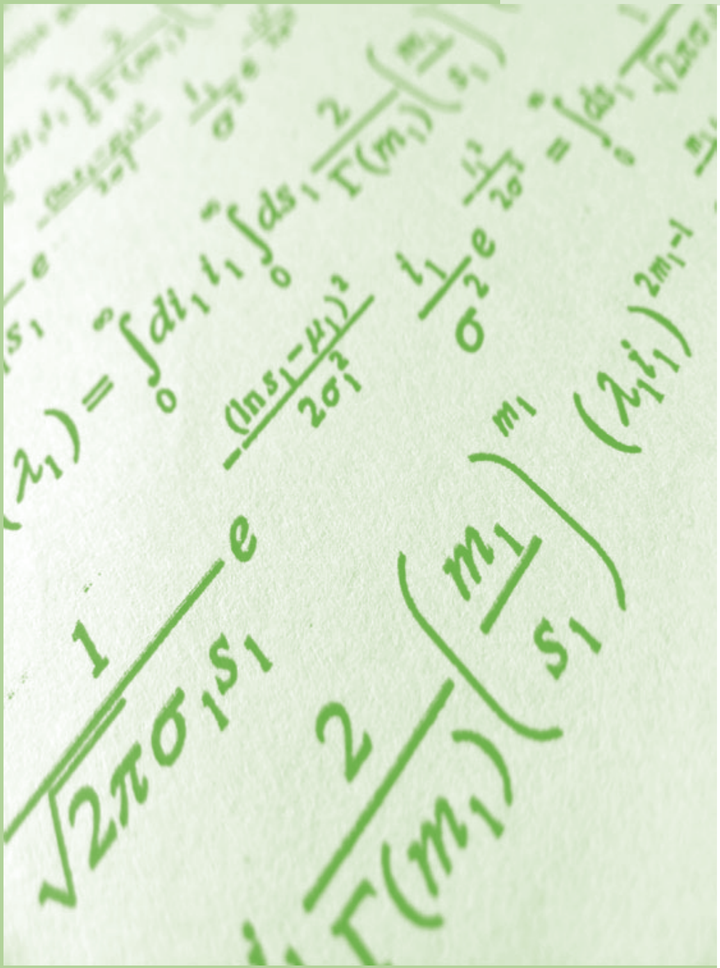
Chapter

7

Progressions



REMEMBER

Before beginning this chapter, you should be able to:

- Understand different types of numbers
- Apply basic operations on numbers

KEY IDEAS

After completing this chapter, you would be able to:

- Understand terms such as sequences, series, and progressions
- Study arithmetic progression (AP) and also obtain arithmetic mean (AM)
- Formulate geometric progression (GP)
- Insert harmonic mean (HM) between two numbers
- To study relation between AM, GM and HM

INTRODUCTION

Let us observe the following pattern of numbers.

1. 5, 11, 17, 23, …
2. 6, 12, 24, 48, …
3. 4, 2, 0, –2, –4, …
4. $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$

In example 1, every number (except 5) is formed by adding 6 to the previous numbers. Hence a specific pattern is followed in the arrangement of these numbers. Similarly, in example 2, every number is obtained by multiplying the previous number by 2. Similar cases are followed in examples 3 and 4.

SEQUENCE

A systematic arrangement of numbers according to a given rule is called a sequence.

The numbers in a sequence are called its terms. We refer the first term of a sequence as T_1, second term as T_2 and so on. The nth term of a sequence is denoted by T_n, which may also be referred to as the general term of the sequence.

Finite and Infinite Sequences

1. A sequence which consists of a finite number of terms is called a finite sequence.

   Example: 2, 5, 8, 11, 14, 17, 20, 23 is the finite sequence of 8 terms.

2. A sequence which consists of an infinite number of terms is called an infinite sequence.

   Example: 3, 10, 17, 24, 31, … is an infinite sequence.

Note If a sequence is given, then we can find its nth term and if the nth term of a sequence is given we can find the terms of the sequence.

Examples:

Find the first four terms of the sequences whose nth terms are given as follows.

1. $T_n = 3n + 1$

   Substituting $n = 1$,

   $T_1 = 3(1) + 1 = 4$

   Similarly, $T_2 = 3(2) + 1 = 7$

   $T_3 = 3(3) + 1 = 10$

   $T_4 = 3(4) + 1 = 13$

   $\therefore$ The first four terms of the sequence are 4, 7, 10, 13.

2. $T_n = 2n^2 - 3$

   Substituting $n = 1$,

   $T_1 = 2(1)^2 - 3 = -1$

Similarly, $T_2 = 2(2)^2 - 3 = 5$

$T_3 = 2(3)^2 - 3 = 15$

$T_4 = 2(4)^2 - 3 = 29$

$\therefore$ The first four terms of the sequence are $-1, 5, 15, 29$.

Series

The sum of the terms of a sequence is called the series of the corresponding sequence.

Example:

$1 + 2 + 3 + \cdots + n$ is a finite series of first n natural numbers.

The sum of first n terms of series is denoted by S_n.

Here, $S_n = T_1 + T_2 + \cdots + T_n$.

Here, $S_1 = T_1$

$S_2 = T_1 + T_2$

$S_3 = T_1 + T_2 + T_3$

$\ldots$

$\ldots$

$S_n = T_1 + T_2 + T_3 + \cdots + T_n$

We have,

$S_2 - S_1 = T_2$

$S_3 - S_2 = T_3$

Similarly,

$S_n - S_{n-1} = T_n$.

EXAMPLE 7.1

In the series, $T_n = 2n + 5$, find S_4.

SOLUTION

$T_n = 2n + 5$

$T_1 = 2(1) + 5 = 7$

$T_2 = 2(2) + 5 = 9$

$T_3 = 2(3) + 5 = 11$

$T_4 = 2(4) + 5 = 13$

$S_4 = T_1 + T_2 + T_3 + T_4 = 7 + 9 + 11 + 13 = 40$.

Sequences of numbers which follow specific patterns are called progressions. Depending on the pattern, the progressions are classified as follows.

1. Arithmetic Progression
2. Geometric Progression
3. Harmonic Progression.

Arithmetic Progression (AP)

Numbers (or terms) are said to be in arithmetic progression when each one, except the first, is obtained by adding a constant to the previous number (or term).

An arithmetic progression can be represented by a, $a + d$, $a + 2d$, …, $[a + (n - 1)d]$. Here, d is added to any term to get the next term of the progression. The term a is the first term of the progression, n is the number of terms in the progression and d is the common difference.

1. The nth term (general term) of an arithmetic progression is $T_n = a + (n - 1)d$.
2. Sum to n terms of an AP $= S_n = \frac{n}{2}[2a + (n - 1)d]$.

The sum to n terms of an AP can also be written in a different manner. That is, sum of n terms $= \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}[a + \{a + (n - 1)d\}]$.

But, when there are n terms in an AP, a is the first term and $\{a + (n - 1)d\}$ is the last term.

Hence, $S_n = \left(\frac{n}{2}\right)$ [first term + last term].

Arithmetic Mean (AM)

The average of all the terms in an AP is called the arithmetic mean (AM) of the AP.

The average of a certain numbers $= \dfrac{\text{Sum of all the numbers}}{\text{number of numbers}}$

$\therefore$ AM of n terms in an AP $= \dfrac{S_n}{n} = \dfrac{1}{n} \times \dfrac{n}{2}$ [first term + last term] $= \dfrac{(\text{first term} + \text{last term})}{2}$,

i.e., the AM of an AP is the average of the first and the last terms of the AP.

The AM of an AP can also be obtained by considering any two terms which are EQUIDISTANT from the two ends of the AP and taking their average, i.e.,

1. the average of the second term from the beginning and the second term from the end is equal to the AM of the AP
2. the average of the third term from the beginning and the third term from the end is also equal to the AM of the AP and so on.

In general, the average of the kth term from the beginning and the kth term from the end is equal to the AM of the AP

If the AM of an AP is known, the sum to n terms of the series (S_n) can be expressed as $S_n = n$ (AM)

In particular, if three numbers are in arithmetic progression, then the middle number is the AM, i.e., if a, b and c are in AP, then b is the AM of the three terms and $b = \dfrac{a + c}{2}$.

If a and b are any two numbers, then their AM $= \dfrac{a + b}{2}$.

Inserting arithmetic mean between two numbers:

When n arithmetic means $a_1, a_2, \ldots, a_3$ are inserted between a and b, then $a, a_1, a_2, \ldots, a_n, b$ are in AP.

$\Rightarrow$ $t_1 = a$ and $t_{n+2} = b$ of AP

The common difference of the AP can be obtained as follows:

Given that, n arithmetic means are there between a and b.

$\therefore$ $a = t_1$ and $b = t_{n+2}$

Let d be the common difference.

$\Rightarrow$ $b = t_1 + (n + 1)d$

$\Rightarrow$ $b = a + (n + 1)d$

$\Rightarrow$ $d = \dfrac{(b - a)}{(n + 1)}$.

Notes

1. If three numbers are in AP, we can take the three terms to be $(a - d)$, a and $(a + d)$.
2. If four numbers are in AP, we can take the four terms to be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$. The common difference in this case is $2d$ and not d.
3. If five numbers are in AP, we can take the five terms to be $(a - 2d)$, $(a - d)$, a, $(a + d)$ and $(a + 2d)$.

Some Important Results

The sum to n terms of the following series are quite useful and hence should be remembered by students.

1. Sum of first n natural numbers $= \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
2. Sum of the squares of first n natural numbers $= \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
3. Sum of the cubes of first n natural numbers $= \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \left[\sum_{1}^{n} i\right]^2$

EXAMPLE 7.2

Find the 14th term of an AP whose first term is 3 and the common difference is 2.

SOLUTION

The nth term of an AP is given by $t_n = a + (n - 1)d$, where a is the first term and d is the common difference.

$\therefore \quad t_{14} = 3 + (14 - 1)\, 2 = 29.$

EXAMPLE 7.3

Find the first term and the common difference of an AP if the 3rd term is 6 and the 17th term is 34.

SOLUTION

If a is the first term and d is the common difference, then we have

$$a + 2d = 6 \tag{1}$$

$$a + 16d = 34 \tag{2}$$

On subtracting Eq. (1) from Eq. (2), we get

$$14d = 28 \quad \Rightarrow \quad d = 2$$

Substituting the value of d in equation (1), we get $a = 2$

$\therefore \quad a = 2$ and $d = 2$.

EXAMPLE 7.4

Find the sum of the first 22 terms of an AP whose first term is 4 and the common difference is $\frac{4}{3}$.

SOLUTION

Given that, $a = 4$ and $d = \frac{4}{3}$.

We have $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{22} = \left(\frac{22}{2}\right)\left[(2)(4) + (22-1)\left(\frac{4}{3}\right)\right] = (11)(8+28) = 369.$$

EXAMPLE 7.5

Divide 124 into four parts in such a way that they are in AP and the product of the first and the 4th part is 128 less than the product of the 2nd and the 3rd parts.

SOLUTION

Let the four parts be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$. The sum of these four parts is 124, i.e., $4a = 124 \quad \Rightarrow \quad a = 31$

$(a - 3d)\ (a + 3d) = (a - d)\ (a + d) - 128$

$\Rightarrow \quad a^2 - 9d^2 = a^2 - d^2 - 128$

$\Rightarrow \quad 8d^2 = 128 \quad \Rightarrow \quad d = \pm 4.$

As $a = 31$, taking $d = 4$, the four parts are 19, 27, 35 and 43.

Note If d is taken as -4, then the same four numbers are obtained, but in decreasing order.

EXAMPLE 7.6

Find the three terms in AP, whose sum is 36 and product is 960.

SOLUTION

Let the three terms of an AP be $(a - d)$, a and $(a + d)$.

Sum of these terms is $3a$.

$3a = 36 \quad \Rightarrow \quad a = 12$

Product of these three terms is

$(a + d)\ a\ (a - d) = 960$

$\Rightarrow \quad (12 + d)\ (12 - d) = 80$

$\Rightarrow \quad 144 - d^2 = 80 \quad \Rightarrow \quad d = \pm 8.$

Taking $d = 8$, we get the terms as 4, 12 and 20.

Note If d is taken as -8, then the same numbers are obtained, but in decreasing order.

EXAMPLE 7.7

Find the sum of all natural numbers and lying between 100 and 200 which leave a remainder of 2 when divided by 5 in each case.

(a) 2990 **(b)** 2847 **(c)** 2936 **(d)** None of these

SOLUTION

The natural number which leaves remainder 2 greater than 100 is 102, and less than 200 is 197.

$\therefore$ The series is 102, 107, 112, 117, 122, …, 197.

Number of terms

$$= \frac{200}{5} - \frac{100}{5} = 40 - 20 = 20.$$

$$S_n = \frac{n}{2}[a + l] = \frac{20}{2}[102 + 197]$$

$$= \frac{20}{2}[299] = 2990.$$

EXAMPLE 7.8

Find the sum of 100 terms of the series 1(3) + 3(5) + 5(7) + …

(a) 1353300 **(b)** 1353400 **(c)** 1353200 **(d)** 1353100

SOLUTION

1.3 + 3.5 + 5.7 + …

$$t_n = t_{n1} \times t_{n2}$$

$$= 1 + (n - 1)2 + 3 + (n - 1)2$$

$$= (2n - 1)(2n + 1)$$

$$t_n = 4n^2 - 1.$$

$$\sum t_n = \sum(4n^2 - 1)$$

$$= 4\sum n^2 - \sum 1$$

$$= 4 \times \frac{n(n+1)(2n+1)}{6} - n$$

$$= \frac{2n(n+1)(2n+1) - 3n}{3}$$

$$= \frac{n}{3}[2(n+1)(2n+1) - 3].$$

$$\sum t_{100} = \frac{100}{3}[2 \times 101 \times 201 - 3]$$

$$= \frac{100}{3}[202 \times 201 - 3]$$

$$= 100[202 \times 67 - 1]$$

$$= 100[13534 - 1] = 100 \times 13533$$

$$= 1353300.$$

GEOMETRIC PROGRESSION (GP)

Numbers are said to be in geometric progression when the ratio of any quantity to the number that follows it is the same. In other words, any term of a GP (except the first one) can be obtained by multiplying the previous term by the same constant.

The constant is called the common ratio and is normally represented by *r*. The first term of a GP is generally denoted by *a*.

A geometric progression can be represented by a, ar, ar^2, … where a is the first term and r is the common ratio of the GP nth term of the GP is ar^{n-1}, i.e., $t_n = ar^{n-1}$.

Sum to n terms $= S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n-1)}{r-1} = \dfrac{r(ar^{n-1})-a}{r-1}$

$\therefore$ The sum to n terms of a geometric progression can also be written as

$$S_n = \frac{r(\text{Last term}) - \text{First term}}{r-1}.$$

Notes

1. If n terms viz., $a_1, a_2, a_3, \ldots, a_n$ are in GP, then the geometric mean (GM) of these n terms is given by $= \sqrt[n]{a_1 a_2 a_3 \ldots a_n}$.
2. If three terms are in geometric progression, then the middle term is the geometric mean of the GP, i.e., if a, b and c are in GP, then b is the geometric mean of the three terms.
3. If there are two terms say a and b, then their geometric mean is given by $\text{GM} = \sqrt{ab}$.
4. When n geometric means are there between a and b, the common ratio of the GP can be derived as follows. Given that, n geometric means are there between a and b.

   $\therefore \quad a = t_1$ and $b = t_{n+2}$.

   Let 'r' be the common ratio

   $\Rightarrow \quad b = (t_1)(r^{n+1}) \quad \Rightarrow \quad b = ar^{n+1}$

   $\Rightarrow \quad r^{n+1} = \dfrac{b}{a} \quad \Rightarrow \quad r = \sqrt[(n+1)]{\dfrac{b}{a}}$
5. For any two positive numbers a and b, their arithmetic mean is always greater than or equal to their geometric mean, i.e., for any two positive numbers a and b, $\dfrac{a+b}{2} \geq \sqrt{ab}$.

   The equality holds if and only if $a = b$.
6. When there are three terms in geometric progression, we can take the three terms to be a/r, a and ar.

Infinite Geometric Progression

If $-1 < r < 1$ (or $|r| < 1$), then the sum of a geometric progression does not increase infinitely but 'converges' to a particular value, no matter how many terms of the GP we take. The sum of an infinite geometric progression is represented by S_∞ and is given by the formula,

$$S_\infty = \frac{a}{1-r}, \text{ if } |r| < 1.$$

EXAMPLE 7.9

Find the 7th term of the GP whose first term is 6 and common ratio is $\dfrac{2}{3}$.

SOLUTION

Given that, $t_1 = 6$ and $r = \dfrac{2}{3}$.

We have $t_n = a \cdot r^{n-1}$

$$t_7 = (6)\left(\frac{2}{3}\right)^6 = \frac{(6)(64)}{729} = \frac{128}{243}.$$

EXAMPLE 7.10

Find the common ratio of the GP whose first and last terms are 25 and $\frac{1}{625}$ respectively and the sum of the GP is $\frac{19531}{625}$.

SOLUTION

We know that the sum of a GP is $\frac{\text{first term} - r(\text{last term})}{1-r}$

or $\frac{19531}{625} = \frac{25 - \left(\frac{r}{625}\right)}{1-r}$

$\therefore \quad r = \frac{1}{5}$.

EXAMPLE 7.11

Find three numbers of a GP whose sum is 26 and product is 216.

SOLUTION

Let the three numbers be a/r, a and ar.

Given that,

$a/r \cdot a \cdot ar = 216;$

$\Rightarrow \quad a^3 = 216; a = 6.$

$a/r + a + ar = 26$

or $\quad 6 + 6r + 6r^2 = 26r$

or, $\quad 6r^2 - 20r + 6 = 0$

or, $\quad 6r^2 - 18r - 2r + 6 = 0$

or, $\quad 6r(r - 3) - 2(r - 3) = 0$

or, $\quad r = 1/3$ (or) $r = 3$.

Hence the three numbers are 2, 6 and 18 (or) 18, 6 and 2.

EXAMPLE 7.12

If $|x| < 1$, then find the sum of the series $2 + 4x + 6x^2 + 8x^3 + \cdots$

SOLUTION

Let $S = 2 + 4x + 6x^2 + 8x^3 + \cdots \quad (1)$

$xS = 2x + 4x^2 + 6x^3 + \cdots \quad (2)$

Eq. (1) – Eq. (2) gives

$S(1 - x) = 2 + 2x + 2x^2 + 2x^3 + \cdots$

$= 2(1 + x + x^2 + \cdots)$

$1 + x + x^2 + \cdots$ is an infinite GP with $a = 1$, $r = x$ and $|r| = |x| < 1$

$\therefore$ Sum of the series $= \frac{1}{1-x}$

$\therefore \quad S(1 - x) = \frac{2}{(1-x)}$

$\therefore \quad S = \frac{2}{(1-x)^2}$.

EXAMPLE 7.13

Find the sum of the series $1, \frac{2}{5}, \frac{4}{25}, \frac{8}{125}, \ldots \infty$.

SOLUTION

Given that, $a = 1, r = \frac{2}{5}$ and $|r| = \left|\frac{2}{5}\right| < 1$

$$\therefore \quad S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}.$$

EXAMPLE 7.14

S_{10} is the sum of the first 10 terms of a GP and S_5 is the sum of the first 5 terms of the same GP. If $\frac{S_{10}}{S_5} = 244$, then find the common ratio.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 2

SOLUTION

$$\frac{S_{10}}{S_5} = \frac{\frac{a\cdot(r^{10}-1)}{(r-1)}}{\frac{a(r^5-1)}{r-1}}$$

$$\text{or} \quad \frac{S_{10}}{S_5} = \frac{r^{10}-1}{r^5-1} = r^5+1$$

$$\frac{S_{10}}{S_5} = \frac{(r^5-1)^2+2r^5-2}{r^5-1}$$

$$244 = \frac{(r^5-1)[r^5-1+2]}{(r^5-1)} \quad \Rightarrow \quad 244 = r^5+1$$

$$\text{or,} \quad r^5 = 243 \quad \Rightarrow \quad r = 3.$$

EXAMPLE 7.15

The difference between two hundred-digit numbers consisting of all 1's and a hundred-digit number consisting of all 2's is equal to

(a) $\underbrace{99\ldots\ldots9}_{100\text{ times}}$

(b) $\left(\frac{333\ldots\ldots3}{80\text{ times}}\right)^2$

(c) $\left(\frac{333\ldots\ldots3}{100\text{ times}}\right)^2$

(d) $\underbrace{99\ldots\ldots9}_{200\text{ times}}$

HINTS

Apply the principle $11 = 1 + 10^{2-1}$, $111 =$ and proceed.

HARMONIC PROGRESSION (HP)

A progression is said to be a harmonic progression if the reciprocal of the terms in the progression form an arithmetic progression.

For example, consider the series $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots$

The progression formed by taking reciprocals of terms of the above series is 2, 5, 8, 11, …. Clearly, these terms form an AP whose common difference is 3.

Hence, the given progression is a harmonic progression.

*n*th term of an HP:

We know that if a, $a + d$, $a + 2d$, … are in AP, then the nth term of this AP is $a + (n - 1)d$. Its reciprocal is $\frac{1}{a+(n-1)d}$.

So, nth term of an HP whose first two terms are and is $\frac{1}{a}$ and $\frac{1}{a+d}$ is $\frac{1}{a+(n-1)d}$.

Note There is no concise general formula for the sum to n terms of an HP.

EXAMPLE 7.16

Find the 10th term of the HP $\frac{3}{2}, 1, \frac{3}{4}, \frac{3}{5}, \ldots$

SOLUTION

The given HP is $\frac{3}{2}, 1, \frac{3}{4}, \frac{3}{5}, \ldots$

The corresponding AP is $\frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, \ldots$

Here $a = \frac{2}{3}; d = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ T_{10} of the corresponding AP is $a + (10 - 1)d = \frac{2}{3} + (9)\frac{1}{3} = \frac{11}{3}$

Hence required term in HP is $\frac{3}{11}$.

Harmonic Mean (HM)

If three terms are in HP, then the middle term is the HM of other two terms.

The harmonic mean of two terms a and b is given by HM $= \frac{2ab}{a+b}$.

Inserting *n* Harmonic Means Between Two Numbers

To insert n harmonic means between two numbers, we first take the corresponding arithmetic series and insert n arithmetic means, and next, we find the corresponding harmonic series.

This is illustrated by the example below:

EXAMPLE 7.17

Insert three harmonic means between $\frac{1}{12}$ and $\frac{1}{20}$

SOLUTION

After inserting the harmonic means

let the harmonic progression be

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \frac{1}{a+4d}$$

Given $\frac{1}{a} = \frac{1}{12}$ and $\frac{1}{a+4d} = \frac{1}{20}$ $\Rightarrow$ $a = 12$ and $d = 2$

$\therefore$ The required harmonic means are $\frac{1}{14}, \frac{1}{16}$ and $\frac{1}{18}$.

Relation between AM, HM and GM of Two Numbers

Let x and y be two numbers.

$\therefore$ $\text{AM} = \frac{x+y}{2}, \text{GM} = \sqrt{xy}$ and $\text{HM} = \frac{2xy}{x+y}$

$\Rightarrow$ $(\text{AM})\ (\text{HM}) = (\text{GM})^2$

EXAMPLE 7.18

The ratio of geometric and arithmetic mean of two real numbers is 3 : 5. Then find the ratio of their harmonic mean and geometric mean.

(a) 3 : 5 **(b)** 9 : 25 **(c)** 9 : 5 **(d)** 5 : 9

SOLUTION

$G^2 = AH \Rightarrow GG = AH$

$\frac{G}{A} = \frac{H}{G}$ It is given, G : A = 3 : 5

$\therefore$ H : G = 3 : 5

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Third term of the sequence whose nth term is $2n + 5$ is ______.
2. If a is the first term and d is the common difference of an AP, then the $(n + 1)$th term of the AP is ______.
3. If the sum of three consecutive terms of an AP is 9, then the middle term is ______.
4. General term of the sequence 5, 25, 125, 625, … is ______.
5. The arithmetic mean of 7 and 8 is ______.
6. The arrangement of numbers $\frac{1}{2}, \frac{-3}{4}, \frac{-5}{6}, \frac{-7}{8}, \ldots$ is an example of sequence. [True/False]
7. If $\frac{a}{2}$ is the first term and d is the common difference of an AP, then the sum of n terms of the AP is ______.
8. In a sequence, if S_n is the sum of n terms and S_{n-1} is the sum of $(n - 1)$ terms, then the nth term is __________.
9. If $T_n = 3n + 8$, then $T_{n-1} =$ ______.
10. The sum of the first $(n + 1)$ natural numbers is ______.
11. For a series in geometric progression, the first term is a and the second term is $3a$. The common ratio of the series is ______.
12. In a series, starting from the second term, if each term is twice its previous term, then the series is in ______ progression.
13. All the multiples of 3 form a geometric progression. [True/False]
14. If a, b and c are in geometric progression then, a^2, b^2 and c^2 are in ______ progression.
15. If every term of a series in geometric progression is multiplied by a real number, then the resulting series also will be in geometric progression. [True/False]
16. Geometric mean of 5, 10 and 20 is ______
17. Sum of the infinite terms of the GP, –3, –6, –12, … is 3. [True/False]
18. The reciprocals of all the terms of a series in geometric progression form a ______ progression.
19. The nth term of the sequence $\frac{1}{100}, \frac{1}{10000}, \frac{1}{1000000}, \ldots$ is ______.
20. In a series, $T_n = x^{2n-2}$ $(x \neq 0)$, then write the infinite series.
21. The harmonic mean of 1, 2 and 3 is $\frac{3}{2}$. [True/False]
22. If a, b, c and d are in harmonic progression, then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ and $\frac{1}{d}$ are in ______ progression.
23. If the AM of two numbers is 9 and their HM is 4, then their GM is 6. [True/False]
24. If a, b and c are the arithmetic mean, geometric mean and harmonic mean of two distinct terms respectively, then b^2 is equal to ______.
25. If the sum of n terms which are in GP is $a(r + 1)$, then the number of terms is ______. (Where a is the first term and r is the common ratio)
26. Write the first three terms of the sequence whose nth term is $T_n = 8 - 5n$.
27. Write the first three terms of the sequence whose nth term is $T_n = 5^{n+1}$
28. If three arithmetic means are inserted between 4 and 5, then the common difference is ______.
29. If the 7th and the 9th terms of a GP are x and y respectively, then the common ratio of the GP is ______.
30. In a series, $T_n = 3 - n$, then $S_5 =$ ______.

Short Answer Type Questions

31. If the 5th term and the 14th term of an AP are 35 and 8 respectively, then find the 20th term of the AP
32. Which term of the series 21, 15, 9, … is −39?
33. If the seventh term of an AP is 25 and the common difference is 4, then find the 15th term of AP.

34. Find the general term of AP whose sum of n terms is given by $4n^2 + 3n$.

35. Find the sum of all three-digit numbers which leave a remainder 2, when divided by 6.

36. If the ratio of the sum of first three terms of a GP to the sum of first six terms is 448 : 455, then find the common ratio.

37. If in a GP, 5th term and the 12th term are 9 and $\frac{1}{243}$ respectively, find the 9th term of GP.

38. A person opens an account with ₹50 and starts depositing every day double the amount he has deposited on the previous day. Then find the amount he has deposited on the 10th day from the beginning.

39. Find the sum of 5 geometric means between $\frac{1}{3}$ and 243, by taking common ratio positive.

40. Using progressions express the recurring decimal $2\cdot\overline{123}$ in the form of $\frac{p}{q}$, where p and q are integers.

41. A ball is dropped from a height of 64 m and it rebounces $\frac{3}{4}$ of the distance every time it touches the ground. Find the total distance it travels before it comes to rest.

42. Find the sum to n terms of the series 5 + 55 + 555 + ….

43. In an HP, if the 3rd term and the 12th term are 12 and 3 respectively, then find the 15th term of the HP.

44. If lth, mth and nth terms of an HP are x, y and z respectively, then find the value of $yz(m - n) + xz(n - l) + xy(l - m)$.

45. The AM of two numbers is 40 more than GM and 64 more than HM Find the numbers.

Essay Type Questions

46. Find the sum to n terms of the series $1 \cdot 2 \cdot 3 + 2 \cdot 4 \cdot 6 + 3 \cdot 6 \cdot 9 + \ldots$

47. One side of an equilateral triangle is 36 cm. The mid-points of its sides are joined to form another triangle. Again another triangle is formed by joining the mid-points of the sides of this triangle and the process is continued indefinitely. Determine the sum of areas of all such triangles including the given triangle.

48. Three numbers form a GP If the third term is decreased by 128 then, the three numbers, thus obtained, will form an AP If the second term of this AP is decreased by 16, a GP will be formed again. Determine the numbers.

49. If A, G and H are AM, GM and HM of any two positive numbers, then prove that $A \geq G \geq H$.

50. The product of three numbers of a GP is $\frac{64}{27}$. If the sum of their products when taken in pairs is $\frac{148}{27}$, then find the numbers.

CONCEPT APPLICATION

Level 1

1. Find t_5 and t_6 of the arithmetic progression $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$.

   (a) $1, \frac{5}{4}$ (b) $\frac{5}{4}, 1$

   (c) $1, \frac{7}{4}$ (d) $\frac{7}{4}, 1$

2. If $t_n = 6n + 5$, then $t_{n+1} =$

   (a) $6n - 1$ (b) $6n + 11$

   (c) $6n + 6$ (d) $6n - 5$

3. Which term of the arithmetic progression 21, 42, 63, 84, … is 420?

   (a) 19 (b) 20

   (c) 21 (d) 22

4. Find the 15th term of the arithmetic progression 10, 4, –2, ….

   (a) –72 (b) –74

   (c) –76 (d) –78

5. If the kth term of the arithmetic progression 25, 50, 75, 100, … is 1000, then k is ______.

(a) 20 (b) 30

(c) 40 (d) 50

6. The sum of the first 20 terms of an arithmetic progression whose first term is 5 and common difference is 4, is

(a) 820 (b) 830

(c) 850 (d) 860

7. Two arithmetic progressions have equal common differences. The first term of one of these is 3 and that of the other is 8, then the difference between their 100th terms is

(a) 4 (b) 5

(c) 6 (d) 3

8. If a, b and c are in arithmetic progression, then $b + c$, $c + a$ and $a + b$ are in

(a) arithmetic progression.

(b) geometric progression.

(c) harmonic progression.

(d) None of these

9. The sum of the first 51 terms of the arithmetic progression whose 2nd term is 2 and 4th term is 8, is

(a) 3774 (b) 3477

(c) 7548 (d) 7458

10. Three alternate terms of an arithmetic progression are $x + y$, $x - y$ and $2x + 3y$, then $x =$

(a) $-y$ (b) $-2y$

(c) $-4y$ (d) $-6y$

11. Find the 15th term of the series 243, 81, 27, ….

(a) $\frac{1}{3^{14}}$ (b) $\frac{1}{3^{8}}$

(c) $\left(\frac{1}{3}\right)^{9}$ (d) $\left(\frac{1}{3}\right)^{10}$

12. If t_8 and t_3 of a geometric progression are $\frac{4}{9}$ and $\frac{27}{8}$ respectively, then find t_{12} of the geometric progression.

(a) $\frac{64}{729}$ (b) $\frac{32}{243}$

(c) $\frac{729}{64}$ (d) $\frac{243}{32}$

13. If $t_n = 3^{n-1}$, then $S_6 - S_5 =$ ______.

(a) 243 (b) 81

(c) 77 (d) 27

14. Find the sum of the first 10 terms of geometric progression 18, 9, 4.5, ….

(a) $9\frac{(2^{10}-1)}{2^8}$ (b) $9\frac{(2^{10}-1)}{2^{10}}$

(c) $36\left(\frac{2^{10}-1}{2^8}\right)$ (d) $8\frac{(2^{10}-1)}{2^8}$

15. If the 3rd, 7th and 11th terms of a geometric progression are p, q and r respectively, then the relation among p, q and r is

(a) $p^2 = qr$ (b) $r^2 = qp$

(c) $q^2 = p^{2r2}$ (d) $q^2 = pr$

16. Evaluate $\Sigma(3 + 2^r)$, where $r = 1, 2, 3, \ldots, 10$.

(a) 2051 (b) 2049

(c) 2076 (d) 1052

17. Find the sum of the series $\frac{27}{8}+\frac{9}{4}+\frac{3}{2}+\ldots\infty$.

(a) $\frac{81}{8}$ (b) $\frac{27}{8}$

(c) $\frac{81}{16}$ (d) $\frac{9}{8}$

18. If $3x - 4$, $x + 4$ and $5x + 8$ are the three positive consecutive terms of a geometric progression, then find the terms.

(a) 2, 8, 32 (b) 2, 10, 50

(c) 2, 6, 18 (d) 12, 6, 3

19. Find the geometric mean of the first twenty five powers of twenty five.

(a) 5^{13} (b) 5^{19}

(c) 5^{24} (d) 5^{26}

20. Find the sum of 3 geometric means between $\frac{1}{3}$ and $\frac{1}{48}(r > 0)$.

(a) $\frac{1}{4}$ (b) $\frac{5}{24}$

(c) $\frac{7}{24}$ (d) $\frac{1}{3}$

21. If the second and the seventh terms of a Harmonic Progression are $\frac{1}{5}$ and $\frac{1}{25}$, then find the series.

(a) $1, \frac{1}{5}, \frac{1}{9}, \ldots$ (b) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots$

(c) $\frac{1}{7}, \frac{1}{5}, \frac{1}{3}, \ldots$ (d) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$

22. The 10th term of harmonic progression $\frac{1}{5}, \frac{4}{19}, \frac{2}{9}, \frac{4}{17}, \ldots$ is

(a) $\frac{11}{4}$ (b) $\frac{13}{4}$

(c) $\frac{4}{13}$ (d) $\frac{4}{11}$

23. If the ratio of the arithmetic mean and the geometric mean of two positive numbers is 3 : 2, then find the ratio of the geometric mean and the harmonic mean of the numbers.

(a) 2 : 3 (b) 9 : 4

(c) 3 : 2 (d) 4 : 9

24. If A, G and H are AM, GM and HM of any two given positive numbers, then find the relation between A, G and H.

(a) $A^2 = GH$ (b) $G^2 = AH$

(c) $H^2 = AG$ (d) $G^3 = A^2H$

25. Find the least value of n for which the sum $1 + 2 + 2^2 + \ldots$ to n terms is greater than 3000.

(a) 8 (b) 10

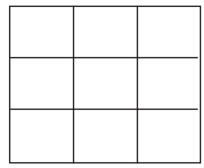
(c) 12 (d) 15

26. Find the HM of $\frac{1}{7}$ and $\frac{1}{12}$.

(a) $\frac{1}{19}$ (b) $\frac{2}{19}$

(c) $\frac{3}{19}$ (d) $\frac{4}{19}$

27. Number of rectangles in the following figure is ______ .



(a) 9 (b) 10

(c) 24 (d) 36

28. In a series, if $t_n = \frac{n^2 - 1}{n + 1}$, then $S_6 - S_3 =$ ______.

(a) 3 (b) 12

(c) 22 (d) 25

29. Find the number of terms to be added in the series 27, 9, 3, … so that the sum is $\frac{1093}{27}$.

(a) 6 (b) 7

(c) 8 (d) 9

30. Find the value of p ($p > 0$) if $\frac{15}{4} + p, \frac{5}{2} + 2p$ and $2 + p$ are the three consecutive terms of a geometric progression.

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$

(c) $\frac{5}{3}$ (d) $\frac{1}{2}$

Level 2

31. If $\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in AP, then a^2, b^2 and c^2 are in

(a) geometric progression.

(b) arithmetic progression.

(c) harmonic progression.

(d) None of these

32. Among the following, which term belongs to the arithmetic progression –5, 2, 9, ...?

(a) 342 (b) 343

(c) 344 (d) 345

33. Five distinct positive integers are in arithmetic progression with a positive common difference. If their sum is 10020, then find the smallest possible value of the last term.

(a) 2002 (b) 2004

(c) 2006 (d) 2008

34. In a right triangle, the lengths of the sides are in arithmetic progression. If the lengths of the sides of the triangle are integers, which of the following could be the length of the shortest side?

(a) 2125 (b) 1700

(c) 1275 (d) 1150

35. If $S_1 = 3, 7, 11, 15, \ldots$ upto 125 terms and $S_2 = 4, 7, 10, 13, 16, \ldots$ upto 125 terms, then how many terms are there in S_1 that are there in S_2?

(a) 29 (b) 30 (c) 31 (d) 32

36. The first term and the mth term of a geometric progression are a and n respectively and its nth term is m. Then its $(m + 1 - n)$th term is ______.

(a) $\frac{ma}{n}$ (b) $\frac{na}{m}$

(c) mna (d) $\frac{mn}{a}$

37. The sum of the terms of an infinite geometric progression is 3 and the sum of the squares of the terms is 81. Find the first term of the series.

(a) 5 (b) $\frac{27}{5}$

(c) $\frac{31}{6}$ (d) $\frac{19}{3}$

38. If $\log_{\sqrt{2}} x + \log_{\sqrt{\sqrt{2}}} x + \log_{\sqrt{\sqrt{\sqrt{2}}}} x + \ldots$ upto 7 terms = 1016, the find the value of x.

(a) 4 (b) 16 (c) 64 (d) 2

39. For which of the following values of x is $8^{1+\sin x+\sin^2 x+\sin^3 x+\ldots\infty} = 64$?

(a) 60° (b) 135° (c) 45° (d) 30°

40. Find the sum of all the multiples of 6 between 200 and 1100.

(a) 96750 (b) 95760

(c) 97560 (d) 97650

41. If the kth term of a HP is λp and the λth term is kp and $k \neq \lambda$, then the pth term is

(a) $k^2\lambda$ (b) k^2p

(c) p^2k (d) λk

42. If six harmonic means are inserted between 3 and $\frac{6}{23}$, then the fourth harmonic mean is

(a) $\frac{6}{11}$ (b) $\frac{6}{17}$ (c) $\frac{3}{7}$ (d) $\frac{3}{10}$

43. If a, b and c are positive numbers in arithmetic progression. and a^2, b^2 and c^2 are in geometric progression then a^3, b^3 and c^3 are in

(A) arithmetic progression.

(B) geometric progression.

(C) harmonic progression.

(a) (A) and (B) only

(b) only (C)

(c) (A), (B) and (C)

(d) only (B)

44. The arithmetic mean A of two positive numbers is 8. The harmonic mean H and the geometric mean G of the numbers satisfy the relation $4H + G^2 = 90$. Then one of the two numbers is ______.

(a) 6 (b) 8

(c) 12 (d) 14

45. The infinite sum $\sum_{n=1}^{\infty}\left(\frac{5^n + 3^n}{5^n}\right)$ is equal to

(a) $\frac{3}{2}$ (b) $\frac{3}{5}$

(c) $\frac{2}{3}$ (d) None of these

46. (i) If $x = 3 + \frac{3}{y} + \frac{3}{y^2} + \frac{3}{y^3} + \ldots\infty$, then, show that $y = \frac{x}{x-3}$. (Where $|y| < 1$). The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $xy - 3y = x$ (B) $x = 3\left(\frac{1}{1-\frac{1}{y}}\right)$

(C) $y(x - 3) = x$ (D) $x = 3\left(\frac{y}{y-1}\right)$

(a) BDCA (b) BDAC

(c) CABD (d) ACBD

47. Find the harmonic mean of 5 and 3.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $\text{HM} = \frac{2 \times 5 \times 3}{5+3}$

(B) We know that the harmonic mean of a, b is $\frac{2ab}{a+b}$

(C) Here $a = 5$ and $b = 3$

(D) $\text{HM} = \frac{30}{8} = \frac{15}{4}$

(a) BCDA (b) BCAD

(c) ABCD (d) BADC

Level 3

48. The numbers $h_1, h_2, h_3, h_4, \ldots, h_{10}$ are in harmonic progression and $a_1, a_2, \ldots, a_{10}$ are in arithmetic progression. If $a_1 = h_1 = 3$ and $a_7 = h_7 = 39$, then the value of $a_4 \times h_4$ is

(a) $\frac{13}{49}$ (b) $\frac{182}{3}$

(c) $\frac{7}{13}$ (d) 117

49. Find the value of

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{16}\right)\left(1+\frac{1}{256}\right)\ldots\infty.$$

(a) 1 (b) 2

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

50. The ratio of the sum of n terms of two arithmetic progressions is given by $(2n+3) : (5n-7)$. Find the ratio of their nth terms.

(a) $(4n + 5) : (10n + 2)$

(b) $(4n + 1) : (10n - 12)$

(c) $(4n - 1) : (10n + 8)$

(d) $(4n - 5) : (10n - 2)$

51. There are n arithmetic means (were $n \in N$) between 11 and 53 such that each of them is an integer. How many distinct arithmetic progressions are possible from the above data?

(a) 7 (b) 8

(c) 14 (d) 16

52. If $x = \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \ldots\infty$, then find the value of $x + \frac{1}{x}$.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $3\sqrt{2}$ (d) $4\sqrt{2}$

53. In a GP of 6 terms, the first and last terms are $\frac{x^3}{y^2}$ and $\frac{y^3}{x^2}$ respectively. Find the ratio of 3rd and 4th terms of that GP.

(a) $x^2 : 1$ (b) $y^2 : x$

(c) $y : x$ (d) $x : y$

54. If $x = 3 + \frac{3}{y} + \frac{3}{y^2} + \frac{3}{y^3} + \ldots\infty$, then y = ______ (where $|y| > 1$).

(a) $\frac{x}{3}$ (b) $\frac{x}{x-3}$

(c) $\frac{1-x}{3}$ (d) $1-\frac{3}{x}$

55. Find the sum of $\frac{0.3}{0.5} + \frac{0.33}{0.55} + \frac{0.333}{0.555} + \ldots$ to 15 terms.

(a) 10 (b) 9

(c) 3 (d) 5

56. In a GP, if the fourth term is the square of the second term, then the relation between the first term and common ratio is ______.

(a) $a = r$ (b) $a = 2r$

(c) $2a = r$ (d) $r^2 = a$

57. For which of the following values of x is $(0° < x < 90°)$ $16^{1+\cos x+\cos^2 x+\cos^3 x+\ldots\infty} = 256$?

(a) 30° (b) 45°

(c) 60° (d) 15°

58. If t_2 and t_3 of a GP are p and q respectively, then t_5 = ______.

(a) $p\left(\frac{q}{p}\right)^3$ (b) $p\left(\frac{q}{p}\right)^2$

(c) $\frac{p^2}{q^3}$ (d) p^2q^2

59. If a, b, c, d are in GP, then $(b + c)^2$ = ______.

(a) $(b + d)(a + d)$ (b) $(a + d)(c + d)$

(c) $(a + b)(c + d)$ (d) $(a + c)(b + d)$

60. a, b, c are in GP If a is the first term and c is the common ratio, then b = ______.

(a) 1 (b) $\frac{1}{a}$

(c) $\frac{1}{c}$ (d) None of these

61. In a GP of 7 terms, the last term is $\frac{64}{81}$ and the common ratio is $\frac{2}{3}$. Find the 3rd term.

(a) 4 (b) 9

(c) 8 (d) 12

PRACTICE QUESTIONS

62. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then find the fourth term.

(a) 2 (b) 3
(c) 5 (d) 6

63. If the sum of 16 terms of an AP is 1624 and the first term is 500 times the common difference, then find the common difference.

(a) 5 (b) $\frac{1}{2}$
(c) $\frac{1}{5}$ (d) 2

64. Find the sum of the series $1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + \cdots + (1 + 2 + 3 + \cdots + 20)$.

(a) 1470
(b) 1540
(c) 1610
(d) 1370

65. Evaluate $\Sigma 2^i$, where $i = 2, 3, 4\ldots 10$.

(a) 2044
(b) 2048
(c) 1024
(d) 1022

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 11
2. $a + nd$
3. 3
4. $5n$
5. 7.5
6. False
7. $\frac{n}{2}[a+(n-1)d]$
8. $S_n - S_{n-1}$
9. $3n + 5$
10. $\frac{(n+1)(n+2)}{2}$
11. 3
12. geometric
13. False
14. geometric
15. True
16. 10
17. False
18. geometric
19. $\frac{1}{100^n}$
20. $1 + x^2 + x^4 + x^6 + \ldots$
21. False
22. arithmetic
23. True
24. ac
25. 2
26. 3, –2, –7.
27. 25, 125, 625.
28. $\frac{1}{4}$
29. $\pm\sqrt{\frac{y}{x}}$
30. 0

Shot Answer Type Questions

31. −10
32. −39
33. 57
34. $8n - 1$
35. 82650
36. $r = \frac{1}{4}$
37. $\frac{1}{9}$
38. ₹25600
39. 121
40. $\frac{707}{333}$
41. 448 m
42. $\frac{50}{81}(10^n - 1) - \frac{5n}{9}$
43. $\frac{12}{5}$
44. 0
45. 180 and 20

Essay Type Questions

46. $\frac{3}{2}n^2(n+1)^2$
47. $432\sqrt{3}\ \text{cm}^2$
48. $\frac{8}{9}, \frac{104}{9}$ and $\frac{1352}{9}$
50. $1, \frac{4}{3}, \frac{16}{9}$

CONCEPT APPLICATION

Level 1

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (d)
11. (c)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (c)
21. (a)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (b)	**30.** (b)

Level 2

31. (b)	**32.** (d)	**33.** (c)	**34.** (c)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (d)
41. (d)	**42.** (c)	**43.** (c)	**44.** (a)	**45.** (d)	**46.** (b)	**47.** (b)			

Level 3

48. (d)	**49.** (b)	**50.** (b)	**51.** (c)	**52.** (b)	**53.** (d)	**54.** (b)	**55.** (b)	**56.** (a)	**57.** (c)
58. (a)	**59.** (c)	**60.** (a)	**61.** (a)	**62.** (a)	**63.** (c)	**64.** (b)	**65.** (a)		

CONCEPT APPLICATION

Level 1

1. The nth term in AP = $t_n = a + (n - 1)\ d$.
2. Substitute $n = n + 1$ in t_n.
3. Use formula to find nth term of an AP.
4. Use the formula to find the nth term of an AP.
5. Use the formula of nth term of an AP.
6. Use the formula to find S_n of an AP.
7. The difference of nth term of two AP's having same common difference is the difference of their first terms.
8. If a, b and c are in AP, then $2b = a + c$.
9. Find a and d using the given data and use the formula to find S_n of AP.
10. The difference between t_3 and t_1 is same as t_5 and t_3.
11. Use the formula to find the nth term of a GP.
12. Find a and r using the given data.
13. $S_6 - S_5 = t_6$.
14. Use the formula of S_n of a GP.
15. $p = ar_1^2, q = ar_1^6$ and $r = ar_1^{10}$.
16. $\Sigma\ (3 + 2r) = 3\Sigma 1 + \sum_{r=1}^{10} 2^r$.
17. Use the formula to find S_∞ of a GP.
18. If a, b and c are in GP, then $b^2 = ac$.
19. Form the series and use the formula to find S_n.
20. The 3 geometric means are t_2, t_3 and t_4 terms.
21. Use the formula of nth term of a HP.
22. Use the formula of the nth term of a HP.
23. Use the relation between AM, GM and HM.
24. (i) Consider two numbers as a and b.
    (ii) Then find AM, GM and HM.
25. (i) $S_n = \dfrac{a(r^n - 1)}{(r - 1)}$
    (ii) $a = 1$ and $r = 2$
    (iii) Given that $S_n > 3000$
    (iv) Then find least possible value of n.
26. HM of a and b is $\dfrac{2ab}{a + b}$.
28. $S_6 - S_3 = T_4 + T_5 + T_6$.
29. Use the formula of S_n of a GP.
30. If a, b and c are in GP, then $b^2 = ac$.

Level 2

31. $t_2 - t_1 = t_3 - t_2$
32. Each term is in the form of $7n + 2$, where $n = -1, 0, 1, 2, 3, \ldots$.
33. Let the five integers be $a - 2d$, $a - d$, a, $a + d$ and $a + 2d$.
34. (i) The sides must be in the ratio 3 : 4 : 5.
    (ii) Shortest side should be a multiple of 3.
35. (i) Common terms in S_1 and S_2 are 7, 19, 31, ….
    (ii) The first term of S_1 is 3. The 125th term is 499.
    The first term of S_2 is 4. The 125th term is 376.
    (iii) The last common term is 367 (and not 499).
    (iv) $7 = 12(1) - 5$, $19 = 12(2) - 5$, ..., $367 = 12(31) - 5$.
36. (i) Use the formula to find nth term of a GP.
    (ii) $ar^{m-1} = n$, $ar^{n-1} = m$.
    (iii) $(m + 1 - n)$th term = ar^{m-n}.
37. (i) Given $S_\infty = \dfrac{a}{1 - r} = 3$.
    (ii) $a^2 + a^2r^2 + a^2r^4 + \cdots = 81$.
38. (i) Use $\log_{b^n} a = \dfrac{1}{n}\log_b a$.
    (ii) $S_n = \dfrac{a(r^n - 1)}{(r - 1)}$
    (iii) If $\log_a x = b$, then $a^b = x$.
39. (i) Use the formula to find S_∞ of a GP.
    (ii) Equate the powers on either sides by making equal bases.

40. Form the series and find the value of n and use the formula $S_n = \frac{n}{2}(2a+(n-1)d)$.

41. $t_n = \frac{1}{a+(n-1)d}$ in HP.

42. (i) $\frac{1}{a} = 3, \frac{1}{a+7d} = \frac{6}{23}$.

(ii) Find a and d by using above relation find t_5 using $\frac{1}{a+4d}$.

43. Use $b = \frac{a+c}{2}$ and $(b^2)^2 = a^2c^2$.

44. Use $G^2 = AH$.

45. (i) $\sum_{n=1}^{\infty}\left(1+\left(\frac{3}{5}\right)^n\right) = 1+\frac{3}{5}+\left(\frac{3}{5}\right)^2+\ldots\infty$

(ii) $S_\infty = \frac{a}{1-r}$, where a *is* the first term and r is the common ratio.

46. BDAC is the sequential order of steps.

47. BCAD is the sequential order of steps.

Level 3

48. (i) In AP, $t_n = a + (n-1)d$ and in HP, $t_n = \frac{1}{a+(n-1)d}$.

(ii) Find the values of a and d by using the data

49. (i) Let $p = \left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\ldots$

(ii) Multiply with $\left(1-\frac{1}{2}\right)$ on both sides.

(iii) $(a+b)(a-b) = a^2 - b^2\ a,\ n - \infty\ \frac{1}{2^n} = 0$.

50. (i) Let a_1 and d_1 be the first term and common difference of the first AP and a_2, d_2 be the corresponding values for the second AP.

(ii) $S_n : S'_n$ has to be converted in the form of $t_n : t'_n$.

51. (i) n arithmetic means are in between a and b; $d = \frac{b-a}{n+1}$.

(ii) evaluate for how many values of $(n+1)$, d is an integer.

52. $x = \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \ldots\infty$

$$x = \frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1}$$

$$x = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1 \quad \Rightarrow \quad \frac{1}{x} = \sqrt{2}-1$$

$$\therefore \quad x + \frac{1}{x} = \sqrt{2}+1+\sqrt{2}-1 = 2\sqrt{2}.$$

53. $a = \frac{x^3}{y^2} = t_1;\ b = \frac{y^3}{x^2} = t_6$

There are 4 GM's between a and b.

$$\therefore \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$r = \left[\frac{\left(\frac{y^3}{x^2}\right)}{\left(\frac{x^3}{y^2}\right)}\right]^{\frac{1}{4+1}} \quad \Rightarrow \quad r = \left[\left(\frac{y}{x}\right)^5\right]^{\frac{1}{5}}$$

$$\Rightarrow \quad r = \frac{y}{x}$$

but the ratio of 3rd and 4th term is $\frac{1}{r}$.

$$\Rightarrow \quad \frac{1}{r} = \frac{x}{y}.$$

There fore the required ratio is $x : y$.

54. $x = 3 + \frac{3}{y} + \frac{3}{y^2} + \frac{3}{y^3} + \ldots\infty$

$$x = 3\left[\frac{1}{1-\frac{1}{y}}\right]$$

$$x = 3\left[\frac{y}{y-1}\right] \quad \Rightarrow \quad xy - x = 3y$$

$$xy - 3y = x$$

$$y(x-3) = x \quad \Rightarrow \quad y = \frac{x}{x-3}.$$

55. $\dfrac{0.3}{0.5} + \dfrac{0.33}{0.55} + \dfrac{0.333}{0.555} \ldots 15$ terms

$$S = \frac{3}{5} + \frac{33}{55} + \frac{333}{555} + \cdots + 15 \text{ terms}$$

$$S = \frac{3}{5}\left[1 + \frac{11}{11} + \frac{111}{111} + \ldots 15 \text{ terms}\right]$$

$$S = \frac{3}{5} \times 15 \quad \Rightarrow \quad S = 9.$$

56. $t_4 = t_2^2$

$\Rightarrow \quad ar^3 = (ar)^2 \quad \Rightarrow \quad ar^3 = a^2r^2 \quad \Rightarrow \quad r = a.$

57. $16^{1+\cos x+\cos^2 x+\cos^3 x+\ldots\infty} = 256$

$$16^{\frac{1}{1-\cos x}} = 256 \quad \Rightarrow \quad 16^{\frac{1}{1-\cos x}} = 16^2$$

$$\Rightarrow \quad \frac{1}{1-\cos x} = 2 \quad \Rightarrow \quad 1 - \cos x = \frac{1}{2}$$

$$1 - \frac{1}{2} = \cos x.$$

$$\frac{1}{2} = \cos x \quad \Rightarrow \quad x = 60°.$$

58. $t_2 = ar = p$

$t_3 = ar^2 = q$

$$\Rightarrow \quad \frac{t_3}{t_2} = \frac{q}{p} = \frac{ar^2}{ar} \quad \Rightarrow \quad r = \frac{q}{p}$$

$ar = p$

$$\Rightarrow \quad a \cdot \frac{q}{p} = p \quad \Rightarrow \quad a = \frac{p^2}{q}$$

$t_5 = a \cdot r^4$

$$= \frac{p^2}{q}\left(\frac{q}{p}\right)^4$$

$$= \frac{p^2}{q} \times \frac{q^4}{p^4} = \frac{q^3}{p^2} = p\left(\frac{q}{p}\right)^3.$$

59. a, b, c, d are in GP

$a = a,\ b = ar,\ c = ar^2,\ d = ar^3$

$b + c = ar + ar^2 = ar(1 + r)$

$(b + c)^2 = a^2r^2\ (1 + r^2 + 2r)$

$= a^2r^2 + a^2r^4 + 2a^2r^3$

$= a \cdot ar^2 + ar \cdot ar^3 + a^2r^3 + a^2r^3$

$= a \cdot c + bd + bc + ad = a(c + d) + b\ (c + d)$

$= (a + b)(c + d).$

60. a, b, c are in GP

$$r = \frac{b}{a} = \frac{c}{b}$$

but given that $r = c$

$$\Rightarrow \quad \frac{c}{b} = c \quad \Rightarrow \quad b = 1.$$

61. Let a be the first term and r the common ratio of the GP.

$$T_\gamma = ar^6 = \frac{64}{81} \quad \Rightarrow \quad a = 9$$

There fore the 3rd term $T_3 = ar^2$

$$T_3 = 9\left(\frac{2}{3}\right)^2 = 4.$$

62. Given, $\dfrac{11}{2}\ [2a + 10d] = 33$

$\Rightarrow \quad a + 5d = 3$

$\Rightarrow \quad a + 3d = 2$

Since alternate terms are integers and the given sum is positive.

63. $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$$\frac{16}{2}[2 \times 500d + (16-1)d] = 1624$$

$$8[1000d + 15d] = 1624$$

$$1015d = \frac{1624}{8}$$

$$1015d = 203$$

$$d = \frac{203}{1015} \quad \Rightarrow \quad d = \frac{1}{5}.$$

64. $1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + \ldots$ 20 terms

$= 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 + 66 + 78 + 91 + 105 + 120 + 136 + 153 + 171 + 190 + 210$

$= 1540.$

Alternative method:

$1 + (1 + 2) + (1 + 2 + 3) + \ldots$

nth term of the series is

$1 + 2 + 3 + 4 + \ldots n = \dfrac{n(n+1)}{2}$

Sum of n terms of the series

$= \sum t_n$

$= \sum \dfrac{n(n+1)}{2} \quad \Rightarrow \quad = \dfrac{1}{2}\sum(n^2 + n)$

$= \dfrac{1}{2}\left(\sum n^2 + \sum n\right)$

$= \dfrac{1}{2}\left[\dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(n+1)}{2}\right]$

$= \dfrac{n(n+1)}{4}\left(\dfrac{2n+1}{3} + 1\right)$

$\Rightarrow \quad \dfrac{n(n+1)(2n+4)}{12} = \dfrac{n(n+1)(n+2)}{6}.$

If $n = 20$

$\sum t_{20} = \dfrac{20(21)(22)}{6}$

$\Rightarrow \quad \dfrac{10 \times 21 \times 22}{3} = 1540.$

65. $\sum 2^i, i = 2, 3, \ldots 10$

$= (2^2 + 2^3 + \cdots + 2^{10})$

$= 2^2 \times \dfrac{(2^9 - 1)}{2 - 1}$

$= \dfrac{2^{11} - 2^2}{1}$

$= 2044.$