12. SURFACE AREAS AND VOLUMES

1. A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the remaining solid

   [**Hint :** Surface Area of the remaining solid = Surface area of the cube – Area of the base of cone + Curved surface area of cone.

   Ans : $(1022+154\sqrt{5})cm^2$]

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel $\left(\text{Use } \pi = \frac{22}{7}\right)$

   [Hint: Inner surface area of the vessel = CSA of hemisphere + CSA is cylinder

   Ans : 572 cm^2**]**

3. A cubical box of side 12 cm has a hemispherical dome like structure on its top having maximum diameter. Find the total surface area of the solid

   [**Hint:** TSA of the solid = Surface area of cube – base area of hemisphere + CSA of hemisphere]

   Ans : $(864+36\pi)cm^2$]

4. A circus tent is made up using two different coloured cloth material. Red coloured material is used to make cylindrical part upto a height of 3 m and green coloured material to make conical part above it. If the diameter of the base is 105 m and slant height of the conical part is 53 m, find the red coloured material and green coloured material required [Assuming no stiching margins].

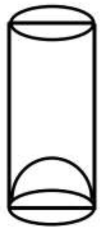
   [Hint : Red coloured materials required $(2\pi rh) = 990 m^2$

   Green coloured material required $(\pi rl) = 8745 m^2$]

5. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 50 cm and the diameter of the cylinder is 28 cm. Find the total surface area of the solid . $\left(\text{Use } \pi = \frac{22}{7}\right)$

[Hint : TSA of solid = CSA of cylinder + 2 × CSA of hemisphere
Ans : 4400 cm^2 **]**

6. Find the actual capacity of a vessel shown in the figure, if the radius of base is 3.5 cm and height of the cylindrical part is 6 cm.



[Hint: Actual capacity = Volume of cylinder – Volume of hemisphere

Ans : $\frac{847}{6} cm^3$ **]**

7. 20 circular plates, each of radius 7 cm and thickness $\frac{1}{2}$ cm, are placed one above another to form a right circular cylinder. Find the volume of the cylinder thus formed. $\left(\text{Use } \pi = \frac{22}{7}\right)$

[Ans : 1540 cm^3**]**

8. If a metallic ball of radius 2.1 cm is put into a cylindrical cup full of water of radius 5 cm and height 7 cm, then how much water is remaining in the cylindrical cup ?

[Hint: Water remaining in the cup = $\pi r_1^2 h - \frac{4}{3}\pi r_2^3$

Ans : $\frac{63899}{125} cm^3$ **]**

9. Marbles of diameter 1.4 cm each are dropped into a cylindrical beaker of radius 7 cm containing some water. How many marbles must be dropped so that water in the beaker rises by 28 cm ?

[Sol. : Let the number of marbles to be dropped be x.

Now, volume of one marble $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \times \left(\frac{1.4}{2}\right)^3 cm^3$$

So, volume of x marbles $= x \times \frac{4}{3}\pi(0.7)^3 cm^3$

Volume of water risen in the beaker

$= \pi r^2 h$

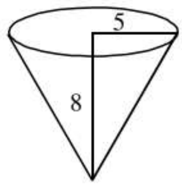
$= \pi \times 7^2 \times 28 cm^3$

So, we get $x \times \frac{4}{3}\pi(0.7)^3 = \pi \times 7^2 \times 28$.

$$\Rightarrow \quad x = \frac{\pi \times 7^2 \times 28 \times 3}{4\pi \times (0.7)^3} = \frac{7 \times 7 \times 7 \times 4 \times 3}{4 \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}}$$

$= 3 \times 10 \times 10 \times 10 = 3000.$]

10. A vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots each of which is a sphere of radius 0.5 cm, are dropped into the vessel , one fourth of the water flows out. Find the number of lead shots dropped in the vessel



[Sol : Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 25 \times 8 cm^3$

Volume of the lead shots $= \frac{1}{4}$ of the volume of the cone

$= \frac{1}{4} \times \frac{1}{3}\pi \times 25 \times 8 cm^3 = \frac{50}{3}\pi cm^3$ ....(1)

Now, volume of one lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.5)^3 cm^3$.

Let the number of lead shots be x.

So, volume of the lead shots $= x \times \frac{4}{3}\pi(0.5)^3 cm^3$ ....(2)

From (1) and (2),

$x \times \frac{4}{3}\pi(0.5)^3 = \frac{50}{3}\pi$.

$$\Rightarrow \quad x = \frac{50\pi \times 3}{3 \times 4\pi(0.5)^3} = \frac{50 \times 10 \times 10 \times 10}{4 \times 5 \times 5 \times 5}$$

$= 50 \times 2 = 100.$]

11. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 2.1 cm and th height of the cone is 4 cm. The solid is placed in a cylindrical bucket full of water in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 9.8 cm find the volume of water left in the cylindrical bucket. $\left(\text{Use } \pi = \frac{22}{7}\right)$

[Ans : Volume of the water left in the cylinder = 732.116 cm^3]

12. A cylindrical jar of radius 12 cm contains orange juice to a depth of 20 cm. A child drops an orange into the jar and the level of the juice rises by 6.75 cm. What is the radius of the orange, if it is of the shape of a complete sphere ? Find the surface area of the orange.

[Sol : Surface area of the orange = 1017.36 cm^2]

13. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 0.5 cm in diameter. A full barrel of ink in the pen can be used for writing 275 words on an average. How many words would be written using a bottle of ink containing one fourth of a litre ?

[Sol. : Volume of the fountain pen $= \pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{0.5}{2}\right)^2 \times 7\, cm^3$$

$$= \frac{22}{7} \times 0.25 \times 0.25 \times 7\, cm^3$$

$$= \frac{22 \times 25 \times 25}{10000} cm^3 = \frac{22}{16} cm^3$$

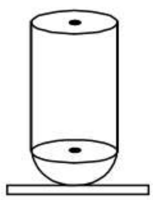
Now, in $\frac{22}{16}$ cm^3, number of words = 275.

So, in $\frac{1}{4}$ litres, i.e., $\frac{1}{4} \times 1000$ cm^3, number of words

$$= \frac{275 \times 16}{22} \times \frac{1000}{4}$$

= 25 × 2 × 1000 = 50000 words.]

14. A trophy awarded to the best student in the class is in the form of a solid cylinder mounted on a solid hemisphere with the same radius and is made from some metal. This trophy is mounted on a wooden cuboid as shown in the figure. The diameter of the hemisphere is 21 cm and the total height of the trophy is 24.5 cm. Find the weight of the metal used in making the trophy, if the weight of 1 cm^3 of the metal is 1.2 g $\left(\text{Use } \pi = \frac{22}{7}\right)$



[Sol. : Total height of the trophy = 24.5 cm

So, height of the cylinder $= 24.5\, cm - \frac{21}{2} cm = 14$ cm

So, volume of the metal used in the trophy

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \left\{\frac{22}{7}\times\left(\frac{21}{2}\times\frac{21}{2}\right)\times 14 + \frac{2}{3}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}\right\} \text{ cm}^3.$$

$$= \left(11\times 21\times 21 + 11\times 21\times\frac{21}{2}\right) \text{cm}^3$$

$$= 11\times 21\left(21 + \frac{21}{2}\right) \text{cm}^3$$

$$= 11\times 21\times 21\times\frac{3}{2} \text{ cm}^3$$

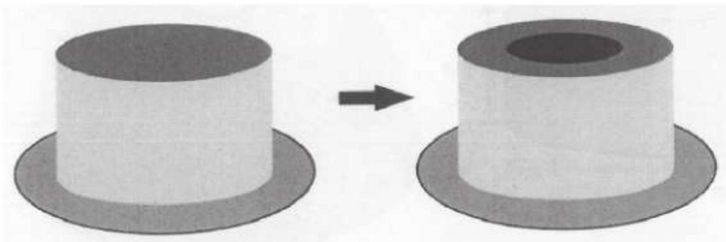
So, weight of the metal $= 11\times 21\times 21\times\frac{3}{2}\times 1.2\, g$

$$= 11\times 21\times 21\times\frac{3}{2}\times\frac{12}{10}\, g$$

$$= \frac{11\times 441\times 18}{10}\, g = 8731.8\, g$$

$= 8.7318$ kg.]

15. Shown below is a cake that Subodh is baking for his brother's birthday. The cake is 21 cm tall and has a radius of 15 cm. He wants to surprise his brother by filling gems inside the cake. In order to do that, he removes a cylindrical portion of cake out of the centre as shown. The piece that is removed is 21 cm tall.



If the cake weighs 0.5 g per cubic cm and the weight of the cake that is left after removing the central portion is 6600 g, find the radius of the central portion that is cut. Show your steps.

(**Note :** Take $\pi = \frac{22}{7}$)

[Sol. : The volume of the cake without the hole = as $(\pi\times 15^2\times 21) cm^3$

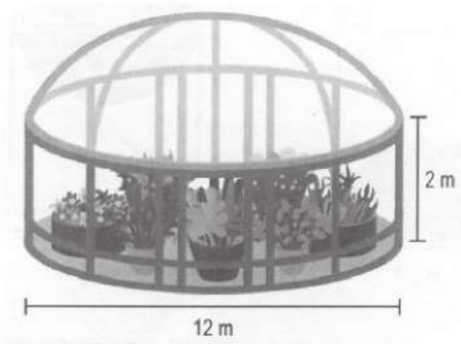
The weight of the cake without hole = $(\pi\times 15^2\times 21\times 0.5) gm$.

Uses the above step and the given information to find the weight of the central portion that is removed as $(\pi\times 15^2\times 21\times 0.5) - 6600 = 825 g$

The volume of the central portion that is removed as 825 + 0.5 = 1650 cm^3.

The radius of the central portion that is removed as $\sqrt{(1650+21\pi)} = 5cm$.]

16. Dinesh is building a greenhouse in his farm as shown below. The base of the greenhouse is circular having a diameter of 12 m and it has a hemispherical dome on top.



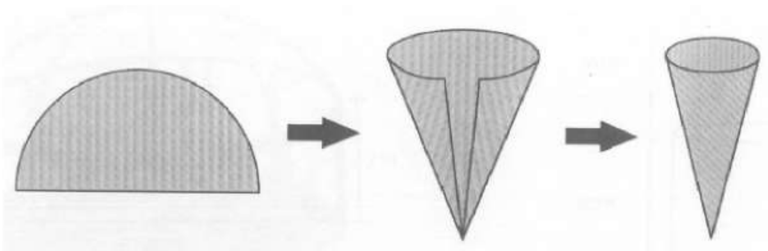
How much will it cost him to cover the walls and top of the greenhouse with transparent plastic, if the plastic sheet costs Rs. 77 per sq m? Show your steps.

(Note : Take $\pi = \frac{22}{7}$)

[Ans : TSA of the green house to be covered $= 96\pi m$

Total cost = Rs. 23,232]

17. A semi - circular waffle sheet of radius 5 cm is folded into an ice - cream cone as shown below.



Due to overlap while folding, the radius of the base of the cone is 80% of what it would be without overlap.

Find the approximate volume of the cone. Show your work.

(Note : Take $\pi = \frac{22}{7}$)

[Sol. : The circumference of the base to the curved surface area of the waffle sheet as $2\pi r = \pi r$

and find the base radius of the cone without overlap as $\frac{5}{2} cm$

The radius of the cone with overlap as $\frac{80}{100} \times \frac{5}{2} = 2cm$.]

18. A 5.54 litre watering can sprinkles water at the rate of 500 mL/min. The can has a diameter of 14 cm and is initially filled to its full capacity.

What is the height of water in the can after it is used for 8 minutes? Show your work.

(**Note :** Take $\pi = \frac{22}{7}$)

[Sol. : The volume of water left after 8 minutes as $5.54 - \left(8 \times \frac{1}{2}\right) = 1.54$ litres or 1540 cm^3.

The equation for the height of the can, h as $\frac{22}{7} \times (7)^2 \times h = 1540$.

The above equation for h as 10 cm.]

19. Two people have an equal amount of moulding clay. They make different solids of the same circular radius out of it - cylinder and hemisphere.
State true or false for the below statements and justify your answer.

i) Simran said, "The curved surface area of the cylinder is larger."

i) Mabnoj said, "Both the solids have the same curved surface area".

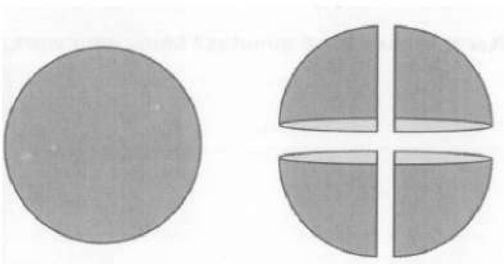
[Sol. : The ratio of curved surface area to volume of a :

cylinder as $\frac{2}{r}$

hemisphere as $\frac{3}{r}$

where r is the circular radius.
Uses the above step to conclude that both Simran's and Manoj's statements are false.]

20. The surface area of a solid spherical ball is S cm^2. It is cut into 4 identical pieces as shown below.



Find the total surface area of 4 identical pieces of the solid spherical ball in terms of S. Show your work.

[Sol. : Assumes the radius of the solid spherical ball as r cm and writes that the total surface area of 4 identical pieces of solid spherical ball $= 4\pi r^2 + 2\pi r^2 + 2\pi r^2 = 8\pi r^2 = (2\times 4\pi r^2)cm^2$

The total surface area of the 4 identical pieces of solid spherical ball as $2 \times S$ cm^2]