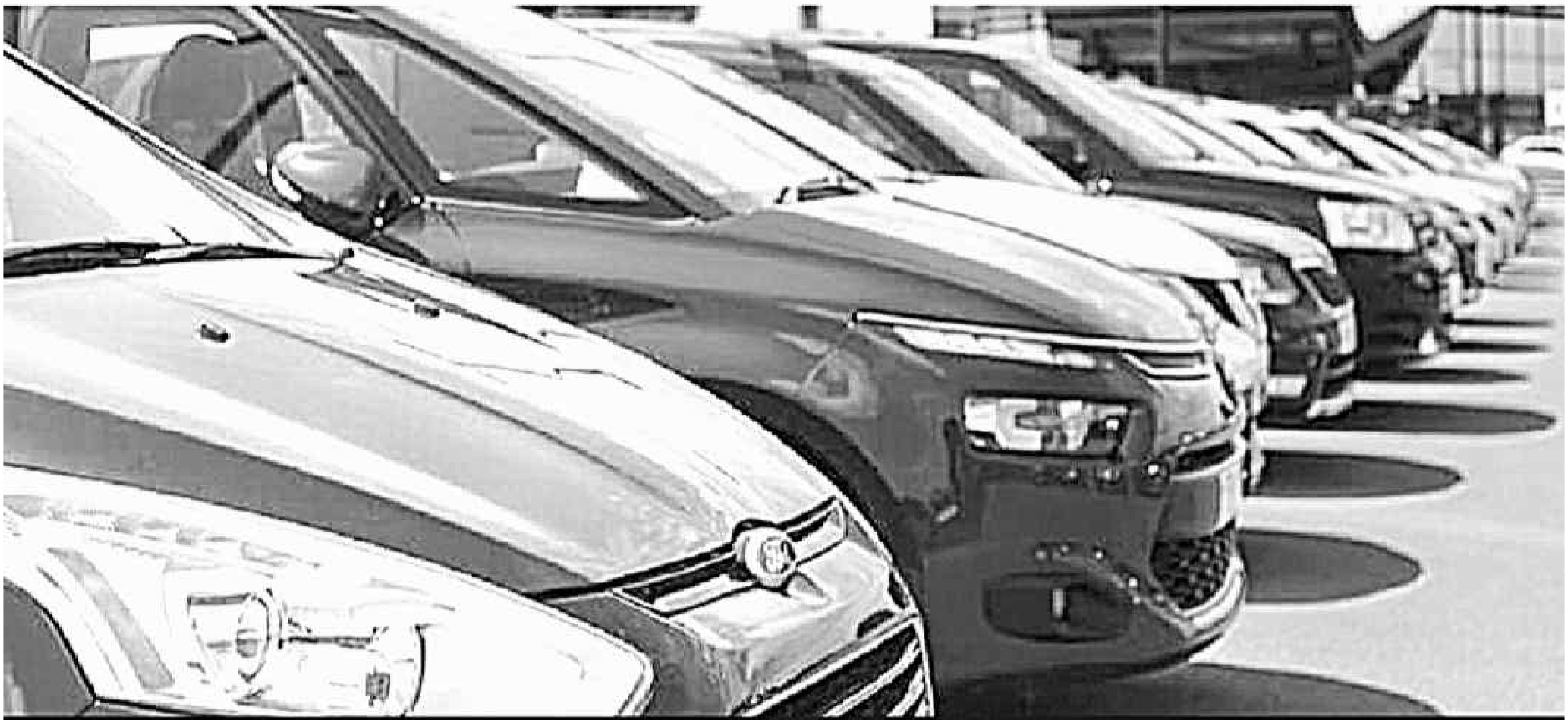
3 Matrices



"Have you ever wondered how the resale value of cars is calculated? By putting the power factors (make and model, colour, mileage, condition etc.) in one column of a matrix, the resale value can be predicted using a matrix operation "Resale value = slope x power factor + intercept". Moreover, matrices like these can be used to calculate sales forecasting and budgeting as well.

Topic Notes

- *Matrix and its Types*
- *Operations on Matrices*
- *Transpose of a Matrix, Symmetric and Skew-Symmetric Matrices*
- *Invertible Matrices*

MATRIX AND ITS TYPES 1

TOPIC 1

MATRIX

Matrices is that topic of mathematics which finds its application in mathematics as well as in most scientific fields, genetics, economics, sociology, modern psychology and industrial management. The concept of matrices evolved due to the need of a compact and simpler way of solving linear equations. Matrices are not only used as a representation of the coefficients in system of linear equations, but utility of matrices far exceeds that use. Matrix notation and operations are used in electronic spreadsheet programs, which in turn is used in different areas of business and science like budgeting, sales projection, cost estimation, analysing the results of an experiment etc. Also, many physical operations such as magnification, rotation and reflection through a plane can be represented mathematically by matrices. Matrices are also used in cryptography. All in all, matrices are one of the most powerful tools that mathematics has provided mankind with.

A matrix is an ordered rectangular array of numbers (real or complex) or functions. The plural of matrix is "matrices". The numbers or functions are called the elements or the entries of a matrix. Thus, a set of mn numbers, real or complex, arranged in a rectangular array of m rows and n columns written as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

is called a $m \times n$ (read as m by n) matrix. The numbers $a_{11}, a_{12}, a_{13}, \ldots a_{mn}$ are the elements of the matrix.

Important

➥ *We shall consider only those matrices whose elements are real numbers or function taking real values.*

Notation of Matrices

Matrices are usually denoted by capital letters A, B, C, etc. and their elements by corresponding small letters. The symbol a_{ij} denotes the element in the i^{th} row and j^{th} column, when $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$.

For the element a_{ij}, first subscript i denotes the row and the second subscript j denotes the column in which the element lies. We can also call a_{ij} as (i, j)th element.

With this notation, we can represent the above matrix as:

$$A = [a_{ij}]_{m \times n}$$

TOPIC 2

ORDER OF A MATRIX

A matrix having m rows and n columns is said to be of order $m \times n$ (read as m by n).

Here are some examples:

(1) $A = \begin{bmatrix} -1 & 2 & 3 \\ 8 & 0 & 6 \end{bmatrix}$ is a matrix with 2 rows and 3 columns, so, its order is 2×3.

Also, $a_{11} = -1, a_{12} = 2, a_{13} = 3, a_{21} = 8, a_{22} = 0$ and $a_{23} = 6$.

(2) $B = \begin{bmatrix} 7 & 1 \\ 8 & -1 \\ 2 & 3 \end{bmatrix}$ is a matrix with 3 rows and 2 columns, so, its order is 3×2.

Also, $a_{11} = 7, a_{12} = 1, a_{21} = 8, a_{22} = -1, a_{31} = 2$ and $a_{32} = 3$.

Example 1.1: In the matrix

$$A = \begin{bmatrix} 2 & 5 & 19 & 7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix} \text{ write}$$

(A) the order of the matrix.

(B) the number of elements

(C) the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$ **[NCERT]**

Ans. (A) The matrix A has 3 rows and 4 columns. So, its order is 3×4.

(B) Since the matrix A is of order 3×4, it has 3×4, *i.e.*, 12 elements.

(C) a_{13} (element of 1^{st} row and 3^{rd} column) = 19

a_{21} (element of 2^{nd} row and 1^{st} column) = 35

a_{33} (element of 3rd row and 3rd column) $= -5$

a_{24} (element of 2nd row and 4th column) $= 12$

a_{23} (element of 2nd row and 3rd column) $= \frac{5}{2}$

Example 1.2: What is the total number of 2 × 3 matrices with each entry 0 or 1?

Ans. Number of elements in a 2 × 3 matrix = 6

Number of ways to write 0 or 1 at one place = 2

⇒ Number of ways to write 0 or 1 at 6 places = 2 × 2 × 2 × 2 × 2 × 2, *i.e.*, 2^6 or 64

Thus, there are 64 matrices of order 2 × 3 with entries 0 or 1.

Order of A Matrix When Number of Elements is Given

As we know that matrix is a rectangular arrangement of elements, so product of number of rows and number of columns is equal to total number of elements.

Example 1.3: If a matrix has 24 elements, what are the possible orders it can have? What if it has 13 elements? **[NCERT]**

Ans. To get all possible orders of a matrix with a given number of elements, say P, we factorise P and get all its possible factors. Possible orders are then formed with of any two of these factors, such that product of them is equal to the total number of elements in the matrix.

All possible factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

So, possible orders are 1 × 24, 2 × 12, 3 × 8, 4 × 6, 6 × 4, 8 × 3, 12 × 2 and 24 × 1.

If it has 13 elements, then posible orders are 1 × 13 and 13 × 1.

⚠ **Caution**

- *In such types of question, careful about the orders of matrix. For example, n × m is not equal to m × n. Here, in the above question, orders 4 × 6 and 6 × 4 are not the same.*

TOPIC 3

FORMATION OF A MATRIX

Sometimes, we are given relation between elements of a matrix and their position in the matrix. On the basis of given relation, we can form a complete matrix by putting the values of *i* and *j* to get the value of general element a_{ij}.

Example 1.4: Construct a 2 × 2 matrix, A = $[a_{ij}]$, whose elements are given by:

(A) $a_{ij} = \frac{(i+j)^2}{2}$

(B) $a_{ij} = \frac{i}{j}$

(C) $a_{ij} = \frac{(i+2j)^2}{2}$

Ans. Here, matrix A = $[a_{ij}]$ is of order 2 × 2.

So, $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(A) Given: $a_{ij} = \frac{(i+j)^2}{2}$, we have

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2;$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2};$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2};$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$$\Rightarrow \quad A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(B) Given: $a_{ij} = \frac{i}{j}$, we have

$$a_{11} = \frac{1}{1} = 1; \; a_{12} = \frac{1}{2};$$

$$a_{21} = \frac{2}{1} = 2; \; a_{22} = \frac{2}{2} = 1$$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

(C) Given: $a_{ij} = \frac{(i+2j)^2}{2}$, we have

$$a_{11} = \frac{(1+2\times1)^2}{2} = \frac{9}{2};$$

$$a_{12} = \frac{(1+2\times2)^2}{2} = \frac{25}{2};$$

$$a_{21} = \frac{(2+2\times1)^2}{2} = \frac{16}{2} = 8;$$

$$a_{22} = \frac{(2+2\times2)^2}{2} = \frac{36}{2} = 18$$

$$\Rightarrow \quad A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

TOPIC 4

TYPES OF MATRICES

Row matrix

A matrix having only one row is called a row matrix.

For example, $A = \begin{bmatrix} 3 & \sqrt{2} & -\frac{3}{4} \end{bmatrix}$ is a row matrix.

In general, $A = [a_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.

Column matrix

A matrix having only one column is called a column matrix.

For example, $B = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}$ is a column matrix.

In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$.

Rectangular matrix

A matrix in which number of rows is not equal to the number of columns or vice-versa is known as rectangular matrix.

For example, $\begin{bmatrix} 1 & 2 \\ 3 & 5 \\ 9 & 8 \end{bmatrix}, \begin{bmatrix} 4 & 4 & 7 & 1 \\ 9 & 5 & 3 & 0 \end{bmatrix}$ are rectangular matrices.

Square matrix

A matrix in which the number of rows is equal to the number of columns is called a square matrix.

For example, $P = \begin{bmatrix} -2 & \sqrt{3} \\ 2 & 3 \end{bmatrix}$ is a square matrix.

In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order $m \times m$.

Important

➥ *A square matrix of order $m \times m$, may be called of order m.*

Diagonal matrix

In a square matrix $A = [a_{ij}]_{m \times m}$, the elements $a_{11}, a_{22}, a_{33}, \ldots, a_{mm}$ constitute the diagonals of the matrix A.

A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a diagonal matrix, if all its non-diagonal elements are zero.

For example, $A = [2]$, $B = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, $C = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ are diagonal matrices of order 1, 2 and 3 respectively.

In general, $A = [a_{ij}]_{m \times m}$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$.

Scalar matrix

A diagonal matrix is said to be a scalar matrix, if its diagonal elements are equal. $A = [2]$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ are scalar matrices of order 1, 2 and 3 respectively.

In general, $A = [a_{ij}]_{m \times m}$ is a scalar matrix if

$$a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ k, & \text{when } i = j \end{cases}, \text{ where } k \text{ is any number.}$$

Identity matrix (or, Unit matrix)

A scalar matrix is said to be an identity matrix, if its diagonal elements are 1.

For example, $A = [1]$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3, respectively.

In general, $A = [a_{ij}]_{m \times n}$ is an identity matrix, if

$$a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ 1, & \text{when } i = j \end{cases}$$

Important

➥ *An identity matrix of order m is denoted by I_m.*

Zero matrix (or Null matrix)

A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a zero matrix, if all its elements are zero.

For example, A = [0], B = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

are zero matrices of order 1 × 1, 2 × 2 and 3 × 3 respectively.

In general, $A = [a_{ij}]_{m \times m}$ is a zero matrix if $a_{ij} = 0$, for all *i* and *j*.

Important

➥ *A zero matrix is denoted by O. Its order will be clear from the context.*

TOPIC 5

EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(1) they are of the same order;

(2) each element of A is equal to the corresponding element of B, *i.e.*, $a_{ij} = b_{ij}$, for all *i* and *j*.

For example, $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ are equal matrices,

but $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$ are not equal matrices.

Symbolically, for two equal matrices A and B, we write A = B.

Illustration: If $\begin{bmatrix} x & y \\ p & q \\ m & n \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 4 & \sqrt{2} \\ 3 & -3 \end{bmatrix}$, then $x = -1$, $y = 0, p = 4, q = \sqrt{2}, m = 3$ and $n = -3$.

Example 1.5: Find the values of *a*, *b*, *c* and *d*, if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ **[NCERT]**

Ans. Since given two matrices are equal, we equate their corresponding elements and get

$a - b = -1$, $2a + c = 5$, $2a - b = 0$, $3c + d = 13$

Solving these equations, we get $a = 1$, $b = 2$, $c = 3$ and $d = 4$.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. If A is a 3 × 3 matrix, B is a 2 × 3 matrix and C is a 2 × 1 matrix, then the number of elements in A, B and C, respectively are:

(a) 6, 2, 9 (b) 9, 2, 6

(c) 3, 2, 1 (d) 9, 6, 2

Ans. *(d)* 9, 6, 2

Explanation: A matrix of order *m* × *n* contains *mn* elements.

∴ Number of elements in A = 3 × 3 = 9

Number of elements in B = 2 × 3 = 6

Number of elements in C = 2 × 1 = 2

2. If a matrix has 18 elements, which of the following will not be a possible order of the matrix?

(a) 2 × 9 (b) 3 × 6

(c) 9 × 9 (d) 1 × 18

Ans. *(c)* 9 × 9

Explanation: A matrix with *mn* elements has order *m* × *n*. So, 9 × 9 cannot be the order of the matrix.

3. In the matrix $\begin{bmatrix} 1 & -3 & 1 \\ 2 & 4 & -1 \\ 3 & 5 & 6 \end{bmatrix}$, the elements of the diagonal are:

(a) 1, 4, 6 (b) 1, 4, 3

(c) 1, – 3, 1 (d) 1, 2, 3

4. A 2 × 2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i-j}{i+j}$ is:

(a) $\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

Ans. *(c)* $\begin{bmatrix} 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 \end{bmatrix}$

Explanation: We have,

$$a_{ij} = \frac{i-j}{i+j}$$

$\because$ A is a matrix of order 2 × 2,

$$\therefore \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\therefore \qquad a_{11} = \frac{1-1}{1+1} = 0$$

$$a_{12} = \frac{1-2}{1+2} = -\frac{1}{3}$$

$$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$a_{22} = \frac{2-2}{2+2} = 0$$

$$\therefore \qquad A = \begin{bmatrix} 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 \end{bmatrix}$$

5. **The matrix** $P = \begin{vmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{vmatrix}$ **is a:**

(a) Square matrix
(b) Diagonal matrix
(c) Unit matrix
(d) None of these **[NCERT Exemplar]**

6. **If** $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, **then the values of** x **and** y **is:**

(a) $x = 3, y = 1$ **(b)** $x = 2, y = 3$
(c) $x = 2, y = 4$ **(d)** $x = 3, y = 3$
[NCERT Exemplar]

Ans. *(b)* $x = 2, y = 3$

Explanation: On comparing two matrices, we get

$$4x = x + 6$$
$$\Rightarrow \quad x = 2$$
$$\text{and} \quad 2x + y = 7$$
$$\Rightarrow \quad y = 7 - 2x = 7 - 4 = 3$$

7. **The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:**

(a) 27 **(b)** 18
(c) 81 **(d)** 512

Ans. *(d) 512*

Explanation: Number of elements in 3 × 3 matrix = 9

Now each element have to be 0 or 1.

$\therefore$ Number of ways to fill a place = 2

$\Rightarrow$ Number of ways to fill 9 places

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^9$ or 512

Thus, there are 512 matrices of order 3 × 3 with entries 0 or 1.

8. **If** $a_{ij} = |2i + 3j^2|$, **then matrix** $A_{2\times2} = [a_{ij}]$ **will be:**

(a) $\begin{vmatrix} 5 & -14 \\ 7 & 16 \end{vmatrix}$ **(b)** $\begin{vmatrix} 5 & 14 \\ -7 & 16 \end{vmatrix}$

(c) $\begin{vmatrix} 5 & 14 \\ 7 & 16 \end{vmatrix}$ **(d)** $\begin{bmatrix} 5 & 14 \\ 7 & -16 \end{bmatrix}$ **[DIKSHA]**

Ans. *(c)* $\begin{bmatrix} 5 & 14 \\ 7 & 16 \end{bmatrix}$

Explanation: Here,

$$a_{ij} = |2i + 3j^2|$$
$$\therefore \quad a_{11} = |2 \times 1 + 3 \times 1^2| = 5$$
$$a_{12} = |2 \times 1 + 3 \times 2^2| = 14$$
$$a_{21} = |2 \times 2 + 3 \times 1^2| = 7$$
$$a_{22} = |2 \times 2 + 3 \times 2^2| = 16$$
$$\therefore \quad A_{2\times2} = \begin{bmatrix} 5 & 14 \\ 7 & 16 \end{bmatrix}$$

9. **Matrix** $A = [a_{ij}]_{p \times q}$ **is a square matrix, if:**

(a) $p > q$ **(b)** $p < q$
(c) $p = q$ **(d) None of these**

10. **The value of θ for which the matrix** $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **is identity matrix is:**

(a) 0° **(b)** 30°
(c) 60° **(d)** 90°

Ans. *(a) 0°*

Explanation: As according to question, given matrix is a identity matrix.

$$\therefore \quad \begin{bmatrix} \cos\theta & -\sin\theta \\ \cos\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing corresponding elements of the matrices, we get

$$\cos\theta = 1$$
$$\Rightarrow \quad \cos\theta = \cos 0°$$
$$\therefore \quad \theta = 0°$$

11. **The matrix** $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **is not a:**

(a) diagonal matrix **(b) square matrix**
(c) unit matrix **(d) scalar matrix**

12. **Identify the type of matrix A = [1 3 5].**

(a) Row matrix **(b) Column matrix**
(c) Square matrix **(d) Scalar matrix**

Ans. *(a) Row matrix*

Explanation: Given matrix has one row only, so it is a row matrix.

13. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then the value of $a+b-c+2d$ is:

(a) 8 (b) 10

(c) 4 (d) − 8

[CBSE Term-1 SQP 2021]

Ans. (a) 8

$$\left.\begin{aligned} 2a+b&=4 \\ a-2b&=-3 \\ 5c-d&=11 \\ 4c+3d&=24 \end{aligned}\right\} \& \quad \begin{aligned} a&=1 \\ b&=2 \\ c&=3 \\ d&=4 \end{aligned}$$

$$\therefore \ a+b-c+2d=8$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation:

$$\because \begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

$$\therefore \quad 2a+b=4 \quad \text{...(i)}$$

$$a-2b=-3 \quad \text{...(ii)}$$

$$5c+d=11 \quad \text{...(iii)}$$

$$4c+3d=24 \quad \text{...(iv)}$$

Solving equations (i) and (ii), we get

$$a=1, b=2$$

Solving equations (iii) and (iv), we get

$$c=3, d=4$$

So, $a+b-c+2d = 1+2-3+8$

$= 8.$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

14. Construct a 2 × 2 matrix, A = $[a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i-j)^2}{2}$.

Ans. Since, A is a matrix of order 2 × 2,

$$\therefore \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Here, $a_{ij} = \frac{(i-j)^2}{2}$

Now, $a_{11} = \frac{(1-1)^2}{2} = 0,$

$$a_{12} = \frac{(1-2)^2}{2} = \frac{1}{2}$$

$$a_{21} = \frac{(2-1)^2}{2} = \frac{1}{2}$$

and $a_{22} = \frac{(2-2)^2}{2} = 0$

So, $A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

15. **If a matrix has 18 elements, then write its possible orders.**

16. Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3.

Ans. In a square matrix of order 2 × 2, there exist 4 entries. Since, each entry has 3 choices {*i.e.*, 1, 2 or 3}, therefore, number of required matrices is 3 × 3 × 3 × 3 *i.e.*, 81.

17. **Find the element a_{32} of a 3 × 3 matrix : if a_{ij} is given by $a_{ij} = \frac{1}{2}|-3i+j|$.** [CBSE 2014]

18. **Write the order of the following matrices :**

(A) $\begin{bmatrix} 1 & 3 & -1 \\ 4 & 5 & 6 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & -1 \\ 4 & 5 \\ 6 & 7 \\ -1 & 0 \end{bmatrix}$

19. Construct a matrix of order 3 × 3, whose elements are given by $a_{ij} = 1$, when $i = j$ and $a_{ij} = 0$, when $i \neq j$. Check whether formed matrix is identity matrix.

Ans. We have $a_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

It means that all diagonal elements are unity and other diagonal elements are zero.

$$\therefore \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, formed matrix is a identity matrix.

20. Write the sum of all diagonal elements of the matrix $P = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

21. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then write the value of $(x + y + z)$. [CBSE 2014]

Ans. We have $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

Comparing the corresponding elements, we get

$xy = 8,\ w = 4,\ z + 6 = 0$ and $x + y = 6$

$\therefore \quad x + y + z = 6 + z = 0$

22. If a matrix $P = [b_{ij}]_{2 \times 2}$, where $b_{ij} = \begin{cases} 3, & i = j \\ 0, & i \neq j \end{cases}$, then write the matrix P.

23. If the matrices $P = \begin{bmatrix} \sin\theta & 1 \\ 2 & \cos\theta \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ are equal, find the angle θ.

24. All shops were closed during lockdown due to the Corona pandemic. When lockdown ended, Anjali and her parents decided to go to the jewellery shop for Anjali's wedding.

A shopkeeper sells 6 rings, 10 bracelets and 12 necklaces in first week and 12 rings, 6 bracelets and 8 necklaces in next week. Represent this information in matrix form.

Ans. Given, information can be written in table form as

	Rings	Bracelets	Necklaces
First week	6	10	12
Next week	12	6	8

This information can be written in matrix form as

$$\begin{matrix} \text{First week} \rightarrow \\ \text{Next week} \rightarrow \end{matrix} \begin{bmatrix} 6 & 10 & 12 \\ 12 & 6 & 8 \end{bmatrix}$$

Rings Bracelets Necklaces

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

25. Construct a matrix B of order 3 × 2, whose elements are given by $b_{ij} = i^2 - j$.

26. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = e^{ix} . \sin jx$. [NCERT Exemplar]

Ans. Matrix to be formed is of order 3 × 2 *i.e.* 3 rows and 2 columns.

$$\therefore \quad A = [a_{ij}]_{3\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times2}$$

where, $a_{ij} = e^{ix} \sin jx$

$\Rightarrow \quad a_{11} = e^{x} \sin x$

$a_{12} = e^{x} \sin 2x$

$a_{21} = e^{2x} \sin x$

$a_{22} = e^{2x} \sin 2x$

$a_{31} = e^{3x} \sin x$

$a_{32} = e^{3x} \sin 2x$

$$\therefore \quad A = \begin{bmatrix} e^{x}\sin x & e^{x}\sin 2x \\ e^{2x}\sin x & e^{2x}\sin 2x \\ e^{3x}\sin x & e^{3x}\sin 2x \end{bmatrix}$$

⚠ Caution

- *Remember the order of the matrix to be formed. Then only we can find the elements of the matrix.*

27. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, find the values of x, y, z and w. [NCERT Exemplar]

28. Find the values of x, y and z, if

$$\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

Ans. We have,

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

Comparing the corresponding elements, we have

$$x + y + z = 10 \quad ...(i)$$
$$x + z = 8 \quad ...(ii)$$
$$y + z = 7 \quad ...(iii)$$

From Eqs. (i) and (ii),

$$y + 8 = 10$$
$$\Rightarrow \quad y = 2$$

From Eqs. (i) and (iii),

$$x + 7 = 10$$
$$\Rightarrow \quad x = 3$$

Put $y = 2$ in Eq. (iii), we get

$$2 + z = 7$$
$$\Rightarrow \quad z = 5$$

Hence, the values of x, y, z are 3, 2, 5 respectively.

29. Find the values of a and b, if A = B, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

[NCERT Exemplar]

Ans. Here, $A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}$

and $B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$

Now, $A = B$ [Given]

$$\Rightarrow \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

$$\therefore \quad a + 4 = 2a + 2$$
$$\Rightarrow \quad -a = -2$$
$$\Rightarrow \quad a = 2$$

Also, $3b = b^2 + 2$

$$\Rightarrow \quad b^2 - 3b + 2 = 0$$
$$\Rightarrow \quad b^2 - 2b - b + 2 = 0$$
$$\Rightarrow \quad (b-2)(b-1) = 0,$$
$$\Rightarrow \quad b = 2, 1$$

and $-6 = b^2 - 5b$

$$\Rightarrow \quad b^2 - 5b + 6 = 0$$
$$\Rightarrow \quad b^2 - 2b - 3b + 6 = 0$$
$$\Rightarrow \quad (b-2)(b-3) = 0$$
$$\Rightarrow \quad b = 2, 3$$

Common value of b is 2.

$\therefore a = 2, b = 2$

⚠ Caution

➥ *Remember that if two matrices are equal, then each element of one matrix is equal to corresponding element of the other matrix.*

30. In the matrix, $A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 5 & 6 \\ 1 & 2 & -1 \end{bmatrix}$, find:

(A) the order of the matrix A

(B) the sum of elements a_{22} and a_{32}.

Ans. We have $A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 5 & 6 \\ 1 & 2 & -1 \end{bmatrix}$

(A) The order of the matrix A is 3 × 3.

(B) The sum of elements a_{22} and a_{32} is 5 + 2 *i.e.,* 7.

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

31. Ⓐ In the matrix $A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2-y \\ 0 & 5 & -\frac{2}{5} \end{bmatrix}$ write

(A) the order of matrix A

(B) the number of elements

(C) elements a_{23}, a_{31} and a_{12}

[NCERT Exemplar]

32. In the matrix $\begin{bmatrix} -1 & 2 & 5 & \sqrt{3} \\ 2 & 6 & 8 & 12 \\ -1 & 3 & 4 & 17 \end{bmatrix}$, write

(A) the order of the matrix

(B) the number of elements

(C) the elements a_{13}, a_{22}, a_{31}

(D) the sum of elements of first row.

Ans. (A) In the given matrix, the number of rows is 3 and the number of columns is 4. Thus, the order of matrix is 3 × 4.

(B) The number of elements in given matrix is $3 \times 4 = 12$.

(C) $a_{13} = 5$, $a_{22} = 6$ and $a_{31} = -1$.

(D) The sum of elements of first row

$$= -1 + 2 + 5 + \sqrt{3}$$
$$= 6 + \sqrt{3}$$

33. Solve the following questions.

(A) Construct a matrix A of order 3×3, whose elements are given by $a_{ij} = [i + j - 3]$, where [.] is a greatest integer.

(B) Find the sum of elements of second row of matrix A.

(C) Find the common element value of second row and second column.

34. Consider the following three matrices

$$A = \begin{bmatrix} a+4 & 3b \\ 7 & -6 \end{bmatrix}, B = \begin{bmatrix} 2a+2 & b+2 \\ 7 & a-8b \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

(A) Find the sum of elements of C other than diagonal elements.

(B) If matrices A and B are equal, determine the values of a and b.

(C) Write the matrix A.

Ans. (A) Given: $C = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

The sum of elements other than diagonal elements $= -2 + 3 = 1$

(B) Given: $A = B$

$$\Rightarrow \begin{bmatrix} a+4 & 3b \\ 7 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 7 & a-8b \end{bmatrix}$$

Comparing the corresponding elements, we get

$a + 4 = 2a + 2$, $3b = b + 2$

$\Rightarrow a = 2, b = 1$

Hence, $a = 2$ and $b = 1$

(C) $\therefore$ Matrix $A = \begin{bmatrix} 2+4 & 3\times 1 \\ 7 & -6 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 3 \\ 7 & -6 \end{bmatrix}$$

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

35. Construct a matrix P of order 3×3, whose elements are defined by $p_{ij} = \sin(30° \times i \times j)$.

(A) Write the total number of elements of matrix P.

(B) If we delete the first row and first column, then find the order of the remaining matrix P.

(C) If matrix $P = \begin{bmatrix} \frac{1}{2} & \cos\theta & \tan\phi \\ \cos\theta & \cos\theta & 0 \\ \tan\phi & 0 & -1 \end{bmatrix}_{3\times 3}$

Then find the angles θ and ϕ.

(D) Write the common element value of third row and third column.

(E) Find the sum of third row elements.

Ans. (A) Here $p_{ij} = \sin(30° \times i \times j)$

Now,

$$p_{11} = \sin(30° \times 1 \times 1) = \sin 30° = \frac{1}{2}$$

$$p_{12} = \sin(30° \times 1 \times 2) = \sin 60° = \frac{\sqrt{3}}{2}$$

$$p_{13} = \sin(30° \times 1 \times 3) = \sin 90° = 1$$

$$p_{21} = \sin(30° \times 2 \times 1) = \sin 60° = \frac{\sqrt{3}}{2}$$

$$p_{22} = \sin(30° \times 2 \times 2) = \sin 120° = \frac{\sqrt{3}}{2}$$

$$p_{23} = \sin(30° \times 2 \times 3) = \sin 180° = 0$$

$$p_{31} = \sin(30° \times 3 \times 1) = \sin 90° = 1$$

$$p_{32} = \sin(30° \times 3 \times 2) = \sin 180° = 0$$

$$p_{33} = \sin(30° \times 3 \times 3) = \sin 270° = -1$$

$$\therefore \quad P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

(B) After deleting the first row and first column, obtained matrix is of order 2 × 2.

(C) Given:

$$P = \begin{bmatrix} \frac{1}{2} & \cos\theta & \tan\phi \\ \cos\theta & \cos\theta & 0 \\ \tan\phi & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \cos\theta & \tan\phi \\ \cos\theta & \cos\theta & 0 \\ \tan\phi & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \quad \text{and} \quad \tan\phi = 1$$

$$\Rightarrow \theta = 30^\circ \quad \text{and} \quad \phi = 45^\circ$$

(D) The common element of third row and third column is a_{33}, *i.e.*, – 1.

(E) The sum of third row elements is 1 + 0 – 1 *i.e.*, 0.

OPERATIONS ON MATRICES 2

TOPIC 1

ADDITION OF MATRICES

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ be two matrices of the same order, then we define the sum of matrices A and B as

$$A + B = [a_{ij} + b_{ij}]_{m \times n}.$$

⚠ Caution

➥ *In a matrix, elements and their position both have their importance; addition in matrix is not always defined as algebra. For addition of two matrices, order of two matrixes must be same. Here, A + B is not defined, if A and B are of different orders.*

For example, consider matrices $A = \begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 0 & -3 \\ 2 & 4 & 1 \end{bmatrix}$

Since A and B are of the same order 2, we can obtain $A + B$ as $\begin{bmatrix} 3+5 & 5-1 \\ 0+0 & 2-2 \end{bmatrix}$, *i.e.*, $\begin{bmatrix} 8 & 4 \\ 0 & 0 \end{bmatrix}$

But A + C is not defined, as A and C are of different orders.

Similarly, B + C is not defined, as B and C are of different orders.

Properties of Matrix Addition

If A, B and C are any three matrices of the same order, then

(1) Matrix addition is commutative,

i.e., $A + B = B + A.$

(2) Matrix addition is associative,

i.e., $(A + B) + C = A + (B + C).$

(3) Existence of additive identity matrix: For every matrix $A = [a_{ij}]_{m \times n}$, there exists a zero matrix O of order $m \times n$, such that

$$A + O = O + A = A$$

Here, the zero matrix O is called the additive identity matrix.

(4) Existence of additive inverse matrix: For every matrix $A = [a_{ij}]_{m \times n}$, there exists a unique matrix $(-A) = [-a_{ij}]_{m \times n}$ such that

$$A + (-A) = O = (-A) + A$$

The matrix (– A) is called the additive inverse of the matrix A or negative of matrix A.

TOPIC 2

DIFFERENCE OF TWO MATRICES

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ be two matrices of the same order, then we define the difference of matrices A and B as

$$A - B = [a_{ij} - b_{ij}]_{m \times n}.$$

⚠ Caution

➥ *In a matrix, elements and their position both have their importance; subtraction in matrix is not always defined as algebra. For subtraction of two matrices, order of two matrices must be same. Here, A – B is not defined, if A and B are of different orders.*

For example, consider matrices $A = \begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 0 & -3 \\ 2 & 4 & 1 \end{bmatrix}$

Since A and B are of the same order 2, we can obtain

$A - B = \begin{bmatrix} 3-5 & 5+1 \\ 0-0 & 2+2 \end{bmatrix}$, *i.e.*, $\begin{bmatrix} -2 & 6 \\ 0 & 4 \end{bmatrix}$

But A – C is not defined, as A and C are of different orders.

Similarly, B – C is not defined, as B and C are of different orders.

TOPIC 3

SCALAR MULTIPLICATION (MULTIPLICATION OF A MATRIX BY A SCALAR)

Let $A = [a_{ij}]_{m \times n}$ be a matrix and let k be a scalar (*i.e.*, real or complex number), then we define multiplication of matrix A by a scalar k as

$$kA = [ka_{ij}]_{m \times n}$$

For example if A = $\begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 & -3 \\ 2 & 4 & 1 \end{bmatrix}$,

then 2A = $\begin{bmatrix} 2\times3 & 2\times5 \\ 2\times0 & 2\times2 \end{bmatrix}$, *i.e.*, $\begin{bmatrix} 6 & 10 \\ 0 & 4 \end{bmatrix}$; and

$$(-4)B = \begin{bmatrix} (-4)\times(-1) & (-4)\times0 & (-4)\times(-3) \\ (-4)\times2 & (-4)\times4 & (-4)\times1 \end{bmatrix}$$

i.e., $\begin{bmatrix} 4 & 0 & 12 \\ -8 & -16 & -4 \end{bmatrix}$

Properties of Scalar Multiplication

Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ be two matrices of the same order. Let p and q be two scalars. Then, we define

(1) $p(A + B) = pA + pB$

(2) $(p + q)A = pA + qA$

Example 2.1: Let A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and C = $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Then find each of the following:

(A) A + B **(B) A – B**

(C) 3A – C **[NCERT]**

Ans. (A) $A + B = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(B) $A - B = \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(C) $3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Example 2.2: Find X, if Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X + Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ **[NCERT]**

Ans. $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ gives $2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y$

$$\Rightarrow \quad 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Example 2.3: Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ **[NCERT]**

Ans. The given equation is

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \quad 2x + 3 = 9,\quad 2z - 3 = 15,\quad 2y + 0 = 12,\quad 2t + 6 = 18$$

$\Rightarrow x = 3, z = 9, y = 6$ and $t = 6$

TOPIC 4

MULTIPLICATION OF TWO MATRICES

Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{jk}]_{n \times p}$ be two matrices, then we define multiplication of matrices A and B as AB = $[c_{ik}]_{m \times p}$, where c_{ik} is obtained by first taking the element wise products of elements of i^{th} row of A and k^{th} column of B and then adding the products.

Important

- *If orders of matrices A and B are m × n and n × p respectively, then the order of matrix AB is m × p.*
- *The product AB is defined only if the number of columns of A is equal to the number of rows of B.*
- *In the product AB, the matrix A is called pre-multiplier matrix and the matrix B is called post-multiplier matrix.*
- *If AB is defined, then BA needs not to be defined.*
- *If A and B both are square matrices of the same order, then both AB and BA are defined.*

Example 2.4: If X, Y, Z, W and P are matrices of orders $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$ respectively, then,

(A) What is the restriction on n, k and p so that PY + WY will be defined?

(B) If $n = p$, then what is the order of the matrix 7X – 5Z? **[NCERT]**

Ans. (A) PY + WY will be defined only when both PY and WY are defined and both are of the same order.

For PY to be defined, k must be 3 {as, number of columns of P must be equal to number of rows of Y} and the order of PY will be $p \times k$ *i.e.,* $p \times 3$.

WY is already defined as number of columns of W is equal to number of rows of Y and the order of WY is $n \times k$.

Now for (PY + WY) to be defined, orders of PY and WY must be same.

i.e., $p \times 3 = n \times k$

And, $p \times 3 = n \times k$ gives $p = n$ and $k = 3$.

(B) The order of 7X is the same as the order of X. Hence, the order of 7X is $2 \times n$.

Similarly, the order of 5Z is the same as the order of Z. Hence, the order of 5Z is $2 \times p$.

It is given that $n = p$, so the order of 7X – 5Z is $2 \times n$, or $2 \times p$.

Understanding the Technique of Matrix Multiplication

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix}_{2\times3}$

Since the number of columns of A (*i.e.,* 2) is equal to the number of rows of B (*i.e.,* 2), the product matrix AB is defined and the order of product matrix $AB = [c_{ik}]_{m \times p}$ is 2×3.

We write AB as $\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}_{2\times3}$

To evaluate c_{11} (*i.e.,* entry in the first row and first column):

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} (2)(-1)+(-1)(4) & * & * \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} -6 & * & * \\ * & * & * \end{bmatrix}$$

To evaluate c_{12} (*i.e.,* entry in the first row and second column):

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} * & (2)(2)+(-1)(0) & * \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} * & 4 & * \\ * & * & * \end{bmatrix}$$

To evaluate c_{13} (*i.e.,* entry in the first row and third column):

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} * & * & (2)(-3)+(-1)(-2) \\ * & * & * \end{bmatrix}$$

$$= \begin{bmatrix} * & * & -4 \\ * & * & * \end{bmatrix}$$

To evaluate c_{21} (*i.e.,* entry in the second row and first column):

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} * & * & * \\ (3)(-1)+(0)(4) & * & * \end{bmatrix}$$

$$= \begin{bmatrix} * & * & * \\ -3 & * & * \end{bmatrix}$$

To evaluate c_{22} (i.e., entry in the second row and second column):

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & (3)(2)+(0)(0) & * \end{bmatrix}$$

$$= \begin{bmatrix} * & * & * \\ * & 6 & * \end{bmatrix}$$

To evaluate c_{23} (i.e., entry in the second row and third column):

$$\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & (3)(-3)+(0)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} * & * & * \\ * & * & -9 \end{bmatrix}$$

Thus, $AB = \begin{bmatrix} -6 & 4 & -4 \\ -3 & 6 & -9 \end{bmatrix}$

Example 2.5: Compute the following matrices:

(A) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

Ans. (A) The product $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ is defined, as the number of columns of $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ is same as the number of rows of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$; and the order of $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ is 2×3.

Now, $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times1+(-2)\times2 & 1\times2+(-2)\times3 & 1\times3+(-2)\times1 \\ 2\times1+3\times2 & 2\times2+3\times3 & 2\times3+3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(B) The product $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ is defined, as the number of columns of $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ is same as the number of rows of $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$; and the order of $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$; is 3×3.

Now, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2\times1+1\times(-1) & 2\times0+1\times2 & 2\times1+1\times1 \\ 3\times1+2\times(-1) & 3\times0+2\times2 & 3\times1+2\times1 \\ (-1)\times1+1\times(-1) & (-1)\times0+1\times2 & (-1)\times1+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(C) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ is defined as each matrix is of order 3×3; and so the product matrices is of order 3×3.

Now, $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2\times1+3\times0+4\times3 & 2\times(-3)+3\times2+4\times0 & 2\times5+3\times4+4\times5 \\ 3\times1+4\times0+5\times3 & 3\times(-3)+4\times2+5\times0 & 3\times5+4\times4+5\times5 \\ 4\times1+5\times0+6\times3 & 4\times(-3)+5\times2+6\times0 & 4\times5+5\times4+6\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Properties of Matrix Multiplication

For any three matrices A, B and C, we have

(1) Matrix multiplication is associative: (AB)C = A(BC), whenever both sides of the equality are defined.

(2) Matrix multiplication is distributive over matrix addition identity: A(B + C) = AB + AC, whenever both sides of the equality are defined.

(3) Existence of multiplicative identity matrix: For every square matrix A, there exists an identity matrix I of the same order such that AI = IA = A.

(4) Matrix multiplication is not commutative in general: For any two matrices A and B, if both AB and BA are defined, it is not necessary that AB = BA (*i.e.*, commutativity may hold in some cases, but may not hold in some other).

Illustration: (1) Let $A = \begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix}$.

Then, $AB = \begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -18 & 6 \\ -6 & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -9 & -15 \end{bmatrix}$

So, $AB \neq BA$

(2) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$.

Then, $AB = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 15 \end{bmatrix}$

and $BA = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 15 \end{bmatrix}$

So, $AB = BA$

Important

- *From the above, we observe that multiplication of diagonal matrices of the same order is commutative.*

Zero Matrix as the Product of Two Non-zero Matrices

We have a result for real numbers, which states that: "Let a, b be any two real numbers. If $ab = 0$, then either $a = 0$ or $b = 0$."

Such a result does not hold true for matrices. For matrices, we have the following:

"If the product of two matrices A and B is a zero matrix, then it is not necessary that one of the matrices is a zero matrix."

For example,

Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 5 \\ 0 & 0 \end{bmatrix}$.

Then, AB = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Here, A ≠ O, B ≠ O but AB = O

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. If A = $[a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{ when } i \neq j \\ 0, \text{ when } i = j \end{cases}$, then A^2 is:

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

[CBSE Term-1 SQP 2021]

Ans. *(d)* $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. If A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and A − 2B = $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A is:

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

3. Given that matrices A and B are of order 3 × *n* and *m* × 5 respectively, then the order of matrix C = 5A + 3B is:

(a) 3 × 5 and *m* = *n* (b) 3 × 5

(c) 3 × 3 (d) 5 × 5

[CBSE Term-1 SQP 2021]

Ans. *(b) 3 × 5*

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: C = $[5A]_{3\times n} + [3B]_{m\times 5}$

We know, addition of two matrices is possible only when their orders are same.

$\therefore \quad 3 \times n = m \times 5$

$\Rightarrow \quad m = 3, n = 5$

So, order of matrix C is 3 × 5.

4. If A and B are two matrices of order 3 × *m* and 3 × *n* respectively and *m* = *n*, then the order of matrix (5A − 2B) is:

(a) *m* × 3 (b) 3 × 3

(c) *m* × *n* (d) 3 × *n*

[NCERT Exemplar]

5. Suppose P, Q and R are different matrices of order 3 × 5, *a* × *b* and *c* × *d* respectively, then value of *ac* + *bd* is, if matrix 2P + 3Q − 4R is defined

(a) 9 (b) 30

(c) 34 (d) 15

[Delhi Gov. 2022]

Ans. *(c) 34*

Explanation: By definition,

3 × 5 = *a* × *b* = *c* × *d*,

Thus *a* = *c* = 3 and *b* = *d* = 5

Thus, *ac* + *bd* = 9 + 25 = 34

6. If $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ + X = $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$, where X = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then *a* + *c* − *b* − *d* =

(a) 13 (b) 5

(c) −8 (d) −3

[Delhi Gov. 2022]

Ans. *(d) −3*

Explanation: $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 3 \end{bmatrix}$

Thus, *a* + *c* − *b* − *d* = 2 + 3 − 5 − 3 = −3.

7. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(a) A (b) I + A

(c) I − A (d) I

[CBSE Term-1 SQP 2021]

Ans. *(d)* *I*

$$(I + A)^3 - 7A = I + A + 3A + 3A - 7A = I$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: We have,

$$(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3IA^2 - 7A$$
$$= I + A^2.A + 3IA + 3IA^2 - 7A \quad [\because I^3 = I^2 = I]$$
$$= I + A.A + 3A + 3A - 7A \quad [\because A^2 = A; IA = A]$$
$$= I + A^2 - A$$
$$= I + A - A \quad [\because A^2 = A]$$
$$= I$$

8. If $A = \begin{bmatrix} \sin 0° & \sin 30° \\ \sin 60° & \sin 90° \end{bmatrix}$, the A^2 is:

(a) $\begin{bmatrix} \frac{\sqrt{3}}{4} & \frac{1}{2} \\ 1 & \frac{\sqrt{3}}{4} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{3}}{4} & \frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{4}+1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{4} & 1 \\ \sqrt{3} & \frac{\sqrt{3}}{4} \end{bmatrix}$ (d) None of these

9. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then $(A - 2I)(A - 3I)$ is equal to:

(a) A (b) I

(c) 5I (d) O

10. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of *k*, *a* and *b* respectively are:

(a) −6, −12, −18 (b) −6, −4, −9

(c) −6, 4, 9 (d) −6, 12, 18

[CBSE Term-1 SQP 2021]

11. If A and B are two square matrices of same order such that, AB = A and BA = B, then (A + B)(A − B) =

(a) $A^2 - B^2$ (b) 2A − 2B

(c) 2A + 2B (d) 0

[Delhi Gov. 2022]

Ans. *(d) 0*

Explanation: $A^2 = A.A = AB.A = A(BA) = A.B = A$

$B^2 = BB = BA.B = B(AB) = B.A = B$

Now,

$$(A + B)(A - B) = A^2 + BA - AB - B^2$$
$$= A + B - A - B = 0$$

12. If $[x - 2 \;\; 5 + y]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = O$, then $x + y =$

(a) 0 (b) −2

(c) −1 (d) −3

[Delhi Gov. 2022]

Ans. *(d) −3*

Explanation: $[x - 2 \;\; 5 + y]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = O$

$\Rightarrow [5 + y \;\; x - 2] = O = [0 \;\; 0]$

On comparing, $y = -5$, $x = 2$, so $x + y = -3$.

13. For any two matrices A and B, we have:

(a) AB = BA (b) AB ≠ BA

(c) AB = O (d) None of these

Ans. *(d) None of these*

Explanation: Given relations are true only in few cases.

14. If AB = A and BA = B, then $B^2 + B$ is equal to:

(a) 2A (b) O

(c) 2I (d) 2B

15. If $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$ and $A^2 - 5A + 10I = O$, then the value of *k* is:

(a) − 6 (b) − 4

(c) 4 (d) 6

Ans. *(c) 4*

Explanation: Here,

$$A^2 = A.A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$
$$= \begin{bmatrix} -5 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix}$$

Now, $A^2 - 5A + 10I = O$

$$\begin{bmatrix} -5 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix} - 5\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} + 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$$

$\Rightarrow -3 - 3k - 5(-3) + 10(0) = 0 \Rightarrow k = 4$

16. If A is a matrix of order 2 × 3 and B is a matrix of order 3 × 2, then:

(a) Only AB is defined.

(b) Only BA is defined.

(c) Both AB and BA are defined.

(d) Neither AB nor BA is defined.

Ans. *(c) Both AB and BA are defined.*

Explanation: $AB = [A]_{2\times3}[B]_{3\times2} = [AB]_{2\times2}$

and $BA = [B]_{3\times2}[A]_{2\times3} = [BA]_{3\times3}$

Hence, both AB and BA are defined.

17. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is:

(a) 2I (b) 3I

(c) I (d) 5I

[CBSE Term-1 2021]

Ans. *(a) 2I*

Explanation: $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$\therefore \quad X^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+1 & 0+0+1 & 0+1+0 \\ 0+0+1 & 1+0+1 & 1+0+0 \\ 0+1+0 & 1+0+0 & 1+1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\therefore X^2 - X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 1-1 & 1-1 \\ 1-1 & 2-0 & 1-1 \\ 1-1 & 1-1 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2I$$

18. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $A^2 + xI = yA$, then the value of $x + y$ is:

(a) 16 (b) – 8

(c) – 16 (d) 0

Ans. *(a) 16*

Explanation: Here,

$$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

So, $A^2 + xI = yA$ implies

$$\begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$\Rightarrow \quad 16 + x = 3y$ and $56 = 7y$

$\Rightarrow \quad y = 8$

So, $16 + x = 3(8)$

$\Rightarrow \quad x = 24 - 16 = 8$

$\therefore \quad x + y = 8 + 8 = 16$

19. Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then :

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$

[CBSE Term-1 SQP 2021]

Ans. *(c) $3 - \alpha^2 - \beta\gamma = 0$*

$$A^2 = 3I$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow \quad 3-\alpha^2-\beta\gamma=0$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: We have, $A^2 = 3I$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\beta\alpha \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \alpha^2+\beta\gamma \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$\Rightarrow \quad \alpha^2 + \beta\gamma = 3$

or $\quad 3 - \alpha^2 - \beta\gamma = 0.$

20. If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then $2x + y - z =$

(a) 2 (b) 1

(c) 3 (d) 5

[Delhi Gov. 2022]

Ans. *(d) 5*

Explanation: As, $\begin{bmatrix} x+y+z \\ y+z \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$,

then $z = 2, y + z = 3, x + y + z = 6$

Thus, $z = 2, y = 1, x = 3$

$\Rightarrow \quad 2x + y - z = 5$

21. **If A is a diagonal matrix of order 3 × 3 such that $A^2 = A$, then number of possible matrices A are:**

(a) 4 **(b) 8**

(c) 16 **(d) 32**

[Delhi Gov. 2022]

Ans. *(b) 8*

Explanation: Let, $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix}$$

As $$A^2 = A \Rightarrow \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix}$$

So, a = 0 or 1, similarly b and c can take 2 values (0 and 1)

Thus, total number of possible matrices are 2 × 2 × 2 = 8

22. **What must be the matrix X, if $\begin{bmatrix} 2 & -5 \\ 4 & 6 \end{bmatrix} + 3X = \begin{bmatrix} 8 & 4 \\ 1 & 0 \end{bmatrix}$.**

(a) $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 3 \\ -1 & -2 \end{bmatrix}$ [DIKSHA]

23. **Ramesh and Mustaq were wholesale grain merchants. Piyush, a student of economics, was doing a research work as a part of his school project. Piyush spoke to both the merchants regarding profit (or loss) earned in the months of March 2022 and April 2022 and tabulated them as given below, represented by matrices A and B respectively:**

	Profit earned by Ramesh in March 2022	Profit earned by Mustaq in March 2022
Wheat	1%	– 2%
Rice	4%	3%
	Profit earned by Ramesh in April 2022	**Profit earned by Mustaq in April 2022**
Wheat	4%	1%
Rice	– 3%	2%

If $A = \begin{bmatrix} 1 & -2 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$, then 3A – 2B is:

(a) $\begin{bmatrix} 5 & -8 \\ -6 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & -8 \\ -6 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ -6 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ 6 & -5 \end{bmatrix}$

Ans. *(b)* $\begin{bmatrix} -5 & -8 \\ -6 & 5 \end{bmatrix}$

Explanation: Here,

$$3A - 2B = 3\begin{bmatrix} 1 & -2 \\ -4 & 3 \end{bmatrix} - 2\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 \\ -12 & 9 \end{bmatrix} - \begin{bmatrix} 8 & 2 \\ -6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 \\ -6 & 5 \end{bmatrix}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

24. **Let A and B be two matrices of order 3 × 2 and 2 × 4 respectively. Write the order of matrix AB.** **[CBSE 2017]**

Ans. Since matrices of A and B are 3 × 2 and 2 × 4. Therefore, matrix AB is order 3 × 4.

25. **If possible, find the sum of the matrices A and B, where**

$$A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}$$

[NCERT Exemplar]

Ans. Here, order of matrix is A is 2 × 2 and B is 2 × 3. We know, addition of matrices with different orders is not possible. So, addition of matrices A and B is not possible.

⚠ **Caution**

➥ *For the addition of two matrices, their order should be same.*

26. **Find the additive inverse of the following matrix** $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}$.

27. **Simplify**

$$\sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} + \cos \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Ans. $\sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} + \cos \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} + \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta + \cos^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

28. **Show by an example that for A ≠ 0, B ≠ 0, AB = 0.** **[NCERT Exemplar]**

Ans. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \neq 0$ and $B = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \neq 0$

$$\therefore \quad AB = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence, proved.

29. **If** $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$, **then find the value of** x.

30. **If** $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ **and find the value of matrix A.** **[CBSE 2019]**

31. **If** $[1 \;\; x \;\; 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$, **then write the value of** x.

Ans. We have, $[1 \;\; x \;\; 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

$$\Rightarrow \quad [1 \;\; x \;\; 1] \begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix} = 0$$

$$\Rightarrow [7 + 2x + x(12 + x) + 21 + 2x] = 0$$

$$\Rightarrow \quad x^2 + 16x + 28 = 0$$

$$\Rightarrow \quad (x + 14)(x + 2) = 0$$

$$\Rightarrow \quad x = -2 \text{ or } -14.$$

32. **If matrix** $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ **and** $A^2 = kA$, **then write the value of** k. **[CBSE 2013]**

33. **If ω is a cube roots of unity, show that**

$$\begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Ans. $\text{LHS} = \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$

$$= \begin{bmatrix} \omega + \omega^3 + \omega^2 \\ \omega^2 + \omega + \omega^3 \\ \omega + \omega^3 + \omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} \omega + 1 + \omega^2 \\ \omega^2 + \omega + 1 \\ \omega + 1 + \omega^2 \end{bmatrix} \qquad [\because \omega^3 = 1]$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \text{RHS} \qquad [\because 1 + \omega + \omega^2 = 0]$$

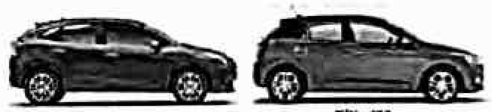
34. **Hatchback cars in India are immensely popular. Their popularity can be attributed primarily to their practicality, simplicity**

and value for money. But of late, feature-laden and powerful premium hatchbacks are getting a lot of slack in the market. One of the oldest contenders in this segment is Korean-marquee, Hyundai's i20, now christened as the Elite i20. In an attempt to give its chief rival a fitting response, Maruti Suzuki resurrected their classic Baleno and gave it a brand-new life. This time it took birth as a contemporary premium hatchback and was all set to lock horns with the successful performing Elite i20. Keya was doing a market research and studied the growth (in percentage) of these two car models from two automobile dealers in the city.

She concluded her study with the following table showing the percentage growth for two consecutive months, which can be represented by 2 × 2 matrices A and B.

1st Month	Growth (in %) of first dealer	Growth (in %) of second dealer
Elite i20	2	3
Baleno	− 1	2

2nd Month	Growth (in %) of first dealer	Growth (in %) of second dealer
Elite i20	− 1	2
Baleno	0	1



Baleno Elite i20

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$, then find $(2A + 7B)$.

Ans. We have, $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

Therefore, $2A + 7B = 2\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 5 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 14 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 20 \\ -2 & 11 \end{bmatrix}$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

35. **Let A and B be square matrices of the order 3 × 3. Is $(AB)^2 = A^2B^2$? Give reasons.** **[NCERT Exemplar]**

Ans. Given: A and B are square matrices of order 3 × 3.

Now, $(AB)^2 = AB.\,AB$

$= ABAB$

$= AABB$ $[\because AB = BA]$

$= A^2B^2$

So, $(AB)^2 = A^2B^2$ is true when AB = BA.

36. Ⓐ **Solve the matrix equation for x.**

$$[x \;\; 1]\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} = O$$ **[CBSE 2014]**

37. **Give an example of matrices A, B, C such that AB = AC, where A is non-zero matrix, but B ≠ C.** **[NCERT Exemplar]**

Ans. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$.

$\therefore$ $AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ...(i)

and $AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get

AB = AC but B ≠ C

⚠ Caution

➥ *There can be other values of A, B and C which satisfy the above condition.*

38. Suppose two matrices X and Y are defined as $X = \begin{bmatrix} 2 & 1 & -3 \\ 4 & -1 & 2 \end{bmatrix}$ and $Y = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 5 \end{bmatrix}$. Find matrix Z, if X + Y + Z = 0.

Ans. We have X + Y + Z = 0

$$\Rightarrow \quad Z = -(X + Y)$$

$$= -\left(\begin{bmatrix} 2 & 1 & -3 \\ 4 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 5 \end{bmatrix}\right)$$

$$= -\begin{bmatrix} 2+3 & 1+1 & -3-2 \\ 4+2 & -1+1 & 2+5 \end{bmatrix}$$

$$= -\begin{bmatrix} 5 & 2 & -5 \\ 6 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 & 5 \\ -6 & 0 & -7 \end{bmatrix}$$

39. Solve for x and y: $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$

[NCERT Exemplar]

40. Find 4A + 3B, if A = diag [3 − 1 4] and B = diag [− 1 0 4].

Ans. A = diag [3 − 1 4]

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

and B = diag [− 1 0 4]

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

So,

$$4A + 3B = 4\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 16 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 12-3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 16+12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 28 \end{bmatrix}$$

= diag [9 − 4 −28]

41. If $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, show that AB ≠ BA.

42. If $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, prove that $PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP$.

[NCERT Exemplar]

43. Find A^2, if $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

Ans. We have $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & i\sin\theta\cos\theta + i\sin\theta\cos\theta \\ i\sin\theta\cos\theta + i\sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos 2\theta \end{bmatrix} \quad [\because i^2 = -1]$$

$$= \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}$$

44. If $[2x \;\; 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$, find x. [CBSE 2015]

Ans. We have,

$$[2x \;\; 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow \quad [2x - 9 \;\; 4x]\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow \quad [2x^2 - 9x + 12x] = 0$$

$$\Rightarrow \quad [2x^2 + 3x] = [0]$$

$$\Rightarrow \quad 2x^2 + 3x = [0]$$

$$\Rightarrow \quad x[2x + 3] = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = \frac{-3}{2}$$

45. Verify that $A^2 = I$ when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

[NCERT Exemplar]

Ans. Here, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -3+3 & 4-4 \\ -12+12 & 4+9-12 & -4-12+16 \\ -12+12 & 3+9-12 & -3-12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, proved.

46. Ⓐ **If A is a square matrix such that $A^2 = A$ then write the value of $7A - (I + A)^3$, where I is an identity matrix. [CBSE 2014]**

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

47. If $A = \begin{bmatrix} 7 & 0 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 4 & 5 \\ -6 & 5 \end{bmatrix}$ find the matrix X such that 2A + 3X = 8B.

Ans. We have 2A + 3X = 8B

$\Rightarrow \quad 3X = 8B - 2A$

or, $\quad X = \frac{1}{3}(8B - 2A)$

$$\Rightarrow \quad X = \frac{1}{3}\left(8\begin{bmatrix} 2 & -3 \\ 4 & 5 \\ -6 & 5 \end{bmatrix} - 2\begin{bmatrix} 7 & 0 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}\right)$$

$$\Rightarrow \quad X = \frac{1}{3}\left(\begin{bmatrix} 16 & -24 \\ 32 & 40 \\ -48 & 40 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 6 & 4 \\ 10 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{3}\left(\begin{bmatrix} 16-14 & -24 \\ 32-6 & 40-4 \\ -48-10 & 40-2 \end{bmatrix}\right)$$

$$= \frac{1}{3}\begin{bmatrix} 2 & -24 \\ 26 & 36 \\ -58 & 38 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3} & -8 \\ \frac{26}{3} & 12 \\ \frac{-58}{3} & \frac{38}{3} \end{bmatrix}$$

48. Ⓐ **A trust fund has ₹ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:**

(A) ₹ 1800

(B) ₹ 2000 [DIKSHA]

49. Solve the matrix equation $\begin{bmatrix} a^2 \\ b^2 \end{bmatrix} - 5\begin{bmatrix} a \\ 2b \end{bmatrix} = \begin{bmatrix} -6 \\ -24 \end{bmatrix}$

Ans. Given: $\begin{bmatrix} a^2 \\ b^2 \end{bmatrix} - 5\begin{bmatrix} a \\ 2b \end{bmatrix} = \begin{bmatrix} -6 \\ -24 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} a^2 \\ b^2 \end{bmatrix} - \begin{bmatrix} 5a \\ 10b \end{bmatrix} = \begin{bmatrix} -6 \\ -24 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a^2 - 5a \\ b^2 - 10b \end{bmatrix} = \begin{bmatrix} -6 \\ -24 \end{bmatrix}$$

Comparing corresponding elements on both sides

$\Rightarrow \quad a^2 - 5a + 6 = 0$

and $\quad b^2 - 10b + 24 = 0$

$\Rightarrow (a-3)(a-2) = 0$ and $(b-6)(b-4) = 0$

$\Rightarrow a = 3, 2$ and $b = 6, 4$

50. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, **find**

(A) X + Y

(B) 2X – 3Y

(C) a matrix Z such that X + Y + Z is a zero matrix. [NCERT Exemplar]

Ans. (A) $X + Y = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$$

(B) $\quad 2X = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix}$$

$$3Y = 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$$\therefore 2X - 3Y = \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix} - \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(C) Given, X + Y + Z = O

Now, $X + Y = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$

So, $\begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix} + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow \quad Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix}$$

51. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, **then find the value of** $(A^2 - 5A)$. **[CBSE 2019]**

52. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, **then show that** $(A + B)(A - B) \neq A^2 - B^2$. **[NCERT Exemplar]**

Ans. Given, $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$\therefore A + B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$$

So, $(A + B)(A - B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \quad ...(i)$$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

and $B^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Then, $A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad ...(ii)$$

From (i) and (ii), we have

$$(A + B)(A - B) \neq A^2 - B^2$$

⚠ Caution

➥ *Multiplication of matrices is only possible when column number of first matrix is equal to the row number of second matrix.*

53. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ **show that** $(A - 2I)(A - 3I) = 0$. **[CBSE 2019]**

Ans. Given: $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

L.H.S. = (A – 2I)(A – 3I)

$$= \left\{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}\left\{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right\}\left\{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Hence, proved.

54. If $A = [3 \;\; 5]$ **and** $B = [7 \;\; 3]$, **then find a non-zero matrix C such that AC = BC.** **[NCERT Exemplar]**

55. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$.

Find a matrix D such that CD – AB = O. **[CBSE 2017]**

Ans. Let matrix D = $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$

Given CD – AB = O

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} - \begin{bmatrix} 10-7 & 4-4 \\ 15+28 & 6+16 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$

Comparing corresponding elements both sides, we get

$$2x + 5z = 3, \quad 3x + 8z = 43$$

and $2y + 5w = 0$, $3y + 8w = 22$

$\Rightarrow x = -191$, $y = -110$, $z = 77$ and $w = 44$

$$\therefore \quad D = \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$$

56. **If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, B = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of *a* and *b*.** **[CBSE 2015]**

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

57. **If A = $\begin{bmatrix} 4 & 3 & -1 \\ 5 & 0 & 2 \\ 2 & -3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 4 \end{bmatrix}$, then compute (A + B) and (B – C). Also, verify that**

A + (B – C) = (A + B) – C.

Ans.

$$A + B = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 0 & 2 \\ 2 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -1 \\ 8 & 2 & 3 \\ 3 & -3 & 3 \end{bmatrix}$$

and

$$B - C = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 2 & 2 & 2 \\ -1 & 1 & -2 \end{bmatrix}$$

Now, we have to verify that

A + (B – C) = (A + B) – C

$$\therefore \quad LHS = A + (B - C) = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 0 & 2 \\ 2 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -3 \\ 2 & 2 & 2 \\ -1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 3 & -4 \\ 7 & 2 & 4 \\ 1 & -2 & -1 \end{bmatrix}$$

and,

$$RHS = (A + B) - C = \begin{bmatrix} 6 & 2 & -1 \\ 8 & 2 & 3 \\ 3 & -3 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & -4 \\ 7 & 2 & 4 \\ 1 & -2 & -1 \end{bmatrix}$$

$\therefore$ LHS = RHS

Hence, verified.

58. **Suppose matrix A of order 3 × 3 is defined by $a_{ij} = \left[\frac{i+j}{2}\right]$, where [ ] is the greatest integer function, and another matrix is B = $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$.**

(A) Determine the product of A and B.

(B) Find the sum of A and B.

(C) Find the difference of A and B.

Ans. We have $a_{ij} = \left[\frac{i+j}{2}\right]$

$$\therefore \quad a_{11} = \left[\frac{1+1}{2}\right] = [1] = 1$$

$$a_{12} = \left[\frac{1+2}{2}\right] = \left[\frac{3}{2}\right] = [1.5] = 1$$

$$a_{13} = \left[\frac{1+3}{2}\right] = \left[\frac{4}{2}\right] = 2$$

$$a_{21} = \left[\frac{2+1}{2}\right] = \left[\frac{3}{2}\right] = [1.5] = 1$$

$$a_{22} = \left[\frac{2+2}{2}\right] = \left[\frac{4}{2}\right] = 2$$

$$a_{23} = \left[\frac{2+3}{2}\right] = \left[\frac{5}{2}\right] = [2.5] = 2$$

$$a_{31} = \left[\frac{3+1}{2}\right] = \left[\frac{4}{2}\right] = 2$$

$$a_{32} = \left[\frac{3+2}{2}\right] = \left[\frac{5}{2}\right] = [2.5] = 2$$

$$a_{33} = \left[\frac{3+3}{2}\right] = \left[\frac{6}{2}\right] = 3$$

$$\therefore \quad A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

(A) $$AB = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2-2 & -1+3+4 & 0+4+0 \\ 1-4-2 & -1+6+4 & 0+8+0 \\ 2-4-3 & -2+6+6 & 0+8+0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 6 & 4 \\ -5 & 9 & 8 \\ -5 & 10 & 8 \end{bmatrix}$$

(B) $$A + B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ -1 & 5 & 6 \\ 1 & 4 & 3 \end{bmatrix}$$

(C) $$A - B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 2 \\ 3 & -1 & -2 \\ 3 & 0 & 3 \end{bmatrix}$$

59. Ⓐ $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ -1 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$,

verify that A(B – C) = AB – AC.

60. For the following matrices, verify the associativity of matrix multiplication $A = \begin{bmatrix} 1 & -2 & -1 \\ 1 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

Ans. Here we have to verify that

$$(AB)C = A(BC)$$

Now, $$AB = \begin{bmatrix} 1 & -2 & -1 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2+0 & 0+4-3 \\ 2+0+0 & 0+0-6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & -6 \end{bmatrix}$$

$$\therefore \quad LHS = (AB)C = \begin{bmatrix} 0 & 1 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 \\ -4-6 \end{bmatrix} = \begin{bmatrix} 1 \\ -10 \end{bmatrix}$$

Also $$BC = \begin{bmatrix} 2 & 0 \\ 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4+0 \\ -2-2 \\ 0+3 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 3 \end{bmatrix}$$

$$\therefore \quad RHS = A(BC) = \begin{bmatrix} 1 & -2 & -1 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} -4 \\ -4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+8-3 \\ -4+0-6 \end{bmatrix} = \begin{bmatrix} 1 \\ -10 \end{bmatrix}$$

Thus, LHS = RHS

Hence, the associative property of matrix multiplication is verified.

61. If $f(x) = 2x^3 + 5x^2 - x$, find $f(A)$ where $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$.

Ans. We have $f(x) = 2x^3 + 5x^2 - x$

$$\Rightarrow \quad f(A) = 2A^3 + 5A^2 - A$$

Now, $$A^2 = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+2 & -1+0-2 & 2-1+0 \\ 2+0+1 & -2+0-1 & 4+0+0 \\ 1-2+0 & -1+0+0 & 2-1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 1 \\ 3 & -3 & 4 \\ -1 & -1 & 1 \end{bmatrix}$$

and $\quad A^3 = A^2.A$

$$= \begin{bmatrix} 1 & -3 & 1 \\ 3 & -3 & 4 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+1 & -1+0-1 & 2-3+0 \\ 3-6+4 & -3+0-4 & 6-3+0 \\ -1-2+1 & 1+0-1 & -2-1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 & -1 \\ 1 & -7 & 3 \\ -2 & 0 & -3 \end{bmatrix}$$

$$\therefore \quad f(A) = 2\begin{bmatrix} -4 & -2 & -1 \\ 1 & -7 & 3 \\ -2 & 0 & -3 \end{bmatrix} + 5\begin{bmatrix} 1 & -3 & 1 \\ 3 & -3 & 4 \\ -1 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -4 & -2 \\ 2 & -14 & 6 \\ -4 & 0 & -6 \end{bmatrix} + \begin{bmatrix} 5 & -15 & 5 \\ 15 & -15 & 20 \\ -5 & -5 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -18 & 1 \\ 15 & -29 & 25 \\ -10 & -4 & -1 \end{bmatrix}$$

62. **If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = 0$.**

Ans. We have $A^2 = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+6+2 & 3+0+4 & 2-3+6 \\ 2+0-1 & 6+0-2 & 4+0-3 \\ 1+4+3 & 3+0+6 & 2-2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

and $A^3 = A^2.A$

$$= \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+14+5 & 27+0+10 & 18-7+15 \\ 1+8+1 & 3+0+2 & 2-4+3 \\ 8+18+9 & 24+0+18 & 16-9+27 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$$

Now,

$LHS = A^3 - 4A^2 - 3A + 11I$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - \begin{bmatrix} 36 & 28 & 20 \\ 4 & 16 & 4 \\ 32 & 36 & 36 \end{bmatrix} - \begin{bmatrix} 3 & 9 & 6 \\ 6 & 0 & -3 \\ 3 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 28-36-3+11 & 37-28-9 & 26-20-6 \\ 10-4-6 & 5-16+11 & 1-4+3 \\ 35-32-3 & 42-36-6 & 34-36-9+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0 = RHS$

Hence, proved.

63. **If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$, find the value of k.** **[CBSE 2016]**

TRANSPOSE OF A MATRIX, SYMMETRIC AND SKEW-SYMMETRIC MATRICS 3

TOPIC 1

TRANSPOSE OF A MATRIX

If $A = [a_{ij}]$ be a $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A.

Transpose of the matrix A is denoted by A′ (or A^T).

Illustration: If $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}_{2\times3}$, then $A' = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}_{3\times2}$

Example 3.1: Find the transpose of matrix: $\begin{bmatrix} 4 & -2 & 1 \\ 6 & 7 & -5 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 4 & -2 & 1 \\ 6 & 7 & -5 \end{bmatrix}$ then, $A' = \begin{bmatrix} 4 & 6 \\ -2 & 7 \\ 1 & -5 \end{bmatrix}$

Properties of Transpose of Matrices

For any matrices A and B of suitable order, we have

(1) **Property 1 [P.1]:** (A′)′ = A

(2) **Property 2 [P.2]:** (kA)′ = kA′ {where k is any constant}

(3) **Property 3 [P.3]:** (A ± B)′ = A′ + B′

(4) **Property 4 [P.4]:** (AB)′ = B′A′

Proof: Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & -2 \\ -1 & 1 & 3 \end{bmatrix}$.

(1) Then, $A' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$

$$(A')' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}' = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \end{bmatrix} = A.$$

Thus, property 1 is verified.

(2) Further, $kA = k\begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} k & 3k & -k \\ 2k & 0 & 4k \end{bmatrix}$

and $A' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$

$$\Rightarrow \quad (kA)' = \begin{bmatrix} k & 2k \\ 3k & 0 \\ -k & 4k \end{bmatrix} = k\begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} = kA'$$

Thus, property 2 is verified.

(3)
$$A + B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 0 & -2 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 & -3 \\ 1 & 1 & 7 \end{bmatrix};$$

$$\Rightarrow \quad (A + B)' = \begin{bmatrix} 4 & 1 \\ 3 & 1 \\ -3 & 7 \end{bmatrix} \quad ...(i)$$

Also, $A' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$ and $B' = \begin{bmatrix} 3 & -1 \\ 0 & 1 \\ -2 & 3 \end{bmatrix}$

$$\Rightarrow \quad A' + B' = \begin{bmatrix} 4 & 1 \\ 3 & 1 \\ -3 & 7 \end{bmatrix} \quad ...(ii)$$

From (i) and (ii), we have (A + B)′ = A′ + B′

Similarly, $(A - B)' = \begin{bmatrix} -2 & 3 \\ 3 & -1 \\ 1 & 1 \end{bmatrix}$

and $A' - B' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 0 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -1 \\ 1 & 1 \end{bmatrix}$

$$\Rightarrow \quad (A - B)' = A' - B'.$$

Thus, property 3 is verified.

(4) For the given matrices A and B, AB and BA both are not defined. So, we take another set of matrices for which AB and BA both are defined.

Let $A = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ and $B = [3 \ \ 0 \ -1]$. Then, AB and BA both are defined.

Now, $AB = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}[3 \ \ 0 \ -1]$

$$= \begin{bmatrix} 2\times3 & 2\times0 & 2\times(-1) \\ (-2)\times3 & (-2)\times0 & (-2)\times(-1) \\ 1\times3 & 1\times0 & 1\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & -2 \\ -6 & 0 & 2 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad (AB)' = \begin{bmatrix} 6 & -6 & 3 \\ 0 & 0 & 0 \\ -2 & 2 & -1 \end{bmatrix} \quad ...(i)$$

Now, $B' = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}; A' = [2 \ -2 \ 1]$

So, $B'A' = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}[2 \ -2 \ 1]$

$$= \begin{bmatrix} 3\times 2 & 3\times(-2) & 3\times 1 \\ 0\times 2 & 0\times(-2) & 0\times 1 \\ (-1)\times 2 & (-1)\times(-2) & (-1)\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -6 & 3 \\ 0 & 0 & 0 \\ -2 & 2 & -1 \end{bmatrix} \quad ...(ii)$$

From (i) and (ii), we have (AB)′ = B′A′.

Thus, property 4 is verified.

Caution

- *Matrices multiplication is not commutative i.e., AB is not equal to BA. Similarly, (AB)′ ≠ A′ B′.*

TOPIC 2

SYMMETRIC AND SKEW-SYMMETRIC MATRICES

A square matrix $A = [a_{ij}]$ is said to be symmetric, if $A' = A$.

A square matrix $A = [a_{ij}]$ is said to be skew-symmetric, if $A' = -A$.

Illustration: The matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ is symmetric, as $A' = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 1 & 3 & 5 \end{bmatrix} = A$.

The matrix $B = \begin{bmatrix} 0 & 5 & 1 \\ -5 & 0 & 3 \\ -1 & -3 & 0 \end{bmatrix}$ is skew-symmetric, as

$$B' = \begin{bmatrix} 0 & -5 & -1 \\ 5 & 0 & -3 \\ 1 & 3 & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & 5 & 1 \\ -5 & 0 & 3 \\ -1 & -3 & 0 \end{bmatrix} = -B$$

Important

- *All the diagonal elements of a skew-symmetric matrix are zero.*

Example 3.2: **Show that the matrix**

(A) $P = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ **is a symmetric matrix.**

(B) $Q = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ **is a skew-symmetric matrix.** [NCERT]

Ans. (A) For the given matrix P, we have

$$P' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = P$$

Thus, P is a symmetric matrix.

(B) For the given matrix Q, we have

$$Q' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}'$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -Q$$

Thus, Q is skew-symmetric matrix.

Important

- *For any square matrix A with real number entries, A + A′ is always a symmetric matrix and A – A′ is a skew-symmetric matrix.*
- *Any square matrix A can be expressed as the sum of a symmetric and skew-symmetric matrices, i.e.,* $A = \frac{1}{2}[(A + A') + (A - A')]$
- *A matrix which is both symmetric and skew-symmetric is a zero matrix.*

Example 3.3: Express $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrices.

[NCERT]

Ans. For any given matrix A, we have

$$A = \frac{1}{2}[(A + A') + (A - A')] \text{ or } \frac{A + A'}{2} + \frac{A - A'}{2}$$

Here, $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

So,

$$A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$\Rightarrow \frac{A + A'}{2} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(As symmetric matrix)

and $A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$\Rightarrow \frac{A - A'}{2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, $\frac{A + A'}{2} + \frac{A - A'}{2}$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$$

Thus, $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is expressible as the sum of a symmetric and skew-symmetric matrices.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, P – Q is

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

[CBSE Term-1 2021]

Ans. *(b)* $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

Explanation: Here $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$$\Rightarrow Q = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

So, $P - Q = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ -1 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

2. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a/an:

(a) identity matrix

(b) symmetric matrix

(c) skew-symmetric matrix

(d) none of these

Ans. *(d) none of these*

Explanation: Given matrix is a diagonal matrix.

3. **If A and B are symmetric matrices of same order, then AB – BA is a :**

(a) Skew-symmetric matrix

(b) Symmetric matrix

(c) Zero matrix

(d) Identity matrix

Ans. *(a) Skew-symmetric matrix*

Explanation: As A and B are symmetric matrices, A′ = A and B′ = B

Now, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\Rightarrow$ AB – BA is a skew-symmetric matrix.

4. **If** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **and A + A′ = I, then the value of α is:**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) π (d) $\frac{3\pi}{2}$

5. **If the order of matrix A is 4 × 3, order of matrix B is 4 × 5 and the order of matrix C is 7 × 3, then the order of the matrix (A′ × B)′ × C′ is:**

(a) 4 × 5 (b) 3 × 7

(c) 4 × 3 (d) 5 × 7

Ans. *(d) 5 × 7*

Explanation: Order of matrix A′ is 3 × 4, order of matrix B′ is 5 × 4 and the order of matrix C′ is 3 × 7

Now, $(A' \times B)' = (B)' \times (A')' = B'A$

The order of B′A is 5 × 3, so, the order of (A′ × B)′ is 5 × 3.

Hence the order of (A′ × B)′ × C′ is 5 × 7.

6. **If the matrix** $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = P + Q$**, where P is a symmetric matrix and Q is a skew symmetric matrix, then Q is:**

(a) $\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

7. **If** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$**, then AA′ is:**

(a) A^2 (b) Null matrix

(c) A (d) Identity matrix

Ans. *(d) Identity matrix*

Explanation: Here,

$$AA' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

8. **If** $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ **and A = A′, then:**

(a) $x = 0, y = 5$ (b) $x = y$

(c) $x + y = 5$ (d) $xy = 8$

9. **If** $A = \begin{bmatrix} 5 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix}$, $B' = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 1 & 3 \end{bmatrix}$ **then BA′ is:**

(a) $\begin{bmatrix} 8 & 7 \\ 5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 7 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 7 \\ 5 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 5 \\ 7 & 8 \end{bmatrix}$ [DIKSHA]

Ans. *(a)* $\begin{bmatrix} 8 & 7 \\ 5 & 3 \end{bmatrix}$

Explanation: Here,

$$A = \begin{bmatrix} 5 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 5 & 6 \\ 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$$\text{And } B' = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 1 & 3 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

So,
$$BA' = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 0 & 3 \end{bmatrix}_{2\times3} \begin{bmatrix} 5 & 6 \\ 1 & 2 \\ 0 & -1 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 10-2+0 & 12-4-1 \\ 5+0+0 & 6+0-3 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} 8 & 7 \\ 5 & 3 \end{bmatrix}$$

10. **If A is a matrix of order** $m \times n$ **and B is a matrix such that AB′ and B′A are both defined, then order of the matrix B is:**

(a) $m \times m$ (b) $n \times n$
(c) $n \times m$ (d) $m \times n$

[NCERT Exemplar]

Ans. *(d)* $m \times n$

Explanation: Let order of matrix B be $p \times q$.

Since AB′ is defined,

$\therefore n = q$.

Further, B′A is defined gives $p = m$

Hence, order of matrix B is $m \times n$.

11. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, then (a, b) is:

(a) (1, –3) (b) (–1, 1)
(c) (–2, 3) (d) (0, 0)

[NCERT Exemplar]

Ans. *(c)* *(–2, 3)*

Explanation: Here,

$$\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}'$$

$$i.e., \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -b \\ -a & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$$

$\Rightarrow a = -2, b = 3$

12. If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then:

(a) skew-symmetric matrix
(b) symmetric matrix
(c) zero matrix
(d) identity matrix

Ans. *(a)* *skew-symmetric matrix*

Explanation: Here, $A = \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$, which is a skew-symmetric matrix.

13. If A is a symmetric matrix then which of the following is not a symmetric matrix?

(a) $A + A^T$ (b) $A.A^T$
(c) $A - A^T$ (d) A^T

[Delhi Gov. 2022]

Ans. *(c)* $A - A^T$

Explanation: As, $(A - A^T)^T = -(A - A^T)$

So, $(A - A^T)$ is not symmetric matrix.

14. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then $(2A + B)'$ is equal to:

(a) 2A′ + B′ (b) 4A′ + B′
(c) 4A′ + B′ (d) 2A + B

Ans. *(a)* *2A′ + B′*

Explanation: Here, $(2A + B)' = (2A)' + (B)'$
$= 2A' + B'$

15. If A and B are matrices of same order, then (AB′ – BA′) is a:

(a) skew-symmetric matrix
(b) null matrix
(c) symmetric matrix
(d) unit matrix

[NCERT Exemplar]

16. If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(a) Diagonal matrix
(b) Zero square matrix
(c) Square matrix
(d) Identity matrix

[CBSE Term-1 2021]

Ans. *(b)* *Zero square matrix*

Explanation: Let A be a square matrix such that A is both symmetric and skew-symmetric matrix.

$\Rightarrow A' = A$ and $A' = -A$

$\Rightarrow A = -A$

$\Rightarrow 2A = O$

$\Rightarrow A = O$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

17. Find the transpose of the matrix $A = \begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ -1 & 2 & 0 \end{bmatrix}$.

Ans. We have, $A = \begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ -1 & 2 & 0 \end{bmatrix}^1$

For finding the transpose of matrix, we have to interchange the rows and columns elements.

$$\therefore A' = \begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ -1 & 2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} -3 & 4 & -1 \\ 2 & 5 & 2 \\ 1 & 6 & 0 \end{bmatrix}$$

18. If $P' = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ then determine PQ.

Ans. $P' = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$

$\Rightarrow \quad P = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}$

$\therefore \quad PQ = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & -3+3 \\ 4+0 & 6+1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 4 & 7 \end{bmatrix}$

19. **If P is a skew-symmetric matrix, then show that P^2 is a symmetric matrix.**

20. If $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ is written as A = P + Q, where P is a symmetric matrix and Q is skew-symmetric matrix, then write matrix P. **[CBSE 2016]**

Ans. We have $A = P + Q$

$\therefore \quad P = \frac{1}{2}(A + A')$

$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}\right)$

$= \frac{1}{2}\left(\begin{bmatrix} 6 & 12 \\ 12 & 18 \end{bmatrix}\right)$

$= \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$

21. **If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}'$, then find the value of x.** **[CBSE 2010]**

22. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, then find the values of a and b. **[CBSE 2016]**

Ans. We have matrix A is symmetric.

$\therefore \quad A = A'$

$\Rightarrow \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix}$

Comparing the corresponding elements, we get,

$2b = 3$ and $-2 = 3a$

$\Rightarrow b = \frac{3}{2}$ and $a = -\frac{2}{3}$

23. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -4 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix? **[DIKSHA]**

Ans. Given: $A = \begin{bmatrix} 0 & 1 & -4 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$

$\Rightarrow \quad A' = \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix}$

Since, A is a skew-symmetric matrix

$\therefore \quad A' = -A$

$\Rightarrow \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -4 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$

$\Rightarrow \quad x = 4$

24. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA' = 9I$, find x. **[CBSE 2018]**

Ans. We have $AA' = 9I$

$\therefore \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2+2x & -2+4-2 \\ 2+2+2x & 4+1+x^2 & -4+2-x \\ -2+4-2 & -4+2-x & 4+4+1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9 & 4+2x & 0 \\ 4+2x & 5+x^2 & -2-x \\ 0 & -2-x & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

Comparing the corresponding elements, we get

$4 + 2x = 0$ and $5 + x^2 = 9$

$\Rightarrow \quad x = -2$ and $x^2 = 4$

$\Rightarrow \quad x = -2$ and $x = \pm 2$

$\therefore x = -2$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

25. If $P' = \begin{bmatrix} -3 & 5 \\ 3 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$, find $(P + 3Q)'$.

Ans. We have $P' = \begin{bmatrix} -3 & 5 \\ 3 & 4 \end{bmatrix}$

$\therefore$ $P = (P')'$

$$= \begin{bmatrix} -3 & 5 \\ 3 & 4 \end{bmatrix}' = \begin{bmatrix} -3 & 3 \\ 5 & 4 \end{bmatrix}$$

Now, $P + 3Q = \begin{bmatrix} -3 & 3 \\ 5 & 4 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 3 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 3 \\ 11 & 13 \end{bmatrix}$$

$\therefore$ $(P + 3Q)' = \begin{bmatrix} -6 & 3 \\ 11 & 13 \end{bmatrix}' = \begin{bmatrix} -6 & 11 \\ 3 & 13 \end{bmatrix}$

26. Show that all the diagonal elements of a skew-symmetric matrix are zero. **[CBSE 2017]**

Ans. Let $P = [p_{ij}]$ be a skew-symmetric matrix. Then by definition

$$p_{ij} = -p_{ji} \; \forall \; i, j$$

Put $i = j$, therefore

$$p_{ii} = -p_{ii}$$

$\Rightarrow$ $2p_{ii} = 0$

$\Rightarrow$ $p_{ii} = 0 \; \forall \; i$

$\therefore$ $p_{11} = p_{22} = p_{33} = \ldots\ldots = 0$

Hence, all diagonal elements of a skew symmetric matrix are zero.

27. Show that A'A and AA' are both symmetric matrices for any matrix A. **[NCERT Exemplar]**

28. For what value of a, the matrix

$$A = \begin{bmatrix} 3a+5 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & -3a-5 \end{bmatrix}$$ **is a skew symmetric matrix?**

Ans. We have, matrix A is skew-symmetric matrix.

$\therefore$ $A = -A^T$

$$\Rightarrow \begin{bmatrix} 3a+5 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & -3a-5 \end{bmatrix} = -\begin{bmatrix} 3a+5 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & -3a-5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3a+5 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & -3a-5 \end{bmatrix} = -\begin{bmatrix} 3a+5 & -3 & -2 \\ 3 & 0 & -5 \\ 2 & 5 & 3a+5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3a+5 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & -3a-5 \end{bmatrix} = \begin{bmatrix} -3a-5 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & -3a-5 \end{bmatrix}$$

Comparing the corresponding elements, we get

$$3a + 5 = -3a - 5$$

$$\Rightarrow \quad 6a = -10 \Rightarrow a = -\frac{5}{3}$$

29. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b, c. **[NCERT Exemplar]**

30. If A is symmetric, then show that B'AB is symmetric matrix. **[NCERT]**

Ans. We have, A is a symmetric matrix

$\therefore$ $A' = A$ ...(i)

Now, $(B'AB)' = (AB)'(B')'$

$= B'A'B$

$= B'AB$ [From eq. (i)]

Hence, B'AB is a symmetric matrix.

31. If A, B are square matrices of same order and B is a skew-symmetric matrix, then show that A' BA is skew-symmetric. **[NCERT Exemplar]**

32. A and B are symmetric matrices. Prove that AB + BA is a symmetric matrix.

Ans. We have, A and B are symmetric matrices.

$\therefore$ A' = A and B' = B

Now, (AB + BA)' = (AB)' + (BA)'

= B'A' + A'B'

= BA + AB

= AB + BA

Hence, AB + BA is a symmetric matrix.

Hence, proved.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

33. **For the following matrices A and B, verify that (AB)' = B'A'.** [CBSE 2010]

$$A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = [-1 \;\; 2 \;\; 1]$$

Ans.

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}[-1 \;\; 2 \;\; 1]$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore \quad \text{L.H.S.} = (AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

and

$$\text{R.H.S.} = B'A' = [-1 \;\; 2 \;\; 1]' \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}[1 \;\; -4 \;\; 3]$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S.,

Hence, verified.

34. **If** $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$**, then verify that:**

(A) (A')' = A

(B) (AB)' = B'A'

(C) (kA)' = kA' [NCERT Exemplar]

35. **If** $A = \begin{bmatrix} 1 & 2 & 3 \\ 8 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} -3 & 2 & -5 \\ 4 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

then verify that (A + B)' = A' + B'.

Ans. We have,

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 8 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 2 & -5 \\ 4 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & -2 \\ 12 & -2 & 0 \\ 5 & -1 & 5 \end{bmatrix}$$

L.H.S. = (A + B)'

$$= \begin{bmatrix} -2 & 4 & -2 \\ 12 & -2 & 0 \\ 5 & -1 & 5 \end{bmatrix}'$$

$$= \begin{bmatrix} -2 & 12 & 5 \\ 4 & -2 & -1 \\ -2 & 0 & 5 \end{bmatrix}$$

R.H.S. = A' + B'

$$= \begin{bmatrix} 1 & 2 & 3 \\ 8 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix}' + \begin{bmatrix} -3 & 2 & -5 \\ 4 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & 8 & 2 \\ 2 & -1 & -3 \\ 3 & 0 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 4 & 3 \\ 2 & -1 & 2 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 12 & 5 \\ 4 & -2 & -1 \\ -2 & 0 & 5 \end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S.,

Hence verified.

36. **If** $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$**, then verify that:**

(A) (2A + B)' = 2A' + B'

(B) (A – B)' = A' – B' [NCERT Exemplar]

Ans. (A)
$$2A + B = 2\begin{bmatrix}1 & 2\\4 & 1\\5 & 6\end{bmatrix} + \begin{bmatrix}1 & 2\\6 & 4\\7 & 3\end{bmatrix}$$
$$= \begin{bmatrix}2 & 4\\8 & 2\\10 & 12\end{bmatrix} + \begin{bmatrix}1 & 2\\6 & 4\\7 & 3\end{bmatrix}$$
$$= \begin{bmatrix}3 & 6\\14 & 6\\17 & 15\end{bmatrix}$$
$$\therefore \quad (2A + B)' = \begin{bmatrix}3 & 14 & 17\\6 & 6 & 15\end{bmatrix} \quad ...(i)$$
Also,
$$2A' + B' = 2\begin{bmatrix}1 & 4 & 5\\2 & 1 & 6\end{bmatrix} + \begin{bmatrix}1 & 6 & 7\\2 & 4 & 3\end{bmatrix}$$
$$= \begin{bmatrix}2 & 8 & 10\\4 & 2 & 12\end{bmatrix} + \begin{bmatrix}1 & 6 & 7\\2 & 4 & 3\end{bmatrix}$$
$$= \begin{bmatrix}2+1 & 8+6 & 10+7\\9+2 & 2+4 & 12+3\end{bmatrix}$$
$$= \begin{bmatrix}3 & 14 & 17\\6 & 6 & 15\end{bmatrix} \quad ...(ii)$$
From (i) and (ii), we get
$$(2A + B)' = 2A' + B'$$
Hence, verified.

(B)
$$A - B = \begin{bmatrix}1 & 2\\4 & 1\\5 & 6\end{bmatrix} - \begin{bmatrix}1 & 2\\6 & 4\\7 & 3\end{bmatrix}$$
$$= \begin{bmatrix}0 & 0\\-2 & -3\\-2 & 3\end{bmatrix}$$
and
$$(A - B)' = \begin{bmatrix}0 & -2 & -2\\0 & -3 & 3\end{bmatrix} \quad ...(i)$$
Also,
$$A' - B' = \begin{bmatrix}1 & 4 & 5\\2 & 1 & 6\end{bmatrix} - \begin{bmatrix}1 & 6 & 7\\2 & 4 & 3\end{bmatrix}$$
$$= \begin{bmatrix}0 & -2 & -2\\0 & -3 & 3\end{bmatrix} \quad ...(ii)$$
From (i) and (ii), we get
$$(A - B)' = A' - B'$$
Hence, verified.

37. **For the matrix $P = \begin{bmatrix}1 & 3\\4 & 5\end{bmatrix}$, verify that**

(A) (P + P') is a symmetric matrix

(B) (P – P') is a skew-symmetric matrix

38. If $B = [b_{ij}]$ is a square matrix such that $b_{ij} = i^3 - j^3$, then check whether B is symmetric or skew-symmetric matrix.

Ans. Consider a square matrix of order 2 *i.e.,*
$$B = [b_{ij}]_{2\times2}$$
We have
$$b_{ij} = i^3 - j^3$$
$$\therefore \quad b_{11} = 1^3 - 1^3 = 0$$
$$b_{12} = 1^3 - 2^3 = -7$$
$$b_{21} = 2^3 - 1 = 7$$
$$b_{22} = 2^3 - 2^3 = 0$$
$$\therefore \quad B = \begin{bmatrix}0 & a & -3\\2 & 0 & -1\\b & 1 & 0\end{bmatrix}$$
Now,
$$B' = \begin{bmatrix}0 & -7\\7 & 0\end{bmatrix}^1$$
$$= \begin{bmatrix}0 & 7\\-7 & 0\end{bmatrix}$$
$$= -\begin{bmatrix}0 & -7\\7 & 0\end{bmatrix}$$
$$= -B$$
Hence, B is skew-symmetric matrix.

39. Prove by mathematical induction that $(A')^n = (A^n)'$, where $n \in N$, for any square matrix A. **[NCERT Exemplar]**

Ans. Let $P(n) : (A')^n = (A^n)'$

So, $P(1) : (A')^1 = A'$

$\Rightarrow \quad A' = A'$

$\therefore$ P(1) is true.

Now, let P(k) be true.

$\therefore \quad P(k) : (A')^k = (A^k)'$, where $k \in N$ ...(i)

Now, we have to prove that

$P(k + 1) : (A')^{k+1} = (A^{k+1})'$ is true

$$\therefore \quad \text{L.H.S.} = (A')^{k+1} = (A')^k . (A')^1$$
$$= (A^k)' . A' \quad \text{[Using (i)]}$$
$$= (A.A^k)' \quad [(AB)' = B'A']$$
$$= (A^{1+k})' = (A^{k+1})'$$
$$= \text{R.H.S.}$$

Hence, P(k + 1) is true, whenever P(k) is true.

Hence, proved.

Caution

➥ *For proving this question, one should know the property of mathematical induction.*

40. **Express the matrix** $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ **as the sum of a symmetric and skew-symmetric matrix.** **[CBSE 2015]**

Ans. We have $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$

For expressing the matrix A as the sum of symmetric and skew-symmetric matrices, we can write

A = P + Q, where

$P = \frac{1}{2}(A + A')$, which is symmetric matrix

and $Q = \frac{1}{2}(A - A')$, which is skew symmetric matrix

Now $P = \frac{1}{2}\left(\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{bmatrix}$

$= \begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix}$

and $Q = \frac{1}{2}(A - A')$

$= \frac{1}{2}\left(\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$

Hence, matrix A is expressed as the sum of symmetric and skew-symmetric matrices

i.e., $A = \begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$

41. **Express the matrix**

$B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

as the sum of a symmetric and skew symmetric matrix. **[DIKSHA]**

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

42. **Suppose A, B and C are three matrices defined as**

$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 8 \\ -1 & 2 & 4 \\ -5 & 0 & 2 \end{bmatrix}$

and $C = \begin{bmatrix} -1 & 3 & 4 \\ 2 & 5 & 6 \\ 3 & 2 & 1 \end{bmatrix}$

Verify the following results.

(A) $(A + B + C)' = A' + B' + C'$

(B) $(AB)' = B'A'$

Ans. (A) A + B + C

$= \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 8 \\ -1 & 2 & 4 \\ -5 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 3 & 4 \\ 2 & 5 & 6 \\ 3 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+2-1 & -1+3+3 & 2+8+4 \\ 2-1+2 & 0+2+5 & 3+4+6 \\ 4-5+3 & -1+0+2 & 2+2+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 & 14 \\ 3 & 7 & 13 \\ 2 & 1 & 5 \end{bmatrix}$

L.H.S. $= (A + B + C)' = \begin{bmatrix} 2 & 5 & 14 \\ 3 & 7 & 13 \\ 2 & 1 & 5 \end{bmatrix}'$

$= \begin{bmatrix} 2 & 3 & 2 \\ 5 & 7 & 1 \\ 14 & 13 & 5 \end{bmatrix}$

R.H.S. = A' + B' + C'

$$= \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & -1 & 2 \end{bmatrix}' + \begin{bmatrix} 2 & 3 & 8 \\ -1 & 2 & 4 \\ -5 & 0 & 2 \end{bmatrix}' + \begin{bmatrix} -1 & 3 & 4 \\ 2 & 5 & 6 \\ 3 & 2 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & -1 \\ 2 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -5 \\ 3 & 2 & 0 \\ 8 & 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 3 & 5 & 2 \\ 4 & 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2-1 & 2-1+2 & 4-5+3 \\ -1+3+3 & 0+2+5 & -1+0+2 \\ 2+8+4 & 3+4+6 & 2+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 2 \\ 5 & 7 & 1 \\ 14 & 13 & 5 \end{bmatrix}$$

∴ LHS = RHS,

Hence verified.

(B) $$AB = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 8 \\ -1 & 2 & 4 \\ -5 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1-10 & 3-2+0 & 8-4+4 \\ 4+0-15 & 6+0+0 & 16+0+6 \\ 8+1-15 & 12-2+0 & 32-4+6 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 1 & 8 \\ -11 & 6 & 22 \\ -6 & 10 & 34 \end{bmatrix}$$

L.H.S. = (AB)'

$$= \begin{bmatrix} -7 & 1 & 8 \\ -11 & 6 & 22 \\ -6 & 10 & 34 \end{bmatrix}'$$

$$= \begin{bmatrix} -7 & -11 & -6 \\ 1 & 6 & 10 \\ 8 & 22 & 34 \end{bmatrix}$$

R.H.S. = B'A'

$$= \begin{bmatrix} 2 & 3 & 8 \\ -1 & 2 & 4 \\ -5 & 0 & 2 \end{bmatrix}'\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & -1 & 3 \end{bmatrix}'$$

$$= \begin{bmatrix} 2 & -1 & -5 \\ 3 & 2 & 0 \\ 8 & 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & -1 \\ 2 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1-10 & 4+0-15 & 8+1-15 \\ 3-2+0 & 6+0+0 & 12-2+0 \\ 8-4+4 & 16+0+6 & 32-4+6 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -11 & -6 \\ 1 & 6 & 10 \\ 8 & 22 & 34 \end{bmatrix}$$

∴ L.H.S. = R.H.S.,

Hence verified.

43. **Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.**

[NCERT Exemplar]

44. **Construct a square matrix A of order 3 × 3 by using the elements $a_{ij} = 2(i + j)$.**

Also express matrix A as sum of symmetric and skew-symmetric matrices.

45. **Express the following matrix as the sum of a symmetric and a skew-symmetric matrices and verify your result.**

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix}$$

Ans. We have, $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix}$

For expressing the matrix A as the sum of symmetric and skew-symmetric matrices, we can write as

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where $\frac{1}{2}(A + A')$ is symmetric and $\frac{1}{2}(A - A')$ is skew-symmetric matrices

Symmetric matrix = $\frac{1}{2}(A + A')$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 \\ 2 & 2 & -1 \\ -3 & 1 & 3 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 1+1 & 2+3 & -3+4 \\ 3+2 & 2+2 & 1-1 \\ 4-3 & -1+1 & 3+3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 5 & 1 \\ 5 & 4 & 0 \\ 1 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{5}{2} & \frac{1}{2} \\ \frac{5}{2} & 2 & 0 \\ \frac{1}{2} & 0 & 3 \end{bmatrix}$$

Skew-symmetric matrix $= \frac{1}{2}(A - A')$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 4 \\ 2 & 2 & -1 \\ -3 & 1 & 3 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 1-1 & 2-3 & -3-4 \\ 3-2 & 2-2 & 1+1 \\ 4+3 & -1-1 & 3-3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -7 \\ 1 & 0 & 2 \\ 7 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{7}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{7}{2} & -1 & 0 \end{bmatrix}$$

Thus, $A = \begin{bmatrix} 1 & \frac{5}{2} & \frac{1}{2} \\ \frac{5}{2} & 2 & 0 \\ \frac{1}{2} & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{7}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{7}{2} & -1 & 0 \end{bmatrix}$

Now, we have to verify that

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\therefore \text{R.H.S.} = \begin{bmatrix} 1 & \frac{5}{2} & \frac{1}{2} \\ \frac{5}{2} & 2 & 0 \\ \frac{1}{2} & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{7}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{7}{2} & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{5}{2}-\frac{1}{2} & \frac{1}{2}-\frac{7}{2} \\ \frac{5}{2}+\frac{1}{2} & 2 & 1 \\ \frac{1}{2}+\frac{7}{2} & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & 1 \\ 4 & -1 & 3 \end{bmatrix} = \text{L.H.S.}$$

Hence, the result is verified.

INVERTIBLE MATRICES 4

TOPIC 1

INVERSE OF A MATRIX

Let A be a square matrix of order n. If there exists a square matrix B of same order n such that AB = BA = I_n, then we say that A is an invertible matrix. The matrix B is called inverse matrix of A and is denoted by A^{-1}.

Important

- *If B is the inverse matrix of A, then A is also the inverse matrix of B.*
- *Only square matrix can possess inverse, and not a rectangular matrix.*

Noteworthy Results

(1) For any invertible square matrix A,

- $(A')^{-1} = (A^{-1})'$
- $(A^{-1})^{-1} = A$
- $AA^{-1} = I = A^{-1}A$

(2) For any invertible square matrix A and non-zero scalar k,

We have $(kA)^{-1} = \frac{1}{k}A^{-1}$.

(3) For two invertible square matrices A and B (for which AB is defined), we have

$$(AB)^{-1} = B^{-1}A^{-1}$$

Uniqueness of Inverse of a Matrix

Theorem: Inverse of a square matrix, if it exists, is unique.

Proof: Let A be a square matrix of order n. If possible, let B and C be the two inverse matrices of A. We will prove that matrix B = matrix C.

$\because$ B is the inverse matrix of A

$\therefore \quad AB = BA = I \quad$ ...(i)

Also, C is the inverse matrix of A.

$\therefore \quad AC = CA = I \quad$ ...(ii)

We know, $B = BI$

$= B(AC)$ [From (ii)]

$= (BA)C$

$= (I)C$ [From (i)]

$= C$

$\Rightarrow \quad B = C$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Matrices A and B will be inverse of each other only if

(a) AB = BA (b) AB = BA = O

(c) AB = BA = I (d) AB = O, BA = I

Ans. *(c) AB = BA = I*

Explanation: Matrices A and B will be inverse of each other only if AB = I = BA.

2. Suppose P and Q are square matrices of the same order and PQ = 8I, then P^{-1} is:

(a) $\frac{1}{6}Q$ (b) $\frac{1}{8}Q$

(c) 8Q (d) $\frac{1}{8}I$

Ans. *(b) $\frac{1}{8}Q$*

Explanation: We have, PQ = 8I

$\Rightarrow \quad \frac{1}{8}(PQ) = I$

or, $\quad P\left(\frac{1}{8}Q\right) = I$

$\Rightarrow \quad P^{-1}P\left(\frac{1}{8}Q\right) = P^{-1}I$

$\Rightarrow \quad I\left(\frac{1}{8}Q\right) = P^{-1}$

or, $\quad P^{-1} = \frac{1}{8}Q$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

3. *A manufacturer produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:*

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2000	18,000
B	6000	20,000	8,000

Unit Sale price of Pencil, Eraser and Sharpener are ₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively, and unit cost of the above three commodities are ₹ 2.00, ₹ 1.00 and ₹ 0.50 respectively.

(A) Total revenue of market A is:
(a) ₹ 64,000 (b) ₹ 60,400
(c) ₹ 46,000 (d) ₹ 40,600

(B) Total revenue of market B is:
(a) ₹ 35,000 (b) ₹ 53,000
(c) ₹ 50,300 (d) ₹ 30,500

(C) Cost incurred in market A is:
(a) ₹ 13,000 (b) ₹ 30,100
(c) ₹ 10,300 (d) ₹ 31,000

(D) Profit in markets A and B respectively are:
(a) ₹ 15,000, ₹ 17,000
(b) ₹ 17,000, ₹ 15,000
(c) ₹ 51,000, ₹ 71,000
(d) ₹ 10,000, ₹ 20,000

(E) Gross profit in both market is:
(a) ₹ 23,000 (b) ₹ 20,300
(c) ₹ 32,000 (d) ₹ 30,200

[CBSE Question Bank 2021]

Ans. (A) *(c) ₹ 46,000*

Explanation: Total Revenue for market A
= 10,000 × ₹ 2.50 + 2,000 × ₹ 1.50 + 18,000 × ₹ 1.00
= ₹ 25,000 + ₹ 3,000 + ₹ 18,000
= ₹ 46,000

(C) *(d) ₹ 31,000*

Explanation: Cost incurred in market A
= 10,000 × ₹ 2.00 + 2,000 × ₹ 1.00 + 18,000 × ₹ 0.50
= ₹ 20,000 + ₹ 2,000 + ₹ 9,000
= ₹ 31,000

4. *A trust care for handicapped children gets ₹ 30,000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them, and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly.*

(A) The expenditure matrix is given by:

(a) $[15000 \quad 15000]$

(b) $\begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 15000 \\ 15000 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 15000 & 0 \\ 0 & 15000 \end{bmatrix}$

(B) Let the rate of interest received from the bank be x%. Then, the interest matrix is given by

(a) $[2\% \quad x\%]$ **(b)** $\begin{bmatrix} 2\% \\ x\% \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2\% \\ x\% & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} x\% & 0 \\ 0 & 2\% \end{bmatrix}$

(C) Matrix of the total earning is:

(a) $[300 \quad 150x]$ **(b)** $\begin{bmatrix} 300 \\ 150x \end{bmatrix}$

(c) $[300 + 150x]$ **(d)** $[300x + 150]$

(D) If the trust gets monthly earning of ₹ 1800, the value of x is:
(a) 6 (b) 8
(c) 10 (d) 12

(E) If the trust spends ₹ 12,000 instead of ₹ 15,000 for medical and educational care, the monthly earning will be:
(a) ₹ 2240 (b) ₹ 2040
(c) ₹ 2400 (d) ₹ 2420

Ans. (A) *(a) [15000 15000]*

(C) *(c) [300 + 150x]*

Explanation: $[15000 \quad 15000] \begin{bmatrix} 2\% \\ x\% \end{bmatrix}$

$= [300 + 150x]$

5. *Amit, Biraj and Chirag were given a task of creating a square matrix of order 2.*

Below are the matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

If $a = 4$ and $b = -2$,

(A) Find A + (B + C).

(B) What is AC − BC?

Ans. (A) A + (B + C)

$$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \left(\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$$

(B) We have,

$$AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$\therefore \ AC - BC = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

6. *A manufacturer produces three products, namely, P, Q and R which he sells in two markets A and B. Annual sales are indicated below:*

Market	***Products***		
	P	***Q***	***R***
A	*5000*	*3000*	*6000*
B	*7000*	*9000*	*5000*

The unit sales price of P, Q and R are ₹ 3, ₹ 2 and ₹ 1 respectively.

(A) Find the total revenue in market B.

(B) If the unit costs of the products P, Q and R are ₹ 2.50, ₹ 1.00 and ₹ 0.50, respectively, calculate the gross profit from both the markets.

Ans. (A) Total revenue in market B =

Products Price

$$[7000 \ \ 9000 \ \ 5000] \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

= ₹ [21000 + 18000 + 5000], *i.e.*, ₹ 44,000

(B) Gross profit from market A

= Revenue – Cost price

$$= [5000 \ \ 3000 \ \ 6000] \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - [5000 \ \ 3000 \ \ 6000] \begin{bmatrix} 2.50 \\ 1 \\ 0.50 \end{bmatrix}$$

= [15000 + 6000 + 6000] – [12500 + 3000 + 3000]

= [27000] – [18500]

= [8500]

Gross profit from market B

= Revenue – cost price

$$= [44000] - [7000 \ \ 9000 \ \ 5000] \begin{bmatrix} 2.50 \\ 1 \\ 0.50 \end{bmatrix}$$

= [44000] – [17500 + 9000 + 2500]

= [44000] – [29000]

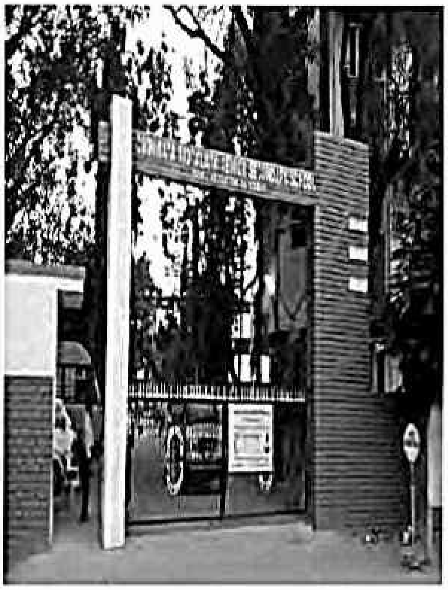
= [15000]

∴ Gross profit from both the markets

= ₹ 85000 + ₹ 15000

= ₹ 23,500

7. *Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each respectively.*



The numbers of articles sold are given as

School/Article	*DPS*	*CVC*	*KVS*
Handmade fans	*40*	*25*	*35*
Mats	*50*	*40*	*50*
Plates	*20*	*30*	*40*

(A) What is the total money (in ₹) collected by the school DPS?

(a) 700 **(b) 7,000**

(c) 6,125 **(d) 7,875**

(B) What is the total amount of money (in ₹) collected by schools CVC and KVS?

(a) 14,000 **(b) 15,725**

(c) 21,000 **(d) 12,125**

(C) What is the total amount of money collected by all three schools DPS, CVC and KVS?

(a) ₹ 15,775 **(b) ₹ 14,000**

(c) ₹ 21,000 **(d) ₹ 17,125**

(D) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

(a) ₹ 18,000 **(b) ₹ 6,750**

(c) ₹ 5,000 **(d) ₹ 21,250**

(E) How many articles (in total) are sold by three schools?

(a) 230 **(b) 130**

(c) 430 **(d) 330**

[CBSE Question Bank 2021]

Ans. (A) *(b)* ₹ *7,000*

Explanation: Let x, y, z be the revenue collected by DPS, CVC and KVS.

The above problem can be represented in the form of matrix as

$$\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore$ Money collected by DPS = ₹ x = ₹ 7,000

(D) *(d) 21,250*

Explanation: Total fans made = 90

Total mats made = 140

Total plates made = 100

$\therefore$ Total money earned

= 90 × ₹ 25 + 140 × ₹ 100 + 100 × ₹ 50

= ₹ 2250 + ₹ 14000 + ₹ 5000

= ₹ 21,250

8. *On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹)*

(A) Find the equations in terms x and y.

(B) Find the number of children who were given some money by Seema.

[CBSE Question Bank 2021]

Ans. (A) Here, number of children is x and amount distributed to one child is y (in ₹).

$\therefore$ Total money distributed = xy

According to the question,

$$(x-8)(y+10) = xy$$

$$\Rightarrow \quad xy - 8y + 10x - 80 = xy$$

$$\Rightarrow \quad 10x - 8y = 80$$

$$\Rightarrow \quad 5x - 4y = 40$$

and $(x+16)(y-10) = xy$

$$\Rightarrow \quad xy + 16y - 10x - 160 = xy$$

$$\Rightarrow \quad -10x + 16y = 160$$

$$\Rightarrow \quad 5x - 8y = -80$$

We can write these equations in matrix form as

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

(B) From part (A), we have

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 5x-4y \\ 5x-8y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

$$5x - 4y = 40 \quad \text{...(i)}$$

and $$5x - 8y = -80 \quad \text{...(ii)}$$

On solving both the equations, we get

$$x = 32, y = 30$$

$\therefore$ Number of children = x = 32.

9. *The bookshop of a particular school has 10 dozens chemistry book, 8 dozens physics book and 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60, ₹ 40 each respectively.*

The shopkeeper allows 10% discount on the sale of each book and charges GST at 5%.

(A) The matrix representing the number of books, is:

(a) $[10 \quad 8 \quad 10]$ **(b)** $[120 \quad 96 \quad 120]$

(c) $\begin{bmatrix} 10 \\ 8 \\ 10 \end{bmatrix}$ **(d)** $\begin{bmatrix} 120 \\ 96 \\ 120 \end{bmatrix}$

(B) The matrix representing the selling price per book, is:

(a) $[80 \quad 60 \quad 40]$ **(b)** $[72 \quad 54 \quad 36]$

(c) $\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$ **(d)** $\begin{bmatrix} 72 \\ 54 \\ 36 \end{bmatrix}$

(C) The matrix representing the net sale price per book, is:

(a) $[80 \quad 60 \quad 40]$ **(b)** $[72 \quad 54 \quad 36]$

(c) $\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$ **(d)** $\begin{bmatrix} 75.60 \\ 56.70 \\ 37.80 \end{bmatrix}$

(D) The matrix indicating the total amount received on selling all books, is:

(a) $[10 \quad 8 \quad 10]\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$

(b) $[120 \quad 96 \quad 120]\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$

(c) $[10 \quad 8 \quad 10]\begin{bmatrix} 75.60 \\ 56.70 \\ 37.80 \end{bmatrix}$

(d) $[120 \quad 96 \quad 120]\begin{bmatrix} 75.60 \\ 56.70 \\ 37.80 \end{bmatrix}$

(E) The total amount received on selling all books, is:

(a) ₹ 1587.60 **(b)** ₹ 1680.20

(c) ₹ 18711.00 **(d)** ₹ 20160.00

Ans. (A) *(b)* $[120 \quad 96 \quad 120]$

(C) *(d)* $\begin{bmatrix} 75.60 \\ 56.70 \\ 37.80 \end{bmatrix}$

Explanation: As on sale of one book, 10% discount is given and 5% GST is charged.

$\therefore$ Discount on chemistry book

$$= \frac{10}{100} \times ₹\ 80 = ₹\ 8$$

$\therefore$ S.P. = ₹ (80 – 8) = ₹ 72

Also, GST is 5%.

$$\therefore \quad \text{GST} = \frac{5}{100} \times ₹\ 72$$

$$= ₹\ 3.60$$

$\therefore$ Final sales price = ₹ 72 + ₹ 3.60

= ₹ 75.60

Similarly, for other books

Sales price of physics book = ₹ 56.70

Sales price of economics book = ₹ 37.80

Hence, final obtained matrix is $\begin{bmatrix} 75.60 \\ 56.20 \\ 37.80 \end{bmatrix}$

VERY SHORT ANSWER Type Questions (VSA)

[**1** mark]

10. If $P = \begin{bmatrix} 4 & -5 \\ -7 & 9 \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} 9 & 5 \\ 7 & 4 \end{bmatrix}$, then verify that $PP^{-1} = I$.

11. If $A = \begin{bmatrix} \frac{2}{19} & \frac{3}{19} \\ \frac{5}{19} & -\frac{2}{19} \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 2 & 3 \\ 5 & x \end{bmatrix}$, find the value of x.

Ans. We know that

$$AA^{-1} = I$$

$$\begin{bmatrix} \frac{2}{19} & \frac{3}{19} \\ \frac{5}{19} & -\frac{2}{19} \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{19}+\frac{15}{19} & \frac{6}{19}+\frac{3x}{19} \\ \frac{10}{19}-\frac{10}{19} & \frac{15}{19}-\frac{2x}{19} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing element a_{12}, we get

$$\frac{6}{19}+\frac{3x}{19} = 0$$

$\Rightarrow$ $3x = -6$

$\Rightarrow$ $x = -2$

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

12. If $P = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ is such that $P^T = P^{-1}$, find α. **[NCERT Exemplar]**

Ans. We have, $P^T = P^{-1}$

$\Rightarrow$ $P^TP = P^{-1}P$

$\Rightarrow$ $P^TP = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\alpha+\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha \\ \cos\alpha\sin\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, $P^TP = I$ is true for all values of α. So α can take any real value.

13. If $B = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$ and its inverse matrix is $B^{-1} = \begin{bmatrix} x & -2 \\ 4 & y \end{bmatrix}$, find the values of x and y.

Ans. We know that

$$BB^{-1} = I$$

$$\frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} x & -2 \\ 4 & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3x+8 & -6+2y \\ -4x+8 & 8+2y \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

On comparing corresponding elements, we get

$3x + 8 = 14$ and $-6 + 2y = 0$

$\Rightarrow$ $3x = 6$ and $2y = 6$

$\Rightarrow$ $x = 2$ and $y = 3$

14. $P = \begin{bmatrix} 1 & 3 & x \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & y & 9 \end{bmatrix}$, find the values of x and y.

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

15. Find x, y and z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$ **[NCERT Exemplar]**

TOPPER'S CORNER

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

1. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

Ans.

$x - y = -1$

$2x - y = 0$

$y = 2x$

$x - 2x = -1$

$x = 1$

$\therefore y = 2$

$\therefore x + y = 3$

[CBSE Topper 2014]

2. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

Ans.

A is a skew symmetric matrix

$\Rightarrow A' = -A$

$A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

$A' = -A$

Now,

$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$

On comparing we get,

$b = 3$

$a = -2$ Ans.

[CBSE Topper 2018]

3. Find the value of $x - y$, if

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Ans.

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

comparing corresponding elements of each matrix,

$2+y=5$ $\quad$ $2x+2=8$

$\boxed{y=3}$ $\quad$ $\boxed{x=3}$

$x-y=3-3$

$\boxed{x-y=0}$

[CBSE Topper 2014]

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

4. Three schools X, Y and Z organized a fair (mela) for collecting funds for flood victims in which they sold hand-held fans, ats and toys made from recycled material, the sale price of each being ₹ 25, ₹ 100 and ₹ 50 respectively. The following table shows the number of articles of each type sold :

School/Article	X	Y	Z
Hand-held fans	30	40	35
Mats	12	15	20
Toys	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the total funds collected. Also write any one value generated by the above situation.

Ans.

Fans, Mats, Toys

$$A = \begin{bmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{bmatrix}_{3\times 3} \begin{matrix} \to \text{school X} \\ \to \text{school Y} \\ \to \text{school Z} \end{matrix}$$

$$B = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \to \text{cost of fans} \\ \to \text{cost of Mats} \\ \to \text{cost of Toys} \end{matrix}$$

$$AB = \begin{bmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$X = AB = \begin{bmatrix} 750+1200+3500 \\ 1000+1500+2750 \\ 875+2000+3750 \end{bmatrix}$$

$$X = \begin{bmatrix} 5450 \\ 5250 \\ 6625 \end{bmatrix} \begin{matrix} \rightarrow \text{fund collected by school } X \\ \rightarrow \text{fund collected by school } Y \\ \rightarrow \text{fund collected by school } Z \end{matrix}$$

fund by X = Rs 5450
fund by Y = Rs 5250
fund by Z = Rs 6625

Total fund = Rs 17325

They are helping victims and hence

[CBSE Topper 2015]