

### 3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. A father is three times as old as his son. After five years, his age will be two and a half times as old as his son. Represent this situation algebraically only.

[Ans :  $5x - 2y + 15 = 0$ ]

2. Is  $5x - 7y = -2$  and  $10x - 14y + 4 = 0$  a pair of dependent linear equations ?

[Sol. : The two given equations are :

$$5x - 7y + 2 = 0$$

$$10x - 14y + 4 = 0$$

Since  $\frac{5}{10} = \frac{-7}{-14} = \frac{2}{4} = \frac{1}{2}$ , therefore it is a pair of dependent linear equations]

3. Find those integral values of  $m$  for which the  $x$ -coordinate of the point of intersection of lines represented by  $y = mx + 1$  and  $3x + 4y = 9$  is an integer.

[Sol. : Point of intersection of given lines  $y = mx + 1$  and  $3x + 4y = 9$  can be obtained as follows :

Putting  $y = mx + 1$  in  $3x + 4y = 9$ , we get

$$3x + 4(mx + 1) = 9$$

$$\text{or } (3 + 4m)x + 4 = 9$$

$$\text{or } (3 + 4m)x = 9 - 4$$

$$\Rightarrow (4m + 3)x = 5$$

$$\text{or } x = \frac{5}{4m + 3}$$

For values of  $x$  as an integer, values of 'm' must be either  $m = -1$  or  $m = -2$ .]

4. Solve graphically the pair of linear equations :  $4x - 3y = 6$  and  $7x + 3y = 27$   
Find the coordinates of the vertices of triangular region formed by these lines and y-axis. Also, calculate the area of this triangle.
5. Solve the following system of linear equations graphically :  
 $x - y = 1$   
 $2x - 5y = -1$   
Do the points (2, 1) and (1, 2) lie on any of the lines ?
6. Draw the graph of the following pair of linear equations :  $3x + 2y = 15$  and  $3x - 4y = -3$   
Also, shade the region bounded by these lines and  $y = 0$ . Write the coordinates of vertices of the triangle.
7. For what value of k does the system of equations  $x + 2y = 3$  and  $5x + ky + 7 = 0$  have a unique solution ?

[Sol. : For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10$$

So, k has any value other than 10]

8. Solve it by substitution method :  
 $0.2x + 0.3y = 1.3$   
 $0.4x + 0.5y = 2.3$   
[Hint : After removing decimals on both sides, we get  $2x + 3y = 13$  and  $4x + 5y = 23$   
By solving, we get  $x = 2$ ,  $y = 3$  is required solution]
9. Find all real values of c for which the pair of linear equations  $2x + y = 8$  and  $5x - 2y = c$  has a unique solution.

[Sol. : Given pair of linear equations  $2x + y = 8$  and  $5x - 2y = c$  has a unique solution.

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ or } \frac{2}{5} \neq \frac{1}{-2}$$

Since  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , so this pair of linear equations will have a unique solution for every value of c.]

10. Solve the following pair of equations :

$$49x + 51y = 499$$

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[Ans : We have :

$$49x + 51y = 499 \quad \dots\dots\dots(1)$$

$$51x + 49y = 501 \quad \dots\dots\dots(2)$$

Adding (1) and (2), we get

$$(49 + 51)x + (51 + 49)y = 499 + 501$$

$$\Rightarrow 100x + 100y = 1000$$

$$\Rightarrow x + y = 10 \quad \text{.....(3)}$$

Subtracting (1) from (2), we get

$$51x - 49x + 49y - 51y = 501 - 499$$

$$\Rightarrow 2x - 2y = 2$$

$$\Rightarrow x - y = 1 \quad \text{.....(4)}$$

Adding (3) and (4), we get

$$2x = 11 \quad \Rightarrow x = \frac{11}{2}$$

Putting  $x = \frac{11}{2}$  in (3), we get

$$\frac{11}{2} + y = 10 \Rightarrow y = 10 - \frac{11}{2} = \frac{9}{2}$$

$$\text{Thus, } x = \frac{11}{2} \text{ and } y = \frac{9}{2}$$

11. A man earns ₹600 per month more than his wife. One-tenth of the man's salary and one-sixth of the wife's salary amount to ₹1500, which is saved every month. Find their incomes.

**[Ans :** Man's income is ₹6000 and wife's income is ₹5400]

12. Aman has certain notes of denominations ₹20 and ₹5 which amount to ₹380. If the number of notes of each kind are interchanged, they amount to ₹60 less than before. Find the number of notes of each denomination.

**[Ans :** There are 16 notes of ₹20 and 12 notes of ₹5]

13. A man bought 4 horses and 9 cows for ₹1340. He sells the horses at a profit of 10% and the cows at a profit of 20% and his whole gain is ₹188. What price did he pay for the horse ?

**[Ans :** Let cost price of one horse = ₹x  
and cost price of one cow = ₹y.

According to question,

By Ist condition,

$$4x + 9y = 1340 \quad \text{.....(i)}$$

By IInd condition,

$$4\left(\frac{10x}{100}\right) + 9\left(\frac{20y}{100}\right) = 188$$

$$\text{or } 4x + 18y = 1880 \quad \text{.....(ii)}$$

Subtracting (i) from (ii), we get

$$9y = 540 \Rightarrow y = \frac{540}{9} = \text{Rs.}60$$

Putting  $x = 60$  in (i), we get

$$4x + 9 \times 60 = 1340$$

$$\text{or } 4x = 1340 - 540$$

$$\text{or } 4x = 800 \quad \Rightarrow \quad x = \frac{800}{4} = \text{Rs.}200$$

So, cost of one horse = ₹200

and cost of one cow = ₹60.]

14. A mobile company charges a fixed amount as monthly rental which include 100 minutes free per month and charges a fixed amount thereafter for ever additinal minute. Abhay paid ₹433 for 370 minutes and Anitha paid ₹398 for 300 minutes. Find the bill amount under the same plan, if Kareena used it for 400 minutes.

[Sol : Let fixed amount be ₹ $x$  and that of each extra minute be ₹ $y$ .

So, as per given consitions, we get :

$$x + (370 - 100) y = 433 \Rightarrow x + 270y = 433 \quad \text{.....(1)}$$

$$\text{and } x + (300 - 100)y = 398 \Rightarrow x + 200y = 398 \quad \text{.....(2)}$$

Subtracting (2) from (1), we get

$$70y = 35 \quad \Rightarrow \quad y = \frac{35}{70} = \frac{1}{2}$$

Putting  $y = \frac{1}{2}$  in (1), we get

$$x + 270 \times \frac{1}{2} = 433 \Rightarrow x + 135 = 433 \Rightarrow x = 433 - 135 = 298$$

So, amount to be paid by Kareena for 400 minutes

$$= 298 + (400 - 100) \times \frac{1}{2}$$

$$= 298 + \frac{300}{2} = 298 + 150 = 448$$

Thus, amount to be paid is ₹448.

15. A man sold a chair and a table together for ₹1520 thereby making a profit of 25% on the chair and 10% on the table. By selling them together for ₹1535, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each.

[Ans : Let the cost price of one chair be ₹ $x$  and that of one table be ₹ $y$ .

As per given conditions, we have :

$$\frac{125x}{100} + \frac{110y}{100} = 1520$$

$$\Rightarrow 125x + 110y = 152000$$

$$\Rightarrow 25x + 22y = 30400 \quad \dots\dots\dots(1)$$

$$\text{and } \frac{110x}{100} + \frac{125y}{100} = 1535$$

$$\Rightarrow 110x + 125y = 153500$$

$$\Rightarrow 22x + 25y = 30700 \quad \dots\dots\dots(2)$$

Adding (1) and (2), we get

$$47x + 47y = 61100$$

$$\Rightarrow x + y = 1300 \quad \dots\dots\dots(3)$$

Subtracting (2) from (1), we get

$$3x - 3y = -300$$

$$\Rightarrow x - y = -100 \quad \dots\dots\dots(4)$$

Adding (3) and (4), we get

$$2x = 1200 \Rightarrow x = \frac{1200}{2} = 600 .$$

Putting  $x = 600$  in (3), we get

$$\Rightarrow 600 + y = 1300$$

$$\Rightarrow y = 1300 - 600 = 700.$$

Thus, cost of one chair is ₹600 and that of one table is ₹700.]

16. In a competitive examination, one mark is awarded for each correct answer, while half mark is deducted for each wrong answer. Rajiv answered 120 questions and got 90 marks. How many questions did he answer correctly ?

[Ans : 100]

17. Students of a class are made to stand in rows. If 4 students are extra in a row there would be 2 rows less. If 4 students are less in a row, there would be 4 more rows. Find the number of students in the class.

[Ans : 96]

18. A new intra-city transportation startup has employed both taxis and auto rickshaws. The night fare for a taxi is Rs.9 for the first half kilometre and Rs. 15 per kilometre thereafter, while the night fare for an auto rickshaw is Rs.20 for the first one kilometre and Rs. 13 per kilometre thereafter.

i) Express the night fare structure for taxis and auto rickshaws in the form of linear equations. Use  $f$  as the fare (in Rs) and  $d$  as the distance travelled (in km).

ii) At what distance is the night fare for a taxi and an auto rickshaw equal? Show your steps.

[Sol. : i) Express the taxi's night fare structure in the form of an equation as

$$f = 9 + 15(d - 0.5)$$

$$\Rightarrow f = 15d + 1.5$$

Express the auto rickshaw's night fare structure in the form of an equation as

$$f = 20 + 13(d - 1)$$

$$\Rightarrow f = 13d + 7$$

- ii) Solve the pair of linear equations obtained in steps (1) and (2) by using an appropriate method to get  $d$  as 2.75 km.]

19. Arvind owns a dry - fruits store. He sells cashew nuts at Rs 600/kg and pistachio nuts at Rs 750 /kg. A customer asks for a mixture of cashew nuts and pistachio nuts with the following conditions :  
 both the items should together weight 500 g.  
 both the items should together cost Rs. 360.
- If Arvind packs  $x$  kg of cashew nuts and  $y$  kg of pistachio nuts for the customer, frame the equations that represent the given context.
  - Find the weights of cashew nuts and pistachio nuts that Arvind packed for the customer. Show your work.

[Sol. : i) Frames the pair of linear equations in two variables representing the given context as :

$$x + y = \frac{1}{2}$$

$$600x + 750y = 360$$

- Solve the above pair of linear equations to find the values of  $x$  and  $y$  as  $\frac{1}{10}$  and  $\frac{2}{5}$  respectively and concludes that Arvind packed  $\frac{1}{10}$  kg of cashew nuts and  $\frac{2}{5}$  kg of pistachionuts for the customer.]

20. The break - even point of a new flavour ice cream is the number of units sold at which the total cost price price of the ice cream is the same as the total selling price. The parlour launches another new flavour whose fixed cost is Rs 10000, variable cost is Rs. 40 and the selling price of each unit Rs.60. Find the number of units at the break - even point of the other new flavour. Show your work.

[Sol. : Assuming  $n$  as the number of units and  $m$  as revenue at break - even point, frames the pair of linear equations that represents the given context as :

$$m = 60 n$$

$$m = 10000 + 40 n$$

Solve the given pair of linear equations by any suitable method to find the values of  $n$  as 500 and concludes that the number of units at the break - even point is 500.]