CHAPTER – 1
REAL NUMBERS

The Fundamental Theorem of Arithmetic
Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

The prime factorisation of a natural number is unique, except for the order of its factors.

❖ Property of HCF and LCM of two positive integers 'a' and 'b':
  ➢ $HCF(a,b) \times LCM(a,b) = a \times b$
  ➢ $LCM(a,b) = \dfrac{a \times b}{HCF(a,b)}$
  ➢ $HCF(a,b) = \dfrac{a \times b}{LCM(a,b)}$

PRIME FACTORISATION METHOD TO FIND HCF AND LCM
HCF(a, b) = Product of the smallest power of each common prime factor in the numbers.
LCM(a, b) = Product of the greatest power of each prime factor, involved in the numbers.

IMPORTANT QUESTIONS

Find the LCM and HCF of 510 and 92 and verify that LCM × HCF = product of the two numbers
Solution: 510 = 2 x 3 x 5 x 17
92 = 2 x 2 x 23 = 2^2 x 23
HCF = 2
LCM = 2^2 x 3 x 5 x 17 x 23 = 23460
Product of two numbers = 510 x 92 = 46920
HCF x LCM = 2 x 23460 = 46920
Hence, product of two numbers = HCF × LCM

Questions for practice
1. Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.
2. Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.
3. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers: (i) 26 and 91 (ii) 336 and 54
4. Find the LCM and HCF of the following integers by applying the prime factorisation method: (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25
5. Explain why 3 × 5 × 7 + 7 is a composite number.
6. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons.
7. Can the number 4^n, n being a natural number, end with the digit 0? Give reasons.
8. Given that HCF (306, 657) = 9, find LCM (306, 657).
9. If two positive integers a and b are written as a = x^3y^2 and b = xy^3; x, y are prime numbers, then find the HCF (a, b).
10. If two positive integers p and q can be expressed as p = ab^2 and q = a^3b; a, b being prime numbers, then find the LCM (p, q).
11. Using prime factorization, find the HCF and LCM of (i) 36, 84 (ii) 23, 31 (iii)96, 404 (iv)144, 198 (v) 396, 1080 (vi)1152, 1664
12. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

13. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, find the other.
14. The HCF of two numbers is 18 and their product is 12960. Find their LCM.
15. Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.

IRRATIONALITY OF NUMBERS

IMPORTANT QUESTIONS

Prove that $\sqrt{5}$ is an irrational number.

Solution: Let $\sqrt{5}$ is a rational number then we have

$\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{5}q$

Squaring both sides, we get

$p^2 = 5q^2$

$\Rightarrow$ p^2 is divisible by 5

$\Rightarrow$ p is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2$

$\Rightarrow q^2 = 5m^2$

$\Rightarrow$ q^2 is divisible by 5

$\Rightarrow$ q is also divisible by 5

From above we conclude that p and q has one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

Questions for practice

1. Prove that $\sqrt{2}$ is an irrational number.
2. Prove that $\sqrt{3}$ is an irrational number.
3. Prove that $\sqrt{7}$ is an irrational number.
4. Prove that $2+5\sqrt{3}$ is an irrational number.
5. Prove that $3-2\sqrt{5}$ is an irrational number.
6. Prove that $2-5\sqrt{3}$ is an irrational number.

MCQ (1 mark)

1. If HCF and LCM of two numbers are 4 and 9696, then the product of the two numbers is:
(a) 9696 (b) 24242 (c) 38784 (d) 4848

2. If a and b are positive integers, then HCF (a, b) x LCM (a, b) =
(a) a x b (b) a + b (c) a – b (d) a/b

3. The HCF of 52 and 130 is
(a) 52 (b) 130 (c) 26 (d) 13

4. If the HCF of two numbers is 1, then the two numbers are called
(a) composite (b) relatively prime or co-prime
(c) perfect (d) irrational numbers

5. Given that HCF(1152, 1664) = 128 the LCM(1152, 1664) is
(a) 14976 (b) 1664 (c) 1152 (d) none of these

6. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, then the other number is
(a) 23 (b) 207 (c) 1449 (d) none of these

7. Which one of the following rational number is a non-terminating decimal expansion:
(a) $\frac{33}{50}$ (b) $\frac{66}{180}$ (c) $\frac{6}{15}$ (d) $\frac{41}{1000}$

8. The product of L.C.M and H.C.F. of two numbers is equal to
(a) Sum of numbers (b) Difference of numbers
(c) Product of numbers (d) Quotients of numbers

9. L.C.M. of two co-prime numbers is always
(a) product of numbers (b) sum of numbers
(c) difference of numbers (d)none

10. What is the H.C.F. of two consecutive even numbers
(a) 1 (b)2 (c) 4 (d) 8

11. What is the H.C.F. of two consecutive odd numbers
(a) 1 (b) 2 (c) 4 (d) 8

12. The missing number in the following factor tree is
(a) 2 (b) 6 (c) 3 (d) 9