04

Laws of Motion

TOPIC 1

Newtons Laws of Motion and Conservation of Momentum

01 A ball of mass 0.15 kg is dropped from a height 10 m, strikes the ground and rebounds to the same height. The magnitude of impulse imparted to the ball is nearly ($g = 10$ m/s^2) **[NEET 2021]**

(a) 0 (b) 4.2 kg-m/s
(c) 2.1 kg-m/s (d) 1.4 kg-m/s

Ans. ***(b)***

Given, the mass of the ball dropped from the height, $m = 0.15$ kg

The height from the ball dropped,

$$h = 10 \text{ m}$$

We know that,

$$|\text{Impulse}| = m|\Delta \bar{v}|$$

where, $\Delta \bar{v} = v_2 - v_1$

Here, v_2 is velocity reaches to the same height,

v_1 is velocity just before striking to the ground.

For case (1), ball dropped from the 10m height and strikes to the ground.

Now, the velocity of the ball just before striking to the ground is

$$v_1 = -\sqrt{2gh}$$

$$\Rightarrow v_1 = -\sqrt{2(10)(10)}$$

$$\Rightarrow v_1 = -10\sqrt{2} \text{ m/s}$$

For case (2), ball rebounds to the same height.

The velocity with which the ball just reaches to the same height,

$$v_2 = \sqrt{2gh}$$

$$\Rightarrow v_2 = \sqrt{2(10)(10)}$$

$$\Rightarrow v_2 = 10\sqrt{2} \text{ m/s}$$

Now, magnitude of the impulse imparted to the ball,

$$|\text{Impulse}| = m|\Delta \bar{v}|$$
$$= 0.15\ |10\sqrt{2} - (-10\sqrt{2})|$$
$$= 0.15|2 \times 10\sqrt{2}|$$
$$= 4.2 \text{ kg-m/s}$$

02 A truck is stationary and has a bob suspended by a light string, in a frame attached to the truck. The truck, suddenly moves to the right with an acceleration of a. The pendulum will tilt **[NEET (Odisha) 2020]**

(a) to the left and the angle of inclination of the pendulum with the vertical is $\sin^{-1}\left(\frac{g}{a}\right)$

(b) to the left and angle of inclination of the pendulum with the vertical is $\tan^{-1}\left(\frac{a}{g}\right)$

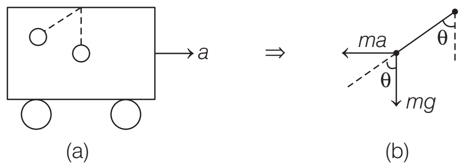
(c) to the left and angle of inclination of the pendulum with the vertical is $\sin^{1-}\left(\frac{a}{g}\right)$

(d) to the left and angle of inclination of the pendulum with the vertical is $\tan^{-1}\left(\frac{g}{a}\right)$

Ans. ***(b)***

As the truck move to the right, so the bob will move to the left due to inertia of rest with acceleration a.

Thus, the given situation can be drawn as

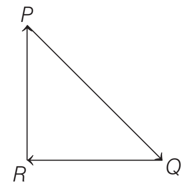


(a) (b)

From the above diagram (b) as the string moves by an angle of θ with the vertical then the tangent angle is

$$\tan\theta = \frac{ma}{mg} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

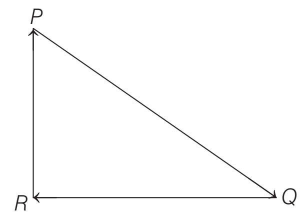
03 A particle moving with velocity **v** is acted by three forces shown by the vector triangle *PQR*.
The velocity of the particle will **[NEET (National) 2019]**



(a) decrease
(b) remain constant
(c) change according to the smallest force *QR*
(d) increase

Ans. ***(b)***

As the three forces are represented by three sides of a triangle taken in order, then they will be in equilibrium.



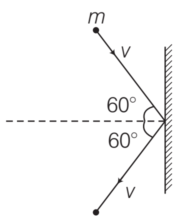
$$\Rightarrow \mathbf{F}_{\text{net}} = \mathbf{F}_{PQ} + \mathbf{F}_{QR} + \mathbf{F}_{RP} = 0$$

$$\mathbf{F}_{\text{net}} = m \times \mathbf{a} = m\frac{d\mathbf{v}}{dt} = 0 \Rightarrow \frac{d\mathrm{v}}{dt} = 0$$

or $\mathbf{v} = $ constant

So, the velocity of particle remain constant.

04 A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure. The value of impulse imparted by the wall on the ball will be **[NEET 2016]**



(a) mv (b) $2mv$ (c) $mv/2$ (d) $mv/3$

Ans. ***(a)***

As we know that, impulse is imparted due to change in perpendicular components of momentum of ball.

$$J = \Delta p = mv_f - mv_i$$
$$= mv\cos 60^\circ - (-mv\cos 60^\circ)$$
$$= 2mv\cos 60^\circ = 2mv \times \frac{1}{2} = mv$$

05 A bullet of mass 10 g moving horizontal with a velocity of 400 m/s strikes a wood block of mass 2 kg which is suspended by light inextensible string of length 5 m. As result, the centre of gravity of the block found to rise a vertical distance of 10 cm. The speed of the bullet after it emerges of horizontally from the block will be **[NEET 2016]**

(a) 100 m/s (b) 80 m/s
(c) 120 m/s (d) 160 m/s

Ans. ***(c)***

According to the law of conservation of momentum, $p_i = p_f$

$\Rightarrow$ $(0.01)\times 400 + 0 = 2v + (0.01)v'$ ...(i)

Also velocity v of the block just after the collision is

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 0.1} = \sqrt{2} \quad \text{...(ii)}$$

$\Rightarrow$ From Eqs. (i) and (ii), we have

$$v' \approx 120 \text{ m/s}$$

06 The force F acting on a particle of mass m is indicated by the force-time graph shown below. The change in momentum of the particle over the time interval from 0 to 8 s is **[CBSE AIPMT 2014]**

(a) 24 N-s (b) 20 N-s (c) 12 N-s (d) 6 N-s

Ans. ***(c)***

The area under F-t graph gives change in momentum.

For 0 to 2 s, $\Delta p_1 = \frac{1}{2}\times 2\times 6$

$$= 6 \text{ kg-m/s}$$

For 2 to 4 s, $\Delta p_2 = 2\times -3 = -6$ kg-m/s

For 4 to 8 s, $\Delta p_3 = 4\times 3 = 12$ kg-m/s

So, total change in momentum for 0 to 8 s

$$\Delta p_{\text{net}} = \Delta p_1 + \Delta p_2 + \Delta p_3$$
$$= (+6 - 6 + 12)$$
$$= 12 \text{ kg-m/s} = 12 \text{ N-s}$$

NOTE

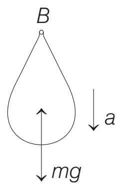
Graphs on negative axis gives – ve momentum.

07 A balloon with mass m is descending down with an acceleration a (where, $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a? **[CBSE AIPMT 2014]**

(a) $\frac{2ma}{g+a}$ (b) $\frac{2ma}{g-a}$ (c) $\frac{ma}{g+a}$ (d) $\frac{ma}{g-a}$

Ans. ***(a)***

When, the balloon is descending down with acceleration a



So, from free body diagram

$$mg - B = m\times a \quad \text{...(i)}$$

[$B \to$ Buoyant force]

Here, we should assume that while removing same mass the volume of balloon and hence,buoyant force will not change.

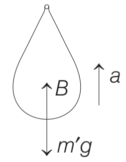
Let the new mass of the balloon be m'.

So, mass removed $(m - m')$

So, $B - m'g = m'\times a$ ...(ii)

Solving Eqs. (i) and (ii),

$$mg - B = m\times a$$
$$B - m'g = m'\times a$$
$$mg - m'g = ma + m'a$$
$$(mg - ma) = m'(g+a)$$
$$\Rightarrow \quad m(g-a) = m'(g+a)$$
$$m' = \frac{m(g-a)}{g+a}$$



So, mass removed $= m - m'$

$$\Rightarrow \quad m\left[1 - \frac{(g-a)}{(g+a)}\right]$$
$$= m\left[\frac{(g+a)-(g-a)}{(g+a)}\right]$$
$$\Rightarrow \quad m\left[\frac{g+a-g+a}{g+a}\right] \Rightarrow \Delta m = \frac{2ma}{g+a}$$

08 An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with 8 ms^{-1} speed. If the third part flies off with 4 ms^{-1} speed, then its mass is **[NEET 2013]**

(a) 3 kg (b) 5 kg
(c) 7 kg (d) 17 kg

Ans. ***(b)***

Concept Momentum is conserved before and after collision.

We have, $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0 \quad [\because p = mv]$

$\therefore \quad 1\times 12\mathbf{i} + 2\times 8\mathbf{j} + \mathbf{p}_3 = 0$

$\Rightarrow \quad 12\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + \mathbf{p}_3 = 0$

$\Rightarrow \quad \mathbf{p}_3 = -(12\mathbf{i} + 16\mathbf{j})$

$\therefore \quad \mathbf{p}_3 = \sqrt{(12)^2 + (16)^2}$

$$= \sqrt{144 + 256}$$
$$= 20 \text{ kg-m/s}$$

Now, $\mathbf{p}_3 = m_3 v_3$

$\Rightarrow \quad m_3 = \frac{\mathbf{p}_3}{v_3} = \frac{20}{4} = 5$ kg

09 Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision, B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction **[CBSE AIPMT 2012]**

(a) same as that of B
(b) opposite to that of B

(c) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the x-axis

(d) $\theta = \tan^{-1}\left(\frac{-1}{2}\right)$ to the x-axis

Ans. ***(c)***

Here, $\mathbf{p}_i = m_2 v\mathbf{i} + m_1 \times 0$

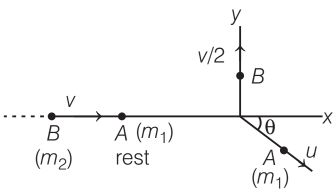
$$\mathbf{p}_f = m_2 \frac{v}{2}\hat{\mathbf{j}} + m_1 \times \mathbf{v}_1$$

Law of conservation of momentum

$$\mathbf{p}_i = \mathbf{p}_f$$

$$m_2 v\hat{\mathbf{i}} = m_2 \frac{v}{2}\hat{\mathbf{j}} + m_1 \times \mathbf{v}_1$$

$$\mathbf{v}_1 = \frac{m_2}{m_1} v\,\hat{\mathbf{i}} + \frac{m_2}{m_1}\frac{v}{2}\hat{\mathbf{j}}$$



From this equation, we can find

$$\tan\theta = \frac{y}{x} = \frac{1}{2}, \theta = \tan^{-1}\left(\frac{1}{2}\right) \text{ to the } x\text{-axis.}$$

10 A man of 50 kg mass is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0.5 kg mass downwards with a speed 2 ms^{-1}. When the stone reaches the floor, the distance of the man above the floor will be **[CBSE AIPMT 2010]**

(a) 9.9 m (b) 10.1 m (c) 10 m (d) 20 m

Ans. ***(b)***

As, mr = constant

$$m_1 r_1 = m_2 r_2$$

$$\Rightarrow \quad r_2 = \frac{m_1 r_1}{m_2} = \frac{0.5 \times 10}{50} = 0.1$$

So, distance travelled by the man will be

$$10 + 0.1 = 10.1 \text{ m}$$

11 A body, under the action of a force $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$, acquires an acceleration of 1 ms^{-2}. The mass of this body must be **[CBSE AIPMT 2009]**

(a) $2\sqrt{10}$ kg (b) 10 kg

(c) 20 kg (d) $10\sqrt{2}$ kg

Ans. ***(d)***

Here, $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$

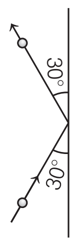
$$|F| = \sqrt{36 + 64 + 100} = 10\sqrt{2} \text{ N}$$

$$a = 1 \text{ ms}^{-2}$$

$$\therefore \quad m = \frac{10\sqrt{2}}{1} \qquad [\because F = ma]$$

$$= 10\sqrt{2} \text{ kg}$$

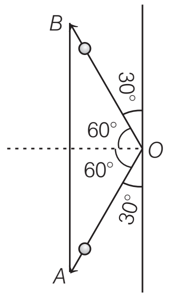
12 A 0.5 kg ball moving with a speed of 12 m/s strikes a hard wall at an angle of 30° with the wall. It is reflected with the same speed and at the same angle. If the ball is in contact with the wall for 0.25 s, the average force acting on the wall is **[CBSE AIPMT 2006]**



(a) 48 N (b) 24 N (c) 12 N (d) 96 N

Ans. ***(b)***

The vector **OA** represents the momentum of the object before the collision, and the vector **OB** that after the collision. The vector **AB** represents the change in momentum of the object $\Delta\mathbf{p}$.



As the magnitudes of **OA** and **OB** are equal, the components of **OA** and **OB** along the wall are equal and in the same direction, while those perpendicular to the wall are equal and opposite. Thus, the change in momentum is only due to the change in direction of the perpendicular components.

Hence,

$$\Delta p = OB \sin 30° - (-OA \sin 30°)$$

$$= mv \sin 30° - (-mv \sin 30°)$$

$$= 2\,mv \sin 30°$$

Its time rate will appear in the form of average force acting on the wall.

$$\therefore \quad F \times t = 2mv \sin 30° \quad \text{or}$$

$$F = \frac{2mv \sin 30°}{t}$$

Given, $m = 0.5$ kg, $v = 12$ m/s, $t = 0.25$ s

$$\theta = 30°$$

Hence, $F = \dfrac{2 \times 0.5 \times 12 \sin 30°}{0.25} = 24$ N

13 A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally, so that the block does not slip on the wedge. The force exerted by the wedge on the block (g is acceleration due to gravity) will be **[CBSE AIPMT 2004]**

(a) $mg\cos\theta$ (b) $mg\sin\theta$

(c) mg (d) $\dfrac{mg\sin\theta}{\cos\theta}$

Ans. ***(d)***

Let an acceleration to the wedge be given towards left, then the block (being in non-inertial frame) has a pseudo acceleration to the right because of which the block is not slipping

$$\therefore \quad mg\sin\theta = a_{\text{pseudo}} \cos\theta$$

$$\Rightarrow \quad a_{\text{pseudo}} = \frac{mg\sin\theta}{\cos\theta}$$

14 An object of mass 3 kg is at rest. If a force $\mathbf{F} = (6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}})$ N is applied on the object, then the velocity of the object at $t = 3$ s is **[CBSE AIPMT 2002]**

(a) $18\hat{i} + 3\hat{j}$ (b) $18\hat{i} + 6\hat{j}$

(c) $3\hat{i} + 18\hat{j}$ (d) $18\hat{i} + 4\hat{j}$

Ans. ***(b)***

According to Newton's 2nd law, force applied on an object is equal to rate of change of momentum.

i.e. $$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

or $$\mathbf{F} = m\frac{dv}{dt} \qquad \text{...(i)}$$

Given, $m = 3$ kg, $t = 3$ s, $\mathbf{F} = (6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}})$ N

Substituting these values in Eq. (i), we get

$$(6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}}) = 3\frac{d\mathbf{v}}{dt}$$

or $$d\mathbf{v} = \frac{1}{3}(6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}})\,dt$$

Now, taking integration of both sides, we get

$$\int d\mathbf{v} = \int_0^t \frac{1}{3}(6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}})\,dt$$

$$\mathbf{v} = \frac{1}{3}\int_0^t (6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}})\,dt$$

but $t = 3\,s$ (given)

$\therefore \quad \mathbf{v} = \frac{1}{3}\int_0^3 (6t^2\hat{\mathbf{i}} + 4t\hat{\mathbf{j}})\,dt$

or $\quad \mathbf{v} = \frac{1}{3}\left[\frac{6t^3}{3}\hat{\mathbf{i}} + \frac{4t^2}{2}\hat{\mathbf{j}}\right]_0^3$

or $\quad \mathbf{v} = \frac{1}{3}[2(3)^3\hat{\mathbf{i}} + 2(3)^2\hat{\mathbf{j}}]$

or $\quad \mathbf{v} = \frac{1}{3}[54\hat{\mathbf{i}} + 18\hat{\mathbf{j}}]$

or $\quad \mathbf{v} = 18\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

15 A player takes 0.1 s in catching a ball of mass 150 g moving with velocity of 20 m/s. The force imparted by the ball on the hands of the player is **[CBSE AIPMT 2001]**

(a) 0.3 N (b) 3 N (c) 30 N (d) 300 N

Ans. ***(c)***

Force imparted = rate of change of momentum

$F = \frac{\Delta p}{\Delta t}$ or $F = \frac{p_1 - p_2}{\Delta t}$ or $F = \frac{m(v_1 - v_2)}{\Delta t}$

Here, mass of body $m = 150$ g $= 0.150$ kg,

$v_1 = 20$ m/s,

$v_2 = 0$

Time taken, $\Delta t = 0.1\,s$

$\therefore \quad F = \frac{0.150 \times (20 - 0)}{0.1} = 30\text{ N}$

16 1 kg body explodes into three fragments. The ratio of their masses is 1 : 1 : 3. The fragments of same mass move perpendicular to each other with speeds 30 m/s, while the heavier part remains in the initial direction. The speed of heavier part is **[CBSE AIPMT 2001]**

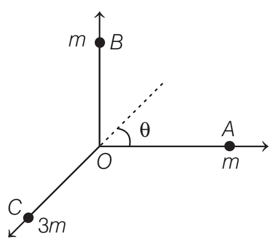
(a) $\frac{10}{\sqrt{2}}$ m/s (b) $10\sqrt{2}$ m/s

(c) $20\sqrt{2}$ m/s (d) $30\sqrt{2}$ m/s

Ans. ***(b)***

Concept Apply conservation of momentum with direction.

Let u be the velocity and θ the direction of the third piece as shown.



Equating the momentum of the system along OA and OB to zero, we get

$m \times 30 - 3m \times v\cos\theta = 0$ ...(i)

and $m \times 30 - 3m \times v\sin\theta = 0$ ...(ii)

From Eqs. (i) and (ii), we get

$3mv\cos\theta = 3mv\sin\theta$ or $\cos\theta = \sin\theta$

$\therefore \quad \theta = 45°$

Thus, $\angle AOC = \angle BOC$

$= 180° - 45° = 135°$

Putting the value of θ in Eq. (i), we get

$30\,m = 3mv\cos 45° = \frac{3mv}{\sqrt{2}}$

$\therefore \quad v = 10\sqrt{2}$ m/s

The third piece will go with a velocity of $10\sqrt{2}$ m/s in a direction making an angle of 135° with either piece.

Alternative

The square of momentum of third piece is equal to sum of squares of momentum first and second pieces.

$p_3^2 = p_1^2 + p_2^2$

or $\quad p_3 = \sqrt{p_1^2 + p_2^2}$

or $\quad 3mv_3 = \sqrt{(m \times 30)^2 + (m \times 30)^2}$

or $\quad v_3 = \frac{30\sqrt{2}}{3} = 10\sqrt{2}$ m/s

17 A particle of mass 1 kg is thrown vertically upwards with speed 100 m/s. After 5 s, it explodes into two parts. One part of mass 400 g comes back with speed 25 m/s, what is the speed of other part just after explosion? **[CBSE AIPMT 2000]**

(a) 100 m/s upwards
(b) 600 m/s upwards
(c) 100 m/s downwards
(d) 300 m/s upwards

Ans. ***(a)***

According to 1st equation of motion, velocity of particle after 5 s

$v = u - gt$

$v = 100 - 10 \times 5$

$= 100 - 50 = 50$ m/s (upwards)

Applying conservation of linear momentum gives

$Mv = m_1v_1 + m_2v_2$ ...(i)

Taking upward direction positive, the velocity v_1 will be negative.

$\therefore \quad v_1 = -25$ m/s, $v = 50$ m/s

Also, $M = 1$ kg, $m_1 = 400$ g $= 0.4$ kg

and $m_2 = (M - m_1) = 1 - 0.4 = 0.6$ kg

Thus, Eq. (i) becomes,

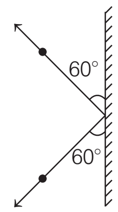
$1 \times 50 = 0.4 \times (-25) + 0.6\,v_2$

or $\quad 50 = -10 + 0.6\,v_2$

or $\quad 0.6\,v_2 = 60$ or $v_2 = \frac{60}{0.6} = 100$ m/s

As v_2 is positive, therefore the other part will move upwards with a velocity of 100 m/s.

18 A ball of mass 3 kg moving with a speed of 100 m/s, strikes a wall at an angle 60° (as shown in figure). The ball rebounds at the same speed and remains in contact with the wall for 0.2 s, the force exerted by the ball on the wall is **[CBSE AIPMT 2000]**



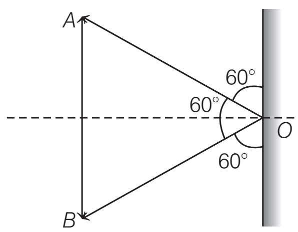
(a) $1500\sqrt{3}$ N (b) 1500 N
(c) $300\sqrt{3}$ N (d) 300 N

Ans. ***(a)***

Concept Apply 2nd law of motion i.e., rate of change of momentum is equal to force applied.

The vector **OA** represents the momentum of the wall, before the collision and the vector **OB** that after the collision. The vector **AB** represents the change in momentum of the ball $\Delta\mathbf{P}$.

As, the magnitude of **OA** and **OB** are equal the components of **OA** and **AB** along the wall are equal and in the same direction, while those perpendicular to the wall are equal and opposite. Thus, the change in momentum is only due to the change in direction of the perpendicular components.



Hence, $\Delta\mathbf{P} = OA\sin 60° - (-OB\sin 60°)$

$= mv\sin 60° + mv\sin 60°$

$= 2mv\sin 60° = 2 \times 3 \times 100 \times \frac{\sqrt{3}}{2}$

$= 300\sqrt{3}$ kg-m/s

The force exerted on the wall

$F = \frac{\Delta\mathbf{P}}{\Delta t} = \frac{300\sqrt{3}}{0.2} = 1500\sqrt{3}$ N

19 The force on a rocket moving with a velocity 300 m/s is 210 N. The rate of consumption of fuel of rocket is **[CBSE AIPMT 1999]**

(a) 0.7 kg/s (b) 1.4 kg/s
(c) 0.07 kg/s (d) 10.7 kg/s

Ans. *(a)*

Concept Whenever there is change in the mass w.r.t. time, apply $F = -v\frac{dm}{dt}$

Thrust force on the rocket

$$F_t = v_r\left(-\frac{dm}{dt}\right) \quad \text{(upwards)}$$

Rate of combustion of fuel

$$-\frac{dm}{dt} = \frac{F_t}{v_r}$$

Given, $F_t = 210$ N

$v_r = 300$ m/s

$\therefore \quad -\frac{dm}{dt} = \frac{210}{300} = 0.7$ kg/s

20 A 5000 kg rocket is set for vertical firing. The exhaust speed is 800 ms^{-1}. To give an initial upward acceleration of 20 m/s^2, the amount of gas ejected per second to supply the needed thrust will be ($g = 10$ ms^{-2}) **[CBSE AIPMT 1998]**

(a) 127.5 kg s^{-1} (b) 187.5 kg s^{-1}
(c) 185.5 kg s^{-1} (d) 137.5 kg s^{-1}

Ans. *(b)*

Thrust force on the rocket

$$F_t = v_r\left(-\frac{dm}{dt}\right) \quad \text{(upwards)}$$

Weight of the rocket

$$w = mg \quad \text{(downwards)}$$

Net force on the rocket

$$F_{net} = F_t - w$$

$$\Rightarrow \quad ma = v_r\left(\frac{-dm}{dt}\right) - mg$$

$$\Rightarrow \quad \left(\frac{-dm}{dt}\right) = \frac{m(g+a)}{v_r}$$

$\therefore$ Rate of gas ejected per second

$$= \frac{5000\,(10+20)}{800} = \frac{5000 \times 30}{800}$$

$$= 187.5 \text{ kg s}^{-1}$$

21 A bullet is fired from a gun. The force on the bullet is given by

$$F = 600 - 2 \times 10^5\, t$$

where, F is in newton and t in second. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet? **[CBSE AIPMT 1998]**

(a) 8 N-s (b) Zero
(c) 0.9 N-s (d) 1.8 N-s

Ans. *(c)*

Concept To calculate impulse first of all calculate the time during which force becomes zero.

We have given,

$$F = 600 - 2 \times 10^5\, t$$

When, bullet leaves the barrel, the force on the bullet becomes zero.

So, $600 - 2 \times 10^5\, t = 0 \Rightarrow t = \frac{600}{2 \times 10^5}$

$$= 3 \times 10^{-3} \text{ s}$$

Then, average impulse imparted to the bullet

$$I = \int_0^t F\,dt$$

$$= \int_0^{3 \times 10^{-3}} (600 - 2 \times 10^5\, t)\,dt$$

$$= \left[600\,t - \frac{2 \times 10^5\, t^2}{2}\right]_0^{3 \times 10^{-3}}$$

$$= 600 \times 3 \times 10^{-3} - 10^5 \times (3 \times 10^{-3})^2$$

$$= 1.8 - 0.9 = 0.9 \text{ N-s}$$

Alternative

As obtained in previous method, the time taken by bullet when it leaves the barrel

$$t = 3 \times 10^{-3} \text{ s}$$

Let F_1 and F_2 denote the forces at the time of firing of bullets i.e. at $t = 0$ and at the time of leaving the bullet i.e. at $t = 3 \times 10^{-3}$ s.

$$F_1 = 600 - 2 \times 10^5 \times 0 = 600 \text{ N}$$

$$F_2 = 600 - 2 \times 10^5 \times 3 \times 10^{-3} = 0$$

Mean value of force

$$F = \frac{1}{2}(F_1 + F_2) = \frac{600 + 0}{2} = 300 \text{ N}$$

Thus, impulse $= F \times t$

$$= 300 \times 3 \times 10^{-3} = 0.9 \text{ N - s}$$

22 A 10 N force is applied on a body produces an acceleration of 1 m/s^2. The mass of the body is **[CBSE AIPMT 1996]**

(a) 5 kg (b) 10 kg
(c) 15 kg (d) 20 kg

Ans. *(b)*

According to second law of motion, magnitude of force can be calculated by multiplying mass of the body and the acceleration produced in it.

or force $F = ma$

Here, $F = 10$ N

$$a = 1 \text{ m/s}^2 \Rightarrow \therefore m = \frac{F}{a} = \frac{10}{1} = 10 \text{ kg}$$

23 A ball of mass 150 g moving with an acceleration 20 m/s^2 is hit by a force, which acts on it for 0.1 s. The impulsive force is **[CBSE AIPMT 1996]**

(a) 0.5 N-s (b) 0.1 N-s
(c) 0.3 N-s (d) 1.2 N-s

Ans. *(c)*

Impulse of a force, which is the product of average force during impact and the time for, which the impact lasts is measured by the total change in linear momentum produced during the impact.

Impulse $I = F_{av} \times t = \mathbf{p}_2 - \mathbf{p}_1$

Here, Mass $= 150$ g $= \frac{150}{1000}$ kg

$\therefore \quad F = \frac{150}{1000} \times 20 = 3$ N

$\therefore \quad I = F \cdot \Delta t = 3 \times 0.1 = 0.3$ N-s

24 If the force on a rocket moving with a velocity of 300 m/s is 345 N, then the rate of combustion of the fuel is **[CBSE AIPMT 1995]**

(a) 0.55 kg/s (b) 0.75 kg/s
(c) 1.15 kg/s (d) 2.25 kg/s

Ans. *(c)*

Thrust on the rocket is the force with which the rocket moves upwards. Thrust on rocket at time t is given by $F = -u\frac{dm}{dt}$

The negative sign indicates that thrust on the rocket is in a direction opposite to the direction of escaping gases.

Here, velocity of the rocket $u = 300$ m/s

and force $F = 345$ N

$\therefore$ Rate of combustion of fuel

$$-\left(\frac{dm}{dt}\right) = \frac{F}{u} = \frac{345}{300} = 1.15 \text{ kg/s}$$

25 A satellite in a force free space sweeps stationary interplanetary dust at a rate. $\left(\frac{dM}{dt}\right) = \alpha v$. The acceleration of satellite is **[CBSE AIPMT 1994]**

(a) $-\frac{2\alpha v^2}{M}$ (b) $-\frac{\alpha v^2}{M}$ (c) $-\frac{\alpha v^2}{2M}$ (d) $-\alpha v^2$

Ans. *(b)*

Thrust on the satellite is the force with which the satellite moves upwards in space. It is given by

$$F = -\ u\frac{dm}{dt}$$

Here, initial velocity

$u = v$, rate of change of mass

$$\frac{dm}{dt} = \alpha v$$

As we know that,

$$F = -v\frac{dm}{dt} = -v(\alpha v) = -\alpha v^2$$

$$\text{Acceleration} = \frac{F}{M} = -\frac{\alpha v^2}{M}$$

26 Physical independence of force is a consequence of **[CBSE AIPMT 1991]**

(a) third law of motion
(b) second law of motion
(c) first law of motion
(d) All of these

Ans. *(c)*

According to Newton's first law of motion, a body continues to be in a state of rest or of uniform motion, unless it is acted upon by an external force to change the state. Hence, Newton's first law of motion is related to physical independence of force.

27 A particle of mass m is moving with a uniform velocity v_1. It is given an impulse such that its velocity becomes v_2. The impulse is equal to **[CBSE AIPMT 1990]**

(a) $m[|v_2| - |v_1|]$ (b) $\frac{1}{2}m(v_2^2 - v_1^2)$
(c) $m(v_1 + v_2)$ (d) $m(v_2 - v_1)$

Ans. *(d)*

Concept Impulse of a force can be calculated as the product of large force applied to the small time to which force act.

i.e. $F = \frac{dp}{dt}$

$\Rightarrow \quad F \cdot dt = dp$

$\Rightarrow \quad \text{impulse} = p_2 = p_1$

Impulse of a force, which is the product of average force during impact and the time for which the impact lasts, is measured by the total change in linear momentum produced during the impact.

Here, $\mathbf{p}_1 = m\mathbf{v}_1$, $\mathbf{p}_2 = m\mathbf{v}_2$

Impulse, $I = m\mathbf{v}_2 - m\mathbf{v}_1 = m(\mathbf{v}_2 - \mathbf{v}_1)$

28 A 600 kg rocket is set for a vertical firing. If the exhaust speed is 1000 ms^{-1}, the mass of the gas ejected per second to supply the thrust needed to overcome the weight of rocket is **[CBSE AIPMT 1990]**

(a) 117.6 kg s^{-1} (b) 58.6 kg s^{-1}
(c) 6 kg s^{-1} (d) 76.4 kg s^{-1}

Ans. *(c)*

Thrust on the rocket is the force with which the rocket moves upwards.

Thrust on the rocket at time t is given by

$$F = -u\frac{dm}{dt}$$

where, u is relative velocity of exhaust gases with respect to the rocket. $\frac{dm}{dt}$ is rate of combustion of fuel at that instant.

$$\therefore \quad F = -u\frac{dm}{dt} = mg \Rightarrow -\frac{dm}{dt} = \frac{mg}{u}$$

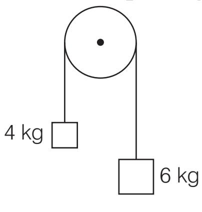
Here, $m = 600$ kg, $u = 1000\ \text{ms}^{-1}$

$$\therefore \quad -\frac{dm}{dt} = \frac{600 \times 10}{1000} = 6\ \text{kg s}^{-1}$$

TOPIC 2

Equilibrium of a Particle and Common Forces in Mechanics

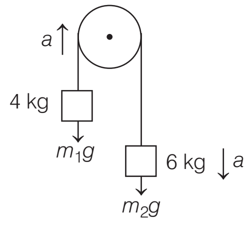
29 Two bodies of mass 4 kg and 6 kg are tied to the ends of a massless string. The string passes over a pulley which is frictionless (see figure). The acceleration of the system in terms of acceleration due to gravity g is **[NEET (Sep.) 2020]**



(a) $g/2$ (b) $g/5$ (c) $g/10$ (d) g

Ans. *(b)*

Given, $m_1 = 4$ kg, $m_2 = 6$ kg and $a = ?$



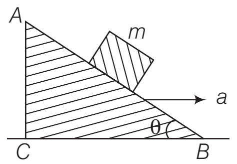
From the above free body diagram, the relation for acceleration of the given system can be given as

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) \cdot g$$

$$= \left(\frac{6-4}{4+6}\right) \times g = \frac{g}{5}$$

Hence, correct option is (b).

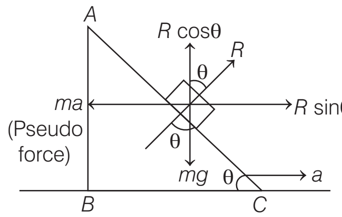
30 A block of mass m is placed on a smooth inclined wedge ABC of inclination θ as shown in the figure. The wedge is given an acceleration a towards the right. The relation between a and θ for the block to remain stationary on the wedge is **[NEET 2018]**



(a) $a = g\cos\theta$ (b) $a = \frac{g}{\sin\theta}$
(c) $a = \frac{g}{\text{cosec}\,\theta}$ (d) $a = g\tan\theta$

Ans. *(d)*

According to the question, the FBD of the given condition will be



Since, the wedge is accelerating towards right with a, thus a pseudo force acts in the left direction in order to keep the block stationary. As, the system is in equilibrium.

$\therefore \quad \Sigma F_x = 0$

or $\quad \Sigma F_y = 0$

$\Rightarrow \quad R\sin\theta = ma$

or $\quad mg\sin\theta = ma$ ...(i)

Similarly, $\quad R\cos\theta = mg$

or $\quad mg\cos\theta = mg$ ...(ii)

Dividing Eq. (i) by Eq (ii), we get

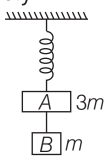
$$\frac{mg\sin\theta}{mg\cos\theta} = \frac{ma}{mg}$$

$$\Rightarrow \quad \tan\theta = \frac{a}{g}$$

or $\quad a = g\tan\theta$

$\therefore$ The relation between a and g for the block to remain stationary on the wedge is $a = g\tan\theta$.

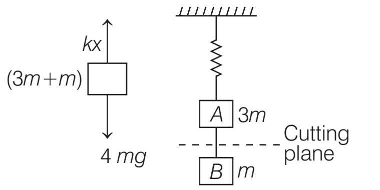
31 Two blocks A and B of masses $3m$ and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively **[NEET 2017]**



(a) $g, \frac{g}{3}$ (b) $\frac{g}{3}, g$ (c) g, g (d) $\frac{g}{3}, \frac{g}{3}$

Ans. ***(b)***

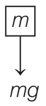
Initially system, is in equilibrium with a total weight of $4mg$ over spring.



$\therefore \quad kx = 4mg$

When string is cut at the location as shown above.

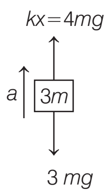
Free body diagram for m is



So, force on mass $m = mg$

$\therefore$ Acceleration of mass, $m = g$

For mass $3m$; free body diagram is



If a = acceleration of block of mass $3m$, then

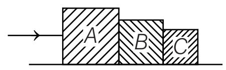
$$F_{net} = 4mg - 3mg$$

$$\Rightarrow \quad 3m\cdot a_A = mg \text{ or } a_A = \frac{g}{3}$$

So, accelerations for blocks A and B are

$$a_A = \frac{g}{3} \text{ and } a_B = g$$

32 Three blocks A, B and C of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface, as shown. If a force of 14 N is applied on the 4 kg block, then the contact force between A and B is **[CBSE AIPMT 2015]**

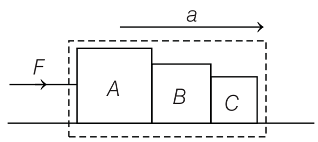


(a) 2 N (b) 6 N (c) 8 N (d) 18 N

Ans. ***(b)***

Given, $m_A = 4\text{ kg}$

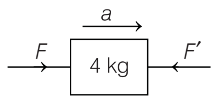
$$m_B = 2\text{ kg} \Rightarrow m_C = 1\text{ kg}$$



So, total mass $(M) = 4 + 2 + 1 = 7\text{ kg}$

Now, $\quad F = Ma \Rightarrow 14 = 7a \Rightarrow a = 2\text{ m/s}^2$

FBD of block A,



$$F - F' = 4a$$

$$\Rightarrow \quad F' = 14 - 4\times 2 \Rightarrow F' = 6\text{ N}$$

33 A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on control panel. The lift starts moving upwards with an acceleration 1.0 m/s^2. If $g = 10\text{ m/s}^2$, the tension in the supporting cable is **[CBSE AIPMT 2011]**

(a) 9680 N (b) 11000 N
(c) 1200 N (d) 8600 N

Ans. ***(b)***

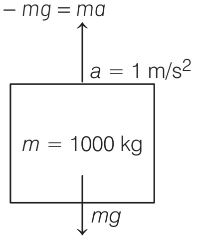
Total mass (m)

= Mass of lift + Mass of person

= 940 + 60 = 1000 kg

So, from the free body diagram

$$T - mg = ma$$



Hence, $T - 1000\times 10 = 1000\times 1$

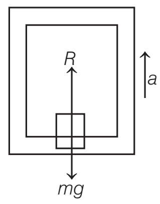
$$T = 11000\text{ N}$$

34 The mass of a lift is 2000 kg. When the tension in the supporting cable is 28000 N, then its acceleration is **[CBSE AIPMT 2009]**

(a) 30 ms^{-2} downwards
(b) 4 ms^{-2} upwards
(c) 4 ms^{-2} downwards
(d) 14 ms^{-2} upwards

Ans. ***(b)***

Here, lift is accelerating upward at the rate of a.



Hence, equation of motion is written as

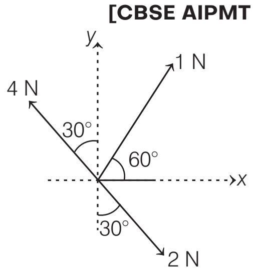
$$R - mg = ma$$

$$28000 - 20000 = 2000a$$

$$[\because g = 10\text{ ms}^{-2}]$$

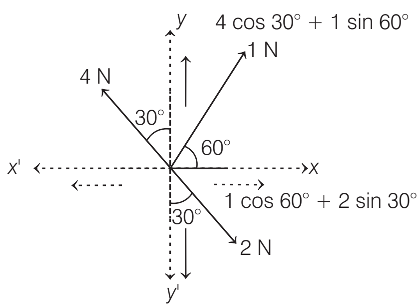
$$\Rightarrow \quad a = \frac{8000}{2000} = 4\text{ ms}^{-2} \text{ upwards}$$

35 Three forces acting on a body are shown in the figure. To have the resultant force only along the y-direction, the magnitude of the minimum additional force needed is **[CBSE AIPMT 2008]**



(a) 0.5 N (b) 1.5 N (c) $\frac{\sqrt{3}}{4}$ N (d) $\sqrt{3}$ N

Ans. ***(a)***



Breaking all the forces in x-y axis.
otal force along $(+x)$ axis

$$=(1\cos 60° + 2\sin 30°)$$

along $(-x)$ axis $=(4\sin 30°)$ along $(+y)$ axis $=(4\cos 30° + 1\sin 60°)$ along $(-y)$ axis $=(2\cos 30°)$

$\Rightarrow$ Net force along x-axis
$=-(1\cos 60° + 2\sin 30°) + 4\sin 30°$

$$\Rightarrow \quad -\left(\frac{1}{2}+2\times\frac{1}{2}\right)+4\times\frac{1}{2}$$

$$\Rightarrow \quad \frac{-3}{2}+2=+\frac{1}{2}$$

Net force along y-axis

$$=4\cos 30° + 1\sin 60° - 2\cos 30°$$

$$\Rightarrow \quad 4\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}-2\times\frac{\sqrt{3}}{2}=\frac{5\sqrt{3}}{2}-\frac{2\sqrt{3}}{2}$$

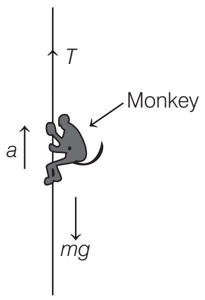
$$=\frac{3\sqrt{3}}{2}$$

To have, resultant only in y-axis we must have $\frac{1}{2}$ N force towards $+x$-axis, so that it can compensate the net force of $-x$ axis.

36 A monkey of mass 20 kg is holding a vertical rope. The rope will not break, when a mass of 25 kg is suspended from it but will break, if the mass exceeds 25 kg. What is the maximum acceleration with which the monkey can climb up along the rope? (Take $g = 10\ \text{m/s}^2$)
[CBSE AIPMT 2003]

(a) $25\ \text{m/s}^2$ (b) $2.5\ \text{m/s}^2$
(c) $5\ \text{m/s}^2$ (d) $10\ \text{m/s}^2$

Ans. ***(b)***
Maximum bearable tension in the rope
$T = 25\times 10 = 250\ \text{N}$



From the figure,

$T - mg = ma$ or acceleration $a=\frac{T-mg}{m}$

Given, mass $m = 20$ kg,
$g = 10\ \text{m/s}^2$,

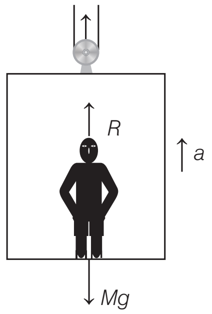
Taking, $T = 250$ N

Hence, $a=\frac{250-20\times 10}{20}=\frac{50}{20}$
$=2.5\ \text{m/s}^2$

37 A man weighs 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5\ \text{m/s}^2$. What would be the reading on the scale? (Take $g = 10\ \text{m/s}^2$)
[CBSE AIPMT 2003]

(a) 800 N (b) 1200 N
(c) Zero (d) 400 N

Ans. ***(b)***
Mass of man $M = 80$ kg
Acceleration of lift, $a = 5\ \text{m/s}^2$
When, lift is moving upwards, the reading of weighing scale will be equal to R.

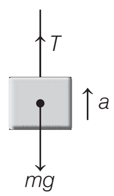


The equation of motion gives
$R - Mg = Ma$ or
$R = Mg + Ma = M(g + a)$
$\therefore \quad R = 80\,(10 + 5) = 80\times 15 = 1200\ \text{N}$

38 A lift of mass 1000 kg is moving upwards with an acceleration of $1\ \text{m/s}^2$. The tension developed in the string, which is connected to lift is ($g = 9.8\ \text{m/s}^2$)
[CBSE AIPMT 2002]

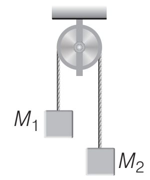
(a) 9800 N (b) 10800 N
(c) 11000 N (d) 10000 N

Ans. ***(b)***
When, lift move upwards with same acceleration, then according to free body diagram of the left



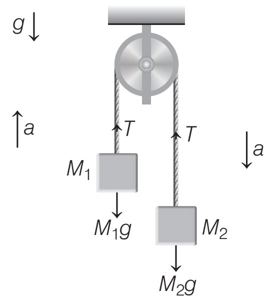
$T - mg = ma$
or $T = m(g + a)$
Given,
$m = 1000$ kg, $a = 1\ \text{m/s}^2$,
$g = 9.8\ \text{m/s}^2$
Thus, $T = 1000\,(9.8 + 1) = 1000\times 10.8$
$= 10800$ N

39 Two masses $M_1 = 5$ kg, $M_2 = 10$ kg are connected at the ends of an inextensible string passing over a frictionless pulley as shown. When masses are released, then acceleration of masses will be
[CBSE AIPMT 2000]



(a) g (b) $\frac{g}{2}$ (c) $\frac{g}{3}$ (d) $\frac{g}{4}$

Ans. ***(c)***
Concept In the case of masses hanging from a pulley by a string, the tension in whole string is same, say equal to T.



As $M_2 > M_1$, so mass M_2 moves down and mass M_1 moves up with the same acceleration a (say). The arrangement of the motion is represented in the figure.

According to free body diagram of mass M_2, is

$$M_2 g - T = M_2 a \quad \text{...(i)}$$

According to free body diagram of mass M_1, is

$$T - M_1 g = M_1 a \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$(M_2 g - T) + (T - M_1 g) = (M_1 + M_2)a$$
$$(M_2 - M_1)g = (M_1 + M_2)a$$
$$\Rightarrow \quad a=\left(\frac{M_2-M_1}{M_1+M_2}\right)g$$

Given, $M_1 = 5$ kg, $M_2 = 10$ kg

Hence, $a=\left(\frac{10-5}{5+10}\right)g=\frac{5}{15}g=\frac{g}{3}\ \text{m/s}^2$

Alternative
Acceleration,

$$a=\frac{(F_{net})\text{system}}{\text{Net mass}}=\frac{(10-5)\times g}{5+10}=\frac{g}{3}\ \text{m/s}^2$$

In a mass-pulley system, the tension in the string is always towards the pulley.

40 A mass of 1 kg is suspended by a thread. It is

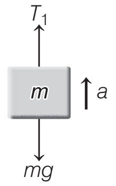
1. lifted up with an acceleration 4.9 m/s^2,
2. lowered with an acceleration 4.9 m/s^2.

The ratio of the tensions is **[CBSE AIPMT 1998]**

(a) 3 : 1 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

Ans. *(a)*

(i) When, mass is lifted upwards with an acceleration *a*, then according to free body diagram



$$T_1 - mg = ma \Rightarrow T_1 = mg + ma$$

$$T_1 = m(g + a)$$

Substituting the values, we obtain

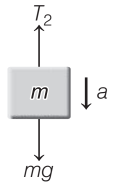
$$\therefore \quad T_1 = (1)(9.8 + 4.9) = 14.7 \text{ N}$$

(ii) When, mass is lowered downwards with an acceleration *a*, then

$$mg - T_2 = ma$$

$$\Rightarrow T_2 = mg - ma = m(g - a)$$

Substituting the values, we have



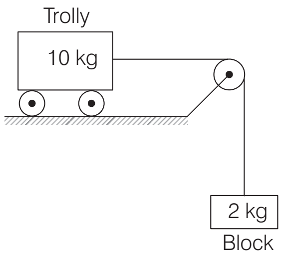
$$T_2 = (1)(9.8 - 4.9) = 4.9 \text{ N}$$

Then, ratio of tensions

$$\frac{T_1}{T_2} = \frac{14.7}{4.9} = \frac{3}{1} \Rightarrow T_1 : T_2 = 3 : 1$$

TOPIC 3
Friction

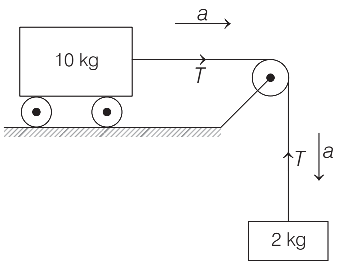
41 Calculate the acceleration of the block and trolly system shown in the figure. The coefficient of kinetic friction between the trolly and the surface is 0.05. ($g = 10$ m/s^2, mass of the string is negligible and no other friction exists). **[NEET (Oct.) 2020]**



(a) 1.25 m/s^2 (b) 1.50 m/s^2
(c) 1.66 m/s^2 (d) 1.00 m/s^2

Ans. *(a)*

The given situation is shown in the following diagram.



If '*a*' be the acceleration of the system then, equation of motion of 10 kg trolly,

$$T - \mu R = 10a$$

$$\Rightarrow \quad T - 0.05 \times 10g = 10a \qquad [\because \mu = 0.05, R = 10g]$$

$$\Rightarrow \quad T - 0.05 \times 10 \times 10 = 10a$$

$$\Rightarrow \quad T - 5 = 10a \qquad \text{...(i)}$$

Equation of motion of 2kg block,

$$2g - T = 2a$$

$$2 \times 10 - T = 2a$$

$$20 - T = 2a \qquad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we have

$$20 - 5 = 12a$$

$$\Rightarrow \quad 15 = 12a$$

$$\Rightarrow \quad a = \frac{15}{12} = \frac{5}{4} = 1.25 \text{ ms}^{-2}$$

42 Two particles *A* and *B* are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of *A* to that of *B* will be **[NEET (National) 2019]**

(a) $v_A : v_B$ (b) $r_B : r_A$ (c) 1 : 1 (d) $r_A : r_B$

Ans. *(c)*

The angular speed of a particle in a uniform circular motion is given by

$$\omega = \frac{\text{angle of circle}}{\text{Time}}$$

$$\omega = \frac{2\pi}{T}, \text{ where } T \text{ is the time period}$$

of rotation

For particle *A*, $\omega_A = \frac{2\pi}{T_A}$

For particle *B*, $\omega_B = \frac{2\pi}{T_B}$

$$\therefore \frac{\omega_A}{\omega_B} = \frac{2\pi}{T_A} \times \frac{T_B}{2\pi} = \frac{T_B}{T_A}$$

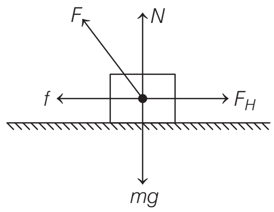
$$= \frac{1}{1} \text{ or } 1 : 1 \qquad [\because T_A = T_B \text{ (given)}]$$

43 A body of mass *m* is kept on a rough horizontal surface (coefficient of friction $= \mu$). Horizontal force is applied on the body, but it does not move. The resultant of normal reaction and the frictional force acting on the object is given F, where F is **[NEET (Odisha) 2019]**

(a) $|F| = mg + \mu\, mg$
(b) $|F| = \mu mg$
(c) $|F| \leq mg\sqrt{1 + \mu^2}$
(d) $|F| = mg$

Ans. *(c)*

The situation can be drawn as



The frictional force, $f = \mu N = \mu\, mg$ $[\because N = mg]$

From Free body diagram (*FBD*), the resultant force is

$$|\mathbf{F}| = \sqrt{N^2 + f^2}$$

$$= \sqrt{(mg)^2 + (\mu mg)^2}$$

$$= mg\sqrt{1 + \mu^2}$$

This is the minimum force required to move the object. But as the body is not moving

$$\therefore \quad |\mathbf{F}| \leq mg\sqrt{1 - \mu^2}$$

44 Which one of the following statements is incorrect? **[NEET 2018]**

(a) Frictional force opposes the relative motion
(b) Limiting value of static friction is directly proportional to normal reaction
(c) Rolling friction is smaller than sliding friction
(d) Coefficient of sliding friction has dimensions of length

Ans. *(d)*

The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction.

The coefficient of sliding is given as

$$\mu_S = \frac{N}{F_{\text{sliding}}}$$

where, N is the normal reaction and F_{sliding} is the sliding force.

As, the dimensions of N and F_{sliding} are same. Thus, μ_S is a dimensionless quantity.

Hence, statement(d) is incorrect.

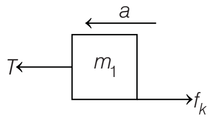
45 A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block A is sliding on the table, the tension in the string is **[CBSE AIPMT 2015]**

(a) $\frac{(m_2+\mu_k m_1)g}{(m_1+m_2)}$ (b) $\frac{(m_2-\mu_k m_1)g}{(m_1+m_2)}$

(c) $\frac{m_1 m_2(1+\mu_k)g}{(m_1+m_2)}$ (d) $\frac{m_1 m_2(1-\mu_k)g}{(m_1+m_2)}$

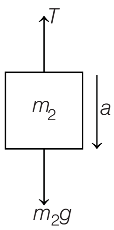
Ans. *(c)*

FBD of block A,



$$T - m_1 a = f_k \quad \text{...(i)}$$

FBD of block B,



$$m_2 g - T = m_2 a \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$m_2 g - m_1 a = m_2 a + f_k$$

$$\Rightarrow \quad m_2 g - m_1 a = m_2 a + \mu_k m_1 g$$

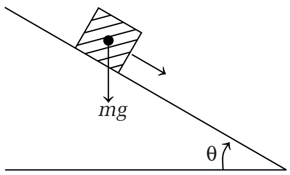
$$\Rightarrow \quad a = \frac{(m_2 - \mu_k m_1)g}{m_1 + m_2}$$

From Eq. (ii), $T = m_2(g-a)$

$$= m_2\left[1 - \frac{(m_2 - \mu_k m_1)}{m_1 + m_2}\right]g$$

$$T = \frac{m_1 m_2 (1+\mu_k)}{m_1 + m_2} g$$

46 A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0 s. The coefficients of static and kinetic friction between the box and the plank will be, respectively **[CBSE AIPMT 2015]**

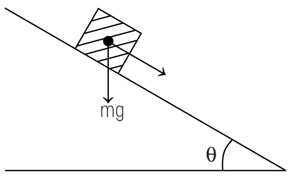


(a) 0.6 and 0.6 (b) 0.6 and 0.5

(c) 0.5 and 0.6 (d) 0.4 and 0.3

Ans. *(b)*

Given a plank with a box on its one end is gradually raised about the end having angle of inclination is 30°, the box starts to slip and slides down 4 m the plank in 4 s as shown in figure.



The coefficient of static friction,

$$\mu_s = \tan 30° = \frac{1}{\sqrt{3}} = 0.6$$

So, distance covered by a plank,

$$s = ut + \frac{1}{2}at^2$$

Here, $u = 0$ and $a = g(\sin\theta - \mu\cos\theta)$

$$\therefore \quad 4 = \frac{1}{2}g(\sin 30° - \mu_k \cos 30°)(4)^2$$

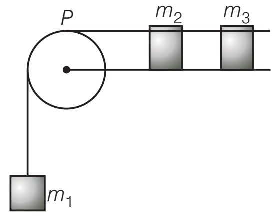
$$\Rightarrow \quad 0.5 = 10 \times \frac{1}{2} - \mu_K \times 10 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad 5\sqrt{3}\,\mu_K = 4.5$$

$$\Rightarrow \quad \mu_K = 0.51$$

Thus, coefficient of kinetic friction between the box and the plank is 0.51.

47 A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction $= \mu$). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is (Assume, $m_1 = m_2 = m_3 = m$) **[CBSE AIPMT 2014]**

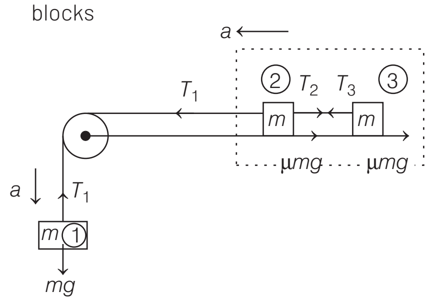


(a) $\frac{g(1-g\mu)}{9}$ (b) $\frac{2g\mu}{3}$

(c) $\frac{g(1-2\mu)}{3}$ (d) $\frac{g(1-2\mu)}{2}$

Ans. *(c)*

First of all consider the forces on the blocks



For the Ist block, $[\because m_1 = m_2 = m_3]$

$$mg - T_1 = m \times a \quad \text{...(i)}$$

Let us consider 2nd and 3rd block as a system.

So, $T_1 - 2\mu mg = 2m \times a$ ...(ii)

Solving Eqs. (i) and (ii),

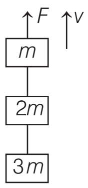
$\Rightarrow \quad mg - T_1 = m \times a$

$\Rightarrow \quad T_1 - 2\mu mg = 2m \times a$

Addiing Eqs. (i) and (ii)

$$mg(1-2\mu) = 3m \times a \Rightarrow a = \frac{g}{3}(1-2\mu)$$

48 Three blocks with masses m, $2m$ and $3m$ are connected by strings, as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass $2m$? (g is the acceleration due to gravity). **[NEET 2013]**



(a) Zero (b) $2\ mg$
(c) $3\ mg$ (d) $6\ mg$

Ans. *(a)*

Since, all the blocks are moving with constant velocity and we know that, if velocity is constant, acceleration of the body becomes zero. Hence, the net force on all the blocks will be zero.

49 The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and lower half of the plane is given by **[NEET 2013]**

(a) $\mu = \dfrac{1}{\tan\theta}$

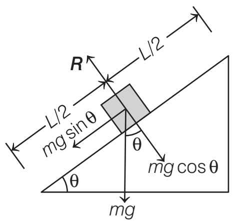
(b) $\mu = \dfrac{2}{\tan\theta}$

(c) $\mu = 2\tan\theta$

(d) $\mu = \tan\theta$

Ans. *(c)*

Concept Net work done by the block in going from top to bottom of the inclined plane, must be equal to the work done by frictional force.



The block may be stationary, when

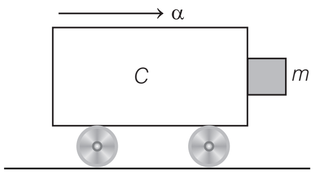
$$mg\sin\theta \cdot L = \mu\ mg\cos\theta \frac{L}{2}$$

or $$\mu = \frac{mg\sin\theta \cdot L}{mg\cos\theta \cdot \frac{L}{2}}$$

$$= 2\frac{\sin\theta}{\cos\theta} = 2\tan\theta$$

$$\mu = 2\tan\theta$$

50 A block of mass m is in contact with the cart C as shown in the figure.



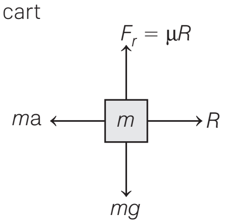
The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies **[CBSE AIPMT 2010]**

(a) $\alpha > \dfrac{mg}{\mu}$ (b) $\alpha > \dfrac{g}{\mu m}$

(c) $\alpha \geq \dfrac{g}{\mu}$ (d) $\alpha < \dfrac{g}{\mu}$

Ans. *(c)*

When, a cart moves with some acceleration towards right, then a pseudo force $(m\alpha)$ acts on block towards left. This force $(m\alpha)$ is action force by a block on cart

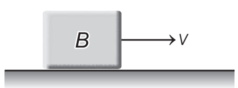


Now, block will remain static w.r.t. cart, if frictional force $\mu R \geq mg$

$\Rightarrow \quad \mu\ m\alpha \geq mg \quad [\text{as } R = m\alpha]$

$\Rightarrow \quad \alpha \geq \dfrac{g}{\mu}$

51 A block B is pushed momentarily along a horizontal surface with an initial velocity v. If μ is the coefficient of sliding friction between B and the surface, block B will come to rest after a time **[CBSE AIPMT 2007]**



(a) $\dfrac{v}{g\mu}$ (b) $\dfrac{g\mu}{v}$ (c) $\dfrac{g}{v}$ (d) $\dfrac{v}{g}$

Ans. *(a)*

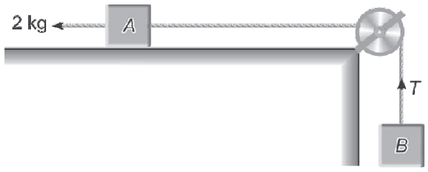
Block B will come to rest, if force applied to it will vanish due to frictional force acting between block B and surface, i.e.

frictional force = force applied

i.e. $\mu mg = ma$

or $\mu mg = m\left(\dfrac{v}{t}\right)$ or $t = \dfrac{v}{\mu g}$

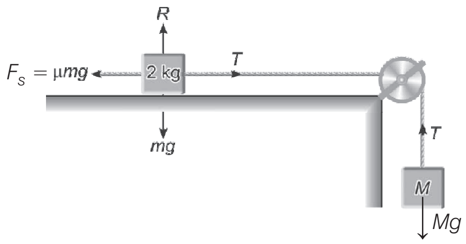
52 The coefficient of static friction, μ_s, between block A of mass 2 kg and the table as shown in the figure, is 0.2. What would be the maximum mass value of block B, so that the two blocks do not move ? The string and the pulley are assumed to be smooth and massless ($g = 10\ \text{m/s}^2$) **[CBSE AIPMT 2004]**



(a) 2.0 kg (b) 4.0 kg
(c) 0.2 kg (d) 0.4 kg

Ans. *(d)*

Let the mass of the block B be M.



In equilibrium,

$T - Mg = 0 \Rightarrow T = Mg$ ...(i)

If blocks do not move, then

$T = f_s$

where, f_s = frictional force

$= \mu_s R = \mu_s\ mg$

$\therefore \quad T = \mu_s\ mg$ ...(ii)

Thus, from Eqs. (i) and (ii), we have

$Mg = \mu_s\ mg$ or $M = \mu_s\ m$

Given, $\mu_s = 0.2$, $m = 2$ kg

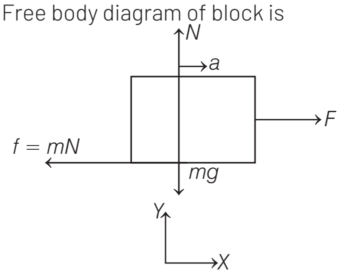
$\therefore \quad M = 0.2 \times 2 = 0.4$ kg

53 A block of mass 10 kg is placed on a rough horizontal surface having coefficient of friction $\mu = 0.5$. If a horizontal force of 100 N is applied on it, then the acceleration of the block will be (Take $g = 10\ \text{m/s}^2$) **[CBSE AIPMT 2002]**

(a) $15\ \text{m/s}^2$ (b) $10\ \text{m/s}^2$
(c) $5\ \text{m/s}^2$ (d) $0.5\ \text{m/s}^2$

Ans. ***(c)***

Free body diagram of block is



From Newton's second law along *X*-axis

$$\Sigma F_x = ma$$

i.e. $F - f = ma$

or $F - \mu mg = ma$

or $a = \frac{F - \mu mg}{m}$

Given, $F = 100\ \text{N}, \mu = 0.5, m = 10\ \text{kg}$,

$g = 10\ \text{m/s}^2$

Substituting, the values in the above relation for acceleration of block,

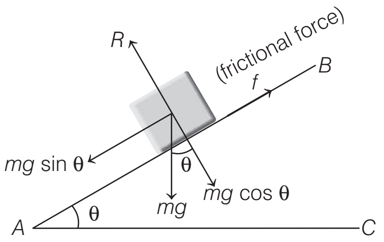
$$a = \frac{(100) - (0.5)(10)(10)}{(10)} = 5\ \text{m/s}^2$$

54 A block has been placed on an inclined plane with the slope angle θ, block slides down the plane at constant speed. The coefficient of kinetic friction is equal to **[CBSE AIPMT 1993]**

(a) $\sin\theta$ (b) $\cos\theta$
(c) g (d) $\tan\theta$

Ans. ***(d)***

Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down.



AB is an inclined plane such that a body placed on it just begins to slide down

$\angle BAC = \theta =$ angle of repose

In equilibrium,

$$f = mg\sin\theta$$

and $R = mg\cos\theta$

$\therefore \frac{f}{R} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$

i.e. $\mu = \tan\theta$

NOTE

Coefficient of kinetic friction between any two surfaces in contact is equal to the tangent of the angle of inclination between them.

55 Consider, a car moving along a straight horizontal road with a speed of 72 km/h. If the coefficient of static friction between the tyres and the road is 0.5, the shortest distance in which the car can be stopped is (Take $g = 10\ \text{m/s}^2$) **[CBSE AIPMT 1992]**

(a) 30 m (b) 40 m (c) 72 m (d) 20 m

Ans. ***(b)***

When, static friction is present, then acceleration of body is given by $a = -\mu g$

Here, initial velocity

$$u = 72\ \text{km h}^{-1} = 72 \times \frac{5}{18} = 20\ \text{m/s}$$

Final velocity $v = 0$

$\therefore a = -\mu g = -0.5 \times 10 = -5\ \text{m/s}^2$

Now, from third equation of motion,

i.e. $v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (20)^2}{2 \times (-5)} = 40\ \text{m}$$

56 A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum friction of the length of the chain that can hang over one edge of the table is **[CBSE AIPMT 1991]**

(a) 20% (b) 25% (c) 35% (d) 15%

Ans. ***(a)***

The force of friction should balance the weight of chain hanging. If *M* is the mass of whole chain of length *L* and *x* is the length of chain hanging to balance, then

$$\mu \frac{M}{L}(L - x)\, g = \frac{M}{L} xg$$

or $\mu (L - x) = x$

or $x = \frac{\mu L}{\mu + 1} = \frac{0.25 L}{1.25}$ (As, $\mu = 0.25$)

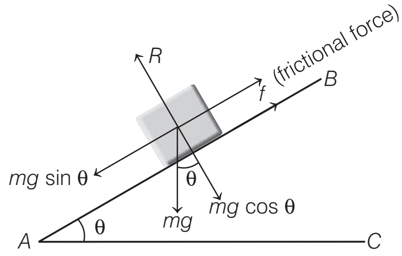
$\therefore x = \frac{L}{5}$ or $\frac{x}{L} = \frac{1}{5} = \frac{1}{5} \times 100 = 20\%$

57 Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is **[CBSE AIPMT 1988]**

(a) 0.80 (b) 0.75
(c) 0.25 (d) 0.33

Ans. ***(b)***

When, a plane is inclined to the horizontal at an angle θ, which is greater than the angle of repose, the body placed on the inclined plane slides down with an acceleration a.



As, it is clear from figure

$$R = mg\cos\theta \quad \text{...(i)}$$

Net force on the body down the inclined plane which means it is sliding downwards

$$F = mg\sin\theta - f \quad \text{...(ii)}$$

i.e. $F = ma = mg\sin\theta - \mu R$ $(f = \mu R)$

$\therefore ma = mg\sin\theta - \mu\, mg\cos\theta$

$= mg(\sin\theta - \mu\cos\theta)$

Hence, $a = g(\sin\theta - \mu\cos\theta)$

$\therefore$ Time taken by body to slide down the plane

$$t_1 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}}$$

When friction is absent, then time taken to slide down the plane

$$t_2 = \sqrt{\frac{2s}{g\sin\theta}} \Rightarrow \because t_1 = 2t_2 \quad \text{(given)}$$

$\therefore t_1^2 = 4t_2^2$

or $\frac{2s}{g(\sin\theta - \mu\cos\theta)} = \frac{2s \times 4}{g\sin\theta}$

or $\sin\theta = 4\sin\theta - 4\mu\cos\theta$

or $\mu = \frac{3}{4}\tan\theta = \frac{3}{4}\tan 45°$

$\therefore \mu = \frac{3}{4} = 0.75$

TOPIC 4

Dynamics of Circular Motion

58 A block of mass 10 kg is in contact against the inner wall of a hollow cylindrical drum of radius 1 m. The coefficient of friction between the block and the inner wall of the cylinder is 0.1. The minimum angular velocity needed for the cylinder to keep the block stationary when the cylinder is vertical and rotating about its axis, will be $(g = 10\,m/s^2)$ **[NEET (National) 2019]**

(a) $\frac{10}{2\pi}$ rad/s (b) 10 rad/s

(c) 10π rad/s (d) $\sqrt{10}$ rad/s

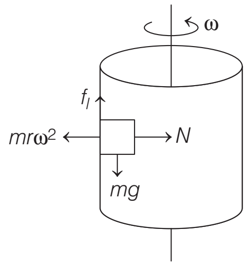
Ans. ***(b)***

Given, mass of cylinder $m = 10$ kg,

radius of cylinder, $r = 1$ m

coefficient of friction, $\mu = 0.1$.

The given situation can be as shown in the figure given below.



From the above figure, it can be concluded that the block will be stationary when the limiting friction (f_l) is equal to or greater than the downward force or weight of block, i.e.

$$f_l \geq mg \quad \text{...(i)}$$

Also, the magnitude of limiting friction between two bodies is directly proportional to the normal reaction (N) between them, i.e.

$$f_l \propto N \text{ or } f_l = \mu N \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\mu N \geq mg \text{ or } \mu(mr\omega^2) \geq mg \quad [\because N = mr\omega^2]$$

$$\Rightarrow \omega \geq \sqrt{\frac{g}{r\mu}}$$

Thus, the minimum angular velocity is

$$\omega_{min} = \sqrt{\frac{g}{r\mu}} = \sqrt{\frac{10}{1 \times 0.1}} = 10 \text{ rad/s}$$

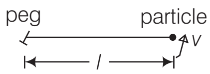
59 One end of the string of length l is connected to a particle of mass m and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed v, the net force on the particle (directed towards center) will be (T represents the tension in the string) **[NEET 2017]**

(a) T (b) $T + \frac{mv^2}{l}$

(c) $T - \frac{mv^2}{l}$ (d) Zero

Ans. ***(a)***

Consider the string of length l connected to a particle as shown in the figure.



Speed of the particle is v. As the particle is in uniform circular motion, net force on the particle must be equal to centripetal force which is provided by the tension (T).

$\therefore$ Net force = Centripetal force

$$\Rightarrow \frac{mv^2}{l} = T$$

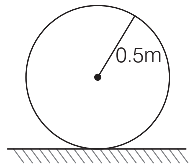
60 A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2.0 s is a approximately **[NEET 2016]**

(a) 7.0 (b) 6.0

(c) 3.0 (d) 8.0

Ans. ***(d)***

According to given question, a uniform circular disc of radius 50 cm at rest is free to turn about an axis having perpendicular to its plane and passes through its centre. This situation can be shown by the figure given below:



$\therefore$ Angular acceleration, $\alpha = 2$ rad s^{-2} (given)

Angular speed, $\omega = \alpha t = 4$ rad s^{-1}

$\because$ Centripetal acceleration, $a_c = \omega^2 r$

$$= (4)^2 \times 0.5$$

$$= 16 \times 0.5$$

$$a_c = 8\,m/s^2$$

$\because$ Linear acceleration at the end of 2s,

$$a_t = \alpha r = 2 \times 0.5 \Rightarrow a_t = 1\,m/s^2$$

Therefore, the net acceleration at the end of 2.0 s is given by

$$a = \sqrt{a_c^2 + a_t^2}$$

$$a = \sqrt{(8)^2 + (1)^2} = \sqrt{65} \Rightarrow a \approx 8\,m/s^2$$

61 A car is negotiating a curved road of radius R. The road is banked at angle θ. The coefficient of friction between the tyres of the car and the road is μ_s. The maximum safe velocity on this road is **[NEET 2016]**

(a) $\sqrt{gR\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$

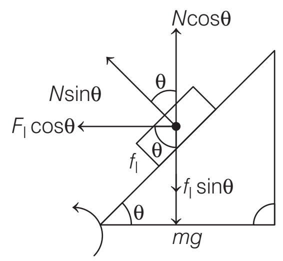
(b) $\sqrt{\frac{g}{R}\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$

(c) $\sqrt{\frac{g}{R^2}\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$

(d) $\sqrt{gR^2\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$

Ans. ***(a)***

According to question, a car is negotiating a curved road of radius R. The road is banked at angle θ and the coefficient of friction between the tyres of car and the road is μ_s. So, this given situation can be drawn as shown in figure below.



Considering the case of vertical equilibrium

$$N\cos\theta = mg + f_l \sin\theta$$

$$\Rightarrow mg = N\cos\theta - f_l \sin\theta \quad \text{...(i)}$$

Considering the case of horizontal equilibrium,

$$N\sin\theta + f_l\cos\theta = \frac{mv^2}{R} \quad \text{...(ii)}$$

Divide eqs. (i) and (ii), we get

$$\frac{v^2}{Rg} = \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} \quad [f_l \propto \mu_s]$$

$$\Rightarrow v = \sqrt{Rg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$$

$$\Rightarrow v = \sqrt{Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}$$

62 A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is 45°, the speed of the car is **[CBSE AIPMT 2012]**

(a) 20 ms^{-1}
(b) 30 ms^{-1}
(c) 5 ms^{-1}
(d) 10 ms^{-1}

Ans. ***(b)***

The angle of banking

$$\tan\theta = \frac{v^2}{rg}$$

Given, $\theta = 45°$

Radius of banked curve road

$r = 90$ m and $g = 10$ m/s^2

$$\Rightarrow \tan 45° = \frac{v^2}{90 \times 10}$$

$$v = \sqrt{90 \times 10 \times \tan 45°}$$

$$= \sqrt{90 \times 10 \times 1} = 30 \text{ m/s}$$

63 A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if **[CBSE AIPMT 2010]**

(a) $r = \mu g \omega^2$ (b) $r < \frac{\omega^2}{\mu g}$
(c) $r \le \frac{\mu g}{\omega^2}$ (d) $r \ge \frac{\mu g}{\omega^2}$

Ans. ***(c)***

When the disc spins, the frictional force between the gramophone record and coin is μmg.

The coin will revolve with record, if

$$F_{\text{frictional}} \ge F_{\text{centripetal}}$$

i.e. $\mu mg \ge m\omega^2 r$

$$\frac{\mu g}{r} \ge \omega^2$$

64 A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved ? **[CBSE AIPMT 1998]**

(a) 14 m/s (b) 3 m/s
(c) 3.92 m/s (d) 5 m/s

Ans. ***(a)***

For a ball to move in horizontal circle, the ball should satisfy the condition

Tension in the string = Centripetal force

$$\Rightarrow T_{\max} = \frac{Mv_{\max}^2}{R}$$

$$\Rightarrow v_{\max} = \sqrt{\frac{T_{\max} \cdot R}{M}} \quad \text{...(i)}$$

Making substitution, we obtain

$$v_{\max} = \sqrt{\frac{25 \times 1.96}{0.25}} = \sqrt{196} = 14 \text{ m/s}$$

In a vertical circle, the tension at the highest point is zero and at lowest point is maximum.

65 What will be the maximum speed of a car on a road turn of radius 30 m, if the coefficient of friction between the tyres and the road is 0.4 ? (Take $g = 9.8$ m / s^2) **[CBSE AIPMT 1995]**

(a) 10.84 m/s
(b) 9.84 m/s
(c) 8.84 m/s
(d) 6.84 m/s

Ans. ***(a)***

When a vehicle goes round a curved road, it requires some centripetal force. While rounding the curve, the wheels of the vehicle have a tendency to leave the curved path and regain the straight line path. Force of friction between the wheels and the road opposes this tendency of the wheels. This force (of friction) therefore, acts towards the centre of the circular track and provides the necessary centripetal force.

If v is the velocity of the vehicle while rounding the curve, the centripetal force required $= \frac{mv^2}{r}$

As, this force is provided only by the force of friction,

$$\frac{mv^2}{r} \le \mu mg \Rightarrow v^2 \le \mu rg \Rightarrow v \le \sqrt{\mu rg}$$

$$\therefore \quad v_{\max} = \sqrt{\mu rg}$$

Here, radius of curved road $r = 30$ m,

coefficient of friction $\mu = 0.4$

$$\therefore \quad v_{\max} = \sqrt{0.4 \times 30 \times 9.8}$$

$$= 10.84 \text{ m/s}$$

66 Two racing cars of masses m and $4m$ are moving in circles of radii r and $2r$ respectively. If their speeds are such that each makes a complete circle in the same time, then the ratio of the angular speeds of the first to the second car is **[CBSE AIPMT 1995]**

(a) 8 : 1 (b) 4 : 1 (c) 2 : 1 (d) 1 : 1

Ans. ***(d)***

As both cars take the same time to complete the circle and as $\omega = \frac{2\pi}{t}$, therefore ratio of angular speeds of the cars will be 1 : 1.