Permutations and Combinations

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

1. Assertion (A): If $5.{}^4P_r = 6.{}^5P_{r-1}$, then r = 3.

Reason (R): If ${}^5P = {}^6P_{r-1}$ then r = 9.

Ans. (c) (A) is true but (R) is false.

Explanation: We have $5.{}^4Pr = 6.{}^5Pr-1$

$$\Rightarrow \quad 5\times\frac{4!}{(4-r)!} = 6\times\frac{5!}{(5-r+1)!}$$

$$\Rightarrow \quad \frac{5!}{(4-r)!} = \frac{6\times5!}{(6-r)(5-r)(4-r)!}$$

$$\Rightarrow \quad (7-r)(6-r) = 6$$

$$\Rightarrow \quad 42-r^2-13r = 6$$

$$\Rightarrow \quad (6-r)(5-r) = 6$$

$$\Rightarrow \quad r^2-11r+24 = 0$$

$$\Rightarrow \quad (r-8)(r-3) = 0$$

$$\Rightarrow \quad r = 8, 3$$

But $r \neq 8$ as $r \leq 4$

$$\therefore \quad r = 3$$

We have, $\quad {}^5P_r = {}^6P_{r-1}$

$$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)(6-r)(5-r)!}$$

$$\Rightarrow \quad r^2-13r+36 = 0$$

$$\Rightarrow \quad (r-4)(r-9) = 0$$

$$\Rightarrow \quad r = 4, 9$$

$$\Rightarrow \quad r = 4 \qquad [\because r \neq 9]$$

2. Assertion (A): The number of permutations of letters of word 'ROOT' are 10.

Reason (R): The number of permutations of letters of word 'INSTITUTE' is

$$\frac{9!}{2!3!}.$$

Ans. (d) (A) is false but (R) is true.

Explanation: There are 4 letters in 'Root' of which there are 20's and rest are different.

Therefore, the required number of arrangements

$$= \frac{4!}{2!} = 12$$

There are 9 letters in 'INSTITUTE' of which there are 2 I's, 3T's and rest are different.

Therefore, the required number of arrangements

$$= \frac{9!}{2!3!}$$

3.

Assertion (A) : $^nP_r = \frac{n!}{(n-r)!}, 0 \le r \le n.$

Reason (R): $^nP_r = n(n-1)(n-2)....(n-r+1),$
$0 \le r \le n.$

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: We know,

$$^nP_r = n(n-1)(n-2)....(n-r+1)$$

$$= \frac{n(n-1)(n-2)...(n-r+1)(n-r)...3\times2\times1}{(n-r)(n-r-1)...3\times2\times1}$$

$$= \frac{n!}{(n-r)!}$$

4. Assertion (A): The value of $^6P_4$ is 360.

Reason (R): $^nP_r = P(n, r) = \frac{n!}{(n-r)!}.$

Ans. (a) Both A and R are true and R is the correct explanation of A.

Explanation: We know that,

$$^nP_r = P(n, r) = \frac{n!}{(n-r)!}$$

$$^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$$

$$= \frac{6\times5\times4\times3\times2!}{2!}$$

$$= 360$$

5. Assertion (A): Number of lines formed by joining n points on a circle $(n \geq 2)$ is $\frac{n(n-1)}{2}$.

Reason (R): $C(n, 3) = \frac{n(n-1)}{2}$.

Ans. (c) (A) is true but (R) is false.

Explanation: Number of lines is $^nC_2 = \frac{n(n-1)}{2}$

$$C(n, 3) = \frac{n!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{6}$$

6. Assertion (A): The product of five consecutive natural numbers is divisible by 4!

Reason (R): Product of n consecutive natural numbers is divisible by (n + 1)!.

Ans. (c) (A) is true but (R) is false.

Explanation: Product of n consecutive natural numbers

$$= (m + 1)(m + 2)(m + 3) \ldots (m + n), m \in W$$

$$= \frac{(m+n)!}{m!} = n! \times \frac{(m+n)!}{m!n!} = n! \times {}^{m+n}C_m$$

Product is divisible by n! and so it is always divisible by (n-1)! but not by (n + 1)!

7. Assertion (A): Number of rectangles on a chess board is $^8C_2$ x $^8C_2$

Reason (R): To form a rectangle, we have to select any two of the horizontal line and any two of the vertical line.

Ans. (d) (A) is false but (R) is true.

Explanation: To form a rectangle, we have to select any two of the horizontal line and any

two of the vertical line.
In a chess board, there are 9 horizontal and 9 vertical lines. Number of rectangles of any size are $^9C_2 \times {}^9C_2$

8. Assertion (A): If n is a positive integer, then $n(n^2 - 1)(n+ 2)$ is divisible by 24.
Reason (R): Product of r consecutive positive integers is divisible by r!.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
Explanation: $n(n^2 - 1)(n+ 2) = (n - 1)n(n + 1)(n + 2)$ is the product of four consecutive positive integers and hence it is divisible by 24.

9. Assertion (A): The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$.
Reason (R): The number of ways of choosing any 3 places, from 9 different places is $^9C_3$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
Explanation: Let the number of ways of distributing n identical objects among r persons such that each person gets at least one object is same as the number of ways of selecting (r-1) places out of (n - 1) different places, i.e.,

$^{n-1}C_{r-1}$.

$\therefore \quad {}^{10-1}C_{4-1}$

The number of ways will become $^9C_3$.