ n = 1, 2, 3 \ldots$$

Corresponding frequencies are

$$f = \frac{nv}{2L}$$

$\therefore$ For fundamental mode or first harmonic, $f_1 = \dfrac{v}{2L}$

For the second harmonic or first overtone, $f_2 = \dfrac{v}{2L} = 2f_1$

Similarly, for the third harmonic or second overtone,

$$f_3 = \frac{3\,v}{2\,L} = 3f_1$$

The above relations, we get

$$f_1 : f_2 : f_3 : \ldots = 1 : 2 : 3 : \ldots$$

i.e. The natural frequencies of oscillations form a harmonic series that includes all integral multiples of fundamental frequency.

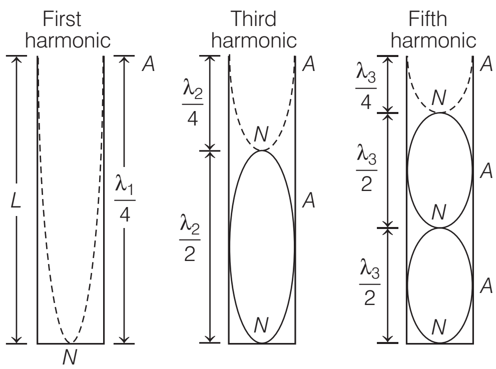
Closed Organ Pipe

In this type of organ pipe, one end is closed and the other end is open. A displacement node is formed at the closed end and an antinode at the open end.

For a organ pipe of length L, possible wavelengths of the stationary waves are

$$\lambda = \frac{4L}{n};\ \text{where } n = 1, 3, 5 \ldots$$

$\therefore$ Corresponding frequencies are, $f = \dfrac{nv}{4L}$



(a) (b) (c)

$\therefore$ Fundamental frequency given as

$$f_1 = \frac{v}{4L}$$

Frequency of third harmonic or first overtone,

$$f_2 = \frac{3v}{4L} = 3f_1$$

Similarly, frequency of fifth harmonic or second overtone,

$$f_3 = \frac{5v}{4L} = = 5f_1$$

From above relations, we get

$$f_1 : f_2 : f_3 \ldots = 1 : 3 : 5 \ldots$$

i.e. The natural frequencies of oscillations form a harmonic series that includes only odd integral multiples of the fundamental frequencies.

Example 14. *A pipe, 30 cm long is open at both ends, harmonic mode of the pipe which resonates at 1.1 kHz source is (Take, speed of sound in air is 330 ms^{-1})*

(a) *first* (b) *second*
(c) *third* (d) *four*

Sol. (*b*) The first harmonic frequency is

$$f_1 = \frac{v}{\lambda_i} = \frac{v}{2L} \quad \text{(open pipe)}$$

where, L is the length of the pipe. The frequency of its nth harmonic is

$$f_n = \frac{nv}{2L} \quad \text{for } n = 1, 2, 3, \ldots \text{(open pipe)}$$

Given, $L = 30$ cm, $v = 330\ \text{ms}^{-1}$

$$f_n = \frac{n(330\ \text{ms}^{-1})}{0.6\ \text{m}} = 550\ \text{ms}^{-1}$$

The frequencies of 2nd harmonic, 3rd hormonic, 4th hormonic, ... are $2 \times 550 = 1100$ Hz, $3 \times 550 = 1650$ Hz, $4 \times 550 = 2200$ Hz.

Clearly, a source of frequency 1.1 kHz will resonate at f_2, *i.e.* the second harmonic.

Example 15. *An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is*

(a) 100 *Hz* (b) 150 *Hz*
(c) 200 *Hz* (d) 250 *Hz*

Sol. (c) For fundamental mode in open pipe,

$$L = \lambda/2$$

$$\Rightarrow \quad \lambda = 2L$$

and

$$f_F = \frac{v}{\lambda} = \frac{v}{2L} \quad \ldots\text{(i)}$$

For third harmonic in closed pipe,

$$L = \frac{3\lambda}{4}$$

$$\Rightarrow \quad \lambda = \frac{4L}{3}$$

$$f_{H_2} = \frac{v}{\lambda} = \frac{v}{\frac{4}{3}L} = \frac{3v}{4L} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{f_H}{f_F} = \frac{3}{2}$$

$$\Rightarrow \quad f_H = f_F \times \frac{3}{2} \quad \ldots\text{(iii)}$$

But $\quad f_H - f_F = 100$

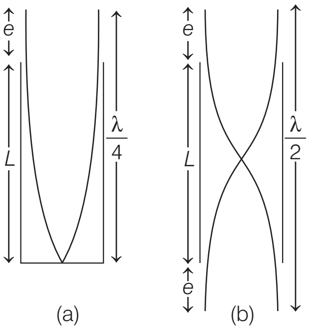
$$\frac{3}{2}f_F - f_F = 100$$

$$\Rightarrow \quad f_F = 200 \text{ Hz}$$

End Correction

It was found that the antinode is not formed exactly at the open end of the *organ pipe* but actually due to finite momentum of the particles the reflection takes place a little above the open end; that is why the antinode is formed a little above the open end.

For this, a correction is applied being known as end correction. This is denoted by *e*. If length of organ pipe is *L* and end correction is *e*, then length of air-column in closed pipe will be $(L + e)$ and in *open pipe*, $(L + 2e)$. Thus, for a closed organ pipe,



$$f_1 = \frac{v}{4(L + e)}$$

and for an open organ pipe,

$$f_2 = \frac{v}{2(L + 2e)}$$

The value of end correction *e* is $0.6r$ for closed organ pipe and $1.2r$ for an open organ pipe, where *r* is the radius of the pipe.

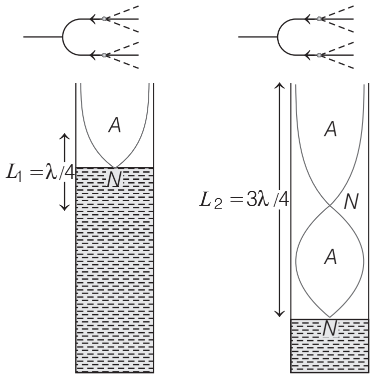
Resonance Tube

It is a closed organ pipe in which length of air-column can be increased or decreased. When a vibrating tuning fork is brought at its mouth as shown in figure, then forced vibrations are set up in its air-column. If we adjust the length of air-column as such its any natural frequency equals to the frequency of tuning fork, then the amplitude of forced vibrations of air-column increases very much. This is the state of *resonance*.

When length of air-column is $L = \lambda/4$, then the first resonance occurs. As shown in Fig. (a), antinode is formed at an open end and a node is formed at the water surface.

Now, when length of air-column is $L_2 = 3\lambda/4$, then second resonance occurs. In this condition, two antinodes and two nodes are formed as shown in Fig. (b).

End correction In resonance tube, antinode is not formed exactly at open end but it is formed a little above the open end known as end correction (e). So, in first and second state of air-column, the lengths are $L_1 + e$ and $L_2 + e$.



(a) (b)

$$L_1 + e = \lambda/4$$

and $$L_2 + e = 3\lambda/4$$

Hence, end correction, $$e = \frac{L_2 - 3L_1}{2}$$

Example 16. *A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air obtained in the experiment is close to* ***[JEE Main 2019]***

(a) $328\, ms^{-1}$ (b) $341\, ms^{-1}$

(c) $322\, ms^{-1}$ (d) $335\, ms^{-1}$

Sol. (a) In first resonance, length of air column $= \dfrac{\lambda}{4}$.

So, $$L_1 + e = \frac{\lambda}{4}$$

or $$11 \times 4 + 4e = \lambda$$

So, speed of sound,

$$v = f_1\lambda = 512(44 + 4e) \quad \ldots\text{(i)}$$

and in second case,

$$L_1' + e = \frac{\lambda'}{4}$$

or $$27 \times 4 + 4e = \lambda'$$

$$\Rightarrow \quad v = f_2\lambda' = 256(108 + 4e) \quad \ldots\text{(ii)}$$

Dividing both Eqs. (i) and (ii), we get

$$1 = \frac{512(44 + 4e)}{256(108 + 4e)}$$

$$\Rightarrow \quad e = 5 \text{ cm}$$

Substituting value of e in Eq. (i), we get

Speed of sound, $v = 512(44 + 4e)$

$= 512(44 + 4 \times 5)$

$= 512 \times 64 \text{ cm s}^{-1}$

$= 327.68 \text{ ms}^{-1}$

$\approx 328 \text{ ms}^{-1}$

Beats

When two sound waves of nearly same frequency are produced simultaneously, then the intensity of resultant sound wave increases and decreases with time. This change in the intensity of sound waves is called as the phenomenon of beats.

The time interval between two successive beats is called *beat period* and the number of beats per second is called the *beat frequency*.

If f_1 and f_2 are the frequencies ($f_1 > f_2$) of the two waves, then the beat frequency, $b = f_1 - f_2$

$\therefore$ Beat period, $T = \dfrac{1}{f_1 - f_2}$

Example 17. *The first overtone of an open pipe and the fundamental note of a pipe closed at one end gives 5 beats s^{-1}, when sounded together. If the length of the pipe closed at one end is 25 cm, what are the possible lengths of the open pipe? (Neglect end corrections and take the velocity of sound in air to be 340 ms^{-1})*

(a) 90.5 *and* 120.5 *cm*

(b) 98.5 *and* 101.5 *cm*

(c) 95.5 *and* 102.5 *cm*

(d) 95.5 *and* 200 *cm*

Sol. (b) Let the fundamental frequency of the closed end pipe of length 25 cm be f_o. Then,

$$f_o = \frac{v}{4l} = \frac{340 \times 100}{4 \times 25} = 340 \text{ Hz}$$

Possible frequencies of first overtone of the required open pipe are 340 ± 5, *i.e.* 345 or 335 Hz.

For the first overtone of an open pipe, the length of the pipe l equals the wavelength of the vibration.

Hence, $$345 = \frac{v}{l}$$

or $$l = \frac{34000}{345} = 98.5 \text{ cm}$$

Other possible length l' is given by

$$335 = \frac{v}{l'}$$

$$l' = \frac{34000}{335} = 101.5 \text{ cm}$$

Hence, possible lengths of the open pipe are 98.5 and 101.5 cm.

Example 18. *Two tuning forks A and B sounded together give 8 beat s^{-1}. With an air resonance tube closed at one end, the two forks give resonance when the two air columns are 32 cm and 33 cm, respectively. The frequencies of forks are*

(a) 260 Hz, 250 Hz (b) 264 Hz, 256 Hz

(c) 274 Hz, 256 Hz (d) 2709 Hz, 250 Hz

Sol. (b) Let the frequency of the first fork be f_1 and that of second be f_2. Then, we have

$$f_1 = \frac{v}{4\times 32} \text{ and } f_2 = \frac{v}{4\times 33}$$

We also see that $f_1 > f_2$

$$\therefore \quad f_1 - f_2 = 8 \quad \ldots\text{(i)}$$

and $$\frac{f_1}{f_2} = \frac{33}{32} \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$f_1 = 264 \text{ Hz}$$

and $$f_2 = 256 \text{ Hz}$$

Doppler's Effect

If a source of wave and the observer/receiver are in relative motion to each other, then the frequency observed (f_a) by the observer is different from the actual source frequency (f_o). This phenomenon is called Doppler's effect.

For different situations of the source and the observer the apparent frequencies observed due to Doppler's effect are tabulated as follows

Observer (o)	Sources (s)	Apparent frequency
$\rightarrow v_o$	Stationary ($v_s = 0$)	$f_a = \left(\frac{v + v_o}{v}\right) f_o$
$v_o \leftarrow$	Stationary ($v_s = 0$)	$f_a = \left(\frac{v - v_o}{v}\right) f_o$
Stationary ($v_o = 0$)	$\rightarrow v_s$	$f_a = \left(\frac{v}{v + v_s}\right) f_o$
Stationary ($v_o = 0$)	$v_s \leftarrow$	$f_a = \left(\frac{v}{v - v_s}\right) f_o$
$\rightarrow v_o$	$\rightarrow v_s$	$f_a = \left(\frac{v + v_o}{v + v_s}\right) f_o$
$v_o \leftarrow$	$\rightarrow v_s$	$f_a = \left(\frac{v - v_o}{v + v_s}\right) f_o$
$\rightarrow v_o$	$v_s \leftarrow$	$f_a = \left(\frac{v + v_o}{v - v_s}\right) f_o$
$v_o \leftarrow$	$v_s \leftarrow$	$f_a = \left(\frac{v - v_o}{v - v_s}\right) f_o$

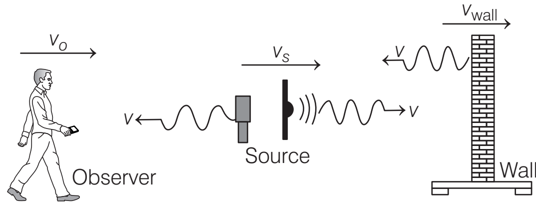
Here, v = speed of sound, v_s = speed of source, v_o = speed of observer, f_o = original frequency of the source.

Note Doppler's effect is a wave phenomenon, it hold not only for sound waves but also for electromagnetic waves.

Special Cases of Doppler's Effect in Sound

1. Doppler's effect in reflected sound

For a source of sound moving with velocity v_s towards a wall which is itself moving in same direction and an observer too moving in the same direction as shown in figure.



A descriptive explanation of this case of Doppler's effect can be understood by example given just below.

Example 19. *Submarine A travelling at 18 km/h is being chased along the line of its velocity by another submarine B travelling at 27 km/h. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to (Speed of sound in water = 1500 ms^{-1})*

[JEE Main 2019]

(a) 504 Hz (b) 507 Hz

(c) 499 Hz (d) 502 Hz

Sol. (d) Given, velocity of submarine (A),

$$v_A = 18 \text{ km/h} = \frac{18000}{3600} \text{ m/s}$$

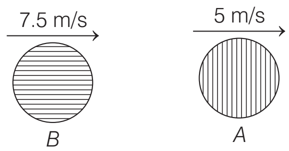
or $$v_A = 5 \text{ m/s} \quad \ldots\text{(i)}$$

and velocity of submarine (B),

$$v_B = 27 \text{ km/h} = \frac{27000}{3600} \text{ m/s}$$

or $$v_B = 7.5 \text{ m/s} \quad \ldots\text{(ii)}$$

Signal sent by submarine (B) is detected by submarine (A) can be shown as



Frequency of the signal, $f_o = 500$ Hz

So, in this relative motion, frequency received by submarine (A) is

$$f_1 = \left(\frac{v_s - v_A}{v_s - v_B}\right) f_o = \left(\frac{1500 - 5}{1500 - 7.5}\right) 500 \text{ Hz}$$

$$\Rightarrow \quad f_1 = \frac{1495}{1492.5} \times 500 \text{ Hz}$$

The reflected frequency f_1 is now received back by submarine (B).

So, frequency received at submarine (B) is

$$f_2 = \left(\frac{v_s + v_B}{v_s + v_A}\right) f_1$$

$$= \left(\frac{1500 + 7.5}{1500 + 5}\right)\left(\frac{1495}{1492.5}\right) 500 \text{ Hz}$$

$$\Rightarrow \quad f_2 = \left(\frac{1507.5}{1505}\right)\left(\frac{1495}{1492.5}\right) 500 \text{ Hz}$$

$$\Rightarrow \quad f_2 = 1.00166 \times 1.00167 \times 500$$

$$\Rightarrow \quad f_2 = 501.67 \text{ Hz}$$

$$\approx 502 \text{ Hz}$$

Example 20. *A siren emitting a sound of frequency 1000 Hz moves away from you towards a cliff with a speed of 10 ms^{-1}. What is the frequency of the sound you hear coming directly from the siren?*

(a) (33/34) $\times 1000$ *Hz*
(b) (34/33) $\times$ 1000 *Hz*
(c) (35/34) $\times$ 1000 *Hz*
(d) (34/35) $\times$ 1000 *Hz*

Sol. (*a*) Sound heard directly,

$$f_1 = f_o\left(\frac{v}{v + v_s}\right)$$

$$v_s = 10 \text{ ms}^{-1}$$

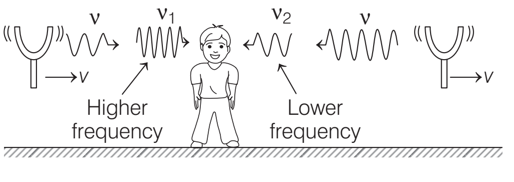
$$\therefore \quad f_1 = \left(\frac{330}{330 + 10}\right) \times 1000$$

$$= \frac{33}{34} \times 1000 \text{ Hz}$$

Example 21. *A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/s. The oscillation frequency of each tuning fork is $\nu_0 = 1400$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to* **[JEE Main 2020]**

(a) $\frac{1}{8}$ *m/s* (b) $\frac{1}{2}$ *m/s* (c) 1 *m/s* (d) $\frac{1}{4}$ *m/s*

Sol. (*d*) Due to Doppler's effect, frequency of approaching tuning fork is higher and receding tuning fork is lower than emitted frequency.



Frequency received from tuning fork moving towards observer,

$$\nu_1 = \nu\left(\frac{v}{v - v_s}\right)$$

where, ν = frequency emitted by tuning fork, v = speed of sound and v_s = speed of tuning fork.

Frequency received from receding tuning fork,

$$\nu_2 = \nu\left(\frac{v}{v + v_s}\right)$$

Beat frequency is the difference of both received frequencies.

So, $\quad \nu_{\text{beat}} = \nu_1 - \nu_2$

$$\Rightarrow \quad \nu_{\text{beat}} = \nu\left(\frac{v}{v - v_s}\right) - \nu\left(\frac{v}{v + v_s}\right)$$

$$= \frac{2\,\nu v_s v}{v^2 - v_s^2} = \frac{2\nu v_s}{v\left(\frac{1 - v_s^2}{v^2}\right)}$$

If $v_s << v$, then $\nu_{\text{beat}} = \frac{2\,f v_s}{v}$

Here, $\nu = 1400$ Hz, $v = 350$ ms^{-1} and $\nu_{\text{beat}} = 2$ s^{-1}

So, $\quad 2 = \frac{2 \times 1400 \times v_s}{350}$

$$\Rightarrow \quad v_s = \frac{1}{4} \text{ ms}^{-1}$$

Example 22. *A source S of acoustic wave of the frequency $\nu_0 = 1700$ Hz and a receiver R are located at the same point. At the instant t = 0, the source starts from rest to move away from the receiver with a constant acceleration a. The velocity of sound in air is 340 m/s. If $a = 10\ m/s^2$, the apparent frequency that will be recorded by the stationary receiver at t = 10 s will be*

(a) 1700 *Hz* (b) 1.35 *Hz* (c) 2.89 *Hz* (d) 1300 *Hz*

Sol. (*b*) Source frequency, $\nu_o = 1700$ Hz. Source (coinciding with observer at $t = 0$) moves away with uniform acceleration a. Consider the wave which is received by the observer at instant $t = \tau$. It will have left the source at an earlier instant of time, say $t\ (< \tau)$, when the distance of source was r (say), if u be velocity of source at instant t, then $r = \left(\frac{1}{2}\right) at^2$ and $u = at$. The relation between τ and t is

$$\tau = t + \frac{r}{v} = t + \frac{at^2}{2v}$$

This is a quadratic equation in t, giving the solution

$$at = \frac{-2v + \sqrt{4v^2 + 8vat}}{2}$$

$$u = at = v\left(\sqrt{1 + \frac{2\,a\tau}{v}} - 1\right)$$

$$u = 340\left(\sqrt{1 + \frac{2 \times 10 \times 10}{340}} - 1\right)$$

$$= 340\left(\frac{\sqrt{27}}{17} - 1\right)$$

Then, apparent frequency is given by

$$\nu_a = \left(\frac{v}{v + u}\right)\nu_o$$

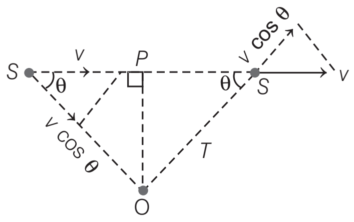
Putting the values $v = 340$ m/s, $\tau = 10$ s, $a = 10$ m/s^2, we have

$$\nu_a = \left(\frac{340}{340 + u}\right) 1700$$

$$\nu_a = 1700\sqrt{\frac{17}{27}} = 1.35 \text{ Hz}$$

2. Doppler's effect when the source and observer are not in same line of motion

From the figure, the position of a source is S and of observer is O. The component of velocity of source towards the observer is $v\cos\theta$. For this situation, the approach frequency is



$$f' = \frac{v}{v - v_s \cos\theta} \times f$$

Similarly, if the source is moving away from the observer as shown above with velocity component $v_s \cos\theta$, then

$$f' = \frac{v}{v + v_s \cos\theta} \times f$$

If $\theta = 90°$, then $v_s \cos\theta = 0$ and there is no shift in the frequency. Thus, at point P, Doppler's effect does not occur.

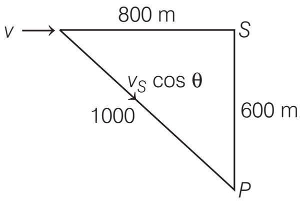
Note If wind blows at a speed v_w from the source to the observer, take $v \to v + v_w$ (both in numerator and denominator) and if in opposite direction (*i.e.* from observer to source), take $v \to v - v_w$. Thus, the modified formula is

$$f' = \left(\frac{v \pm v_w \pm v_o}{v \pm v_w \pm v_s}\right) f$$

Example 23. *A person P is 600 m away from the station, when train is approaching station with 72 km/h, it flows a whistle of frequency 800 Hz when 800 m away from the station. Frequency heard by the person is*
(Take, speed of sound = 340 ms^{-1})

(a) 800 *Hz* (b) 839.5 *Hz*
(c) 829.5 *Hz* (d) 834.5 *Hz*

Sol. (*b*) From Doppler's effect, frequency,



$$f_{app} = \frac{v}{v - v_S \cos\theta} f$$

$$= \frac{340}{340 - 16} \times 800$$

$$= 839.5 \text{ Hz}$$

Practice Exercise

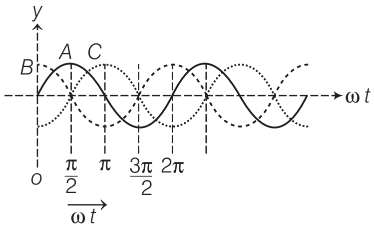
ROUND I Topically Divided Problems

Wave Motion and Progressive Waves

1. Which of the following statements are true for wave motion? **[NCERT Exemplar]**

(a) Mechanical transverse waves can propagate through all mediums.
(b) Longitudinal waves can propagate through solids only.
(c) Mechanical transverse waves can propagate through solids only.
(d) Longitudinal waves can propagate through vacuum.

2. The figure shows three progressive waves A, B and C moving in the same direction. What can be concluded with respect to wave A from the figure?



(a) The wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle $\pi/2$
(b) The wave C is ahead by a phase angle of $\pi/2$ and the wave B lags behind by a phase angle $\pi/2$
(c) The wave C is ahead by a phase angle of π and the wave B lags behind by a phase angle π
(d) The wave C lags behind by a phase angle π and the wave B is ahead by a phase angle π

3. The equation of wave is represented by $y = 10^{-4} \sin\left[100t - \frac{x}{10}\right]$ m, then the velocity of wave will be

(a) 100 ms^{-1}
(b) 4 ms^{-1}
(c) 1000 ms^{-1}
(d) zero

4. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$, where x and y are in cm and t is in second. The positive direction of x is from left to right.

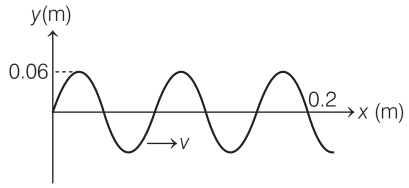
Then, which amongst the following option is incorrect?

(a) The wave is travelling from right to left.
(b) The speed of the wave is 20m/s.
(c) Frequency of the wave is 5.7 Hz.
(d) The least distance between two successive crests in the wave is 2.5 cm.

5. A transverse wave is described by the equation $y = y_0 \sin 2\pi\left[ft - \frac{x}{\lambda}\right]$. The maximum particle velocity is equal to four times the wave velocity, if

(a) $\lambda = \pi y_0/4$
(b) $\lambda = 2\pi y_0$
(c) $\lambda = \pi/y_0$
(d) $\lambda = \pi y_0/2$

6. For the wave shown in figure, write the equation of this wave, if its position is shown at $t = 0$. Speed of wave is $v = 300$ m/s.



(a) $y = (0.06)\cos[(78.5\text{m}^{-1})x + (23562\text{s}^{-1})t]$ m
(b) $y = (0.06)\sin[(78.5\text{ m}^{-1})x - (23562\text{ s}^{-1})t]$ m
(c) $y = (0.06)\sin[(78.5\text{ m}^{-1})x + (28562\text{ s}^{-1})t]$ m
(d) $y = (0.06)\cos[(78.5\text{m}^{-1})x - (28562\text{s}^{-1})t]$ m

7. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50t + 2x)$, where x and y are in metre and t is in second. Which of the following is a correct statement about the wave? **[JEE Main 2019]**

(a) The wave is propagating along the negative X-axis with speed 25 ms^{-1}.
(b) The wave is propagating along the positive X-axis with speed 25 ms^{-1}.
(c) The wave is propagating along the positive X-axis with speed 100 ms^{-1}.
(d) The wave is propagating along the negative X-axis with speed 100 ms^{-1}.

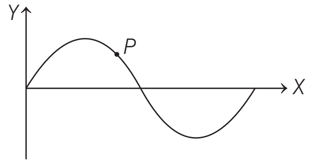
8. In a transverse wave the distance between a crest and neighbouring trough at the same instant is 4.0 cm and the distance between a crest and trough at the same place is 1.0 cm. The next crest appears at the same place after a time interval of 0.4 s. The maximum speed of the vibrating particles in the medium is **[JEE Main 2013]**

(a) $\frac{3\pi}{2}$ cm/s (b) 5π cm/s
(c) $\frac{\pi}{2}$ cm/s (d) 2π cm/s

9. A transverse mechanical harmonic wave is travelling on a string. Maximum velocity and maximum acceleration of a particle on the string are 3 m/s and 90 m/s^2, respectively. If the wave is travelling with a speed of 20 m/s on the string. The wave function describing the wave is

(a) $0.1 \cos (30t \pm 1.5x)$ (b) $0.1 \sin (30t \pm 1.5x)$
(c) $0.2 \sin (1.5t \pm 30x)$ (d) $0.2 \cos (1.5t \pm 30x)$

10. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snapshot of the wave is shown in the figure. The velocity of the point P when its displacement is 5 cm, is



(a) $\frac{\sqrt{3}\pi}{50}\hat{\mathbf{j}}\,\text{ms}^{-1}$ (b) $-\frac{\sqrt{3}\pi}{50}\hat{\mathbf{j}}\,\text{ms}^{-1}$
(c) $\frac{\sqrt{3}\pi}{50}\hat{\mathbf{i}}\,\text{ms}^{-1}$ (d) $-\frac{\sqrt{3}\pi}{50}\hat{\mathbf{i}}\,\text{ms}^{-1}$

11. From a point source, if amplitude of waves at a distance r is A, its amplitude at a distance $2r$ will be

(a) A (b) $2A$ (c) $A/2$ (d) $A/4$

12. A simple harmonic progressive wave is represented by the equation $y = 8 \sin 2\pi (0.1x - 2t)$ where x and y are in cm and t is in seconds. At any instant, the phase difference between two particles separated by 2.0 cm in the x-direction is

(a) 18° (b) 54° (c) 36° (d) 72°

13. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in **[NCERT Exemplar]**

(a) 1 s (b) 0.5 s
(c) 2 s (d) Data is insufficient

14. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to $v/2$. The value of T is close to **[JEE Main 2020]**

(a) 10.2×10^2 N (b) 5.15×10^3 N
(c) $2 .50 \times 10^4$ N (d) 30.5×10^4 N

15. The amplitude of wave disturbance propagating in positive direction of X-axis is given by $y = \frac{1}{1+x^2}$ at $t = 0$ and by $y = \frac{1}{1+(x-1)^2}$ at $t = 2$ s, where x and y are in metres. The shape of the wave disturbance does not change during propagation. The velocity of the wave is

(a) 0.5 ms^{-1} (b) 2.0 ms^{-1}
(c) 1.0 ms^{-1} (d) 4.0 ms^{-1}

16. A 100 Hz sinusoidal wave is travelling in the positive x-direction along a string with a linear mass density of 3.5×10^{-3} kg/m and a tension of 35 N. At time $t = 0$, the point $x = 0$, has maximum displacement in the positive y-direction. Next when this point has zero displacement, the slope of the string is $\pi/20$. Which of the following expression represent(s) the displacement of string as a function of x (in metre) and t (in second)?

(a) $y = 0.025 \cos (200\pi t - 2\pi x)$
(b) $y = 0.5 \cos (200\pi t - 2\pi x)$
(c) $y = 0.025 \cos (100\pi t - 10\pi x)$
(d) $y = 0.5 \cos (100\pi t - 10\pi x)$

Sound Waves

17. Speed of sound waves in a fluid depends upon

(a) directly on density of the medium
(b) square of bulk modulus of the medium
(c) inversely of the square root of density
(d) inversely of the square root of bulk modulus of the medium

18. v_1 and v_2 are the velocities of sound at the same temperature in two monoatomic gases of densities ρ_1 and ρ_2, respectively. If $\frac{\rho_1}{\rho_2} = \frac{1}{4}$, then the ratio of velocities v_1 and v_2 will be

(a) 1: 2 (b) 4 :1 (c) 2 : 1 (d) 1: 4

19. If the temperature is raised by 1 K from 300 K, the percentage change in the speed of sound in the gaseous mixture is (R = 8.31 J/mol-K)

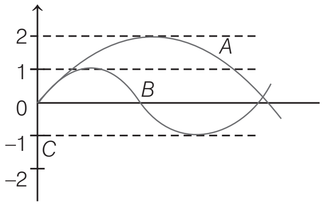
(a) 0.167% (b) 0.334% (c) 1% (d) 2%

20. The speed of sound in a mixture of 1 mole of helium and 2 mol of oxygen at 27°C is

(a) 800 ms^{-1} (b) 400.8 ms^{-1}
(c) 600 ms^{-1} (d) 1200 ms^{-1}

21. In brass, the velocity of longitudinal wave is 100 times the velocity of the transverse wave. If $Y = 1\times 10^{11}$ Nm^{-2}, then stress in the wire is
(a) $1\times10^{13}\,\text{Nm}^{-2}$ (b) $1\times10^{9}\,\text{Nm}^{-2}$
(c) $1\times10^{11}\,\text{Nm}^{-2}$ (d) $1\times10^{7}\,\text{Nm}^{-2}$

22. The displacement-time graphs for two sound waves A and B are shown in figure, then the ratio of their intensities I_A/I_B is equal to



(a) 1 : 4 (b) 1 : 16
(c) 1 : 2 (d) 1 : 1

23. A sound wave of frequency 245 Hz travel with the speed of 300 ms^{-1} along the positive X-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave? **[JEE Main 2021]**
(a) $y(x, t) = 0.03\sin[5.1x - (0.2\times10^3)t]$
(b) $y(x, t) = 0.06\sin[5.1x - (1.5\times10^3)t]$
(c) $y(x, t) = 0.06\sin[0.8x - (0.5\times10^3)t]$
(d) $y(x, t) = 0.03\sin[5.1x - (1.5\times10^3)t]$

24. A plane longitudinal wave of angular frequency 1000 rad s^{-1} is travelling along negative x-direction in a homogeneous gaseous medium of density $\rho = 1$ kg m^{-3}. Intensity of the wave is $I = 10^{-10}$ Wm^{-2} and the maximum pressure change is 2×10^{-4} Nm^{-2}. Then, displacement equation is
(a) $y = 10^{-9}\sin\left(1000\,t - 5x + \dfrac{\pi}{2}\right)$
(b) $y = 10^{-9}\cos(1000\,t + 5x)$
(c) $y = 10^{-9}\tan(1000\,t - 5x)$
(d) $y = 10^{-9}\cos(1000\,t - 5x)$

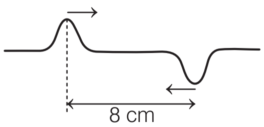
Stationary Waves in Strings and in Organ Pipes

25. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \dfrac{x}{2}\right)$. On reflection from a denser medium, its amplitude becomes 2/3 of the amplitude of the incident wave. The equation of the reflected wave is **[NCERT Exemplar]**
(a) $y = 0.6\sin 2\pi\left(t + \dfrac{x}{2}\right)$ (b) $y = -0.4\sin 2\pi\left(t + \dfrac{x}{2}\right)$
(c) $y = 0.4\sin 2\pi\left(t + \dfrac{x}{2}\right)$ (d) $y = -0.4\sin 2\pi\left(t - \dfrac{x}{2}\right)$

26. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000\,t)$. This expression may be considered to be a result of superposition of
(a) two waves (b) three waves
(c) four waves (d) five waves

27. Four simple harmonic vibrations,
$$y_1 = 8\cos\omega t$$
$$y_2 = 4\cos\left(\omega t + \frac{\pi}{2}\right)$$
$$y_3 = 2\cos(\omega t + \pi)$$
$$y_4 = \cos\left(\omega t + \frac{3\pi}{2}\right)$$
are superimposed on one another. The resulting amplitude and phase are respectively
(a) $\sqrt{45}$ and $\tan^{-1}\left(\dfrac{1}{2}\right)$ (b) $\sqrt{45}$ and $\tan^{-1}\left(\dfrac{1}{3}\right)$
(c) $\sqrt{75}$ and $\tan^{-1}(2)$ (d) $\sqrt{75}$ and $\tan^{-1}\left(\dfrac{1}{3}\right)$

28. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm s^{-1}. After 2 s, the total energy of the pulses will be



(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

29. When two sound waves travel in the same direction in a medium, the displacement of a particle located at X at time t is given by
$$y_1 = 0.05\cos(0.50\pi x - 100\pi t)$$
$$y_2 = 0.05\cos(0.46\pi x - 92\pi t)$$
where, y_1, y_2 and x are in metres and t in seconds. The speed of sound in the medium is **[JEE Main 2013]**
(a) 92 m/s (b) 200 m/s
(c) 100 m/s (d) 332 m/s

30. The following equations represent progressive transverse waves
$Z_1 = A\cos(\omega t - kX)$, $Z_2 = A\cos(\omega t + kX)$
$Z_3 = A\cos(\omega t - kY)$, $Z_4 = A\cos(2\omega t - 2kY)$
A stationary wave will be formed by superposing
(a) Z_1 and Z_2
(b) Z_1 and Z_4
(c) Z_2 and Z_3
(d) Z_3 and Z_4

31. A wave represented by the given equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for the other wave is

(a) $y = a\sin(kx + \omega t)$ (b) $y = -a\cos(kx + \omega t)$
(c) $y = -a\cos(kx - \omega t)$ (d) $y = -a\sin(kx - \omega t)$

32. A wave of wavelength 2 m is reflected from a surface, if a node is formed at 3 m from the surface, then at what distance from the surface another node will be formed?

(a) 3 m (b) 2 m
(c) 3 m (d) 4 m

33. The displacement of a string is given by $y(x, t) = 0.06\sin(2\pi x/3)\cos(120\pi t)$, where x and y are in metre and t in second. The length of the string is 1.5m and its mass is 3.0×10^{-2} kg.

(a) It represents a progressive wave of frequency 60 Hz.
(b) It represents a stationary wave of frequency 120 Hz.
(c) It is the result of superposition of two waves of wavelength 3 m, frequency 60 Hz each travelling with a speed of 180 m/s in opposite direction.
(d) Amplitude of this wave is constant.

34. Amongst the following statements which is correct for stationary waves?

(a) Nodes and antinodes are formed in case of stationary transverse wave only
(b) In case of longitudinal stationary wave, compressions and rarefactions are obtained in place of nodes and antinodes respectively
(c) Suppose two plane waves, one longitudinal and the other transverse having same frequency and amplitude are travelling in a medium in opposite directions with the same speed, by superposition of these waves, stationary waves cannot be obtained
(d) None of the above

35. Two identical sinusoidal waves travel in opposite direction in a wire 15 m long and produce a standing wave in the wire. If the speed of the waves is 12 ms^{-1} and there are 6 nodes in the standing wave. Find the frequency.

(a) 4 Hz (b) 2 Hz
(c) 6 Hz (d) 9 Hz

36. The transverse displacement of a string (clamped at its both ends) is given by

$$y(x, t) = 0.06\sin(2\pi x/3)\cos(120\pi t)$$

All the points on the string between two consecutive nodes vibrate with

I. same frequency
II. same phase
III. same energy
IV. different amplitude

(a) I, II, III (b) I, III only
(c) I, II, IV (d) II, IV only

37. Two wires made up of same material are of equal lengths but their radii are in the ratio 1 : 2. On stretching each of these two wires by the same tension, the ratio between the fundamental frequencies is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

38. n waves are produced on a string in one second. When the radius of the string is doubled and the tension is maintained the same, the number of waves produced in one second for the same harmonic will be

(a) $\frac{n}{2}$ (b) $\frac{n}{3}$ (c) $2n$ (d) $\frac{n}{\sqrt{2}}$

39. A string in a musical instrument is 50 cm long and its fundamental frequency is 800 Hz. If a fundamental frequency of 1000 Hz is to be produced, then required length of string is

(a) 62.5 cm (b) 50 cm (c) 40 cm (d) 37.5 cm

40. Two identical strings X and Z made of same material have tensions T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz respectively, then the ratio of T_X / T_Z is **[JEE Main 2020]**

(a) 1.25 (b) 2.25 (c) 0.44 (d) 1.5

41. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to **[JEE Main 2019]**

(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

42. Choose the correct statement from the following options.

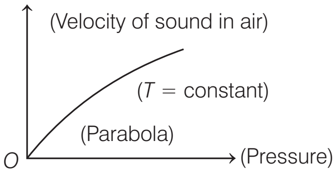
(a) Under identical conditions of pressure and density, the speed of sound is highest in a monoatomic gas.
(b) A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of 100.
(c) A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compression and rarefactions, there is no transfer of heat.
(d) All of the above

43. A person blows into the open end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

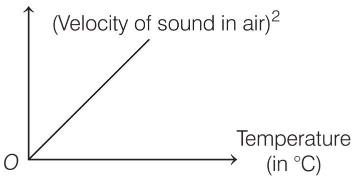
(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed
(d) Both (a) and (b)

44. Which of the following is/are correct?

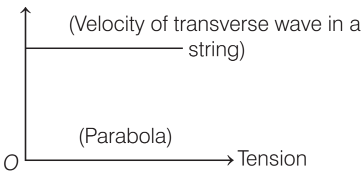
(a)



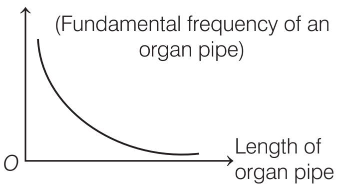
(b)



(c)



(d)



45. If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by

(a) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$ (b) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

(c) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$ (d) $n = n_1 + n_2 + n_3$

46. A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are 420 Hz and 490 Hz, then L in metres is **[JEE Main 2020]**

(a) 8.1 (b) 2.1
(c) 5.1 (d) 1.1

47. A heavy uniform rope hangs vertically from the ceiling with its lower end free. A disturbance on the rope travelling upwards from the lower end has a velocity v at a distance x from the lower end such that

(a) $v \propto x$ (b) $v \propto \sqrt{x}$ (c) $v \propto \frac{1}{x}$ (d) $v \propto \frac{1}{\sqrt{x}}$

48. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is (All quantities are in SI units) **[JEE Main 2019]**

(a) 60 m (b) 40 m (c) 80 m (d) 20 m

49. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **[JEE Main 2018]**

(a) 5 kHz (b) 2.5 kHz
(c) 10 kHz (d) 7.5 kHz

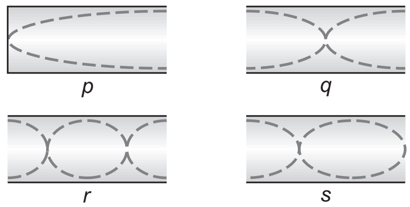
50. An organ pipe open at one end is vibrating in first overtone and is in resonance with another pipe open at both ends and vibrating in third harmonic. The ratio of length of two pipes is

(a) 3 : 8 (b) 8 : 3
(c) 1 : 2 (d) 4 : 1

51. A wave of frequency 100 Hz is sent along a string towards a fixed end. When this wave travels back after reflection, a node is formed at a distance of 10 cm from the fixed end of the string. The speeds of incident (and reflected) waves are

(a) 5 ms^{-1} (b) 10 ms^{-1}
(c) 20 ms^{-1} (d) 40 ms^{-1}

52. The vibrating four air columns are represented in the figure. The ratio of frequencies $n_p : n_q : n_r : n_s$ is



(a) 12 : 6 : 3 : 5
(b) 1 : 2 : 4 : 3
(c) 4 : 2 : 3 : 1
(d) 6 : 2 : 3 : 4

53. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now, one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option.

(a) $n = 3, f_2 = \frac{3}{4} f_1$ (b) $n = 3, f_2 = \frac{5}{4} f_1$

(c) $n = 5, f_2 = \frac{5}{4} f_1$ (d) $n = 5, f_2 = \frac{3}{4} f_1$

54. Six antinodes are observed in the air column when a standing wave forms in a closed (at both ends) tube. A steel bar (in its fundamental mode) of length 1m and clamped at the middle is in unison with the air column. The speed of sound in steel is 5250 ms^{-1} and in air, it is 343 ms^{-1}. Length of the air column is

(a) 79 cm (b) 49 cm
(c) 39 cm (d) 29 cm

55. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2, respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency.
The length of the open organ pipe is

(a) $\frac{L}{3}$ (b) $\frac{4L}{3}$

(c) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (d) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

56. A glass tube of length 1.0 m is completely filled with water. A vibrating tuning fork of frequency 500 Hz is kept over the mouth of the tube and the water is drained out slowly at the bottom of the tube. If the velocity of sound in air is 330 ms^{-1}, then the total number of resonances that occur will be

(a) 2 (b) 3
(c) 1 (d) 5

57. A metre-long tube open at one end with a movable piston at the other end, shows resonance with a fixed frequency source (a tunning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected. **[NCERT]**

(a) 336 m/s (b) 331 m/s
(c) 356 m/s (d) 366 m/s

58. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20000 Hz) **[JEE Main 2019]**

(a) 7 (b) 4 (c) 5 (d) 6

59. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now **[JEE Main 2016]**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

60. In a resonance column first and second resonance are obtained at depths 22.7 cm and 70.2 cm. The third resonance will be obtained at a depth of

(a) 117.7 cm (b) 92.9 cm
(c) 115.5 cm (d) 113.5 cm

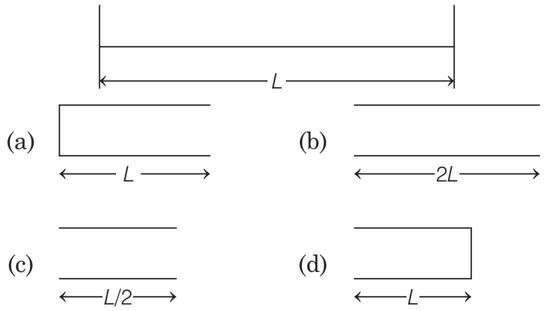
61. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[JEE Main 2014]**

(a) 12 (b) 8
(c) 6 (d) 4

62. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column $l_1 = 30$ cm and $l_2 = 70$ cm. Then, v is equal to

(a) 332 ms^{-1} **[JEE Main 2019]**
(b) 384 ms^{-1}
(c) 338 ms^{-1}
(d) 379 ms^{-1}

63. Figure shows a stretched string of length L and pipes of length L, $2L$, $L/2$ and $L/2$ in options (a), (b), (c) and (d), respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in the air. The fundamental mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance?



64. In a resonance tube experiment, when the tube is filled with water upto a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised, the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is **[JEE Main 2020]**

(a) 1100 Hz (b) 3300 Hz
(c) 2200 Hz (d) 550 Hz

Beats

65. When 2 tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats s^{-1} are heard. Now, some tape is attached on the prong of fork 2. When the tuning forks are sounded again, 6 beats s^{-1} are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2 ?

(a) 196 Hz (b) 200 Hz
(c) 202 Hz (d) 204 Hz

66. In two similar wires of tensions 16 N and T, 3 beats are heard, then T is equals to

(a) 49 N (b) 25 N
(c) 64 N (d) None of these

67. A source of frequency n gives 5 beats s^{-1}, when sounded with a source of frequency 200 s^{-1}. The second harmonic ($2n$) gives 10 beats s^{-1}, when sounded with a source of frequency 420 s^{-1}, then n is equal to
(a) 200 s^{-1} (b) 205 s^{-1} (c) 195 s^{-1} (d) 210 s^{-1}

68. Ten tuning forks are arranged in increasing order of frequency in such a way that any two nearest tuning forks produce 4 beats s^{-1}. The highest frequency is twice that of the lowest. Possible highest and lowest frequencies are
(a) 80 and 40 (b) 100 and 50
(c) 44 and 32 (d) 72 and 36

69. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, ν, $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) 3 (b) 2 (c) 1 (d) 4

70. Two waves $y_1 = A\cos(0.5\,\pi x - 100\,\pi t)$ and
$y_2 = A\cos(0.46\,\pi x - 92\pi t)$ are travelling in a pipe along X-axis.

How many times in a second does a stationary any observer hear loud sound (maximum intensity)?
(a) 4 (b) 8 (c) 10 (d) 12

71. Two forks A and B when sounded together produce four beats s^{-1}. The fork A is in unison with 30 cm length of a sonometer wire and B is in unison with 25 cm length of the same wire at the same tension. The frequencies of the forks are
(a) 24 Hz, 28 Hz (b) 20 Hz, 24 Hz
(c) 16 Hz, 20 Hz (d) 26 Hz, 30 Hz

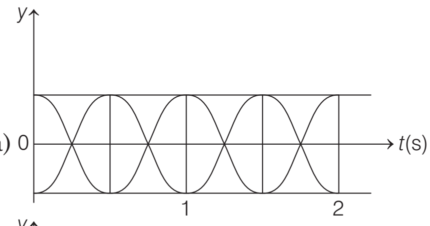
72. Two organ pipes, each closed at one end, give 5 beats s^{-1} when emitting their fundamental notes. If their lengths are in the ratio 50 : 51, their fundamental frequencies (in Hz) are
(a) 250, 255 (b) 255, 260
(c) 260, 265 (d) 265, 270

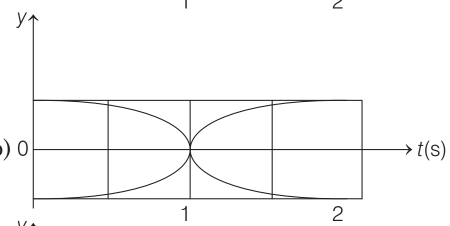
73. A source of sound gives 5 beats s^{-1} when sounded with another source of frequency 100 Hz. The second harmonic of the source together with a source of frequency 205 Hz gives 5 beats s^{-1}. What is the frequency of the source?
(a) 105 Hz (b) 205 Hz
(c) 95 Hz (d) 100 Hz

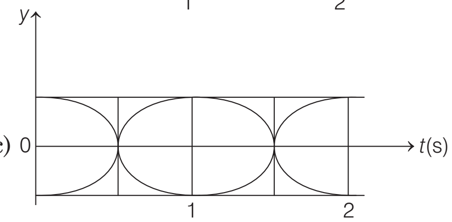
74. Two uniform wires are vibrating simultaneously in their fundamental notes. The tension, lengths diameters and the densities of the two wires are in the ratio 8 : 1, 36 : 35, 4 : 1 and 1 : 2, respectively. If the note of the higher pitch has a frequency 360 Hz, the number of beats produced per second is
(a) 5 (b) 15 (c) 10 (d) 20

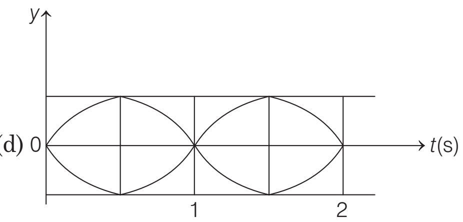
75. A tuning fork of frequency 250 Hz produces a beat frequency of 10 Hz when sounded with a sonometer vibrating as its fundamental frequency. When the tuning fork is filled, the beat frequency decreases. If the length of sonometer wire is 0.5 m, the speed of transverse wave is
(a) 260 ms^{-1} (b) 250 ms^{-1}
(c) 240 ms^{-1} (d) 500 ms^{-1}

76. The correct figure that shows schematically, the wave pattern produced by superposition of two waves of frequencies 9Hz and 11 Hz, is
[JEE Main 2019]

(a)


(b)


(c)


(d)


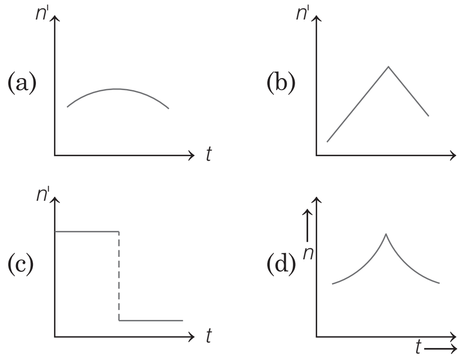
77. Two sounding bodies are producing progressive waves given by $y_1 = 2\sin(400\pi t)$ and $y_2 = 1\sin(404\,\pi t)$, where t is in second, which superpose near the ears of a person? The person will hear
(a) 2 beats/s with intensity ratio 9/4 between maxima and minima
(b) 2 beats/s with intensity ratio 9 between maxima and minima
(c) 4 beats/s with intensity ratio 16 between maxima and minima
(d) 4 beats/s with intensity ratio 16/9 between maxima and minima

Doppler's Effect

78. A source and an observer move away from each other with a velocity of 10 m/s with respect to ground. If the observer finds the frequency of sound coming from the source as 1950 Hz, then actual frequency of the source is (velocity of sound in air = 340 m/s)

(a) 1950 Hz (b) 2068 Hz
(c) 2132 Hz (d) 2486 Hz

79. A train whistling at constant frequency is moving towards a station at a constant speed v. The train goes past a stationary observer on the station. The frequency n' of the sound as heard by the observer is plotted as a function of time t (figure). Identify the expected curve. **[NCERT Exemplar]**



80. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (Take, speed of light = 3×10^8 ms^{-1}) **[JEE Main 2017]**

(a) 12.1 GHz (b) 17.3 GHz
(c) 15.3 GHz (d) 10.1 GHz

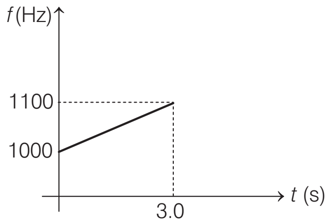
81. A train is moving on a straight track with speed $20\,\text{ms}^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (Take, speed of sound = 320 ms^{-1}) **[JEE Main 2015]**

(a) 6% (b) 12% (c) 18% (d) 24%

82. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (Take, speed of sound in air = 340 ms^{-1}) **[JEE Main 2019]**

(a) 2060 Hz (b) 2250 Hz
(c) 2300 Hz (d) 2150 Hz

83. A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is ($g = 10\text{ m/s}^2$)



(a) 330 m/s (b) 350 m/s
(c) 300 m/s (d) 310 m/s

84. A motor cycle starts from rest and accelerates along a straight path at 2 ms^{-2}. At the starting point of the motor cycle, there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Take, speed of sound = 330 ms^{-1})

(a) 98 m (b) 147 m
(c) 196 m (d) 49 m

85. A bus is moving with a velocity of 5 ms^{-1} towards a huge wall. The driver sounds a horn of frequency 165 Hz. If the speed of sound in air is 335 ms^{-1}, the number of beats heard per second by the passengers in the bus will be

(a) 3 (b) 4
(c) 5 (d) 6

86. Two sound sources emitting sound each of wavelength λ are fixed at a given distance apart. A listener moves with a velocity u along the line joining the two sources. The number of beats heard by him per second is

(a) $2u/\lambda$ (b) u/λ
(c) $\dfrac{u}{3\lambda}$ (d) $\dfrac{2\lambda}{u}$

87. A sound wave of frequency n travels horizontally to the right. It is reflected from a large vertical plane surface moving to the left with speed v. The speed of the sound in the medium is c, then

(a) the frequency of the reflected wave is $\left[\dfrac{c+v}{c-v}\right]$

(b) the wavelength of the reflected wave is $\left[\dfrac{c}{n}\right]\left[\dfrac{c+v}{c-v}\right]$

(c) the number of waves striking the surface per second is $\left[\dfrac{c+v}{c}\right]n$

(d) the number of beats heard by a stationary listener to the left to the reflecting surface is $\dfrac{nv}{c-v}$

88. A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If v is the velocity of sound, the velocity of the car is

(a) $v/\sqrt{2}$ (b) $v/2$
(c) $v/3$ (d) $v/4$

89. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are in ms^{-1} (Take, speed of sound = 300 m/s) **[JEE Main 2019]**

(a) 12, 16 (b) 12, 18
(c) 16, 14 (d) 8, 18

90. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is **[JEE Main 2019]**

(a) $\frac{19}{18}$ (b) $\frac{21}{20}$ (c) $\frac{20}{19}$ (d) $\frac{18}{17}$

91. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take, velocity of sound in air is 350 m/s) **[JEE Main 2019]**

(a) 807 Hz (b) 1143 Hz (c) 750 Hz (d) 857 Hz

92. A source of sound emitting a tone of frequency 200 Hz moves towards an observer with a velocity v equal to the velocity of sound. If the observer also moves away from the source with the same velocity v, the apparent frequency heard by the observer is

(a) 50 Hz (b) 100 Hz (c) 150 Hz (d) 200 Hz

93. A and B are two sources generating sound waves. A listener is situated at C. The frequency of the source at A is 500 Hz. A now moves towards C with a speed 4 m/s. The number of beats heard at C is 6. When A moves away from C with speed 4 m/s, the number of beats heard at C is 18. The speed of sound is 340 m/s. The frequency of the source at B is **[JEE Main 2013]**

(a) 500 Hz (b) 506 Hz
(c) 512 Hz (d) 494 Hz

94. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz.
A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equal to **[JEE Main 2019]**

(a) 5.5 m/s (b) 15.0 m/s
(c) 2.5 m/s (d) 10.0 m/s

95. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz, when he hears it after it gets reflected from the wall. Find the speed of the bus, if speed of the sound is 330 ms^{-1}. **[JEE Main 2020]**

(a) 81 kmh^{-1} (b) 91 kmh^{-1}
(c) 71 kmh^{-1} (d) 61 kmh^{-1}

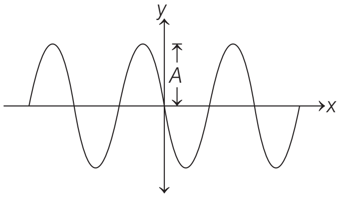
96. A vibrating tuning fork tied to the end of a string 1.988 m long is whirled round a circle. If it makes the revolutions in a second, calculate the ratio of the frequencies of the highest and the lowest notes heard by a stationary observer situated in the plane of rotation of tuning fork at a large distance from the tuning fork. Speed of sound is 350 ms^{-1}. (In both, the observer is assumed to be along the line of velocity sound)

(a) 1.732 (b) 1.154
(c) 1.278 (d) 1.000

ROUND II Mixed Bag

Only One Correct Option

1. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure. **[JEE Main 2019]**



For this wave, the phase ϕ is

(a) $-\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

2. The frequency of a tuning fork A is 2% more than the frequency of a standard tuning fork. The frequency of the same standard tuning fork is 3 % more than the frequency of tuning fork B. If 6 beats s^{-1} are heard when the two tuning forks A and B are excited, the frequency of A is

(a) 120 Hz (b) 122.4 Hz
(c) 116.4 Hz (d) 130 Hz

3. Two uniform strings A and B made of steel are made to vibrate under the same tension. If the first overtone of A is equal to the second overtone of B and if the radius of A is twice that of B, the ratio of the lengths of the strings is

(a) 2 : 1 (b) 3 : 4 (c) 3 : 2 (d) 1 : 3

4. A closed organ pipe and an open organ pipe of same length produce 2 beats s^{-1} when they are set into vibrations together in fundamental mode. The length of open pipe is now halved and that of closed pipe is doubled. The number of beats produced will be

(a) 7 (b) 4 (c) 8 (d) 2

5. A string is under tension, so that its length is increased by $1/n$ times its original length. The ratio of fundamental frequency of longitudinal vibrations and transverse vibrations will be

(a) $1 : n$ (b) $n^2 : 1$ (c) $\sqrt{n} : 1$ (d) $n : 1$

6. Standing waves are produced by the superposition of two waves

$$y_1 = 0.05 \sin(3\pi t - 2x)$$
$$y_2 = 0.05 \sin(3\pi t + 2x)$$

where, x and y are in metres and t is in second. What is the amplitude of the particle at $x = 0.5$ m? (Take, $\cos 57.3° = 0.54$)

(a) 2.7 cm (b) 5.4 cm
(c) 8.1 cm (d) 10.8 cm

7. The ends of a stretched wire of length L are fixed at $X = 0$ and $X = L$. In one experiment, the displacement of the wire is $Y_1 = A \sin\left(\frac{\pi x}{L}\right) \sin \omega t$ and energy is E_1 and in another experiment, its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then,

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

8. In an experiment, it was found that string vibrates in n loops when a mass M is placed on the pan. What mass should be placed on the pan to make it vibrate in $2n$ loops, with same frequency. Neglect the mass of the pan.

(a) $M/4$ (b) $4M$
(c) $2M$ (d) $M/2$

9. A string of mass 0.2 kg/m has length $L = 0.6$ m. It is fixed at both ends and stretched such that it has a tension of 80 N. The string vibrates in three segments with amplitude = 0.5 cm. The amplitude of transverse velocity is

(a) 9.42 ms^{-1} (b) 3.14 ms^{-1}
(c) 1.57 ms^{-1} (d) 6.28 ms^{-1}

10. When a train approaches a stationary observer, the apparent frequency of the whistle is n' and when the same train recedes away from the observer, the apparent frequency is n''. Then, the apparent frequency n when the observer moves with the train is

(a) $n = \frac{n' + n}{2}$ (b) $n = \sqrt{n'n''}$
(c) $n = \frac{2n'n''}{n' + n''}$ (d) $n = \frac{2n'n''}{n' - n''}$

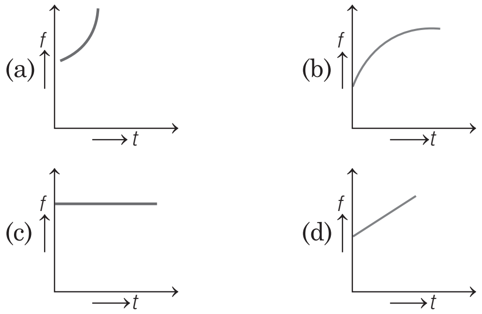
11. A table is revolving on its axis at 5 revolutions per second. A sound source of frequency 1000 Hz is fixed on the table at 70 cm from the axis. The minimum frequency heard by a listener standing at a distance from the table will be (Take, speed of sound = 352 ms^{-1})

(a) 1000 Hz (b) 1066 Hz
(c) 941 Hz (d) 352 Hz

12. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and the plate is allowed to fall freely. Eight complete oscillations are counted when the plate falls through 10 cm. What is the frequency of the tuning fork?

(a) 112 Hz (b) 56 Hz
(c) $\frac{8}{7}$ Hz (d) $\frac{7}{8}$ Hz

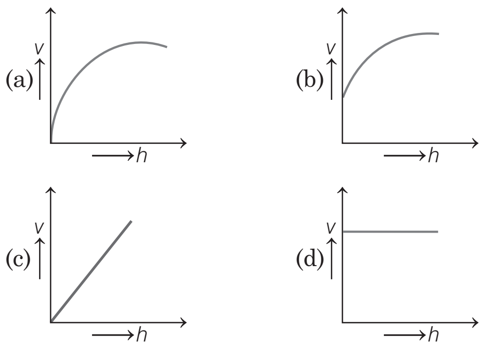
13. An observer starts moving with uniform acceleration a towards a stationary sound source of frequency f_0. As the observer approaches the source, the apparent frequency (f) heard by the observer varies with time (t) as



14. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope? **[JEE Main 2020]**

(a) 3 (b) 6 (c) 12 (d) 9

15. A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of wave pulse varies with height h from the lower end as



16. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is **[JEE Main 2020]**

(a) 0.01 mm (b) 0.04 mm
(c) 0.03 mm (d) 0.02 mm

17. Equations of a stationary wave and a travelling wave are $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin(\omega t - kx)$. The phase difference between two points $x_1 = \dfrac{\pi}{3k}$ and $x_2 = \dfrac{3\pi}{2k}$ are ϕ_1 and ϕ_2 respectively for the two waves. The ratio ϕ_1/ϕ_2 is

(a) 1 (b) 5/6 (c) 3/4 (d) 6/7

18. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is speed of sound, the expression for the beat frequency heard by the motorist is

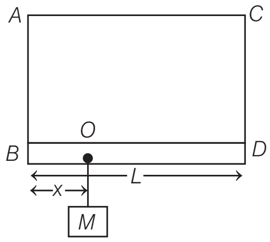
(a) $\dfrac{(v+v_m)f}{v+v_b}$ (b) $\dfrac{(v+v_m)f}{v-v_b}$

(c) $\dfrac{2v_b(v+v_m)f}{v^2-v_b^2}$ (d) $\dfrac{2v_m(v+v_b)f}{v^2-v_b^2}$

19. The frequency of a sonometer wire is 100 Hz. When the weights producing the tension are completely immersed in water, the frequency becomes 80 Hz and on immersing the weights in a certain liquid, the frequency becomes 60 Hz. The specific gravity of the liquid is

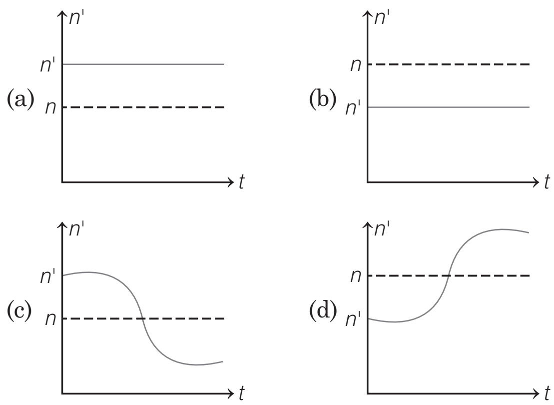
(a) 1.42 (b) 1.77 (c) 1.21 (d) 1.82

20. A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to (x) further it is observed that the frequency of 1st harmonic (fundamental frequency) in AB is equal to 2nd harmonic frequency in CD. Then, length of BO is



(a) $\dfrac{L}{5}$ (b) $\dfrac{4L}{5}$ (c) $\dfrac{3L}{4}$ (d) $\dfrac{L}{4}$

21. Source and observer, both start moving simultaneously from origin, one along X-axis and the other along Y-axis with speed of source equal to twice the speed of observer. The graph between the apparent frequency (n') observed by observer and time t in figure would be



22. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies, ω_1 and ω_2 respectively, where $\omega_2 - \omega_1 = 10^3$ Hz. A detector is receiving signals from the two stations simultaneously. It can only detect signals of intensity $> 2A^2$. The time interval between successive maxima of the intensity of the signal received by the detector is

(a) 10^3 s (b) 10^{-3} s

(c) 10^{-4} s (d) 10^4 s

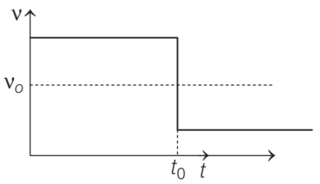
23. A musician produce the sound of second harmonics from open end flute of 50 cm. The other person moves toward the musician with speed 10 km/h from the second end of room. If the speed of sound 330 m/s, the frequency heard by running person will be **[JEE Main 2019]**

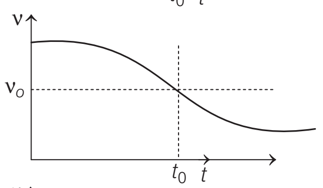
(a) 666 Hz (b) 500 Hz

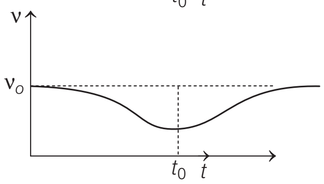
(c) 753 Hz (d) 333 Hz

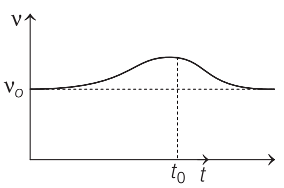
24. A sound source S is moving along a straight track with speed v and is emitting sound of frequency ν_o (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by

(Here, t_0 represents the instant when the distance between the source and observer is minimum.) **[JEE Main 2020]**

(a)


(b)


(c)


(d)


25. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in metre) of the waves are **[JEE Main 2020]**

(a) 1, 2, 3,...... (b) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \ldots$

(c) 1, 3, 5,..... (d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots$

Numerical Value Questions

26. The mass of 1 mol of air is 29×10^{-3} kg, then the speed of sound (in m/s) in air at standard temperature and pressure is ....

27. A whistle of frequency 540 Hz rotates in a circle of radius 2 m at a linear speed of 30 m/s. What is the difference of lowest and highest frequency heard by an observer a long distance away at rest with respect to the centre of circle (in Hz)? (Take, speed of sound in air = 330 m/s)

28. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2, respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency.

The length of the open pipe is $\frac{x}{3} L \sqrt{\frac{\rho_1}{\rho_2}}$, where x is .... (Round off to the nearest integer) **[JEE Main 2021]**

29. Four harmonic waves of equal frequencies and equal intensities (I_0) have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is ....

30. A stretched rope having linear mass density 5×10^{-2} kg m^{-1} is under a tension of 80 N. The power (in watt) that has to be supplied to the rope to generate harmonic waves at a frequency of 60 Hz and an amplitude of 6 cm is ....

31. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ...... Hz. **[JEE Main 2020]**

32. An aluminium wire is clamped at each end and under zero tension at room temperature. Reducing the temperature, which results in a decrease in the wire's equilibrium length, increases the tension in the wire. The strain $(\Delta L/L)$ that results in a transverse wave speed of 100 m/s is 3.86×10^{-n}, then the value of n is ....

(Take, the cross-sectional area of the wire to be 5.00×10^{-6} m^2, the density to be 2.70×10^3 kg/m^3, and Young's modulus to be 7.00×10^{10} N/m^2)

Answers

Round I

1. (c)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (c)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (b)
21. (d)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (a)
31. (b)	**32.** (d)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (a)	**39.** (c)	**40.** (b)
41. (d)	**42.** (d)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (c)	**49.** (a)	**50.** (c)
51. (c)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (c)	**56.** (b)	**57.** (d)	**58.** (d)	**59.** (d)	**60.** (a)
61. (c)	**62.** (b)	**63.** (b)	**64.** (c)	**65.** (a)	**66.** (a)	**67.** (b)	**68.** (d)	**69.** (c)	**70.** (a)
71. (b)	**72.** (a)	**73.** (a)	**74.** (c)	**75.** (a)	**76.** (a)	**77.** (b)	**78.** (b)	**79.** (c)	**80.** (b)
81. (b)	**82.** (b)	**83.** (c)	**84.** (a)	**85.** (c)	**86.** (a)	**87.** (c)	**88.** (c)	**89.** (b)	**90.** (a)
91. (c)	**92.** (d)	**93.** (c)	**94.** (c)	**95.** (b)	**96.** (b)				

Round II

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (c)
11. (c)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (a)
21. (b)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** 332.5		**27.** 99		
28. 4	**29.** 3	**30.** 518	**31.** 106.06		**32.** 4				

Solutions

Round I

1. Mechanical transverse waves can propagate through solids only as solids have elasticity of shape.

2. In the figure, C reaches the position, where A already reached, after $\omega t = \frac{\pi}{2}$ and A reaches the position, where B already reached, after $\omega t = \pi/2$.

3. Given, $y = 10^{-4} \sin\left[100t - \frac{x}{10}\right]$

Comparing it with the standard equation of wave motion

$$y = A\sin\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right], \text{ we get}$$

$$\frac{2\pi}{T} = 100$$

or $$T = \frac{2\pi}{100} = \frac{\pi}{50}\text{ s}$$

$$\frac{2\pi}{\lambda} = \frac{1}{10}$$

or $$\lambda = 20\pi$$

and velocity, $v = \frac{\lambda}{T} = \frac{20\pi}{\pi/50} = 1000 \text{ ms}^{-1}$

4. The given equation is

$$y(x, t) = 3.0 \sin\left[36t + 0.018x + \frac{\pi}{4}\right]$$

As positive direction is from left to right and x is positive, therefore wave is travelling from right to left.

Compare the given equation with the standard from

$$y = r\sin\left[\frac{2\pi t}{T} + \frac{2\pi x}{\lambda} + \phi\right]$$

$$\frac{2\pi}{T} = 36$$

$$\frac{2\pi}{\lambda} = 0.018$$

Speed of wave, $v = \frac{\lambda}{T} = \frac{36}{0.018} = 2000 \text{ cm/s} = 20 \text{ m/s}$

Again, $T = \frac{2\pi}{36} = \frac{\pi}{18}$

Frequency, $\nu = \frac{1}{T} = \frac{18}{\pi}\text{ Hz} = 5.7 \text{ Hz}$

5. As, $\frac{dy}{dt} = y_0 \cos 2\pi\left[ft - \frac{x}{\lambda}\right] \times 2\pi f$

Maximum particle velocity $= \left(\frac{dy}{dt}\right)_{\max} = 2\pi f y_0 \times 1$

Wave velocity $= f\lambda$

As, $$2\pi f y_0 = 4 f\lambda$$

$\therefore$ $$\lambda = \frac{2\pi y_0}{4} = \frac{\pi y_0}{2}$$

6. From the wave shown in the given figure.

The amplitude, $A = 0.06$ m

$$\frac{5}{2}\lambda = 0.2 \text{ m}$$

$\therefore$ $$\lambda = 0.08 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{300}{0.08} = 3750 \text{ Hz}$$

$$k = \frac{2\pi}{\lambda} = 78.5 \text{ m}^{-1}$$

and $\omega = 2\pi f = 23562$ rad/s

At $t = 0, x = 0, \frac{dy}{dx} =$ positive

and the given curve is a sine curve.

Hence, equation of wave travelling in positive x-direction should have the form

$$y(x, t) = A \sin(kx - \omega t)$$

Substituting the values, we have

$$y = (0.06) \sin[(78.5 \text{ m}^{-1})\, x - 23562 \text{ s}^{-1})t] \text{ m}$$

7. Wave equation is given by, $y = 10^{-3} \sin(50\,t + 2x)$

Speed of wave is obtained by differentiating phase of wave.

Now, phase of wave from given equation is

$$\phi = 50\,t + 2x = \text{constant}$$

Differentiating 'ϕ' w.r.t. 't', we get

$$\frac{d}{dx}(50\,t + 2x) = \frac{d}{dt}(\text{constant})$$

$$\Rightarrow \quad 50 + 2\left(\frac{dx}{dt}\right) = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{-50}{2} = -25 \text{ ms}^{-1}$$

So, wave is propagating in negative x-direction with a speed of 25 ms^{-1}.

Alternate Solution

The general equation of a wave travelling in negative x-direction is given as

$$y = a \sin(\omega t + kx) \quad \ldots\text{(i)}$$

Given equation of wave is

$$y = 10^{-3} \sin(50t + 2x) \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\omega = 50 \text{ and } k = 2$$

Velocity of the wave, $v = \frac{\omega}{k} = \frac{50}{2} = 25$ m/s

8. Given, $\frac{\lambda}{4} = 4$ cm

$\therefore \quad \lambda = 16$ cm and $T = 0.4$ s

As, $\quad f\lambda \times T = 2\pi$

$$\Rightarrow \quad f = \frac{2\pi}{16 \times 0.4} = \frac{5\pi}{16} \text{ s}^{-1}$$

Now, $\quad v = f\,\lambda = \frac{5\pi}{16} \times 16 = 5\pi$ cm/s

9. Maximum particle velocity, $u_{\max} = \omega A \quad \ldots\text{(i)}$

Maximum particle acceleration, $a_{\max} = \omega^2 A \quad \ldots\text{(ii)}$

Dividing Eq. (ii) by Eq. (i), we get

Angular frequency, $\omega = \frac{a_{\max}}{u_{\max}} = \frac{90}{3} = 30$ rad/s

From Eq. (i), amplitude, $A = \frac{u_{\max}}{\omega} = \frac{3}{30} = 0.1$ m

Propagation constant, $k = \frac{\omega}{v} = \frac{30}{20} = 1.5 \text{ m}^{-1}$

Equation of wave is $y = A \sin(\omega t \pm kx)$ or wave function $y = 0.1 \sin(30t \pm 1.5\,x)$ wave x is in metres and t in seconds.

Positive sign is for wave propagation along negative X-axis and negative sign for wave propagating along positive X-axis.

10. Here, $y = A \sin(\omega t \pm \phi)$

Given, $A = 10$ cm and $y = 5$ cm,

$$\therefore \quad \sin(\omega t \pm \phi) = \frac{1}{2} = \sin 30°$$

$$\Rightarrow \quad \omega t \pm \phi = 30°$$

Here, $\omega = \frac{2\pi}{T} = 2\pi\left(\frac{v}{\lambda}\right) = 2\pi\left(\frac{10 \text{ cm s}^{-1}}{50 \text{ cm}}\right) = \frac{2\pi}{5} \text{ rad s}^{-1}$

$\therefore$ The velocity of the point P is

$$v_P = \frac{dy}{dt} = A\omega \cos(\omega t \pm \phi) = (10 \text{ cm})\left(\frac{2\pi}{5} \text{s}^{-1}\right)(\cos 30°)$$

$$= 2\pi\sqrt{3} \text{ cms}^{-1} = \frac{\pi\sqrt{3}}{50} \text{ m s}^{-1}.$$

$$\therefore \mathbf{v}_P = \frac{\pi\sqrt{3}}{50} \hat{\mathbf{j}} \text{ m s}^{-1}$$

11. As, intensity = power/area

From a point source, energy spreads over the surface of a sphere of radius r.

$$\therefore \quad \text{Intensity} = \frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

But $\quad$ intensity = (amplitude)^2

$$\therefore \quad (\text{Amplitude})^2 \propto \frac{1}{r^2}$$

or $\quad \text{Amplitude} \propto \frac{1}{r}$

Clearly, at distance $2r$, amplitude becomes $A/2$.

12. Given, $y = 8 \sin 2\pi\,(0.1\,x - 2\,t)$

Compare it with the equation of wave motion

$$y = r \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$\frac{1}{\lambda} = 0.1 = 10 \text{ cm}$$

From $\quad \phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{10} \times 2 = 0.4 \times 180° = 72°$

13. Here, $m = \frac{2.5}{20}$ kg/m, $T = 200$N

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200}{2.5/20}}$$

$$v = \sqrt{\frac{200 \times 200}{25}}$$

$$= \frac{200}{5} = 40 \text{ m/s}$$

Time taken, $\quad t = \frac{L}{v} = \frac{20}{40} = 0.5$ s

14. Transverse wave speed over a string is given by

$$v = \sqrt{\frac{T}{\mu}} \quad \text{... (i)}$$

where, T = tension in string

and μ = mass per unit length of string.

Here, when velocity is v, then tension, $T_1 = 2.06 \times 10^4$ N

Let when velocity is $\frac{v}{2}$, then tension is T, hence from Eq. (i), we get

$$\frac{v}{\frac{v}{2}} = \sqrt{\frac{T_1}{T}}$$

$$\Rightarrow \quad T = \frac{T_1}{4} = \frac{2.06 \times 10^4}{4}$$

or $\quad T = 5.15 \times 10^3$ N

15. In a wave equation, x and t must be related in the form $(x - vt)$. Therefore, we rewrite the given equation as

$$y = \frac{1}{1 + (x - vt)^2}$$

For $t = 0$, it becomes $y = \frac{1}{1 + x^2}$

and for $t = 2$, it becomes

$$y = \frac{1}{[1 + (x - 2v)^2]} = \frac{1}{1 + (x - 1)^2}$$

$\therefore \quad 2v = 1$

or $\quad v = 0.5 \text{ ms}^{-1}$

16. $\because \quad \omega = 2\pi f = (2\pi)(100) = (200\pi)$ rad/s

$$\therefore \quad v = \frac{\omega}{k}$$

$$\Rightarrow \quad k = \frac{\omega}{v} = \omega\sqrt{\frac{\mu}{T}}$$

$$= (200\pi)\sqrt{\frac{3.5 \times 10^{-3}}{35}} = 2\pi \text{ m}^{-1}$$

At zero displacement,

$$v_P = \omega A = -v\frac{\partial y}{\partial x}$$

$$\therefore \quad |A| = \frac{v\left(\frac{\partial y}{\partial x}\right)}{\omega} = \frac{\left(\sqrt{\frac{T}{\mu}}\right)\text{(slope)}}{\omega}$$

$$= \frac{\left(\sqrt{\frac{35}{3.5 \times 10^{-3}}}\right)(\pi/20)}{200\pi}$$

$$\Rightarrow \quad |A| = 0.025 \text{ m}$$

Hence, $y = 0.025\cos(200\pi t - 2\pi x)$

17. According to Newton's formula for velocity of sound in a fluid,

$$v = \sqrt{\frac{B_a}{\rho}}$$

$$v \propto \sqrt{B_a}$$

and $$v \propto \frac{1}{\sqrt{\rho}}$$

18. At given temperature and pressure,

$$v \propto \frac{1}{\sqrt{\rho}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{4}{1}} = 2:1$$

19. From $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{\Delta v}{v} = \frac{1}{2}\left(\frac{\Delta T}{T}\right)$$

$$\Rightarrow \quad \frac{\Delta v}{v} \times 100 = \frac{1}{2}\left(\frac{1}{T}\right) \times 100$$

$$= \frac{1}{2} \times \frac{1}{300} \times 100 = 0.167\%$$

20. Molecular weight of mixture,

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

$$= \frac{1 \times 4 + 2 \times 32}{1 \times 2} = \frac{68}{3}$$

$$= \frac{68}{3} \times 10^{-3} \text{ kg mol}^{-1}$$

For helium, $C_{V_1} = \frac{3}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$

$$(C_V)_{\text{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{1 \times \frac{3R}{2} + 2 \times \frac{5R}{2}}{1 + 2} = \frac{13R}{6}$$

Now, $\quad (C_p)_{\text{mix}} = (C_V)_{\text{mix}} + R$

$$= \frac{13R}{6} + R = \frac{19R}{6}$$

$$\Rightarrow \quad \gamma_{\text{mix}} = \frac{(C_p)_{\text{mix}}}{(C_V)_{\text{mix}}} = \frac{19}{13}$$

Speed of sound, $v = \sqrt{\frac{\gamma_{\text{mix}} RT}{M_{\text{mix}}}} = \sqrt{\frac{19}{13} \times \frac{8.31 \times 300}{\frac{68}{3} \times 10^{-3}}}$

$$= 400.8 \text{ ms}^{-1}$$

21. As, $v_L = \sqrt{\frac{Y}{\rho}}$ and $v_T = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 \rho}}$

$$\because \quad \frac{v_L}{v_T} = \sqrt{\frac{Y}{\rho} \times \frac{\pi r^2 \rho}{T}} = \sqrt{\frac{Y}{T/\pi r^2}} = \sqrt{\frac{Y}{\text{stress}}}$$

$$\therefore \text{ Stress} = \frac{Y}{(v_L/v_T)^2} = \frac{1 \times 10^{11}}{(100)^2} = 1 \times 10^7 \text{ Nm}^{-2}$$

22. As, intensity $\propto a^2\omega^2$

Here, $\quad \frac{a_A}{a_B} = \frac{2}{7}$ and $\frac{\omega_A}{\omega_B} = \frac{1}{2}$

$$\Rightarrow \quad \frac{I_A}{I_B} = \left(\frac{2}{1}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{1}$$

23. Amplitude, $A = \frac{6}{2} = 3 \text{ cm} = 0.03 \text{ cm}$

Angular frequency, $\omega = 2\pi f = 2\pi \times 245$
$= 1538.6 \text{ rad s}^{-1}$
$= 1.5 \times 10^3 \text{ rad s}^{-1}$

Wave number, $k = \dfrac{\omega}{v} = \dfrac{1538.6}{300} = 5.1 \text{ radm}^{-1}$

$\therefore \quad y(x, t) = 0.03 \sin[5.1x - (1.5 \times 10^3)t]$

24. $I = \dfrac{(\Delta p)^2_{\max}}{2\rho V}$

$\Rightarrow \quad v = \dfrac{(\Delta p)^2_{\max}}{2\rho I} = (2 \times 10^{-4})^2/2 \times 1 \times 10^{-10}$

$= 200 \text{ ms}^{-1}$

Amplitude of wave is

$A = \dfrac{(\Delta p)_{\max}}{\omega \rho v} = \dfrac{2 \times 10^{-4}}{10^3 \times 1 \times 200} = 10^{-9} \text{ m}$

$\therefore \quad k = \dfrac{\omega}{v} = \dfrac{1000}{200} = 5 \text{ m}^{-1}$

As, the wave is travelling in $-x$-direction, the only equation with positive sign between ωt and kx, with suitable ω and k values is option (b).

25. On reflection from a denser medium, there is a phase reversal of 180°.

New amplitude $= \dfrac{2}{3} \times 0.6 = 0.4$

$\therefore$ Equation of reflected wave,

$$y = 0.4 \sin 2\pi \left[t + \frac{x}{2} + 180° \right]$$

$$= -0.4 \sin 2\pi \ (t + x/2)$$

26. As, $\quad y = 4\cos^2(t/2)\sin(1000t)$

$= 2[2\cos^2(t/2)\sin(1000t)]$

$= 2[2(1 \ \cos t)\sin(1000t)]$

$= 2\sin 1000t + 2\sin 1000t \cos t$

$y = 2\sin 1000t + \sin(1001)t + \sin(999t)$

$\therefore$ The given wave equation represents the superposition of three waves.

27. Resultant displacement along X-axis,

$x = y_1 - y_3 = 8 - 2 = 6$

Resultant displacement along Y-axis,

$y = y_2 - y_4 = 4 - 1 = 3$

Net displacement,

$r = \sqrt{x^2 + y^2} = \sqrt{6^2 + 3^2} = \sqrt{45}$

Also, $\tan\theta = \dfrac{y}{x} = \dfrac{3}{6} = \dfrac{1}{2}$

$\theta = \tan^{-1}(1/2)$

28. After 2 s, the pulses will overlap completely. The string becomes straight and therefore, does not have any potential energy. Its entire energy must be purely kinetic.

29. $y_1 = 0.05\cos(0.50\pi x - 100\pi t)$

and $y_2 = 0.05\cos(0.46\pi x - 92\pi t)$

Comparing these two equations are

$y = A\sin(kx - \omega t)$, we get

$\omega_1 = 100\pi \quad \text{and} \quad \omega_2 = 92\pi$

Now, speeds, $v_1 = \dfrac{A}{100\pi} = \dfrac{0.05}{100\pi}$

and $\quad v_2 = \dfrac{A}{92\pi} = \dfrac{0.05}{92\pi}$

Now, the resultant speed,

$$v = \sqrt{\left(\frac{0.05}{92\pi}\right)^2 + \left(\frac{0.05}{100\pi}\right)^2}$$

$$= 200 \text{ m/s}$$

30. Since, amplitude at all waves are same but Z_1 and Z_2 are displacements of two waves of same frequency travelling in opposite directions.They will form a stationary wave.

31. Since, the point $x = 0$ is a node and reflection is taking place from point $x = 0$.This means that reflection must be taking place from the fixed end, hence the reflected ray must suffer an additional phase change of π or a path change of $\dfrac{\lambda}{2}$.

So, if $y_{\text{incident}} = a\cos(kx - \omega t)$

$\Rightarrow \quad y_{\text{reflected}} = a\cos(-kx - \omega t + \pi) = -a\cos(\omega t + kx)$

32. Distance between two consecutive node is $\dfrac{\lambda}{2}$.

$\therefore \quad \dfrac{\lambda}{2} = \dfrac{2}{2} \text{ m} = 1 \text{ m}$

So, the distance of another node from the surface will be

$$3 + \frac{\lambda}{2} = 3 + 1 = 4 \text{ m}$$

33. The given equation is

$$y(x, t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$$

As terms involving x and t are independent of each other, the given equation represents a stationary wave.

Compare the given equation with the standard form of equation of stationary wave

$$y(x, t) = 2r\sin kx \cos\omega t$$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{3}$

$\therefore \quad \lambda = 3\text{m}$

$\omega = 120\pi$

$\therefore \quad \nu = \dfrac{\omega}{2\pi} = \dfrac{120\pi}{2\pi} = 60 \text{ Hz}$

and $\quad v = \nu\lambda = 60 \times 3 = 180 \text{ m/s}$

Hence, the given stationary wave is the result of superposition of two waves of wavelength 3 m and frequency 60 Hz each, travelling with a velocity of 180 m/s in opposite directions.

34. Node means a point at which medium particles do not displace from its mean position and antinode mean a point at which particles oscillate with maximum possible amplitude. Nodes and antinodes are obtained for both types of stationary waves, transverse and longitudinal. Hence, options (a) and (b) both are wrong. To obtain a stationary wave, two waves travelling in opposite directions, having same amplitude, same frequency are required. They must have same nature, means either both of the waves should be longitudinal or both of them should be transverse. Hence, option (c) is correct.

35. Given, $L = 15.0$ m, $v = 12\ \text{ms}^{-1}$

Since, there are 6 nodes, with the ends as nodes, there will be five half wavelength in the string.

So, $$\frac{5\lambda}{2} = L = 15$$

$\Rightarrow$ $$\lambda = 6.0\ \text{m}$$

Using $f = \dfrac{v}{\lambda} = \dfrac{12}{6} = 2.0$ Hz

36. The given equation is

$y(x, t) = 0.06 \sin\left(\dfrac{2\pi x}{3}\right) \cos(120\ \pi t)$

It represents a stationary wave. Therefore, all the points between two consecutive nodes.

I. vibrate with same frequency
II. in same phase, but
IV. different amplitudes. The amplitude is zero at nodes and maximum at antinodes (between the nodes).

37. Here, $\rho_1 = \rho_2$; $\dfrac{r_1}{r_2} = \dfrac{1}{2}$, $T_1 = T_2$

$\Rightarrow$ $$n_1 = \frac{1}{2Lr_1}\sqrt{\frac{T_1}{\pi\rho_1}}$$

and $$n_2 = \frac{1}{2Lr_2}\sqrt{\frac{T_2}{\pi\rho_2}}$$

$\therefore$ $$\frac{n_1}{n_2} = \frac{r_1}{r_2} = 2 : 1$$

38. From $n = \dfrac{L}{LD}\sqrt{\dfrac{T}{\pi\rho}}$

When radius of string is doubled, diameter D becomes twice. As T and ρ are same , n becomes $1/2$, *i.e.* $n/2$.

39. According to the law of length,

$$n_1 l_1 = n_2 l_2$$

$\therefore$ $$l_2 = \frac{n_1 l_1}{n_2} = \frac{800 \times 50}{1000} = 40\ \text{cm}$$

40. Fundamental frequency of oscillation of a taut string is

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where, L = length, T = tension

and μ = mass per unit length.

Now, given X and Z are identical strings.

$\therefore$ L and μ are same for both X and Z.

$\Rightarrow$ $$f \propto \sqrt{T}$$

$\Rightarrow$ $$\frac{f_X}{f_Z} = \sqrt{\frac{T_X}{T_Z}}$$

$\Rightarrow$ $$\frac{T_X}{T_Z} = \frac{f_X^2}{f_Z^2}$$

$$= \left(\frac{450}{300}\right)^2 = \frac{9}{4} = 2.25$$

41. Velocity 'v' of the wave on the string $= \sqrt{\dfrac{T}{\mu}}$

where, T = tension and μ = mass per unit length.
Substituting the given values, we get

$$v = \sqrt{\frac{8}{5} \times 1000} = 40\ \text{ms}^{-1}$$

Wavelength of the wave on the string, $\lambda = \dfrac{v}{f}$

where, f = frequency of wave.

$\Rightarrow$ $$\lambda = \frac{40}{100}\ \text{m} = 40\ \text{cm}$$

$\therefore$ Separation between two successive nodes,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0\ \text{cm}$$

42. Speed of sound in gas, $v = \sqrt{\dfrac{\gamma p}{\rho}}$, γ is highest for monoatomic gas.

Since, $\Delta B = 10 \log_{10}\left(\dfrac{I_1}{I_2}\right)$

$\Rightarrow$ $20 = 10 \log_{10}\left(\dfrac{I_1}{I_2}\right) \Rightarrow \dfrac{I_1}{I_2} = 100$

Passing of sound wave is considered as adiabatic process.

43. When reflection of a sound wave occurs at a rigid boundary (like the closed end of a pipe), the particles at the boundary are unable to vibrate. The reflected wave thus generated interferes with the incident wave to produce zero displacement (or node). At this displacement node exists the pressure antinode. Thus, reflected pressure wave has the same phase as the incident wave and a high pressure compression pulse gets reflected as a compression pulse.

Similarly, for reflection from the open end of the pipe, the particles vibrate with increased amplitude (displacement antinode) and pressure remains at the average value (pressure node). The reflected pressure wave interferes destructively with the incident wave, so that a phase change of π occurs from the open end.

Hence, a high-pressure compression pulse reflects as a low-pressure refraction pulse.

44. (a) Velocity of sound wave in air independent of pressure, if T = constant

(b) $v = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{\gamma R(t + 273)}{M}}$

or $$v^2 \propto (t + 273)$$

(c) $v = \sqrt{T/\mu}$

$\therefore$ $$v \propto \sqrt{T}$$

or $$v^2 \propto T$$

(d) $f_0 = \frac{v}{2L}$ or $\frac{v}{4L}$

$\Rightarrow \quad f_0 \propto \frac{1}{L}$

Hence, option (d) is correct.

45. As, $n \propto \frac{1}{L}$ or $L \propto \frac{1}{n}$

and $\quad L = L_1 + L_2 + L_3$

$\Rightarrow \quad \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

46. Resonant frequency for a stretched string is given by

$$f_n = nf_1 = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

Given, two consecutive resonant frequencies are 420 Hz and 490 Hz.

So, $\quad \frac{n}{2L}\sqrt{\frac{T}{\mu}} = 420 \quad$ ...(i)

and $\quad \frac{n+1}{2L}\sqrt{\frac{T}{\mu}} = 490 \quad$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$$\frac{1}{2L}\sqrt{\frac{T}{\mu}} = 490 - 420 = 70$$

Here, $\quad T = 540\text{ N}, \mu = 6 \times 10^{-3}\text{ kg m}^{-1}$

So, $\quad L = \frac{1}{2 \times 70}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 70} \times \sqrt{\frac{540}{6 \times 10^{-3}}}$

$= 2.14\text{ m} \simeq 2.1\text{ m}$

47. Let m = mass per unit length of rope,

T = tension in the rope at a distance x from the lower end

$\therefore \quad T = (mg)\,x$ = weight of x metre of rope

As $\quad v = \sqrt{\frac{T}{m}}$, therefore $v = \sqrt{\frac{mgx}{m}} = \sqrt{gx}$

i.e. $v \propto \sqrt{x}$

48. Given equation of stationary wave is

$$y = 0.3 \sin (0.157x) \cos (200\pi t)$$

Comparing it with general equation of stationary wave, *i.e.* $y = a \sin kx \cos \omega t$, we get

$k = \left(\frac{2\pi}{\lambda}\right) = 0.157$

$\Rightarrow \quad \lambda = \frac{2\pi}{0.157} = 4\pi^2 \quad \left(\because \frac{1}{2\pi} \approx 0.157\right)$ ...(i)

and $\quad \omega = 200\,\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{20}$ s

As the possible wavelength associated with nth harmonic of a vibrating string, *i.e.* fixed at both ends is given as

$$\lambda = \frac{2L}{n} \text{ or } L = n\left(\frac{\lambda}{2}\right)$$

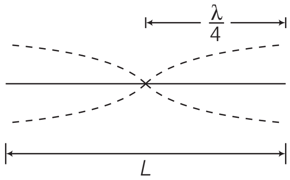
Now, according to question, string is fixed from both ends and oscillates in 4th harmonic, so

$$4\left(\frac{\lambda}{2}\right) = L \Rightarrow 2\lambda = L$$

or $\quad L = 2 \times 4\pi^2 = 8\pi^2 \quad$ [using Eq.(i)]

Now, $\quad \pi^2 \approx 10 \Rightarrow L \approx 80$ m

49.



From vibration mode,

$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$\therefore$ Wave speed, $v = \sqrt{\frac{Y}{\rho}}$

So, frequency $f = \frac{v}{\lambda}$

$\Rightarrow \quad f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$

$= \frac{1}{2 \times 60 \times 10^{-2}}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}} \approx 5000$ Hz

$f = 5$ kHz

50. For an organ pipe open at one end,

frequency of 1st overtone, $n_1 = \frac{3\,v}{4\,L_1}$

For the organ pipe open at both ends,

frequency of 3rd harmonic, $n_2 = \frac{3\,v}{2\,L_2}$

As, $\quad n_1 = n_2$

$\therefore \quad \frac{3\,v}{4\,L_1} = \frac{3\,v}{2\,L_2}$

or $\quad \frac{L_1}{L_2} = \frac{2}{4} = \frac{1}{2}$

51. As fixed end is a node, therefore distance between two consecutive nodes $= \frac{\lambda}{2} = 10$ cm

$\lambda = 20\text{ cm} = 0.2\text{ m}$

Now, $\quad v = \nu\,\lambda$

$\therefore \quad v = 100 \times 0.2 = 20\text{ ms}^{-1}$

52. As is clear from figure of question,

$L = \frac{\lambda_p}{4}, \; \lambda_p = 4\,L, \; n_p = \frac{v}{\lambda_p} = \frac{v}{4\,L}$

$L = \frac{\lambda_q}{2}, \; \lambda_q = 2\,L, \; n_q = \frac{v}{\lambda_q} = \frac{v}{2\,L}$

$L = \lambda r, \lambda_r = 1, n_r = \frac{v}{\lambda_r} = \frac{v}{L}$

$L = \frac{3\lambda_s}{4}, \; \lambda_s = \frac{4\,L}{3}, \; n_s = \frac{v}{\lambda_s} = \frac{3\,v}{4\,L}$

$L = \frac{3\,\lambda s}{4}, \lambda_s = \frac{4\,L}{3}$

$$h_s = \frac{v}{\lambda_s} = \frac{3v}{4L}$$

$\therefore \quad n_p : n_q : n_r : n_s = \frac{v}{4L} : \frac{v}{2L} : \frac{v}{L} : \frac{3v}{4L} = 1 : 2 : 4 : 3$

53. In case of open pipe, the frequency of second harmonic is,

$$f_1 = 2v/2L = v/L$$

In case of closed pipe, the frequency of nth harmonic is

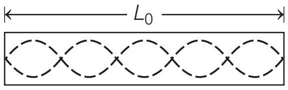
$$f_2 = nv/4L = nf_1/4$$

where, $n = 1, 3, 5, \ldots$, *i.e.* n is odd and $f_1 > f_2$

It will be so if $n = 5$

$\therefore \quad f_2 = \frac{5}{4} f_1$

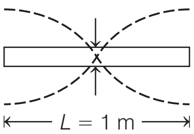
54. $L_0 = \frac{5\lambda_{\text{air}}}{2} \Rightarrow \lambda_{\text{air}} = \frac{2L_0}{5}$



$$f = v_{\text{air}}/\lambda_{\text{air}} = \frac{343 \times 3}{2L_0} = \frac{1029}{2L_0}$$

$$\frac{\lambda}{4} = 1 \text{ m} \Rightarrow \lambda = 4 \text{ m}$$

$\therefore \quad f = v_{\text{steel}}/\lambda = \frac{5250}{4} = 1312.5 \text{ Hz}$



As both of them are in unison, $\frac{1029}{2L_0} = 1312.5$ Hz

$\therefore \quad L_0 = 0.392 \text{ m} \simeq 39 \text{ cm}$

55. As, $f_c = f_o$ (both first overtone)

or $\quad 3\left(\frac{v_c}{4L}\right) = 2\left(\frac{v_o}{2l_o}\right)$

$\therefore \quad l_o = \frac{4}{3}\left(\frac{v_o}{v_c}\right) L$

$$= \frac{4}{3} \frac{\sqrt{B/\rho_2}}{\sqrt{B/\rho_1}} L$$

$$= \frac{4}{3} \sqrt{\frac{\rho_1}{\rho_2}} L$$

56. As, $\lambda = \frac{v}{n} = \frac{330}{500} = 0.66 \text{ m} = 66 \text{ cm}$

The successive resonance lengths are at

$$\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4} \text{ and so on.}$$

Within one metre length of the tube, total number of resonances is 3(as, $\frac{7\lambda}{4}$ is more than 1.0 m).

57. Length at which first resonance occurs

$$L_1 = 25.5 \text{ cm}$$

Length at which second resonance occurs $L_2 = 79.3$ cm

$\therefore \quad$ Wavelength, $\lambda = 2(L_2 - L_1)$

$= 2(79.3 - 25.5)$

$= 2 \times 53.8$

$= 107.6$ cm

$= 1.076$ m

Using $v = \nu\lambda$

$\therefore$ Speed of sound in air $v = 340(107.6)$ m/s ($\because \nu = 340$ Hz)

$= 365.84$ m/s

$\simeq 366 \text{ ms}^{-1}$

58. Fundamental frequency of closed organ pipe is given by $f_0 = v/4L$, where v is the velocity of sound in it and L is the length of the pipe.

Also, overtone frequencies are given by

$$f = (2n + 1)\frac{v}{4L}$$

or $\quad f = (2n + 1) f_0$

Given, $f_0 = 1500$ Hz and $f_{\max} = 20000$ Hz

This means, $f_{\max} > f$

So, $\quad f_{\max} > (2n + 1) f_0$

$\Rightarrow \quad 20000 > (2n + 1)\, 1500$

$\Rightarrow \quad 2n + 1 < 13.33$

$\Rightarrow \quad 2n < 13.33 - 1$

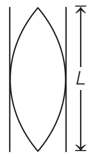
$\Rightarrow \quad 2n < 12.33$

or $\quad n < 6.16$

or $n = 6$ (integer number)

Hence, total six overtones will be heard.

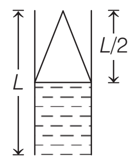
59. For open ends, fundamental frequency f in air, we have



$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$$v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{v}{2L} \qquad \ldots\text{(i)}$$

When a pipe is dipped vertically in water, so that half of it is in water, we have



$$\frac{\lambda}{4} = \frac{L}{2}$$

$\Rightarrow \quad \lambda = 2L$

$\Rightarrow \quad v = f'\lambda$

$$\Rightarrow \qquad f' = \frac{v}{\lambda} = \frac{v}{2L} = f \qquad \ldots\text{(ii)}$$

Thus, the fundamental frequency of the air column is now,

$$f = f'$$

60. As, $\quad L_1 + x = \frac{\lambda}{4} = 22.7$

$$L_2 + x = \frac{3\lambda}{4} = 70.2$$

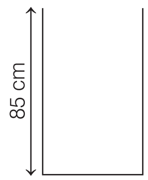
and $\quad L_3 + x = \frac{5\lambda}{4}$

$$\therefore \quad x = \frac{L_2 - 3L_1}{2} = \frac{70.2 - 68.1}{2} = \frac{2.1}{2} = 1.05 \text{ cm}$$

Now, $\quad \frac{L_3 + x}{L_1 + x} = 5$

$$\Rightarrow \quad L_3 = 5L_1 + 4x = 5 \times 22.7 + 4 \times 1.05 = 117.7 \text{ cm}$$

61. For closed organ pipe $= \frac{(2n+1)v}{4l}$ $\quad (n = 0, 1, 2 \ldots\ldots)$



$$\frac{(2n+1)v}{4l} < 1250$$

$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

$$(2n+1) < 12.5$$

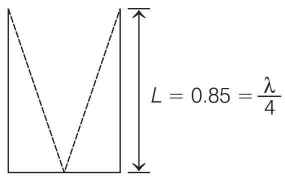
$$\Rightarrow \quad 2n < 11.50$$

$$n < 5.25$$

So, $\quad n = 0, 1, 2, 3, \ldots 5$

So, we have 6 possibilities.

Alternate method In closed organ pipe, fundamental node



i.e. $\quad \frac{\lambda}{4} = 0.85$

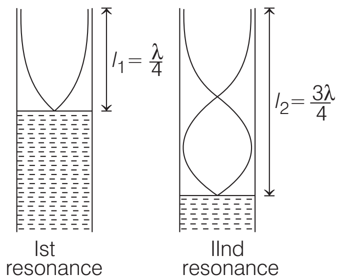
$$\lambda = 4 \times 0.85$$

As, we know, $\nu = \frac{v}{\lambda}$

$$= \frac{340}{4 \times 0.85} = 100 \text{ Hz}$$

$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

62. In a resonance tube apparatus, first and second resonance occur as shown below



As in a stationary wave, distance between two successive nodes is $\frac{\lambda}{2}$ and distance of a node and an antinode is $\frac{\lambda}{4}$.

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

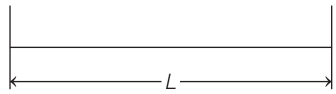
So, speed of sound, $v = f\lambda$

$$= f \times 2(l_2 - l_1)$$

$$= 480 \times 2 \times (70 - 30) \times 10^{-2}$$

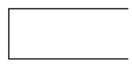
$$= 384 \text{ ms}^{-1}$$

63. Given,



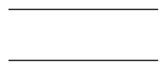
fundamental frequency of wire (f_{wire}) = $v/2L$.

(a)



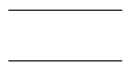
$f = \frac{v}{4L}, \frac{3v}{4L}, \frac{5v}{4L}$ cannot match with f_{wire}.

(b)



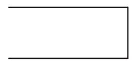
$f = \frac{v}{2(2L)}, \frac{2v}{2(2L)}, \frac{3v}{2(2L)}$ its second harmonic $\frac{2v}{2(2L)}$ matches with f_{wire}.

(c)



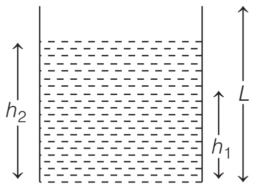
$f = \frac{v}{2(L/2)}, \frac{2v}{2(L/2)}$ cannot match with f_{wire}.

(d)



$f = \frac{v}{4(L/2)}, \frac{3v}{4(L/2)}, \ldots$ cannot match with f_{wire}.

64. Let λ be the wavelength and L be the length of the tube.



The resonance conditions for the given two cases are

$$L - h_1 = \frac{n\lambda}{2} + \frac{\lambda}{4} \qquad \text{...(i)}$$

$$L - h_2 = (n-1)\frac{\lambda}{2} + \frac{\lambda}{4} \qquad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii), we get

$$(L-h_1)-(L-h_2) = \frac{n\lambda}{2} + \frac{\lambda}{4} - (n-1)\frac{\lambda}{2} - \frac{\lambda}{4}$$

$$h_2 - h_1 = \frac{\lambda}{2}$$

Here, $h_1 = 17.0$ cm and $h_2 = 24.5$ cm

Substituting these values in above equation, we get

$$24.5 - 17.0 = \frac{\lambda}{2}$$

$$\Rightarrow \qquad \frac{\lambda}{2} = 7.5 \text{ cm}$$

$$\Rightarrow \qquad \lambda = 15.0 \text{ cm}$$

Now, the tuning fork frequency,

$$f = \frac{v}{\lambda} = \frac{330}{15 \times 10^{-2}} = 2200 \text{ Hz}$$

65. Here, $n_1 = 200$ Hz

Number of beats s^{-1}; $m = 4$

$\therefore \qquad n_2 = 200 \pm 4 = 204$ or 196 Hz

On loading fork 2, its frequency decreases. And number of beats per second increases to 6. Therefore, m is negative.

$$n_2 = 200 - 4 = 196 \text{ Hz}$$

66. Here, $T_1 = 16$ N, $T_2 = T = ?$

As per the choice given, $T_2 > T_1$

$\therefore \quad n_2 > n_1$ and given that $(n_2 - n_1) = 3 \qquad$ ...(i)

Now, $\qquad n \propto \sqrt{T}$

$$\therefore \qquad \frac{n_2}{n_1} = \sqrt{\frac{T}{16}} = \frac{\sqrt{T}}{4}$$

If n_1 corresponds to 4, then n_2 corresponds to $3 + 4 = 7$, which is $\sqrt{T}$. Therefore, $T = 49$ N

67. Here, $n = 200 \pm 5$ and $2n = 420 \pm 10$. This is possible only when $n = 200 + 5 = 205$ Hz or s^{-1}.

68. If n is frequency of first fork, then frequency of the last (10th fork) $= n + 4(10-1) = 2n$

$\therefore \qquad n = 36$ and $2n = 72$

69. If we assume that all the three waves are in same phase at $t = 0$, we shall hear only one beat per second.

70. Here, $\omega_1 = 100\pi$ and $\omega_2 = 92\pi$

Hence $\nu_1 = \dfrac{100\pi}{2\pi} = 50$ Hz

and $\qquad \nu_2 = \dfrac{92\pi}{2\pi} = 46$ Hz

$\therefore$ Number of beats per second $= \nu_1 - \nu_2 = 50 - 46 = 4$

71. $\dfrac{n_1}{n_2} = \dfrac{L_2}{L_1} = \dfrac{25}{30} = \dfrac{5}{6}$

and $\qquad n_2 - n_1 = 4$

On solving, we get

$$n_2 = 24 \text{ Hz}$$
$$n_1 = 20 \text{ Hz}$$

72. As, $\dfrac{n_1}{n_2} = \dfrac{L_2}{L_1} = \dfrac{51}{50}$

and $\qquad n_1 - n_2 = 5$

On solving, we get

$$n_2 = 250 \text{ Hz}, \; n_1 = 255 \text{ Hz}$$

73. Two possible frequencies of source are $= 100 \pm 5$ $= 105$ or 95 Hz

Frequencies of 2nd harmonic $= 210$ or 190 Hz

5 beats with source of frequency 205 Hz are possible only when 2nd harmonic has frequency $= 210$ Hz

$\therefore$ Frequency of source $= 105$ Hz

74. Here, $\dfrac{T_1}{T_2} = \dfrac{8}{1}, \dfrac{L_1}{L_2} = \dfrac{36}{35}, \dfrac{D_1}{D_2} = \dfrac{4}{1}$

$$\frac{\rho_1}{\rho_2} = \frac{1}{2}$$

and $\qquad n_1 = 360$ Hz, $n_2 = ?$

Now, $\qquad \dfrac{n_2}{n_1} = \dfrac{L_1 D_1}{L_2 D_2}\sqrt{\dfrac{T_2 \rho_1}{\rho_2 T_1}}$

$$\frac{n_2}{n_1} = \frac{36}{35} \times \frac{4}{1}\sqrt{\frac{1}{8} \times \frac{1}{2}} = \frac{36}{35}$$

Clearly, $\qquad n_2 > n_1$.

When $\qquad n_2 = 360$ Hz, $n_1 = 350$ Hz

Number of beats per second $= n_2 - n_1 = 360 - 350 = 10$

75. Frequency of fork $= 250$ Hz.

Possible frequencies of sonometer wire $= (250 \pm 10)$ Hz

On filling the fork, number of beats per second decreases.

$\therefore$ Frequency of sonometer wire, $n = 260$ Hz

$$v = n\lambda = 260\,(2L) = 260\,(2 \times 0.5)$$
$$= 260 \text{ ms}^{-1}$$

76. When waves of nearby frequencies overlaps, beats are produced.

Beat frequency is given by, $f_{\text{beat}} = f_1 - f_2$

Here, $f_1 = 11$ Hz and $f_2 = 9$ Hz

$\Rightarrow$ Beat frequency is, $f_{\text{beat}} = 11 - 9 = 2$ Hz

Hence, time period of beats or time interval between beats,

$$T = \frac{1}{f_{\text{beat}}}$$

$$\Rightarrow \qquad T = \frac{1}{2} = 0.5 \text{ s}$$

So, resultant wave has a time period of 0.5 s which is correctly depicted in option (a) only.

77. $$f_1 = \frac{\omega_1}{2\pi} = \frac{(400\pi)}{2\pi} = 200 \text{ Hz}$$

$$f_2 = \frac{\omega_2}{2\pi} = \frac{404\,\pi}{2\pi} = 202 \text{ Hz}$$

$$\therefore \qquad f_b = f_2 - f_1 = 2 \text{ Hz}$$

$$A_1 = 2 \quad \text{and} \quad A_2 = 1$$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{9}{1}$$

78. From Doppler's effect,

$$n' = n\left(\frac{v - v_o}{v + v_s}\right) = n\left(\frac{340 - 10}{340 + 10}\right) = 1950 \text{ Hz} \quad \text{(given)}$$

$$n = 2068 \text{ Hz}$$

79. Whistling train is the source of sound, $v_s = V$. Before crossing a stationary observer on station, frequency heard is $n' = \frac{vn}{(v - v_s)} = \frac{vn}{v - V} =$ constant and $n' > n$.

Here, v is velocity of sound in air and n is actual frequency of whistle.

After crossing the stationary observer, frequency heard is $n' = \frac{vn}{(v + v_s)} = \frac{vn}{v + V} =$ constant and $n' < n$.

Therefore, the expected curve is given in option (c).

80. As the observer is moving towards the source, so frequency of waves emitted by the source will be given by the formula

$$f_{\text{observed}} = f_{\text{actual}} \cdot \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Here, frequency, $\frac{v}{c} = \frac{1}{2}$

So, $$f_{\text{observed}} = f_{\text{actual}} \left(\frac{3/2}{1/2}\right)^{1/2}$$

$$\therefore \qquad f_{\text{observed}} = 10 \times \sqrt{3} = 17.3 \text{ GHz}$$

81. Apparent frequency heard by the person before crossing the trains,

$$f_1 = \left(\frac{c}{c - v_s}\right) f_0 = \left(\frac{320}{320 - 20}\right) 1000$$

Similarly, apparent frequency heard after crossing the train,

$$f_2 = \left(\frac{c}{c + v_s}\right) f_0 = \left(\frac{320}{320 + 20}\right) 1000$$

(c = speed of sound)

$$\Delta f = f_1 - f_2 = \left(\frac{2\,cv_s}{c^2 - v_s^2}\right) f_0$$

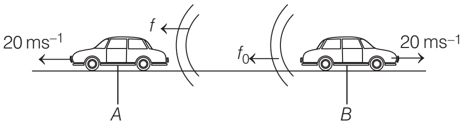
or $$\frac{\Delta f}{f_0} \times 100 = \left(\frac{2\,cv_s}{c^2 - v_s^2}\right) \times 100$$

$$= \frac{2 \times 320 \times 20}{300 \times 340} \times 100$$

$$= \frac{2 \times 32 \times 20}{3 \times 34} = 12.54\%$$

$$= 12\%$$

82. The given condition can be shown below as



Here, source and observer both are moving away from each other. So, by Doppler's effect, observed frequency is given by

$$f = f_0\left(\frac{v + v_o}{v - v_s}\right) \qquad \ldots \text{(i)}$$

where, v = speed of sound = 340 ms^{-1},

v_o = speed of observer = $-$ 20 ms^{-1},

v_s = speed of source = $-$ 20 ms^{-1},

f_0 = true frequency

and f = apparent frequency = 2000 Hz

Substituting the given values in Eq. (i), we get

$$2000 = \left(\frac{340 - 20}{340 + 20}\right) \times f_0$$

$$\Rightarrow \qquad f_0 = \frac{2000 \times 360}{320} = 2250 \text{ Hz}$$

83. $$f = f_0\left(\frac{v + v_o}{v}\right) = 10^3\left(1 + \frac{10t}{v}\right) \qquad (\text{as } v_o = gt)$$

Hence, f *versus* t graph is a straight line of slope $\frac{10^4}{v}$.

$$\therefore \qquad \frac{10^4}{v} = \text{slope} = \frac{100}{3}$$

$$\therefore \qquad v = 300 \text{ m/s}$$

84. As the listener on motor cycle is moving away from the source (siren), therefore $\frac{n'}{n} = \frac{v - v_L}{v} = \frac{94}{100}$

$$\Rightarrow \qquad 1 - \frac{v_L}{v} = \frac{94}{100}$$

$$\Rightarrow \qquad \frac{v_L}{v} = 1 - \frac{94}{100} = \frac{6}{100}$$

$$\Rightarrow \qquad v_L = \frac{6 \times v}{100} = \frac{6 \times 330}{100} = 19.8 \text{ ms}^{-1}$$

Distance covered $= \frac{v_L^2}{2a} = \frac{19.8 \times 19.8}{2 \times 2} = 98$ m

85. $\nu = 165$ Hz and

$$\nu' = \frac{335 + 5}{335} \times \frac{335}{330} \times 165 = 170 \text{ Hz}$$

$\therefore$ Number of beats per second $= \nu' - \nu = 170 - 165 = 5$

86. Number of extra waves received per second

$$= \frac{u}{\lambda} - (-u/\lambda) = \frac{2u}{\lambda}$$

= Number of beats heard per second

87. Large vertical plane acts as listener moving per second.

$$\therefore \qquad n' = \frac{(c+v)n}{c}$$

This is the number of waves striking the surface per second.

88. Let n be the actual frequency of sound of horn.

If v_s is velocity of car, then frequency of sound striking the cliff (source moving towards listener)

$$n' = \frac{(v+v_s)n'}{v} = \frac{(v+v_s)}{v} \times \frac{v \times n}{(v-v_s)}$$

or
$$\frac{n''}{n} = \frac{v+v_s}{v-v_s} = 2$$

$$v + v_s = 2v - 2v_s$$

$$3v_s = v, v_s = \frac{v}{3}$$

89. Given, frequency of sound source $(f_0) = 500$ Hz

Apparent frequency heard by observer 1, $f_1 = 480$ Hz and apparent frequency heard by observer 2, $f_2 = 530$ Hz.

Let v_o be the speed of sound.

When observer moves away from the source,

$$\text{Apparent frequency, } f_1 = f_o \left(\frac{v - v_o'}{v}\right) \qquad \dots \text{(i)}$$

When observer moves towards the source,

$$\text{Apparent frequency, } f_2 = f_o \left(\frac{v + v_o''}{v}\right) \qquad \dots \text{(ii)}$$

Substituting the values in Eq. (i), we get

$$480 = 500 \left(\frac{300 - v_o'}{300}\right)$$

$$\Rightarrow \qquad 96 \times 3 = 300 - v_o'$$

$$\Rightarrow \qquad v_o' = 12 \text{ m/s}$$

Substituting the values in Eq. (ii), we get

$$530 = 500 \left(\frac{330 + v_o''}{300}\right)$$

$$\Rightarrow \qquad 106 \times 3 = 300 + v_o''$$

$$\Rightarrow \qquad v_o'' = 18 \text{ m/s}$$

Thus, their respective speeds (in m/s) is 12 and 18.

90. When a source is moving towards a stationary observer, observed frequency is given by

$$f_{\text{observed}} = f\left(\frac{v}{v + v_s}\right)$$

where, f = frequency of sound from the source, v = speed of sound and v_s = speed of source.

Now, applying above formula to two different conditions given in problem, we get

$$f_1 = \text{Observed frequency initially}$$

$$= f\left(\frac{340}{340 - 34}\right)$$

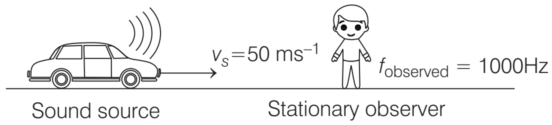
$$= f\left(\frac{340}{306}\right)$$

and f_2 = Observed frequency when speed of source is reduced

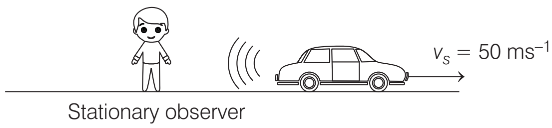
$$= f\left(\frac{340}{340 - 17}\right) = \frac{340}{323}$$

So, the ratio $f_1 : f_2$ is, $\dfrac{f_1}{f_2} = \dfrac{323}{306} = \dfrac{19}{18}$.

91. Initially,



After sometime,



When source is moving towards stationary observer, frequency observed is more than source frequency due to Doppler's effect, it is given by

$$f_{\text{observed}} = f\left(\frac{v}{v - v_s}\right)$$

where, f = source frequency,

f_o = observed frequency = 1000 Hz,

v = speed of sound in air = 350 ms^{-1}

and v_s = speed of source = 50 ms^{-1}

So, $f = \dfrac{f_{\text{obs}}(v - v_s)}{v} = \dfrac{1000(350 - 50)}{350} = \dfrac{6000}{7}$ Hz

When source moves away from stationary observer, observed frequency will be lower due to Doppler's effect and it is given by

$$f_0 = f\left(\frac{v}{v + v_s}\right)$$

$$= \frac{6000 \times 350}{7 \times (350 + 50)}$$

$$= \frac{6000 \times 350}{7 \times 400}$$

$$= 750 \text{ Hz}$$

92. As source and observer both are moving in the same direction with the same velocity, their relative velocity is zero. Therefore, $n' = n = 200$ Hz.

93. Here, frequency of source = 500 Hz

Speed of source $A = 4$ m/s $= u$

Then, source is moving towards stationary observer,

$$\nu' = \frac{v}{v-u}\nu_0 \qquad \text{(where } v = \text{speed of sound)}$$

$$= \frac{340}{340-4} \times 500$$

$$\Rightarrow \nu' = \frac{340}{336} \times 500 \text{ Hz} = 506 \text{ Hz}$$

Now, when source is reciding from the observer,

$$\nu' = \frac{v}{v+u}\nu_o$$

$$\Rightarrow \qquad = \frac{340}{344} \times 500 \text{ Hz}$$

$$\therefore \qquad \nu' = 494 \text{ Hz}$$

According to question,

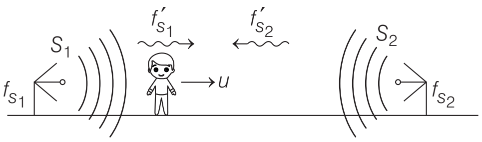
Let frequency of source B is Z Hz.

$\therefore \qquad Z = 506 \pm 6 \;\Rightarrow\; Z = 500$ or 512

and $\qquad Z = 494 \pm 18 \Rightarrow Z = 512$ or 476

Thus, required frequency = 512 Hz

94. When observer moves away from S_1 and towards S_2,



then due to Doppler's effect observed frequencies of sources by observer are

$$f'_{S_1} = \frac{v - v_o}{v} \cdot f_{S_1}$$

(observer moving away from source)

and $$f'_{S_2} = \left(\frac{v + v_o}{v}\right) \cdot f_{S_2}$$

(observer moving towards source)

(where, v = speed of sound, v_o = speed of observer)

So, beat frequency heard by observer is

$$f_b = f'_{S_2} - f'_{S_1}$$

Here, $\qquad v_o = u,\; v = 330 \text{ ms}^{-1}$

$$f_b = 10 \text{ Hz},\; f_{S_1} = f_{S_2} = 660 \text{ Hz}$$

On putting the values, we get

$$f_b = f'_{S_2} - f'_{S_1}$$

$$= \left(\frac{v+v_o}{v}\right) \cdot f_{S_2} - \left(\frac{v-v_o}{v}\right) f_{S_1}$$

$$= f_{S_1}\left(\frac{v+v_o}{v} - \frac{v-v_o}{v}\right)$$

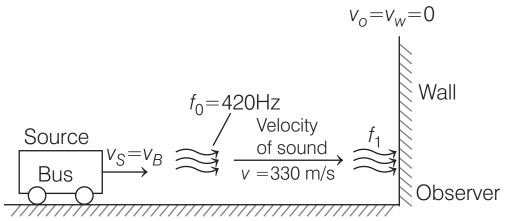
$$= f_{S_1} \cdot \frac{2v_o}{v}$$

$$\Rightarrow \qquad 10 = \frac{660 \times 2u}{330} \qquad (\because v_o = u)$$

$$\Rightarrow \qquad u = \frac{330 \times 10}{2 \times 660}$$

$$\Rightarrow \qquad u = 2.5 \text{ ms}^{-1}$$

95. Let v_B be the speed of the bus. Before reflection, the frequency of the horn is 420 Hz.

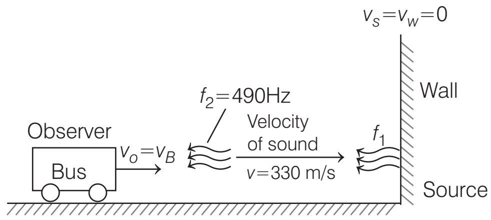


Using Doppler's effect,

$$f_1 = \left(\frac{v}{v - v_s}\right) f_0$$

$$\Rightarrow \qquad f_1 = \left(\frac{330}{330 - v_B}\right) \times 420 \qquad \ldots\text{(i)}$$

After reflection from the wall, the frequency of the horn becomes 490 Hz.



Again using Doppler's effect ,

$$f_2 = \left(\frac{v + v_o}{v}\right) f_1$$

$$\Rightarrow \qquad 490 = \left(\frac{330 + v_B}{330}\right) \times \left(\frac{330}{330 - v_B}\right) \times 420$$

$$\Rightarrow \qquad \frac{490}{420} = \frac{330 + v_B}{330 - v_B}$$

$$\Rightarrow \qquad \frac{7}{6} = \frac{330 + v_B}{330 - v_B}$$

On applying Componendo-Dividendo rule, we get

$$\frac{7+6}{7-6} = \frac{(330 + v_B) + (330 - v_B)}{(330 + v_B) - (330 - v_B)}$$

$$\Rightarrow \qquad \frac{13}{1} = \frac{2 \times 330}{2v_B}$$

$$\Rightarrow \qquad v_B = \frac{330}{13} \text{ m/s}$$

$$= \frac{330}{13} \times \frac{18}{5} \text{ km/h}$$

$$= 91.38 \text{ km/h} \simeq 91 \text{ km/h}$$

96. Number of revolutions per second = 2

Radius of the circle = 1.988 m

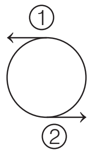
Linear velocity of the tuning fork is

$$v = (2\pi R) \times 2 = 4 \times \frac{22}{7} \times 1.988 = 25 \text{ ms}^{-1}$$

Let the listener is located on the left side at a large distance in the diagram.

$\therefore$ Apparent frequency, when the tuning fork is approaching the listener is

$$f_1 = \left(\frac{v}{v - v_s}\right) f_0 = \left(\frac{350}{350-25}\right) f_0 = 1.077\, f_0$$



Apparent frequency, when the tuning fork is moving away from the listener is

$$f_2 = \left(\frac{v}{v + v_s}\right) f_0$$

$$f_2 = \left(\frac{350}{350+25}\right) \cdot f_0 = 0.933\, f_0$$

f_1 is the highest note and f_2 is the lowest note.

Now, $\dfrac{f_1}{f_2} = \dfrac{1.077}{0.933} = 1.154$

Round II

1. From the given snapshot at $t = 0$,

$$y = 0 \text{ at } x = 0$$

and $y = -$ ve when x increases from zero.

Standard expression of any progressive wave is given by $y = A\sin(kx - \omega t + \phi)$

Here, ϕ is the phase difference, we need to get

at $t = 0$, $y = A\sin(kx + \phi)$

Clearly $\phi = \pi$, so that

$$y = A\sin(kx + \pi)$$
$$y = -A\sin(kx)$$

$\Rightarrow$ $y = 0$ at $x = 0$

and $y = -$ve at $x > 0$

which satisfies the given snapshot.

2. Let the frequency of standard tuning fork $= x$

$\therefore$ $n_A = \dfrac{102}{100}x$ and $n_B = \dfrac{97}{100}x$

Number of beats per second $= n_A - n_B = 6$

$\because$ $\dfrac{102}{100}x - \dfrac{97}{100}x = 6$

$\Rightarrow$ $x = \dfrac{6 \times 100}{5} = 120$ Hz

So, frequency of $A = \dfrac{102}{100} \times 120$ Hz $= 122.4$ Hz

3. Frequency of 1st overtone of A,

$$n_1 = \frac{2}{2L_1}\sqrt{\frac{T}{m}} = \frac{2}{L_1 D_1}\sqrt{\frac{T}{\pi\rho}}$$

Frequency of 2nd overtone of B,

$$n_2 = \frac{3}{2L_2}\sqrt{\frac{T}{m}} = \frac{2}{L_2 D_2}\sqrt{\frac{T}{\pi\rho}}$$

As, $n_1 = n_2$ (resonance condition)

$\therefore$ $\dfrac{2}{L_1 D_1}\sqrt{\dfrac{T}{m}} = \dfrac{2}{L_1 D_2}\sqrt{\dfrac{T}{\pi\rho}}$

$$\frac{L_1 D_1}{L_1 D_2} = \frac{2}{3}$$

$$\frac{L_1}{L_2} = \frac{2D_2}{D_2} = \frac{2}{3} \quad \text{or} \quad \frac{L_1}{L_2} = 1:3$$

4. $n_c = \dfrac{v}{4L}$ and $n_o = \dfrac{v}{2L}$

Now, $n_o - n_c = 2$

$\therefore$ $\dfrac{v}{2L} - \dfrac{v}{4L} = 2$

or $\dfrac{v}{L} = 8$

Also, $n'_o = \dfrac{v}{2L/2} = \dfrac{v}{L}$

and $n'_c = \dfrac{v}{4(2L)} = \dfrac{v}{8L}$

Number of beats per second $= n'_o - n'_c$

$$= \frac{v}{L} - \frac{v}{8L} = \frac{7v}{8L}$$

$$= \frac{7}{8} \times 8 = 7$$

5. Velocity of longitudinal waves,

$$v_1 = \sqrt{\frac{Y}{\rho}}$$

and velocity of transverse waves,

$$v_2 = \sqrt{\frac{T}{m}}$$

If a is area of cross-section of string, then

$$m = \frac{\text{mass}}{\text{length}} = \frac{\text{mass}}{\text{volume}} \times \text{area} = \rho a$$

$\therefore$ $v_2 = \sqrt{\dfrac{T}{\rho a}}$

$\because$ $\dfrac{v_1}{v_2} = \sqrt{\dfrac{Y}{\rho} \cdot \dfrac{\rho a}{T}} = \sqrt{\dfrac{Ya}{T}}$

As, $Y = \dfrac{F}{a\Delta l/l} = \dfrac{T}{a(\Delta l/l)}$

$\Rightarrow$ $\dfrac{v_1}{v_2} = \sqrt{\dfrac{T}{a\left(\dfrac{\Delta l}{l}\right)} \dfrac{a}{T}} = \left(\dfrac{\Delta l}{l}\right)^{-1/2}$

We are given, $\dfrac{\Delta l}{l} = \dfrac{1}{n}$

$\therefore$ $\dfrac{v_1}{v_2} = \left(\dfrac{1}{n}\right)^{-1/2} = \sqrt{n}$

If f_1 and f_2 are the corresponding fundamental frequencies of longitudinal and transverse vibrations, then

$$v_1 = f_1 \lambda$$

and $v_2 = f_2 \lambda$

$\therefore$ $\dfrac{v_1}{v_2} = \dfrac{f_1}{f_2} = \sqrt{n} : 1$

6. Here, $y_1 = 0.05\sin(3\pi t - 2x)$

$$y_2 = 0.05\sin(3\pi t + 2x)$$

According to superposition principle, the resultant displacements is

$$y = y_1 + y_2$$
$$= 0.05[\sin(3\pi t - 2x) + \sin(3\pi t + 2x)]$$
$$y = 0.05 \times 2 \sin 3\pi t \cos 2x$$
$$y = (0.1\cos 2x)\sin 3\pi t = R\sin 3\pi t$$

where, $R = 0.1\cos 2x =$ amplitude of the resultant standing wave.

At $\quad x = 0.5$ m,

$$R = 0.1\cos 2x = 0.1\cos 2\times 0.5$$
$$= 0.1\cos 1(\text{radian}) = 0.1\cos\frac{180^\circ}{\pi}$$
$$= 0.1\cos 57.3^\circ$$

or $\quad R = 0.1\times 0.54 \text{ m} = 0.054 \text{ m} = 5.4 \text{ cm}$

7. As, energy $E \propto (\text{amplitude})^2 (\text{frequency})^2$

Amplitude is same in both the cases; but frequency 2ω in the second case is two times the frequency (ω) in the first case.

Hence, $\quad E_2 = 4E_1$

8. As the string vibrates in n loops, therefore

$$L = \frac{n\lambda}{2}$$

Therefore, v would become $\frac{1}{2}$ time.

As, $\quad v \propto \sqrt{T}$

Therefore, to make v half-time, T must be made $\frac{1}{4}$ time, $i.e. M/4$.

9. As the string is vibrating in three segments, therefore,

$$L = \frac{3\lambda}{2}$$

or $\quad \lambda = \frac{2L}{3} = \frac{2(0.6)}{3} = 0.4 \text{ m}$

Now, $\quad v = \sqrt{\frac{T}{m}}$

$$v = \sqrt{\frac{80}{2.0}} = 20 \text{ ms}^{-1}$$

$\therefore \quad n = \frac{v}{\lambda} = \frac{20}{0.4} = 50 \text{ Hz}$

Amplitude of particle velocity

$$= \left(\frac{dy}{dt}\right)_{\max} = (a_{\max})\,\omega = a_{\max}(2\pi n)$$
$$= (0.5\times 10^{-2})\times 2\pi\times 50 = 1.57 \text{ ms}^{-1}$$

10. As, $n' = \frac{vn}{v - v_s}$ and $n'' = \frac{vn}{v + v_s}$

$\therefore \quad \frac{n}{n'} = 1 - \frac{v_s}{v}$,

$$\frac{n}{n''} = 1 + \frac{v_s}{v}$$

Adding the two, we get

$$\frac{n}{n'} + \frac{n}{n''} = 2$$

$\therefore \quad n = \frac{2n'n''}{n' + n''}$

11. As, $v_s = r\omega = r\times 2\pi\nu = \frac{70}{100}\times 2\times\frac{22}{7}\times 5 = 22 \text{ ms}^{-1}$

Frequency is minimum when source is moving away from listenery.

Therefore, from Doppler's effect,

$$\nu' = \frac{u\times v}{u + u_s} = \frac{352\times 1000}{352 + 22} = 941 \text{ Hz}$$

12. The time taken by the plate falling through a distance y is given by

$$t = \sqrt{(2yg)} = \sqrt{\left(\frac{2\times 10}{980}\right)} = \left(\frac{1}{7}\right) \text{s}$$

The number of oscillations completed in $\frac{1}{7}$ s is 8.

$\therefore$ Frequency = Number of oscillations completed in one second

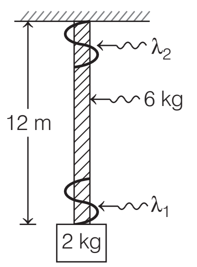
$$= \frac{8}{1/7} = 56 \text{ Hz}$$

13. Let v be the speed of sound in air, v_L velocity of observer at time t. As, the observer approaches the source, therefore apparent frequency,

$$f = \frac{(v + v_L)}{v} f_0 = \left[\frac{v + (0 + at)}{v}\right] f_0 = f_0 + \left(\frac{f_0 at}{v}\right)$$

This is the equation of a straight line with a positive intercept (f_0) and positive slope $\left(\frac{f_0 a}{v}\right)$. Therefore, option (d) is correct.

14. As wavetrain moves up over string, tension and hence, wave speed also changes. Also, frequency (f) of wavetrain remains constant as it depends only on the source.



Now, using $v = \sqrt{\frac{T}{\mu}}$

where, v = wave speed, T = tension in string and μ = mass per unit length of string.

Tension T_1 at bottom $= 2\,g$

Tension T_2 at top $= (2 + 6)\,g = 8\,g$

If λ_1 and λ_2 are wavelengths at bottom and top, then

$$\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\sqrt{\frac{T_1}{\mu}}}{\sqrt{\frac{T_2}{\mu}}}$$

$$\Rightarrow \qquad \lambda_2 = \lambda_1 \sqrt{\frac{T_2}{T_1}}$$

$$= 6 \times \sqrt{\frac{8\,g}{2\,g}} = 12 \text{ cm}$$

15. Let m be the total mass of the rope of length l. Tension in the rope at a height h from lower end = weight of rope of length h,

i.e. $$T = \frac{mg}{L}(h)$$

As, $$v = \sqrt{\frac{T}{(m/L)}}$$

$\therefore$ $$v = \sqrt{\frac{mg(h)}{L(m/L)}} = \sqrt{gh}$$

$$v^2 = gh$$

which is a parabola. Therefore, h *versus* v graph is a parabola, hence option (a) is correct.

16. Speed of transverse wave over a string,

$$v = \sqrt{\frac{T}{\mu}} \qquad \text{...(i)}$$

where, T = tension or force on string

and $\mu = \frac{m}{L}$ = mass per unit length.

Also, Young's modulus of string,

$$Y = \frac{TL}{A\Delta L}$$

$$\Rightarrow \qquad T = \frac{YA\Delta L}{L} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$v^2 = \frac{YA\Delta L}{\mu L}$$

or $$\Delta L = \frac{mv^2}{YA} \qquad \text{...(iii)}$$

Here, $m = 6 \text{ g} = 6 \times 10^{-3}$ kg, $L = 60$ cm
$= 60 \times 10^{-2}$ m, $A = 1 \text{ mm}^2$
$= 1 \times 10^{-6} \text{ m}^2$,
$Y = 16 \times 10^{11} \text{ Nm}^{-2}$ and $v = 90 \text{ ms}^{-1}$

Substituting these given values in Eq. (iii), we get

$$\Delta L = \frac{6 \times 10^{-3} \times (90)^2}{16 \times 10^{11} \times 1 \times 10^{-6}}$$

$$= 3.03 \times 10^{-5} \text{ m}$$

$$\approx 30 \times 10^{-6} \text{ m} = 0.03 \text{ mm}$$

17. Equation of stationary wave is

$$y_1 = a \sin kx \cos \omega t$$

and equation of progressive wave is

$$y_2 = a \sin(\omega t - kx)$$

$$= a(\sin \omega t \cos kx - \cos \omega t \sin kx)$$

At $$x_1 = \frac{\pi}{3k} \text{ and } x_2 = \frac{3\pi}{2k}$$

$\sin kx_1$ or $\sin kx_2$ is zero.

So, neither x_1 nor x_2 is node.

$$\Delta x = x_2 - x_1 = \frac{3\pi}{2k} = \frac{\pi}{3k} = \frac{7\pi}{6k}$$

As, $$\Delta x = \frac{7\pi}{6x}$$

Therefore, $$\frac{2\pi}{k} > \Delta x > \frac{\pi}{k}$$

But $$\frac{2\pi}{k} = \lambda$$

So, $$\lambda > \Delta x > \frac{\lambda}{2}$$

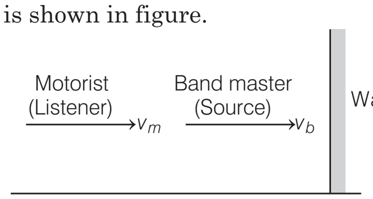
In case of a stationary wave, phase difference between any two points is either zero or π.

$\therefore$ $$\phi_1 = \pi$$

and $$\phi_2 = k\Delta x = k\frac{7\pi}{6k} = \frac{7}{6}\pi$$

$\therefore$ $$\frac{\phi_1}{\phi_2} = \frac{\pi}{\frac{7}{6}\pi} = \frac{6}{7}$$

18. The motorist receives two sound waves, one directly from the band and second reflected from the wall which is shown in figure.



For direct sound waves, apparent frequency,

$$f' = \frac{(v + v_m)f}{v + v_b}$$

For reflected sound waves,
frequency of sound wave, reflected from the wall,

$$f'' = \frac{v \times f}{v - v_b}$$

Frequency of reflected waves as received by the moving motorist,

$$f'' = \frac{(v + v_m)f''}{v} = \frac{(v + v_m)f}{v - v_b}$$

$\therefore$ Beat frequency $= f''' - f'$

$$= \frac{(v + v_m)f}{v - v_b} - \frac{(v + v_m)f}{v + v_b}$$

$$= \frac{2\,v_b(v + v_m)f}{v^2 - v_b^2}$$

19. As is known, frequency of vibration of a stretched string,

$$n \propto \sqrt{T} \propto \sqrt{mg} \propto \sqrt{g}$$

As, $$n_\omega = \frac{80}{100} n_a = 0.8\, n_a$$

$\therefore$ $$\frac{g'}{g} = \left(\frac{n_\omega}{n_a}\right)^2 = (0.8)^2 = 0.64$$

If ρ_ω = relative density of water (= 1),
ρ_m = relative density of mass,
ρ_t = relative density of liquid, then

$$\frac{g'}{g} = \left(1 - \frac{\rho_\omega}{\rho_\mu}\right) = 0.64$$

$$\frac{\rho_\omega}{\rho_m} = 1 - 0.64 = 0.36 \quad \ldots\text{(i)}$$

Similarly, in the liquid,

$$\frac{g'}{g} = \left(\frac{n_L}{n_a}\right)^2 = (0.6)^2 = 0.36$$

$$\frac{g'}{g} = \left(1 - \frac{\rho_L}{\rho_m}\right) = 0.36$$

$$\frac{\rho_L}{\rho_m} = 1 - 0.36 = 0.64 \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

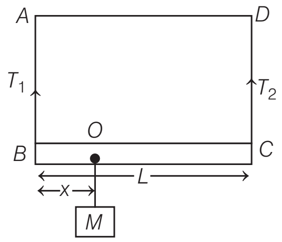
$$\frac{\rho_L}{\rho_\omega} = \frac{0.64}{0.34} = 1.77$$

Hence, specific gravity of liquid = 1.77

20. According to the question,

$$\frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{1}{l}\sqrt{\frac{T_2}{\mu}}$$

$$\Rightarrow \quad T_2 = T_1/4$$

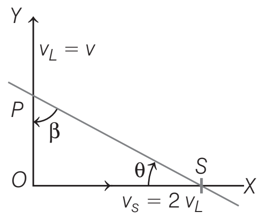


For rotational equilibrium, net torque should be equal to zero

$$\Rightarrow \quad T_1 x = T_2 (L - x)$$

$$x = \frac{L}{5}$$

21. Let speed of observer be $v_L = v$ along Y-axis and speed of source be $v_s = 2v_L = 2v$ along X-axis



$$\therefore \quad PS = 2\,(OL)$$

$$\cos\alpha = \frac{2}{\sqrt{5}}$$

and $$\cos\beta = \frac{2}{\sqrt{5}}$$

Now, apparent frequency n' is given by

$$n' = \frac{(v - v_L \cos\beta)n}{(v + v_L \cos\alpha)}$$

where, v is velocity of sound.

$$n' = \frac{(v - v\sqrt{5})n}{(v + 4v\sqrt{5})}$$

Clearly, n' is constant but $n' < n$. This is shown in curve (b).

22. Here, $A_1 = A_2$; $n_1 = \omega$, $n_2 = \omega_2$

$\therefore \quad y_1 = A \sin 2\pi\omega_1 t,$

and $\quad y_2 = A \sin 2\pi\omega_2 t$

Now, $\quad y = y_1 + y_2$ (from superposition principle)

$$= 2A \frac{\cos 2\pi\,(\omega_2 - \omega_1)\,t}{2} \sin \frac{2\pi\,(\omega_2 + \omega_1)\,t}{2}$$

$$= A' \sin \pi\,(\omega_2 + \omega_1)\,t$$

where, $\quad A' = 2A\cos\pi(\omega_2 - \omega_1)t$

Sound heard will be of maximum intensity ($> 2A^2$)

when $\quad \cos\pi(\omega_2 - \omega_1)\,t = \max = \pm 1$

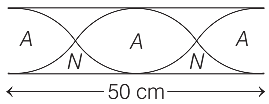
$$\pi(\omega_2 - \omega_1)t = 0, \pi, 2\pi$$

$$t = 0, \frac{1}{\omega_2 - \omega_1}; \frac{2}{\omega_2 - \omega_1}; \ldots$$

Time interval between two successive maxima

$$= \frac{1}{\omega_2 - \omega_1} = \frac{2}{10^3} = 10^{-3}\text{ s}$$

23. According to the question, the musician uses a open flute of length 50 cm and produce second harmonic sound waves.



When the flute is open from both ends and produce second harmonic, then

$$L = \lambda_2 \Rightarrow f_2 = \frac{v}{L}$$

where, λ_2 = wavelength for second harmonic,

f_2 = frequency for second harmonic

and $\quad v$ = speed of wave.

For given question, $f_2 = \dfrac{v}{L} = \dfrac{330}{50 \times 10^{-2}}$

$f_2 = 660$ Hz (frequency produce by source)

Now, a person runs towards the musician from another end of a hall,

$$v_{\text{observer}} = 10\text{ km/h (towards source)}$$

There is apparent change in frequency, which heard by person and given by Doppler's effect formula,

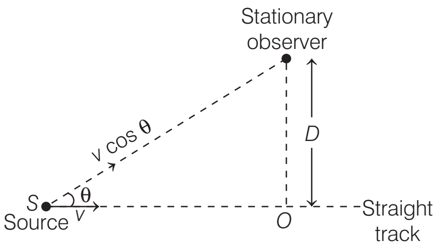
$$\nu' = f' = \nu\left[\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}}\right]$$

$$f' = f_2\left[\frac{v_s + v_o}{v_{\text{sound}}}\right]$$

$$f' = 660\left[\frac{330 + \frac{50}{18}}{330}\right]$$

$$f' = 666\text{ Hz}$$

24. The situation can be shown for a stationary observer of height D,



Source frequency = ν_o

Using the concept of Doppler's effect,

Observed frequency,

$$\nu_{\text{observed}} = \left(\frac{v_{\text{sound}}}{v_{\text{sound}} - v\cos\theta}\right)\nu_o$$

When $t < t_0 (0° < \theta < 90°)$

Initially, θ is less but increasing with time, so $\cos\theta$ decreases continuously and ν_{observed} also decreases.

When $t = t_0 (\theta = 90°)$

$$\cos\theta = \cos 90° = 0, \nu_{\text{observed}} = \nu_o$$

When $t > t_0$ $(0° < q < 90°)$

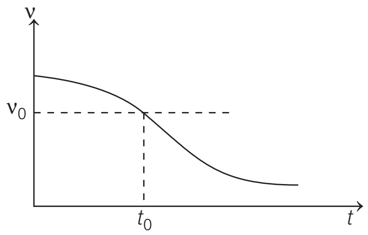
i.e. sound source is moving away from observer.

So, in this case, expression of observed frequency will be

$$\nu_{\text{observed}} = \frac{v_{\text{sound}}}{v_{\text{sound}} + v\cos\theta} \cdot \nu_o$$

With time θ decrease and $\cos\theta$ increases, so ν_{observed} decreases continuously.

So, correct graph is



25. Distance between any two crests,

$$n\lambda = 5 \qquad (n \text{ is an integer}) \quad \ldots\text{(i)}$$

Distance between any crest and any trough,

$$(2m+1)\frac{\lambda}{2} = 1.5 \qquad (m \text{ is an integer})$$

$$\Rightarrow \qquad (2m+1)\lambda = 3 \qquad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{n\lambda}{(2m+1)\lambda} = \frac{5}{3}$$

$$3n = 5(2m+1)$$

$$3n = 10m + 5 \qquad \ldots\text{(iii)}$$

Now, putting the values of m and n in Eq. (iii), which satisfy the equation (only integral values of m and n are acceptable).

(I) $m = 1$ and $n = 5$

Now, on putting these values in Eqs. (i) and (ii), we get

$$\lambda = 1$$

(II) $m = 4$ and $n = 15$

Now, on putting these values in Eqs. (i) and (ii), we get

$$\lambda = \frac{1}{3}$$

(III) $m = 7$ and $n = 25$

Now, on putting these values in Eqs. (i) and (ii), we get

$\lambda = \frac{1}{5}$ and so on.

So, possible values of $\lambda = 1, \frac{1}{3}, \frac{1}{5}, \ldots..$

Hence, correct option is (b).

26. Here, $p = 1$ atmospheric pressure

$$= 1.01 \times 10^5 \text{ N/m}^2$$

Density of air, $\rho = \dfrac{\text{Mass of one mole}}{\text{Volume of one mole}}$

$$= \frac{29.0 \times 10^{-3} \text{ (kg)}}{22.4 \times 10^{-3} \text{ (m}^3)}$$

$$\rho = 1.29 \text{ kgm}^{-3}$$

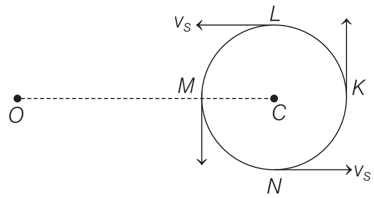
For air, $\gamma = 1.41$

According to corrected Newton's formula,

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

$$v = \sqrt{\frac{1.41 \times 1.01 \times 10^5}{1.29}} = 332.5 \text{ ms}^{-1}$$

27. Apparent frequency will be minimum when the source is at N and moving away from the observer.



$$f_{\min} = \left(\frac{v}{v + v_s}\right) f = \left(\frac{330}{330 + 30}\right)(540) = 495 \text{ Hz}$$

Frequency will be maximum when source is at L and approaching the observer.

$$f_{\max} = \left(\frac{v}{v - v_s}\right) f$$

$$= \left(\frac{330}{330 - 30}\right)(540) = 594 \text{ Hz}$$

$$\therefore \quad f_{\max} - f_{\min} = 594 - 495 = 99 \text{ Hz}$$

28. Here, $f_c = f_o$

$$\frac{3v_c}{4L} = \frac{2v_o}{2L'}$$

$$L' = \frac{4L}{3}\frac{v_o}{v_c} = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

$$\therefore \qquad x = 4$$

29. The first and the fourth waves have phase difference π, so they interfere destructively leading to zero intensity.

The second and the third waves differ in phase by $\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = \frac{\pi}{3}$. So the net intensity,

$$I_{\text{net}} = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\frac{\pi}{3} = 3I_0$$

$\therefore \quad n = 3$

30.
$$P = \frac{1}{2}\rho\omega^2 A^2 sv = \frac{1}{2}\mu\omega^2 A^2 v$$
$$= \frac{1}{2}\mu\omega^2 A^2 \sqrt{\frac{T}{\mu}}$$

or
$$P = \frac{1}{2}\omega^2 A^2 \sqrt{\mu T}$$
$$= \frac{1}{2}(2\pi \times 60)^2 (6 \times 10^{-2})^2 \sqrt{5 \times 10^{-2} \times 80}$$
$$\simeq 518 \text{ W}$$

31. Speed of sound in air by Laplace formula,

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

So, ratio of speed of sound in a gas to that of air,

$$\frac{v_{\text{gas}}}{v_{\text{air}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_{\text{gas}}}}$$

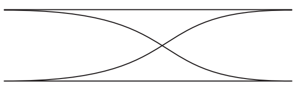
Here, $\quad v_{\text{air}} = 300 \text{ ms}^{-1}$ and $\frac{\rho_{\text{air}}}{\rho_{\text{gas}}} = \frac{1}{2}$

So, we have

$$\frac{v_{\text{gas}}}{300} = \frac{1}{\sqrt{2}}$$

$\Rightarrow$
$$v_{\text{gas}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \text{ ms}^{-1}$$

Now, for a tube open at both ends.

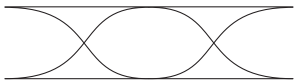


$L = \lambda/2$

$\therefore$ Fundamental frequency,

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

Second harmonic frequency f_2 in tube,



$L = \lambda$

$$f_2 = \frac{v}{\lambda},\ f_2 = v/L$$

So, frequency difference,

$$\Delta f = f_2 - f_1 = \frac{v}{L} - \frac{v}{2L} = \frac{v}{2L} = \frac{150\sqrt{2}}{2 \times 1} \qquad (\because L = 1 \text{ m})$$
$$= 75\sqrt{2} = 106.06 \text{ Hz}$$

32. The expression for the elastic modulus, $Y = \frac{F/A}{\Delta L/L}$

$\Rightarrow$
$$\frac{\Delta L}{L} = \frac{F/A}{Y} \qquad \text{...(i)}$$

As, the wave speed,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{F}{\mu}}$$

where, tension T is the same as stretching force F.

$$\mu = \frac{m}{L} = \frac{\rho(AL)}{L} = \rho A$$

Substituting these values, we get

$$v^2 = \frac{F}{\mu} = \frac{1}{\rho}\left(\frac{F}{A}\right)$$

or
$$\left(\frac{F}{A}\right) = \rho v^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\Delta L}{L} = \frac{\rho v^2}{Y}$$
$$= \frac{2.70 \times 10^3 \times 100}{7.00 \times 10^{10}}$$
$$= 3.86 \times 10^{-4}$$

$\therefore \quad n = 4$