Shortcuts and Important Results to Remember

1 **Symmetric Determinant** The elements situated at equal distance from the diagonal are equal both in magnitude and sign. i.e.

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

2 **Skew-symmetric Determinant** All the diagonal elements are zero and the elements situated at equal distance from the diagonal are equal in magnitude but opposite in sign. The value of skew-symmetric determinant of even order is always a perfect square and that of odd order is always zero i.e.

$$\begin{vmatrix} 0 & a \\ -a & 0 \end{vmatrix} = a^2 \text{ and } \begin{vmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{vmatrix} = 0$$

3 **Circulant Determinant** The elements of the rows (or columns) are in cyclic order. i.e.,

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

$= (a-b)(b-c)(c-a)(ab+bc+ca)$

(iii) $\begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(v) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

Remark

These results direct applicable in lengthy questions as behaviour of standard results.

4 (i) If $\Delta = 0$, then $\Delta^c = 0$, where Δ^c denotes the determinant of cofactors of elements of Δ.

(ii) If $\Delta \neq 0$, then $\Delta^c = \Delta^{n-1}$, where n is order of Δ.

(iii) Let $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

The sum of products of the elements of any row or column with the corresponding cofactors is equal to the value of determinant, i.e.

$$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$$
$$= a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33} = \Delta$$

and sum of products of the elements of any row or column with the cofactors of the corresponding elements of any other row or column is zero, i.e.,

$$a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$$
$$= 0$$

5 A homogeneous system of equations is never consistent.

6 **Conjugate of a Determinant** If a_i, b_i and $c_i \in C$ $(i = 1, 2, 3)$

and $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\overline{\Delta} = \begin{vmatrix} \overline{a_1} & \overline{b_1} & \overline{c_1} \\ \overline{a_2} & \overline{b_2} & \overline{c_2} \\ \overline{a_3} & \overline{b_3} & \overline{c_3} \end{vmatrix}$

(i) If Δ is purely real, then $\Delta = \Delta$

(ii) If Δ is purely imaginary, then $\overline{\Delta} = -\Delta$

7 (i) If $x_1, x_2, x_3, \ldots$ are in AP or $a^{x_1}, a^{x_2}, a^{x_3}, \ldots$ are in GP,

then $\begin{vmatrix} x_1 & x_2 & x_3 \\ x_{n+1} & x_{n+2} & x_{n+3} \\ x_{2n+1} & x_{2n+2} & x_{2n+3} \end{vmatrix} = 0$

(ii) If $a_1, a_2, a_3, \ldots$ are in GP and $a_i > 0, i = 1, 2, 3, \ldots,$

then $\begin{vmatrix} \log a_n & \log a_{n+1} & \log \alpha_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} = 0$