02

Vector Analysis

IN THIS CHAPTER ....

Scalars and Vectors

On the basis of magnitude, direction and rules of addition, all physical quantities are classified into two groups as scalars and vectors.

A *scalar quantity* is one whose specification is completed with its magnitude only. For example, mass, distance, speed, energy, electric flux, current electricity, etc.

A *vector quantity* is one whose specification is completed with its magnitude and direction both. For example, displacement, velocity, acceleration, force, electric field intensity, current density, etc.

Types of Vectors

Some important types of vectors are given in the table below

S.No.	Types of Vectors	Description
(i)	**Polar vectors**	These are the vectors which have a starting point or a point of application. *e.g.* Displacement, force, etc.
(ii)	**Axial vectors**	These are the vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule. *e.g.* Angular velocity, angular acceleration, etc.
(iii)	**Equal vectors**	These are the vectors which have equal magnitude and same direction.
(iv)	**Like parallel vectors**	These are the vectors which have same direction and magnitude may equal or different.
(v)	**Anti-parallel vectors**	These are the vectors which have opposite direction and magnitude may equal or different.
(vi)	**Collinear vectors**	These are the vectors which act along same line, *i.e.* vectors lying in the same line. Angle between them can be zero or 180°.
(vii)	**Zero vector**	A vector having zero magnitude is known as zero vector. Its direction is not specified and hence it is arbitrary. It is represented by 0.

S.No.	Types of Vectors	Description
(viii)	**Unit vector**	A vector whose magnitude is unit and points in a particular direction is called unit vector. It is represented by $\hat{A}$ (A cap or A hat). The unit vector along the direction of A is $\hat{A} = \frac{A}{\lvert A \rvert}$, $\therefore$ $A = \hat{A}\lvert A \rvert$, where $A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\lvert A \rvert = \sqrt{A_x^2 + A_y^2 + A_z^2}$.
(ix)	**Coplanar vectors**	These are the vectors which always lie in the same plane.
(x)	**Negative vector**	If the direction of a vector is reversed, the sign of the vector is reversed. It is called negative vector of the original vector.

Example 1. *A vector may change, if*

(*a*) *frame of reference is translated*
(*b*) *vector is rotated*
(*c*) *frame of reference is rotated*
(*d*) *vector is translated parallel to itself*

Sol. (*b*) Vector will change, if it is rotated because its direction changes.

Example 2. *The expression* $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ *is a*

(*a*) *unit vector*
(*b*) *null vector*
(*c*) *vector of magnitude* $\sqrt{2}$
(*d*) *scalar*

Sol. (*a*) We have $|\mathbf{R}| = \left[\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2\right]^{1/2} = 1$

Multiplication of a Vector by a Real Number

The multiplication of a vector by a scalar quantity n gives a new vector whose magnitude is n times the magnitude of the given vector. Its direction is same as that of the given vector, if n is a positive real number and reverses, if n is a negative real number.

Example 3. *If* $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, *is multiplied by a number 5, then the vector along y-direction is*

(*a*) $-15\hat{\mathbf{j}}$
(*b*) $5\hat{\mathbf{j}}$
(*c*) $-5\hat{\mathbf{j}}$
(*d*) $15\hat{\mathbf{j}}$

Sol. (*a*) As, $n \times \mathbf{A} = n\mathbf{A}$,

So, $5 \times (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = 10\hat{\mathbf{i}} - 15\hat{\mathbf{j}} + 20\hat{\mathbf{k}}$

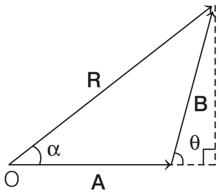
$\therefore$ Vector along y-direction $= -15\hat{\mathbf{j}}$

Addition of Vectors

Vectors can be added by following laws

1. Triangle Law of Vector Addition

If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by the third side of the triangle taken in reverse order as shown below



or $\mathbf{R} = \mathbf{A} + \mathbf{B}$

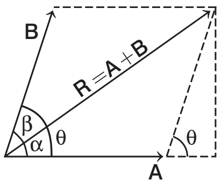
If θ is the angle between $\mathbf{A}$ and $\mathbf{B}$, then the magnitude of resultant, $\mathbf{R}$ can be given by

$$R = |\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\mathbf{R}$ makes an angle α with $\mathbf{A}$, then $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$.

2. The Parallelogram Law

If two non-zero vectors $\mathbf{A}$ and $\mathbf{B}$ are represented by the two adjacent sides of a parallelogram then, the resultant is given by the diagonal of the parallelogram in magnitude and direction both passing through the point of intersection of the two vectors.



The magnitude of $\mathbf{R}$ is

$$R = |\mathbf{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{...(i)}$$

where, θ is the angle between $\mathbf{A}$ and $\mathbf{B}$.

Here, $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

and $\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$ ...(ii)

where, $\alpha + \beta = \theta$.

Special Cases

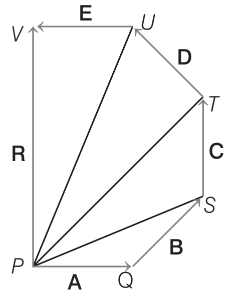
If $\theta = 0°$, $R_{max} = A + B$

If $\theta = 180°$, $R_{min} = A - B$ and if $\theta = 90°$, $R = \sqrt{A^2 + B^2}$

In all other cases, Eqs. (i) and (ii) can be used to calculate magnitude and direction of $\mathbf{R}$.

3. Polygon Law of Vector Addition

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then the resultant vector is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.



$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E}$$

Properties of Vector Addition

(i) Vector addition is commutative,
i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

(ii) Vector addition is associative,
i.e. $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$

(iii) Vector addition is distributive,
i.e. $\lambda(\mathbf{A} + \mathbf{B}) = \lambda\mathbf{A} + \lambda\mathbf{B}$

Example 4. *If* $\mathbf{A} = \mathbf{B} + \mathbf{C}$ *and the magnitude of* $\mathbf{A}$, $\mathbf{B}$ *and* $\mathbf{C}$ *are 5, 4 and 3 minutes respectively, then angle between* $\mathbf{A}$ *and* $\mathbf{C}$ *is*

(a) $\cos^{-1}(4/5)$ (b) $\cos^{-1}(3/5)$
(c) $\tan^{-1}(3/4)$ (d) $\sin^{-1}(3/5)$

Sol. (b) Given, $\mathbf{A} = \mathbf{B} + \mathbf{C}$

$$\Rightarrow \mathbf{B} = \mathbf{A} - \mathbf{C} \Rightarrow B = A - C$$
$$\Rightarrow B^2 = (A - C)^2$$
$$\Rightarrow \mathbf{B}\cdot\mathbf{B} = (\mathbf{A} - \mathbf{C})\cdot(\mathbf{A} - \mathbf{C})$$
$$\Rightarrow \mathbf{B}\cdot\mathbf{B} = \mathbf{A}\cdot\mathbf{A} - \mathbf{A}\cdot\mathbf{C} - \mathbf{C}\cdot\mathbf{A} + \mathbf{C}\cdot\mathbf{C}$$
$$\Rightarrow B^2 = A^2 - AC\cos\theta - AC\cos\theta + C^2$$
$$\Rightarrow 2AC\cos\theta = A^2 + C^2 - B^2$$
$$\Rightarrow 2\times5\times3\times\cos\theta = 5^2 + 3^2 - 4^2$$
$$\Rightarrow \cos\theta = \frac{18}{30} = \frac{3}{5}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right)$$

Example 5. *The resultant of two forces acting at an angle of 150° is 10 N and is perpendicular to one of the forces. One of the two other forces is*

(a) $20/\sqrt{3}$ *N* (b) $10/\sqrt{3}$ *N*
(c) 20 *N* (d) $20/\sqrt{3}$ *N*

Sol. (c) We have, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$(10)^2 = A^2 + B^2 + 2AB\cos150°$$
$$= A^2 + B^2 + 2AB(-\sqrt{3}/2)$$

or $$100 = A^2 + B^2 - \sqrt{3}\,AB \quad \text{...(i)}$$

Again $$\tan 90° = \frac{B\sin150°}{A + B\cos150°}$$
$$= \frac{B\times1/2}{A + B(-\sqrt{3}/2)}$$
$$= \frac{B}{2A - \sqrt{3}B}$$

or $$\infty = \frac{B}{2A - \sqrt{3}B}$$

or $$2A - \sqrt{3}B = 0$$

or $$A = \frac{\sqrt{3}}{2}B$$

Putting the value of A in Eq. (i), we get

$$100 = \frac{3}{4}B^2 + B^2 - \sqrt{3}\times\frac{\sqrt{3}}{2}B\times B = \frac{1}{4}B^2$$

or $$B^2 = 4\times100$$

or $$B = 20\text{ N}$$

and $$A = \frac{\sqrt{3}}{2}20 = 10\sqrt{3}\text{ N}$$

Example 6. *Three vectors each of magnitude A are acting at a point such that angle between any two vectors is 60°. The magnitude of their resultant is*

(a) zero (b) $\sqrt{2}\,A$
(c) $\sqrt{3}\,A$ (d) $\sqrt{6}\,A$

Sol. (d) We have, $R = |\mathbf{A} + \mathbf{B} + \mathbf{C}|$

$$= [A^2 + B^2 + C^2 + 2\,\mathbf{A}\cdot\mathbf{B} + 2\,\mathbf{B}\cdot\mathbf{C} + 2\,\mathbf{C}\cdot\mathbf{A}]^{1/2}$$

Given, $A = B = C$

and $\theta = 60°$

$$R = [3A^2 + 2\,\mathbf{A}\cdot\mathbf{B} + 2\,\mathbf{B}\cdot\mathbf{C} + 2\,\mathbf{C}\cdot\mathbf{A}]^{1/2}$$
$$= [3A^2 + (2A\cdot A\cos60°)\times3]^{1/2}$$
$$= \sqrt{6}\,A$$

Example 7. *A particle has two equal accelerations in two given directions. If one of the accelerations is halved, then the angle which the resultant makes with the other is also halved. The angle between the accelerations is*

(a) 120° (b) 90°
(c) 60° (d) 45°

Sol. (a) $$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$
$$= \frac{A\sin\theta}{A + A\cos\theta} \quad (\because A = B)$$
$$= \frac{\sin\theta}{(1 + \cos\theta)} \quad \text{...(i)}$$
$$\tan\frac{\beta}{2} = \frac{(A/2)\sin\theta}{A + (A/2)\cos\theta}$$
$$= \frac{\sin\theta}{2 + \cos\theta} \quad \text{...(ii)}$$

We know that, $\tan\beta = \dfrac{2\tan\dfrac{\beta}{2}}{1-\tan^2\dfrac{\beta}{2}}$

$$\Rightarrow \quad \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\theta}{(2+\cos\theta)\left[1-\left(\dfrac{\sin\theta}{2+\cos\theta}\right)^2\right]}$$

$$\Rightarrow \quad \frac{1}{1+\cos\theta} = \frac{2(2+\cos\theta)}{(2+\cos\theta)^2-\sin^2\theta}$$

$$= \frac{2(2+\cos\theta)}{4+\cos^2\theta+4\cos\theta-(1-\cos^2\theta)}$$

$$\Rightarrow \quad 2\cos^2\theta + 4\cos\theta + 3 = 2(2+\cos\theta)(1+\cos\theta)$$

$$\Rightarrow \quad 2\cos^2\theta + 4\cos\theta + 3 = 2\cos^2\theta + 6\cos\theta + 4$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2} \Rightarrow \cos\theta = \cos 120^\circ \Rightarrow \theta = 120^\circ$$

Example 8. *The sum of magnitudes of two forces acting at a point is* 16 *and magnitude of their resultant is* $8\sqrt{3}$. *If the resultant is at* 90° *with the force of smaller magnitude, their magnitudes are*

(a) 3, 13 (b) 2, 14
(c) 5, 11 (d) 4, 12

Sol. (b) Given $A + B = 16$

$$R = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$$

$$8\sqrt{3} = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$$

and $\quad \tan 90^\circ = \dfrac{B\sin\theta}{A+B\cos\theta}$ or $\infty = \dfrac{B\sin\theta}{A+B\cos\theta}$

or $\quad A + B\cos\theta = 0$

or $\quad B\cos\theta = -A$

$\therefore \quad 8\sqrt{3} = [A^2 + B^2 + 2A(-A)]^{1/2}$

or $\quad 192 = B^2 - A^2 = (B-A)(B+A) = (B-A)\times 16$

or $\quad B - A = 192/16 = 12$

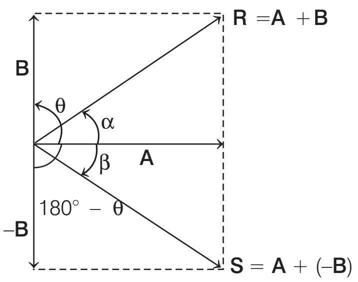
On solving, $A = 2$ and $B = 14$

Subtraction of Vectors

Negative of a vector (–**A**) is a vector of the same magnitude as vector **A** but pointing in a direction opposite to that of **A**.

Therefore, $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$.

Let the angle between vectors **A** and **B** be θ, then the angle between **A** and $-\mathbf{B}$ will be $180^\circ - \theta$.



Magnitude of $\mathbf{S} = \mathbf{A} - \mathbf{B}$ will be given by

$$|\mathbf{S}| = |\mathbf{A} - \mathbf{B}|$$

$$= \sqrt{A^2 + B^2 + 2AB\cos(180^\circ - \theta)}$$

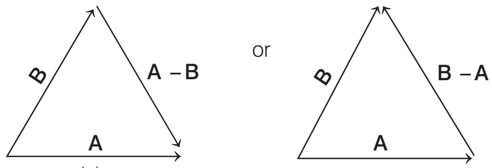
or $\quad S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

For direction of **S**, we will either calculate angle α or β,

$$\tan\alpha = \frac{B\sin(180^\circ-\theta)}{A+B\cos(180^\circ-\theta)} = \frac{B\sin\theta}{A-B\cos\theta}$$

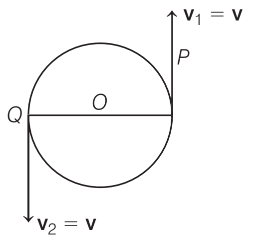
$$\tan\beta = \frac{A\sin(180^\circ-\theta)}{B+A\cos(180^\circ-\theta)} = \frac{A\sin\theta}{B-A\cos\theta}$$

Note (i) $\mathbf{A}-\mathbf{B}$ or $\mathbf{B}-\mathbf{A}$ can also be found by making triangles as shown in figure.



(a) (b)

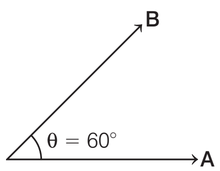
(ii) Change in velocity of a particle moving along circular path with a constant speed



When a particle moves along a circular path with a constant speed, then its velocity changes due to change in direction.

$\therefore$ Change in velocity, $\Delta\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2 = \mathbf{v} - (-\mathbf{v}) = 2\mathbf{v}$

Example 9. *Find* **A** – **B** *from the diagram shown in figure. Given A* = 4 *units and B* = 3 *units*



(a) $\sqrt{18}$ *units* (b) $\sqrt{17}$ *units*
(c) $\sqrt{14}$ *units* (d) $\sqrt{13}$ *units*

Sol. (d) Subtraction, $S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

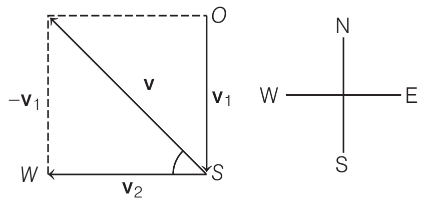
$$= \sqrt{16 + 9 - 2\times 4\times 3\cos 60^\circ}$$

$$= \sqrt{13} \text{ units}$$

Example 10. *A car moving towards south changes its direction towards west moving with the same speed. Find the change in the direction of velocity of the car.*

(a) *North-West* (b) *North-East*
(c) *South-East* (d) *South-West*

Sol. (a)



Here, $|v_1| = |v_2| = v$ (say)

$\therefore$ Change in velocity of car, $\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$

Magnitude of the change in velocity,

$$|\Delta \mathbf{v}| = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos 90^\circ}$$

$$= \sqrt{v^2 + v^2 - 0} = \sqrt{2v^2} = \sqrt{2}\, v$$

The direction of change in velocity,

$$\tan\theta = \frac{|\mathbf{v}_1|}{|\mathbf{v}_2|} = \frac{v}{v} = 1$$

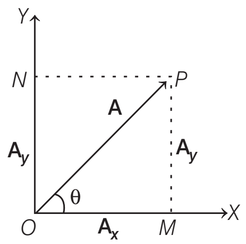
or $\theta = 45^\circ$

The change in velocity of the car is along north-west direction.

Resolution of Vectors

When a vector is resolved into its components and the components are right angles to each other, then such components are called rectangular components of given vector along two perpendicular directions.

The rectangular components of a vector lying in the plane



$$\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$$

If θ is the angle subtended by vector $\mathbf{A}$ with X-axis, then $A_x = A\cos\theta$ and $A_y = A\sin\theta$ represented the rectangular components of A along two perpendicular directions.

$\therefore$ $$A^2 = A_x^2 + A_y^2$$

or $$A = \sqrt{A_x^2 + A_y^2}$$

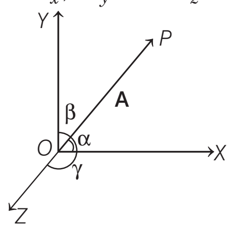
For the directions of vectors

$$\tan\theta = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

The rectangular components of a vector lying in the space

Suppose there is a vector $\mathbf{A}$ in space as shown in the figure. Let the rectangular components of $\mathbf{A}$ along X-axis, Y-axis and Z-axis be $\mathbf{A}_x$, $\mathbf{A}_y$ and $\mathbf{A}_z$ respectively.



$\therefore$ $$\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$\therefore$ $$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

This vector makes angle

$$\alpha = \cos^{-1}\left(\frac{A_x}{A}\right) \text{ with } X\text{-axis}$$

$$\beta = \cos^{-1}\left(\frac{A_y}{A}\right) \text{ with } Y\text{-axis}$$

$$\gamma = \cos^{-1}\left(\frac{A_z}{A}\right) \text{ with } Z\text{-axis}$$

Example 11. *A force of 8N makes an angle 30° with X-axis. Find the x and y components of the force.*

(a) $F_x = 4\sqrt{3}\ N, F_y = 4\ N$ (b) $F_x = 4\ N, F_y = 4\sqrt{3}\ N$
(c) $F_x = 2\ N, F_y = 2\sqrt{3}\ N$ (d) $F_x = 2\sqrt{3}\ N, F_y = 2\ N$

Sol. (a) Here, $F = 8$ N, $\theta = 30^\circ$

x-component of force, $F_x = F\cos 30^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$ N

y-component of force, $F_y = F\sin 30^\circ = 8 \times \frac{1}{2} = 4$ N

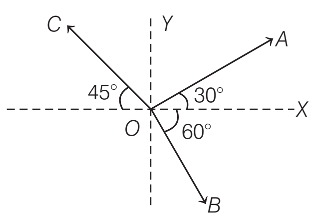
Example 12. *A force of 10.5 N acts on a particle along a direction making an angle of 37° with the vertical. The component of force in the vertical direction is*

(a) 4.5 *N* (b) 8.4 *N*
(c) 6.9 *N* (d) 3.2 *N*

Sol. (b) The component of the force in the vertical direction will be

$$F_v = F\cos\theta = 10.5\cos 37^\circ = 10.5 \times \frac{4}{5} = 8.4\text{ N}$$

Example 13. *The magnitudes of vectors* **OA**, **OB** *and* **OC** *as in figure are equal. The direction of* **OA** + **OB** − **OC** *is*



(a) $\tan^{-1}\left[\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}\right]$ (b) $\tan^{-1}\left[\frac{(1+\sqrt{3}+\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}\right]$

(c) $\cot^{-1}\left[\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}\right]$ (d) $\cot^{-1}\left[\frac{(1+\sqrt{3}+\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}\right]$

Sol. (a) Let, $OA = OB = OC = F$

x-component of $\mathbf{OA} = F\cos 30° = F\frac{\sqrt{3}}{2}$

x-component of $\mathbf{OB} = F\cos 60° = \frac{F}{2}$

x-component of $\mathbf{OC} = F\cos 135° = -\frac{F}{\sqrt{2}}$

$\therefore$ x-component of $\mathbf{OA} + \mathbf{OB} - \mathbf{OC}$

$$= \left(\frac{F\sqrt{3}}{3}\right) + \left(\frac{F}{2}\right) - \left(-\frac{F}{\sqrt{2}}\right)$$

$$= \frac{F}{2}(\sqrt{3} + 1 + \sqrt{2})$$

y-component of $\mathbf{OA} = F\sin 30° = \frac{F}{2}$

y-component of $\mathbf{OB} = -F\sin 60° = -\frac{F\sqrt{3}}{2}$.

y-component of $\mathbf{OC} = F\sin 45° = \frac{F}{\sqrt{2}}$

$\therefore$ y-component of $\mathbf{OA} + \mathbf{OB} - \mathbf{OC}$

$$= \left(\frac{F}{2}\right) + \left(-\frac{F\sqrt{3}}{2}\right) - \left(\frac{F}{\sqrt{2}}\right) = \frac{F}{2}(1-\sqrt{3}-\sqrt{2})$$

Angle of $\mathbf{OA} + \mathbf{OB} - \mathbf{OC}$ with x-axis

$$= \tan^{-1}\left[\frac{\frac{F}{2}(1-\sqrt{3}-\sqrt{2})}{\frac{F}{2}(1+\sqrt{3}+\sqrt{2})}\right] = \tan^{-1}\left[\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}\right]$$

Scalar Product or Dot Product

The scalar product of two vectors **A** and **B** is defined as the product of magnitude of **A** and **B** multiplied by the cosine of the smaller angle between them.

i.e. $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

or $\mathbf{A}\cdot\mathbf{B} = A(B\cos\theta)$ = (magnitude of **A**) (component of **B** in the direction of **A**).

Dot product or scalar product of two vectors gives the scalar quantity.

Some Important Properties of Dot Product

- Dot product is commutative $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$ and $\mathbf{B}\cdot\mathbf{A} = BA\cos\theta = AB\cos\theta$

  $\therefore \mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$, which is commutative law.
- Dot product is distributive over the addition of vectors *i.e.*, $\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$
- Dot product of two like parallel vectors

  Here, $\theta = 0°$

  $\therefore\ A\cdot B = AB\cos 0° = AB$ $\quad (\because \cos 0° = 1)$
- Dot product of two antiparallel vectors

  Here, $\theta = 180°$

  $\therefore\ \mathbf{A}\cdot\mathbf{B} = AB\cos 180° = -AB$ $\quad (\because \cos 180° = -1)$
- Dot product of orthogonal unit vectors

  $$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

  $$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$
- Dot product of two vectors in terms of their components

  $$\mathbf{A}\cdot\mathbf{B} = (x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}})\cdot(x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}})$$

  or $\mathbf{A}\cdot\mathbf{B} = x_1x_2 + y_1y_2 + z_1z_2$

Example 14. *If* **A** *and* **B** *are perpendicular vectors, where* $\mathbf{A} = 5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ *and* $\mathbf{B} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - a\hat{\mathbf{k}}$, *then the value of a is*

(a) -2 (b) 8 (c) -7 (d) -8

Sol. (d) For perpendicular vectors, $\mathbf{A}\cdot\mathbf{B} = 0$

So, $(5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - a\hat{\mathbf{k}}) = 0$

or $10 + 14 + 3a = 0$ or $3a = -24$

or $a = -8$

Example 15. *The angle between the two vectors* $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ *and* $\mathbf{B} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ *will be*

(a) *zero* (b) 45° (c) 90° (d) 180°

Sol. (c) $\mathbf{A}\cdot\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) = 9 + 16 - 25 = 0$

or $AB\cos\theta = 0$

or $\cos\theta = 0$

or $\theta = 90°$

Example 16. *The vectors* $\mathbf{P} = a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ *and* $\mathbf{Q} = a\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ *are perpendicular to each other. The positive value of a is*

(a) 3 (b) 2 (c) 1 (d) 0

Sol. (a) For perpendicular vectos $\mathbf{P}\cdot\mathbf{Q} = 0$

So, $(a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\cdot(a\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$

or $a^2 - 2a - 3 = 0$

On solving $a = 3$ or -1

Example 17. *What is the angle* ϕ *between* $\mathbf{a} = 3.0\,\hat{\mathbf{i}} - 4.0\,\hat{\mathbf{j}}$ *and* $\mathbf{b} = 2.0\,\hat{\mathbf{i}} - 3.0\,\hat{\mathbf{k}}$ *?*

(a) 60° (b) 53°

(c) 71° (d) 75°

Sol. (c) Angle between **a** and **b** is given as

$$\cos\phi = \frac{\mathbf{a}\cdot\mathbf{b}}{ab} = \frac{3\times 2 + (-4\times 0) + 0\times(-3)}{\sqrt{3^2 + (-4)^2 + 0^2}\cdot\sqrt{2^2 + 0^2 + (-3)^2}}$$

$$\Rightarrow \cos\phi = \frac{6}{5\sqrt{13}} \Rightarrow \cos\phi = 0.333$$

$$\Rightarrow \phi = \cos^{-1}(0.333) = 70.55° \simeq 71°$$

Vector Product or Cross Product

The vector product or cross product of two vectors is a single vector whose magnitude is equal to the product of the magnitudes of two given vectors multiplied by the sine of the smaller angle between the two given vectors.

i.e. $\mathbf{A} \times \mathbf{B} = (AB \sin \theta)\, \hat{\mathbf{n}}$

The unit vector normal to the plane containing vectors A and $\mathbf{B}$, is given by $\hat{\mathbf{n}} = \dfrac{\mathbf{A} \times \mathbf{B}}{AB \sin \theta} = \dfrac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$

Some Important Properties of Cross Product

- Cross product of two vectors does not obey the commutative law.

  i.e. $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$

  Here, $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$
- Cross product of two vectors is distributive over vector addition

  i.e. $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$
- Cross product of two like parallel vectors is zero. In this case, the angle between vectors will be zero degree.

  $\therefore \quad \mathbf{A} \times \mathbf{B} = (AB \sin 0^\circ)\, \hat{\mathbf{n}} = 0 \qquad [\because \sin 0^\circ = 0]$
- Cross product of two perpendicular vectors : In this case, $\theta = 90^\circ$

  $$\mathbf{A} \times \mathbf{B} = (AB \sin 90^\circ)\, \hat{\mathbf{n}} = (AB)\, \hat{\mathbf{n}}$$
- Cross product of orthogonal unit vectors

  $$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = (1 \times 1 \times \sin 0^\circ)\, \hat{\mathbf{n}} = 0,$$

  Similarly, $\hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$ and $\hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$

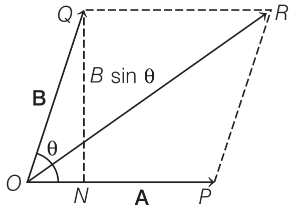
  Also, $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = 1 \times 1 \times \sin 90^\circ\, \hat{\mathbf{k}} = \hat{\mathbf{k}}$

  Similarly, $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}, \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$

  Now, $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}, \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$ and $\hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}$
- Cross product of two vectors in terms of their rectangular components

  $$\mathbf{A} \times \mathbf{B} = (x_1 \hat{\mathbf{i}} + y_1 \hat{\mathbf{j}} + z_1 \hat{\mathbf{k}}) \times (x_2 \hat{\mathbf{i}} + y_2 \hat{\mathbf{j}} + z_2 \hat{\mathbf{k}})$$

  $$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$
- Magnitude of cross product of two vectors $\mathbf{A}$ and $\mathbf{B}$ represents the area of the parallelogram.



$\therefore |\mathbf{A} \times \mathbf{B}| = AB \sin \theta = A(B \sin \theta) = OP \times QN$ = Area of the parallelogram.

Triple Product

Scalar triple product of three vectors is given by

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

where $\mathbf{A} = a_1 \hat{\mathbf{i}} + a_2 \hat{\mathbf{j}} + a_3 \hat{\mathbf{k}}$,

$$\mathbf{B} = b_1 \hat{\mathbf{i}} + b_2 \hat{\mathbf{j}} + b_3 \hat{\mathbf{k}} \text{ and } \mathbf{C} = c_1 \hat{\mathbf{i}} + c_2 \hat{\mathbf{j}} + c_3 \hat{\mathbf{k}}$$

It gives volume of parallelopiped formed with **A, B** and **C** as adjacent sides.

Vector triple product is given by

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

It is worth noting that

(i) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$ implies that vectors are coplanar.

(ii) In scalar triple product, dot and cross can be interchanged provided that their cyclic order is maintained.

(iii) Four points *A, B, C* and *D* are coplanar, if $\mathbf{AB} \cdot (\mathbf{BC} \times \mathbf{CD}) = 0$.

Example 18. *The vector* **A** *has a magnitude of 5 unit,* **B** *has a magnitude of 6 unit and the cross product* **A** *and* **B** *has the magnitude of 15 unit. The angle between* **A** *and* **B** *is*

(a) 90° (b) 60° (c) 30° (d) 120°

Sol. (c) If the angle between **A** and **B** is θ, the cross product will have a magnitude,

$$|\mathbf{A} \times \mathbf{B}| = AB \sin \theta \text{ or } 15 = 5 \times 6 \sin \theta \text{ or } \sin \theta = \frac{1}{2}$$

$$\therefore \quad \theta = 30^\circ$$

Example 19. *If* $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{k}}$ *and* $\mathbf{B} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$, *then,* $\mathbf{A} \times \mathbf{B}$ *is equal to*

(a) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (b) $-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

Sol. (b) We have, $\mathbf{A} \times \mathbf{B} = (\hat{\mathbf{i}} + 2\hat{\mathbf{k}}) \times (\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(0 - 2) + \hat{\mathbf{j}}(0 + 1) + \hat{\mathbf{k}}(1 - 0) = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Example 20. *The area of a parallelogram whose adjacent sides are* $\mathbf{P} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ *and* $\mathbf{Q} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ *is*

(a) 5 *square units* (b) 15 *square units*
(c) 20 *square units* (d) 25 *square units*

Sol. (a) $\mathbf{P} \times \mathbf{Q} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0 - 0) - \hat{\mathbf{j}}(0 - 0) + \hat{\mathbf{k}}(8 - 3) = 5\hat{\mathbf{k}}$$

Area of parallelogram $= |\mathbf{P} \times \mathbf{Q}| = 5$ square units

Practice Exercise

ROUND I Topically Divided Problems

Addition and Subtraction of Vectors

1. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are **[NCERT Exemplar]**

(a) Impulse, pressure and area
(b) Impulse and area
(c) Area and gravitational potential
(d) Impulse and pressure

2. What is the numerical value of the vector $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+5\hat{\mathbf{k}}$?

(a) $3\sqrt{2}$ (b) $5\sqrt{2}$
(c) $7\sqrt{2}$ (d) $9\sqrt{2}$

3. If, $0.5\,\hat{\mathbf{i}}+0.8\,\hat{\mathbf{j}}+c\,\hat{\mathbf{k}}$ is a unit vector, then the value of c is

(a) $\sqrt{0.11}$ (b) $\sqrt{0.22}$
(c) $\sqrt{0.33}$ (d) $\sqrt{0.89}$

4. Which one of the following statements is true? **[NCERT Exemplar]**

(a) A scalar quantity is the one that is conserved in a process.
(b) A scalar quantity is the one that can never take negative values.
(c) A scalar quantity is the one that does not vary from one point to another in space.
(d) A scalar quantity has the same value for observers with different orientations of the axes.

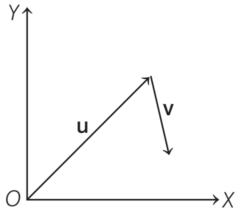
5. Given, vector, $\mathbf{A}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\,\hat{\mathbf{k}}$ and vector $\mathbf{B}=3\,\hat{\mathbf{i}}-3\hat{\mathbf{j}}+6\,\hat{\mathbf{k}}$, then which one of the following statements is true?

(a) A is perpendicular to B.
(b) A is parallel to B.
(c) Magnitude of A is half of that of B.
(d) Magnitude of B is equal to that of A.

6. What is the unit vector along $\hat{\mathbf{i}}+\hat{\mathbf{j}}$?

(a) $\dfrac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ (b) $\sqrt{2}\,(\hat{\mathbf{i}}+\hat{\mathbf{j}})$
(c) $\hat{\mathbf{i}}+\hat{\mathbf{j}}$ (d) $\hat{\mathbf{k}}$

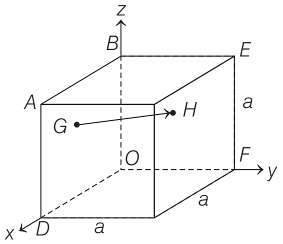
7. Figure shows the orientation of two vectors $\mathbf{u}$ and $\mathbf{v}$ in the XY-plane. **[NCERT Exemplar]**



If $\mathbf{u}=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}$ and $\mathbf{v}=p\hat{\mathbf{i}}+q\hat{\mathbf{j}}$, which of the following is correct?

(a) a and p are positive while b and q are negative
(b) a, p and b are positive while q is negative
(c) a, q and b are positive while p is negative
(d) a, b, p and q are all positive

8. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **[JEE Main 2019]**



(a) $\dfrac{1}{2}a\,(\hat{\mathbf{i}}-\hat{\mathbf{k}})$ (b) $\dfrac{1}{2}a\,(\hat{\mathbf{j}}-\hat{\mathbf{i}})$
(c) $\dfrac{1}{2}a\,(\hat{\mathbf{j}}-\hat{\mathbf{k}})$ (d) $\dfrac{1}{2}a\,(\hat{\mathbf{k}}-\hat{\mathbf{i}})$

9. Two forces, each equal to $\dfrac{p}{2}$ act at right angles. Their effect may be neutralised by a third force acting along their bisector in the opposite direction with a magnitude of

(a) p (b) $\dfrac{p}{2}$ (c) $\dfrac{p}{\sqrt{2}}$ (d) $\sqrt{2}\,p$

10. Given $\mathbf{A}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\,\hat{\mathbf{k}}$. When a vector $\mathbf{B}$ is added to $\mathbf{A}$, we get a unit vector along X-axis, then $\mathbf{B}$ is

(a) $-2\hat{\mathbf{j}}+3\,\hat{\mathbf{k}}$ (b) $-\hat{\mathbf{i}}-2\hat{\mathbf{j}}$
(c) $-\hat{\mathbf{i}}+3\,\hat{\mathbf{k}}$ (d) $2\hat{\mathbf{j}}-3\,\hat{\mathbf{k}}$

11. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are acting at right angles to each other, then their resultant is

(a) $F_1 + F_2$ (b) $\sqrt{F_1^{\,2} + F_2^{\,2}}$

(c) $\sqrt{F_1^{\,2} - F_2^{\,2}}$ (d) $\dfrac{F_1 + F_2}{2}$

12. Given, $\mathbf{R} = \mathbf{A} + \mathbf{B}$ and $R = A = B$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is

(a) 60° (b) 90°

(c) 120° (d) 180°

13. The resultant of two forces, each $\mathbf{P}$, acting at an angle θ is

(a) $2P\sin\dfrac{\theta}{2}$ (b) $2P\cos\dfrac{\theta}{2}$

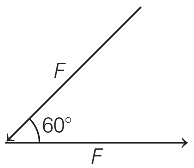
(c) $2P\cos\theta$ (d) $P\sqrt{2}$

14. The resultant of two vectors of magnitudes $2A$ and $\sqrt{2}\,A$ acting at an angle θ is $\sqrt{10}\,A$. The correct value of θ is

(a) 30° (b) 45°

(c) 60° (d) 90°

15. Two forces, each equal to F, act as shown in figure. Their resultant is



(a) $\dfrac{F}{2}$ (b) F (c) $\sqrt{3}\,F$ (d) $\sqrt{5}\,F$

16. If $\mathbf{P} = 4\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} + 6\,\hat{\mathbf{k}}$ and $\mathbf{Q} = \hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} - 3\,\hat{\mathbf{k}}$, then the angle which $\mathbf{P} + \mathbf{Q}$ makes with X-axis is

(a) $\cos^{-1}\left(\dfrac{3}{\sqrt{50}}\right)$ (b) $\cos^{-1}\left(\dfrac{4}{\sqrt{50}}\right)$

(c) $\cos^{-1}\left(\dfrac{5}{\sqrt{50}}\right)$ (d) $\cos^{-1}\left(\dfrac{12}{\sqrt{50}}\right)$

17. If $\mathbf{A} + \mathbf{B} = \mathbf{C}$ and $A = \sqrt{3}, B = \sqrt{3}$ and $C = 3$, then the angle between $\mathbf{A}$ and $\mathbf{B}$ is

(a) 0° (b) 30° (c) 60° (d) 90°

18. If the magnitude of the sum of the two vectors is equal to the difference of their magnitudes, then the angle between vectors is

(a) 0° (b) 45° (c) 90° (d) 180°

19. If, the resultant of two forces $(A + B)$ and $(A - B)$ is $\sqrt{A^2 + B^2}$, then the angle between these forces is

(a) $\cos^{-1}\left[-\dfrac{(A^2 - B^2)}{A^2 + B^2}\right]$ (b) $\cos^{-1}\left[-\dfrac{(A^2 + B^2)}{(A^2 - B^2)}\right]$

(c) $\cos^{-1}\left[-\dfrac{A^2 + B^2}{2\,(A^2 - B^2)}\right]$ (d) $\cos^{-1}\left[-\dfrac{2\,(A^2 + B^2)}{A^2 - B^2}\right]$

20. If the resultant of $\mathbf{A}$ and $\mathbf{B}$ makes angle α with $\mathbf{A}$ and β with $\mathbf{B}$, then

(a) $\alpha < \beta$, always (b) $\alpha < \beta$, if $A < B$

(c) $\alpha < \beta$, if $A > B$ (d) $\alpha < \beta$, if $A = B$

21. If the resultant of the vectors $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}), (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ and $\mathbf{C}$ is a unit vector along the y-direction, then $\mathbf{C}$ is

(a) $-2\hat{\mathbf{i}} - \hat{\mathbf{k}}$ (b) $-2\hat{\mathbf{i}} + \hat{\mathbf{k}}$ (c) $2\hat{\mathbf{i}} - \hat{\mathbf{k}}$ (d) $-2\hat{\mathbf{i}} + \hat{\mathbf{k}}$

22. $(\mathbf{P} + \mathbf{Q})$ is a unit vector along X-axis. If $\mathbf{P} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then what is the value of vector $\mathbf{Q}$?

(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (b) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$ (c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$

23. What vector must be added to the sum of two vectors $2\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$ and $3\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}$ so that the resultant is a unit vector along Z-axis?

(a) $5\hat{\mathbf{i}} + \hat{\mathbf{k}}$ (b) $-5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ (c) $3\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}$ (d) $-3\,\hat{\mathbf{j}} + 2\,\hat{\mathbf{k}}$

24. Two vectors $\mathbf{a}$ and $\mathbf{b}$ are at an angle of 60° with each other. Their resultant makes an angle of 45° with $\mathbf{a}$. If $|\mathbf{b}| = 2$ units, then $|\mathbf{a}|$ is

(a) $\sqrt{3}$ (b) $\sqrt{3} - 1$ (c) $\sqrt{3} + 1$ (d) $\sqrt{3}/2$

25. A vector $\mathbf{A}$ when added to the vector $\mathbf{B} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$ yields a resultant vector that is in the positive y-direction and has a magnitude equal to that of $\mathbf{B}$. Find the magnitude of $\mathbf{A}$.

(a) $\sqrt{10}$ (b) 10 (c) 5 (d) $\sqrt{15}$

26. It is found that $|\mathbf{A} + \mathbf{B}| = |\mathbf{A}|$. This necessarily implies, **[NCERT Exemplar]**

(a) $\mathbf{B} = 0$ (b) $\mathbf{A}, \mathbf{B}$ are parallel

(c) $\mathbf{A}, \mathbf{B}$ are perpendicular (d) $\mathbf{A} \cdot \mathbf{B} \le 0$

27. The simple sum of two co-initial vectors is 16 units. Their vector sum is 8 units. The resultant of the vectors is perpendicular to the smaller vector. The magnitudes of the two vectors are

(a) 2 units and 14 units (b) 4 units and 12 units

(c) 6 units and 10 units (d) 8 units and 8 units

28. The resultant of two forces at right angle is 5N. When the angle between them is 120°, the resultant is $\sqrt{13}$. Then, the forces are

(a) $\sqrt{12}$ N, $\sqrt{13}$ N (b) $\sqrt{20}$ N, $\sqrt{5}$ N

(c) 3 N, 4 N (d) $\sqrt{40}$ N, $\sqrt{15}$ N

29. Given, $\mathbf{P} = \mathbf{A} + \mathbf{B}$ and $P = A + B$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is

(a) 0° (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) π

30. What is the angle between $\mathbf{P}$ and the resultant of $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$?

(a) zero (b) $\tan^{-1}(P/Q)$

(c) $\tan^{-1}(Q/P)$ (d) $\tan^{-1}(P - Q)/(P + Q)$

31. Two vectors **a** and **b** are such that $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$. What is the angle between **a** and **b**?
(a) 0° (b) 90° (c) 60° (d) 180°

32. The vectors $\mathbf{A}_1$ and $\mathbf{A}_2$ each of magnitude A are inclined to each other such that their resultant is equal to $\sqrt{3}A$. Then the resultant of $\mathbf{A}_1$ and $-\mathbf{A}_2$ is
(a) $2A$ (b) $\sqrt{3}A$ (c) $\sqrt{2}A$ (d) A

33. If $\mathbf{A}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{B}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then the magnitude of $2\mathbf{A}-3\mathbf{B}$ is
(a) $\sqrt{90}$ (b) $\sqrt{50}$
(c) $\sqrt{190}$ (d) $\sqrt{30}$

34. The vector which can give unit vector along X-axis with $\mathbf{A}=2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+7\hat{\mathbf{k}}$, $\mathbf{B}=7\hat{\mathbf{i}}+2\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $\mathbf{C}=-4\hat{\mathbf{i}}+7\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is
(a) $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ (b) $-5\hat{\mathbf{i}}-5\hat{\mathbf{j}}+5\hat{\mathbf{k}}$
(c) $-5\hat{\mathbf{i}}-5\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ (d) $4\hat{\mathbf{i}}-5\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

Resolution of Vectors

35. The x and y components of a force are 2 N and – 3N. The force is
(a) $2\hat{\mathbf{i}}-3\hat{\mathbf{j}}$ (b) $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}$
(c) $-2\hat{\mathbf{i}}-3\hat{\mathbf{j}}$ (d) $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}$

36. The magnitude of the X and Y components of **A** are 7 and 6. Also the magnitudes of X and Y components of $\mathbf{A}+\mathbf{B}$ are 11 and 9, respectively. What is the magnitude of **B**?
(a) 5 (b) 6
(c) 8 (d) 9

37. One of the rectangular components of a velocity of 60 kmh^{-1} is 30 kmh^{-1}. The other rectangular component is
(a) 30 km h^{-1} (b) $30\sqrt{3}$ km h^{-1}
(c) $30\sqrt{2}$ km h^{-1} (d) zero

38. A force is inclined at 60° to the horizontal. If its rectangular component in the horizontal direction is 50 N, then magnitude of the force in the vertical direction is
(a) 25 N (b) 75 N
(c) 87 N (d) 100 N

39. The component of a vector **r** along X-axis will have maximum value, if **[NCERT Exemplar]**
(a) **r** is along positive Y-axis
(b) **r** is along positive X-axis
(c) **r** makes an angle of 45° with the X-axis
(d) **r** is along negative Y-axis

40. Consider a vector $\mathbf{F}=4\hat{\mathbf{i}}-3\hat{\mathbf{j}}$. Another vector that is perpendicular to **F** is
(a) $4\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ (b) $6\hat{\mathbf{j}}$ (c) $7\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}}-4\hat{\mathbf{j}}$

41. The angle subtended by the vector, $\mathbf{A}=4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+12\hat{\mathbf{k}}$ with the X-axis is
(a) $\sin^{-1}\left(\frac{3}{13}\right)$ (b) $\sin^{-1}\left(\frac{4}{13}\right)$
(c) $\cos^{-1}\left(\frac{4}{13}\right)$ (d) $\cos^{-1}\left(\frac{3}{13}\right)$

Scalar and Vector Product of Vectors

42. The angle between the Z-axis and the vector $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\sqrt{2}\,\hat{\mathbf{k}}$ is
(a) 30° (b) 45°
(c) 60° (d) 90°

43. The angle between $\mathbf{A}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\mathbf{B}=\hat{\mathbf{i}}-\hat{\mathbf{j}}$ is **[NCERT Exemplar]**
(a) 45° (b) 90° (c) –45° (d) 180°

44. Given, $\mathbf{P}=3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $\mathbf{Q}=2\hat{\mathbf{i}}+5\hat{\mathbf{k}}$. The magnitude of the scalar product of these vectors, is
(a) 20 (b) 23
(c) 26 (d) $5\sqrt{33}$

45. If $\mathbf{P}=2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{Q}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}$, then $\mathbf{P}\cdot\mathbf{Q}$ is
(a) zero (b) 6
(c) 12 (d) 15

46. If $\mathbf{AB}=AB$, then the angle between **A** and **B** is
(a) 0° (b) 45°
(c) 90° (d) 180°

47. Projection of **P** on **Q** is
(a) $\mathbf{P}\cdot\hat{\mathbf{Q}}$ (b) $\hat{\mathbf{P}}\cdot\mathbf{Q}$
(c) $\mathbf{P}\times\hat{\mathbf{Q}}$ (d) $\mathbf{P}\times\mathbf{Q}$

48. **A** and **B** are two vectors given by $\mathbf{A}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ and $\mathbf{B}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$. The magnitude of the components of **A** along **B** is
(a) $\frac{5}{\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$
(c) $\frac{7}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

49. If, $\mathbf{A}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $\mathbf{B}=4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, then angle between **A** and **B** is
(a) $\sin^{-1}\left(\frac{25}{29}\right)$ (b) $\sin^{-1}\left(\frac{29}{25}\right)$
(c) $\cos^{-1}\left(\frac{25}{29}\right)$ (d) $\cos^{-1}\left(\frac{29}{25}\right)$

50. What is the angle between $(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ and $\hat{\mathbf{i}}$?
(a) 0° (b) $\pi/6$
(c) $\pi/3$ (d) None of these

51. If $\mathbf{A}=\mathbf{B}$, then which of the following is not correct?
(a) $\hat{\mathbf{A}}=\hat{\mathbf{B}}$ (b) $\hat{\mathbf{A}}\cdot\hat{\mathbf{B}}=AB$
(c) $|\mathbf{A}|=|\mathbf{B}|$ (d) $A\hat{\mathbf{B}}\,||\,B\hat{\mathbf{A}}$

52. For what value of a, $\mathbf{A} = 2\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}$ will be perpendicular to $\mathbf{B} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$?

(a) 4 (b) zero
(c) 3 (d) 1

53. The sum of two vectors **A** and **B** is at right angles to their difference, then

(a) $A = B$
(b) $A = 2B$
(c) $B = 2A$
(d) **A** and **B** have the same direction

54. Given, θ is the angle between $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$. Then, $|\hat{\mathbf{A}} \times \hat{\mathbf{B}}|$ is equal to

(a) $\sin\theta$ (b) $\cos\theta$
(c) $\tan\theta$ (d) $\cot\theta$

55. If $\mathbf{P}\cdot\mathbf{Q} = 0$, then $|\mathbf{P}\times\mathbf{Q}|$ is

(a) $|\mathbf{P}||\mathbf{Q}|$ (b) zero
(c) 1 (d) $\sqrt{PQ}$

56. Given, $\mathbf{C} = \mathbf{A}\times\mathbf{B}$. The angle which **A** makes with **C** is

(a) 0° (b) 45°
(c) 90° (d) 180°

57. The adjacent sides of a parallelogram are represented by co-initial vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. The area of the parallelogram is

(a) 5 units along Z-axis (b) 5 units in XY-plane
(c) 3 units in XZ-plane (d) 3 units in YZ-plane

58. The magnitudes of the two vectors **a** and **b** are a and b, respectively. The vector product of **a** and **b** cannot be

(a) equal to zero (b) less than ab
(c) equal to ab (d) greater than ab

59. Given, $\mathbf{A} = 4\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. Which of the following is correct?

(a) $\mathbf{A}\times\mathbf{B} = 0$
(b) $\mathbf{A}\cdot\mathbf{B} = 24$
(c) $\dfrac{|\mathbf{A}|}{|\mathbf{B}|} = \dfrac{1}{2}$
(d) **A** and **B** are anti-parallel

60. If $\mathbf{A}\cdot\mathbf{B} = 0$ and $\mathbf{A}\times\mathbf{B} = 1$, then **A** and **B** are

(a) perpendicular unit vectors
(b) parallel unit vectors
(c) parallel
(d) perpendicular

61. The area of a parallelogram formed by the vectors $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ as its adjacent sides, is

(a) $8\sqrt{3}$ units (b) 64 units
(c) 32 units (d) $4\sqrt{6}$ units

62. The magnitude of the vectors product of two vectors is $\sqrt{3}$ times their scalar product. The angle between the two vectors is

(a) 90° (b) 60°
(c) 45° (d) 30°

63. Three vectors **A**, **B** and **C** satisfy the relation $\mathbf{AB} = 0$ and $\mathbf{AC} = 0$. If **B** and **C** are not lying in the same plane, then **A** is parallel to

(a) **B** (b) **C** (c) $\mathbf{B}\times\mathbf{C}$ (d) **BC**

64. A vector $\mathbf{F}_1$ is along the positive Y-axis. If its vector product with another vector $\mathbf{F}_2$ is zero, then $\mathbf{F}_2$ could be

(a) $4\hat{\mathbf{j}}$ (b) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$
(c) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $-4\hat{\mathbf{i}}$

65. The area of the parallelogram represented by the vectors, $\mathbf{A} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ is

(a) 14 units (b) 7.5 units
(c) 10 units (d) 5 units

66. If **A** and **B** denote the sides of a parallelogram and its area is $\frac{1}{2}AB$ (A and B are the magnitude of **A** and **B** respectively), the angle between **A** and **B** is

(a) 30° (b) 60° (c) 45° (d) 120°

67. Two vectors **A** and **B** are inclined to each other at an angle θ. Which of the following is the unit vector perpendicular to both **A** and **B**?

(a) $\dfrac{\mathbf{A}\times\mathbf{B}}{\mathbf{AB}}$ (b) $\dfrac{\hat{\mathbf{A}}\times\hat{\mathbf{B}}}{\sin\theta}$
(c) $\dfrac{\mathbf{A}\times\mathbf{B}}{AB\sin\theta}$ (d) $\dfrac{\mathbf{A}\times\mathbf{B}}{AB\cos\theta}$

68. Angle between **A** and **B** is θ. What is the value of $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A})$?

(a) $A^2B\cos\theta$ (b) $A^2B\sin\theta\cos\theta$
(c) $A^2B\sin\theta$ (d) zero

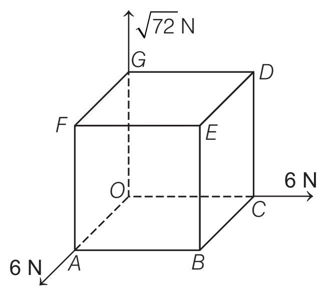
69. Given, $\mathbf{C} = \mathbf{A}\times\mathbf{B}$ and $\mathbf{D} = \mathbf{B}\times\mathbf{A}$. What is the angle between **C** and **D**?

(a) 30° (b) 60°
(c) 90° (d) 180°

ROUND II Mixed Bag

Only One Correct Option

1. Three forces of magnitudes 6 N, 6 N and $\sqrt{72}$ N act at a corner of a cube along three sides as shown in figure. Resultant of these forces is



(a) 12 N along OB (b) 18 N along OA
(c) 18 N along OC (d) 12 N along OE

2. The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these forces is perpendicular to the smaller force has a magnitude of 8 N. If the smaller force has magnitude x, then the value of x is
(a) 2 N (b) 4 N
(c) 6 N (d) 7 N

3. The magnitude of resultant of three vectors of magnitude 1, 2 and 3 whose directions are those of the sides of an equilateral triangle taken in order is
(a) zero (b) $2\sqrt{2}$ units
(c) $4\sqrt{3}$ units (d) $\sqrt{3}$ units

4. Two vectors $\mathbf{A}$ and $\mathbf{B}$ have equal magnitudes. The magnitude of $(\mathbf{A}+\mathbf{B})$ is n times the magnitude of $(\mathbf{A}-\mathbf{B})$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is **[JEE Main 2019]**
(a) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (b) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$
(c) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (d) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

5. There are two forces each of magnitude 10 units. One inclined at an angle of 30° and the other at an angle of 135° to the positive direction of X-axis. The x and y components of the resultant are respectively,
(a) $1.59\,\hat{\mathbf{i}}$ and $12.07\,\hat{\mathbf{j}}$ (b) $10\,\hat{\mathbf{i}}$ and $10\,\hat{\mathbf{j}}$
(c) $1.59\,\hat{\mathbf{i}}$ (d) $15.9\,\hat{\mathbf{i}}$ and $12.07\,\hat{\mathbf{j}}$

6. What is the angle between $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}$?
(a) 0° (b) $\pi/6$
(c) $\pi/3$ (d) None of these

7. If $\mathbf{A}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ and $\mathbf{B}=3\hat{\mathbf{i}}-6\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, then vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$ has magnitude k times that of $(6\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}})$. The value of k is equal to
(a) 1 (b) 4
(c) 7 (d) 9

8. If $\mathbf{A}_1$ and $\mathbf{A}_2$ are two non-collinear unit vectors and if $|\mathbf{A}_1+\mathbf{A}_2|=\sqrt{3}$, then the value of $(\mathbf{A}_1-\mathbf{A}_2)\cdot(2\mathbf{A}_1+\mathbf{A}_2)$ is
(a) 1 (b) 1/2
(c) 3/2 (d) 2

9. If vectors $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A}=5\hat{\mathbf{i}}+6\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\mathbf{B}=6\hat{\mathbf{i}}-2\hat{\mathbf{j}}-6\hat{\mathbf{k}}$. Which is/are of the following correct?
(a) $\mathbf{A}$ and $\mathbf{B}$ are mutually perpendicular
(b) Product of $\mathbf{A}\times\mathbf{B}$ is the same $\mathbf{B}\times\mathbf{A}$
(c) The magnitude of $\mathbf{A}$ and $\mathbf{B}$ are equal
(d) The magnitude of $\mathbf{A}\cdot\mathbf{B}$ is non-zero

10. Which of the following statement(s) is/are correct?
(a) The magnitude of the vector $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$ is 10.
(b) A force $(3\hat{\mathbf{i}}+4\hat{\mathbf{j}})$ N acting on a particle causes a displacement $6\hat{\mathbf{j}}$. The work done by the force is 30 N.
(c) If $\mathbf{A}$ and $\mathbf{B}$ represent two adjacent sides of a parallelogram, then $|\mathbf{A}\times\mathbf{B}|$ give the area of that parallelogram.
(d) A force has magnitude 20 N. Its component in a direction making an angle 60° with the force is $10\sqrt{3}$ N.

11. The component of vector $\mathbf{A}=a_x\hat{\mathbf{i}}+a_y\hat{\mathbf{j}}+a_z\hat{\mathbf{k}}$ along the direction of $(\hat{\mathbf{i}}-\hat{\mathbf{j}})$ is
(a) $(a_x-a_y+a_z)$ (b) (a_x+a_y)
(c) $(a_x-a_y)/\sqrt{2}$ (d) $(a_x-a_y+a_z)$

12. If $\mathbf{A}\times\mathbf{B}=\mathbf{B}\times\mathbf{A}$, then the angle between A and B is
(a) π (b) $\pi/3$
(c) $\pi/2$ (d) $\pi/4$

13. Given that, $\mathbf{A}+\mathbf{B}+\mathbf{C}=0$. Out of three vectors, two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then, the angles between vectors are given by
(a) 45°, 45°, 90° (b) 90°, 135°, 135°
(c) 30°, 60°, 90° (d) 45°, 60°, 90°

14. If the vectors $\mathbf{A}=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}$ and $\mathbf{B}=5\hat{\mathbf{i}}-p\hat{\mathbf{j}}$ are parallel to each other, the magnitude of $\mathbf{B}$ is
(a) $5\sqrt{5}$ (b) 10
(c) 15 (d) $2\sqrt{5}$

15. Two vectors **A** and **B** are inclined at an angle θ. Now if the vectors are interchanged, then the resultant turns through an angle α. Which of the following relation is true?

(a) $\tan\frac{\alpha}{2} = \left(\frac{A-B}{A+B}\right)^2 \tan\frac{\theta}{2}$ (b) $\tan\frac{\alpha}{2} = \left(\frac{A-B}{A+B}\right) \tan\frac{\theta}{2}$

(c) $\tan\frac{\alpha}{2} = \left(\frac{A-B}{A+B}\right) \cot\frac{\theta}{2}$ (d) $\tan\frac{\alpha}{2} = \left(\frac{A-B}{A+B}\right) \cot\frac{\theta}{2}$

16. Let $|\mathbf{A}_1| = 3$, $|\mathbf{A}_2| = 5$ and $|\mathbf{A}_1 + \mathbf{A}_2| = 5$. The value of $(2\mathbf{A}_1 + 3\mathbf{A}_2)\cdot(3\mathbf{A}_1 - 2\mathbf{A}_2)$ is **[JEE Main 2019]**

(a) -106.5 (b) -112.5
(c) -99.5 (d) -118.5

17. The vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $5\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are coplanar when a is

(a) -9 (b) 9
(c) -18 (d) 18

18. Three vectors **A**, **B** and **C** add up to zero. Find which is false? **[NCERT Exemplar]**

(a) $(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$ is not zero unless **B**, **C** are parallel
(b) $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$ is zero unless **B**, **C** are parallel
(c) If **A**, **B**, **C** defined a plane, $(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$ is in that plane
(d) $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = |\mathbf{A}||\mathbf{B}||\mathbf{C}| \to C^2 = A^2 + B^2$

Numerical Value Questions

19. When two non-zero vectors **a** and **b** are perpendicular to each other, the magnitude of their resultant is R. When they are opposite to each other, the magnitude of their resultant is $\frac{R}{\sqrt{2}}$. The value of $\left(\frac{a}{b} + \frac{b}{a}\right)$ is ……… .

20. The sum of two forces **P** and **Q** is **R** such that $|\mathbf{R}| = |\mathbf{P}|$. The angle θ (in degree) that the resultant of $2\mathbf{P}$ and **Q** will make with **Q** is ……… . **[JEE Main 2020]**

21. If $\mathbf{a} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{c} = 3\hat{\mathbf{i}} - y\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ are coplanar, then the value of y is ……… .

22. A triangle ABC has its vertices at $A(2, 1, 1)$, $B(1, -3, -5)$ and $C(4, -4, -4)$. If $\angle BAC = \theta$ and $\cos\theta = \frac{8n}{\sqrt{1537}}$. The value of n is ……… .

23. The volume of a parallelopiped, whose edges are represented by
$\mathbf{a} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ m, $\mathbf{b} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$ m and $\mathbf{c} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ m is ……… m^3.

Answers

Round I

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (a)
11. (b)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (c)
21. (a)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (a)	**30.** (a)
31. (b)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (c)
41. (c)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (a)	**46.** (a)	**47.** (a)	**48.** (a)	**49.** (c)	**50.** (d)
51. (b)	**52.** (c)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (d)	**59.** (a)	**60.** (a)
61. (d)	**62.** (b)	**63.** (c)	**64.** (a)	**65.** (c)	**66.** (a)	**67.** (b)	**68.** (d)	**69.** (d)	

Round II

1. (d)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (d)	**19.** 4	**20.** 90
21. 4	**22.** 6	**23.** 7							

Solutions

Round I

1. Out of the given quantities, impulse and area are vector quantities and other are scalar quantities.

2. Required numerical value is $\sqrt{3^2+4^2+5^2}$, *i.e.* $\sqrt{50}$ or $5\sqrt{2}$.

3. Clearly, $(0.5)^2+(0.8)^2+c^2=1$

$$0.25+0.64+c^2=1$$

or $\quad c^2=1-0.25-0.64=0.11$

or $\quad c=\sqrt{0.11}$

4. A scalar quantity has the same value for observers with different orientations of the axes.

5. A vector **A** is parallel to that of vector **B**, if it can be written as

$$A=mB$$

Here, $\quad A=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\,\hat{\mathbf{k}})=\dfrac{1}{3}(3\hat{\mathbf{i}}-3\hat{\mathbf{j}}+6\,\hat{\mathbf{k}})$

$$A=\frac{1}{3}B$$

This implies A is parallel to B and magnitude of A is $\dfrac{1}{3}$ times the magnitude of B.

6. We have $\mathbf{A}=A\hat{\mathbf{A}}$ or $\hat{\mathbf{A}}=\dfrac{\mathbf{A}}{A}$

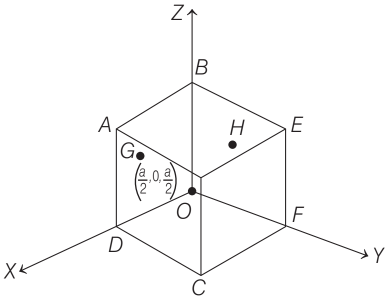
$\therefore$ Required unit vector is $\dfrac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{|\hat{\mathbf{i}}+\hat{\mathbf{j}}|}=\dfrac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{\mathbf{2}}}$

7. As per figure, in $\mathbf{u}=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}$, both a and b are positive. In $\mathbf{v}=p\hat{\mathbf{i}}+q\hat{\mathbf{j}}$, p is positive and q is negative.

Thus, a, p and b are positive and q is negative.

8. In the given cube, coordinates of point G(centre of face $ABOD$) are $x_1=\dfrac{a}{2}$, $y_1=0, z_1=\dfrac{a}{2}$

where, a = side of cube



and coordinates of point H are

$$x_2=0,\ y_2=\frac{a}{2},\ z_2=\frac{a}{2}$$

So, vector GH is

$$GH=(x_2-x_1)\hat{\mathbf{i}}+(y_2-y_1)\hat{\mathbf{j}}+(z_2-z_1)\ \hat{\mathbf{k}}$$

$$=-\frac{a}{2}\ \hat{\mathbf{i}}+\frac{a}{2}\ \hat{\mathbf{j}}=\frac{a}{2}\ (\hat{\mathbf{j}}-\hat{\mathbf{i}})$$

9. As, $\quad R=\sqrt{a^2+b^2+2\,ab\cos\theta}$

$\Rightarrow \quad R=\sqrt{\left(\dfrac{p}{2}\right)^2+\left(\dfrac{p}{2}\right)^2+2\left(\dfrac{p}{2}\right)\left(\dfrac{p}{2}\right)\cos 90^\circ}$

$\Rightarrow \quad R=\sqrt{2}\cdot\dfrac{p}{2}=\dfrac{p}{\sqrt{2}}$

10. As, $\mathbf{B}+(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\,\hat{\mathbf{k}})=\hat{\mathbf{i}}$ or $\mathbf{B}=-2\,\hat{\mathbf{j}}+3\,\hat{\mathbf{k}}$

11. As, resultant of two vectors is given by,

$$R=\sqrt{A^2+B^2+2AB\cos\theta}$$

$\therefore \quad F^2=F_1^2+F_2^2+2\,F_1F_2\cos 90^\circ$

Given, $\theta=90^\circ$

or $\quad F^2=F_1^2+F_2^2 \;\Rightarrow\; F=\sqrt{F_1^2+F_2^2}$

12. For the resultant,

$$R^2=R^2+R^2+2\,R^2\cos\theta$$

or $\quad R^2=2\,R^2+2\,R^2\cos\theta$

$$\frac{1}{2}=1+\cos\theta$$

or $\quad \cos\theta=-\dfrac{1}{2}$ or $\theta=120^\circ$

13. As, $R^2=P^2+P^2+2\,P^2\cos\theta$

or $\quad R^2=2\,P^2+2\,P^2\cos\theta$

or $\quad R^2=2\,P^2\,(1+\cos\theta)$

or $\quad R^2=2\,P^2\left(2\cos^2\dfrac{\theta}{2}\right)$

or $\quad R^2=4\,P^2\cos^2\dfrac{\theta}{2}$

or $\quad R=2\,P\cos\dfrac{\theta}{2}$

14. Resultant, $R=\sqrt{A^2+B^2+2AB\cos\theta}$

$\Rightarrow \quad 10\,A^2=4\,A^2+2\,A^2+2\times 2\,A\times\sqrt{2}\,A\times\cos\theta$

or $\quad 4\,A^2=4\sqrt{2}A^2\cos\theta$

or $\quad \cos\theta=\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta=45^\circ$

15. Note that the angle between two forces is 120° and not 60°.

$$R^2=F^2+F^2+2\cdot F\cdot F\cos 120^\circ$$

$\Rightarrow \quad R^2=F^2+F^2-F^2$

$\Rightarrow \quad R^2=F^2$

$\Rightarrow \quad R=F$

16. As, $\mathbf{P}+\mathbf{Q}=5\,\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\,\hat{\mathbf{k}}$

$\Rightarrow \quad \cos\alpha=\dfrac{5}{\sqrt{5^2+4^2+3^2}}=\dfrac{5}{\sqrt{50}}$

or $\quad \alpha=\cos^{-1}\left(\dfrac{5}{\sqrt{50}}\right)$

17. As, $\mathbf{A}+\mathbf{B}=\mathbf{C}$ (Given)

So, it is given that $\mathbf{C}$ is the resultant of $\mathbf{A}$ and $\mathbf{B}$

$\therefore \quad C^2 = A^2 + B^2 + 2AB\cos\theta$

or $\quad 3^2 = 3+3+2\times 3\times\cos\theta$

or $\quad 3 = 6\cos\theta$

or $\quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

18. According to question, $\sqrt{P^2+Q^2+2PQ\cos\theta} = (P-Q)$

$\Rightarrow \quad P^2+Q^2+2PQ\cos\theta = P^2+Q^2-2PQ$

$\Rightarrow \quad 2PQ(1+\cos\theta)=0$

But, $\quad 2PQ \neq 0$

$\therefore \quad 1+\cos\theta = 0$

or $\quad \cos\theta = -1$

or $\quad \theta = 180°$

19. Here, $\quad P=(A+B),\ Q=(A-B)$

and $\quad R=\sqrt{A^2+B^2}$

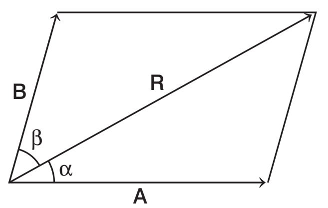
$$\Rightarrow \quad \cos\theta = \frac{R^2-P^2-Q^2}{2PQ}$$

$$= \frac{(A^2+B^2)-(A+B)^2-(A-B)^2}{2(A+B)(A-B)}$$

$$= -\left[\frac{A^2+B^2}{2(A^2-B^2)}\right]$$

$$\therefore \quad \theta = \cos^{-1}\left[-\frac{A^2+B^2}{2(A^2-B^2)}\right]$$

20. We can make the diagram as below



Clearly, $\tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta}$ and $\tan\beta = \dfrac{A\sin\theta}{B+A\cos\theta}$

From the above equation, it is clear that when $\alpha < \beta$, then $B < A$.

21. Given, $(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})+(\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})+\mathbf{C}=\hat{\mathbf{j}}$

$\therefore \quad \mathbf{C} = \hat{\mathbf{j}}-(\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}})-(\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

$\quad = -2\hat{\mathbf{i}}-\hat{\mathbf{k}}$

22. Given, $\mathbf{P}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then we have

$\mathbf{P}+\mathbf{Q}=\hat{\mathbf{i}}$

$\mathbf{Q}=\hat{\mathbf{i}}-\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}=\hat{\mathbf{j}}-\hat{\mathbf{k}}$

23. As, $\mathbf{A}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$, $\mathbf{B}=3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{C}=?$

$\mathbf{R}=\hat{\mathbf{k}}=\mathbf{A}+\mathbf{B}+\mathbf{C}$

$\hat{\mathbf{k}}=(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})+(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}})+\mathbf{C}$

$\quad =5\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}+\mathbf{C}$

$\therefore \quad \mathbf{C}=-5\hat{\mathbf{i}}+3\hat{\mathbf{j}}$

24. Here, $\tan 45° = \dfrac{2\sin 60°}{a+2\cos 60°} = \dfrac{\sqrt{3}}{a+1}$

$$1 = \frac{\sqrt{3}}{a+1}$$

or $\quad a+1=\sqrt{3}$

$\quad a=\sqrt{3}-1$

25. Given, $\mathbf{C}=|\mathbf{B}|\hat{\mathbf{j}} \Rightarrow \mathbf{C}=5\hat{\mathbf{j}}$

Let, $\quad \mathbf{C}=\mathbf{A}+\mathbf{B}=A+3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$

$\quad 5\hat{\mathbf{j}}=A+3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$

$\Rightarrow \quad \mathbf{A}=-3\hat{\mathbf{i}}+\hat{\mathbf{j}}$

$\quad |\mathbf{A}|=\sqrt{(3)^2+(1)^2}=\sqrt{10}$

26. If $|\mathbf{A}+\mathbf{B}|=|\mathbf{A}|$, then either $|\mathbf{B}|=0$ or $\mathbf{A}$ and $\mathbf{B}$ will be antiparallel, where $|\mathbf{B}|=2|\mathbf{A}|$.

27. According to question, $P+Q=16$ ...(i)

$\Rightarrow \quad P^2+Q^2+2PQ\cos\theta=64$ ...(ii)

$$\therefore \quad \tan 90° = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\text{or} \quad \infty = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$\Rightarrow \quad P+Q\cos\theta=0$

or $\quad Q\cos\theta=-P$ ...(iii)

From Eqs. (ii) and (iii), we get

$\quad P^2+Q^2+2P(-P)=64$

or $\quad Q^2-P^2=64$

or $\quad (Q-P)(Q+P)=64$ ...(iv)

Now from Eqs. (i) and (iv), we get

or $\quad Q-P=\dfrac{64}{16}=4$ ...(v)

Adding, Eqs. (i) and (v), we get

$\quad 2Q=20$

or $\quad Q=10$ units

From Eq. (i), $\quad P+10=16$

or $\quad P=6$ units

28. Let, A and B be the two forces.

As per question $\quad \sqrt{A^2+B^2}=5$

or $\quad A^2+B^2=25$ ...(i)

and $\quad A^2+B^2+2AB\cos 120°=13$

or $\quad 25+2AB\times(-1/2)=13$

or $\quad AB=25-13=12$

or $\quad 2AB=24$ ...(ii)

Solving Eqs. (i) and (ii), we get

$\quad A=3\,\text{N}$ and $B=4\,\text{N}$

29. Given, $|\mathbf{P}|=A+B$

$\Rightarrow \quad |\mathbf{P}|^2=(A+B)^2$

or $\quad |\mathbf{A}+\mathbf{B}|^2=(A+B)^2$

or $\quad A^2+B^2+2AB\cos\theta=A^2+B^2+2AB$

or $\quad \cos\theta=1 \Rightarrow \theta=0°$

30. Resultant, $\mathbf{R} = (\mathbf{P}+\mathbf{Q}) + (\mathbf{P}-\mathbf{Q}) = 2\,\mathbf{P}$. Thus, angle between $\mathbf{R}$ and $\mathbf{P}$ is 0°.

31. From the condition given in question,

$$a^2 + b^2 + 2\,ab\cos\theta = a^2 + b^2 - 2\,ab\cos\theta$$

or $\quad 4\,ab\cos\theta = 0$

But $\quad 4\,ab \neq 0$

$\therefore \quad \cos\theta = 0$

or $\quad \theta = 90°$

32. Let θ be the angle between $\mathbf{A}_1$ and $\mathbf{A}_2$, then

$$A_1^2 + A_2^2 + 2A_1A_2\cos\theta = R^2$$

or $\quad A^2 + A^2 + 2AA\cos\theta = 3\,A^2$

or $\quad \cos\theta = \dfrac{1}{2} = \cos 60°$

or $\quad \theta = 60°$

The angle between $\mathbf{A}_1$ and $-\mathbf{A}_2$ is $(180° - 60°) = 120°$

$\therefore$ Resultant of $\mathbf{A}_1$ and $-\mathbf{A}_2$ is

$$R' = [A_1^2 + A_2^2 + 2A_1A_2\cos(180° - 60°)]^{1/2}$$
$$= [A^2 + A^2 + 2AA\cos 120°]^{1/2} = A$$

33. Clearly, $2\,\mathbf{A} - 3\,\mathbf{B} = 2\,(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}) - 3\,(2\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$$= -4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}} - 7\,\hat{\mathbf{k}}$$

$\therefore$ Magnitude of $\;2\mathbf{A} - 3\mathbf{B} = \sqrt{(-4)^2 + (5)^2 + (-7)^2}$

$$= \sqrt{16 + 25 + 49} = \sqrt{90}$$

34. The vector is $\hat{\mathbf{i}} - (\mathbf{A} + \mathbf{B} + \mathbf{C})$

$$= \hat{\mathbf{i}} - [(2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}) + (7\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} - 5\,\hat{\mathbf{k}}) + (-4\,\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}})]$$
$$= -5\,\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 5\,\hat{\mathbf{k}}$$

35. Here, $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}}$ or $\quad \mathbf{F} = 2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}$

36. Let, $\mathbf{A} + \mathbf{B} = \mathbf{R}$. Given, $A_x = 7$ and $A_y = 6$

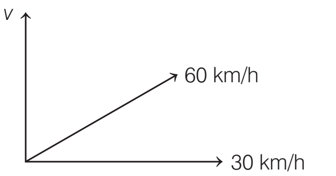
Also, $\quad R_x = 11 \quad$ and $\quad R_y = 9$

Therefore, $\quad B_x = R_x - A_x = 11 - 7 = 4$

and $\quad B_y = R_y - A_y = 9 - 6 = 3$

Hence, magnitude of $\;\mathbf{B} = \sqrt{B_x^2 + B_y^2} = \sqrt{4^2 + 3^2} = 5$

37.



$$60^2 = 30^2 + v^2$$

or $\quad v^2 = 90 \times 30$

or $\quad v = 30\sqrt{3}\ \text{kmh}^{-1}$

38. Given, $A_x = 50$ and $\theta = 60°$

Then $\quad \tan\theta = A_y / A_x$

or $\quad A_y = A_x\tan\theta$

or $\quad A_y = 50\tan 60° = 50 \times \sqrt{3} = 87\ \text{N}$

39. If $\mathbf{r}$ makes an angle θ with X-axis, then component of $\mathbf{r}$ along X-axis $= r\cos\theta$.

It will be maximum, if $\cos\theta = \max = 1$ or $\theta = 0°$. *i.e.* $\mathbf{r}$ is along positive X-axis.

40. Since $\mathbf{F} = 4\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}$ is lying in XY-plane, hence the vector perpendicular to $\mathbf{F}$ must be lying perpendicular to XY-plane or along Z-axis, *i.e.* $7\hat{\mathbf{k}}$.

41. Let θ be the angle which $\mathbf{A}$ make with X-axis, then

$$\cos\theta = \frac{A_x}{A} = \frac{4}{\sqrt{4^2 + 3^2 + 12^2}} = \frac{4}{13}$$

$\therefore \quad \theta = \cos^{-1}\left(\dfrac{4}{13}\right)$

42. From, $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}$

or $\quad \cos\theta = \dfrac{(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \sqrt{2}\,\hat{\mathbf{k}})\cdot\hat{\mathbf{k}}}{1\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}$

or $\quad \cos\theta = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

43. $\cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{(\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{1^2 + 1^2} \times \sqrt{1^2 + (-1)^2}} = \dfrac{1-1}{2} = 0 = \cos 90°$

$\therefore \quad \theta = 90°$

44. $|\mathbf{P}\cdot\mathbf{Q}| = (3\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} + 5\,\hat{\mathbf{k}}) = 6 + 20 = 26$

45. Here, $\mathbf{P}\cdot\mathbf{Q} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 6\,(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}) - 6\,(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}) = 0$

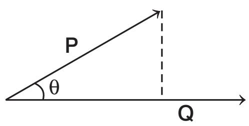
46. We know that, $[\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0,\ \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1]$

As, $\quad AB\cos\theta = AB$

or $\quad \cos\theta = 1$

or $\quad \theta = 0°$

47. Projection of $\mathbf{P}$ on $\mathbf{Q}$ is $P\cos\theta$



Here, $\quad P\cos\theta = \dfrac{PQ\cos\theta}{Q} = \dfrac{\mathbf{P}\cdot\mathbf{Q}}{Q} = \mathbf{P}\cdot\hat{\mathbf{Q}}$

48. Magnitude of component of $\mathbf{A}$ along $\mathbf{B}$

$$= \frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{B}|} = \frac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{1^2 + 1^2}} = \frac{5}{\sqrt{2}}$$

49. As, $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}})\cdot(4\,\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\,\hat{\mathbf{k}})}{\sqrt{4 + 9 + 16}\cdot\sqrt{16 + 9 + 4}}$

$$= \frac{8 + 9 + 8}{29} = \frac{25}{29}$$

$\Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{25}{29}\right)$

50. Using, $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}$

or $\quad \cos\theta = \dfrac{(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})\cdot\hat{\mathbf{i}}}{(1^2+2^2+2^2)^{1/2}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3} = 0.33$

$\Rightarrow \quad \theta = 70°30'$

51. Here $\hat{\mathbf{A}}\cdot\hat{\mathbf{B}} = (1)\,(1)\cos 0° = 1 \neq AB.$

52. $\mathbf{A}\perp\mathbf{B}$, if $\mathbf{A}\cdot\mathbf{B} = AB\cos 90° = 0$

$(2\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 0$

or $\quad 8 - 2a - 2 = 0 \text{ or } a = 3$

53. Using, $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$, given $\theta = 90° \Rightarrow \cos 90° = 0$

Then, $\quad (\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B}) = 0$

$A^2 - B^2 = 0 \text{ or } A = B$

54. The vector product,

$|\mathbf{A}\times\mathbf{B}| = (1)\,(1)\sin\theta = \sin\theta$

55. Since, $\mathbf{P}\cdot\mathbf{Q} = PQ\cos\theta$, then

$\mathbf{P}\cdot\mathbf{Q} = 0$

$\Rightarrow \quad \mathbf{P}\perp\mathbf{Q} \text{ or } \theta = 90°$

$|\mathbf{P}\times\mathbf{Q}| = PQ\sin 90° = PQ \text{ or } |\mathbf{P}||\mathbf{Q}|$

56. The direction of the vector given by the cross product of the two vectors is perpendicular to the plane containing the two vectors, *i.e.* $\mathbf{A}\times\mathbf{B} = (AB\sin\theta) = \mathbf{C}$. Therefore, the angle which $\mathbf{A}$ makes with $\mathbf{C}$ is 90°.

57. The required area, $\mathbf{A}\times\mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\times(\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$= 8\,(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) + 3(\hat{\mathbf{j}}\times\hat{\mathbf{i}}) = 8\,\hat{\mathbf{k}} - 3\hat{\mathbf{k}} = 5\,\hat{\mathbf{k}}$

58. As, $|\mathbf{a}\times\mathbf{b}| = ab\sin\theta$

Since, $\sin\theta$ cannot be greater than 1.

$\therefore |\mathbf{a}\times\mathbf{b}|$ cannot be greater than ab.

59. We have $\mathbf{A}\times\mathbf{B} = (4\,\hat{\mathbf{i}} + 6\hat{\mathbf{j}})\times(2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}})$

$= 12\,(\hat{\mathbf{i}}\times\hat{\mathbf{j}})\times 12(\hat{\mathbf{j}}\times\hat{\mathbf{i}})$

$= 12\,(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) - 12\,(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) = 0$

60. Given, $\mathbf{A}\cdot\mathbf{B} = 0$

$\Rightarrow \quad \mathbf{A}\perp\mathbf{B} \qquad$ (Since, $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$)

Now, $\quad \mathbf{A}\times\mathbf{B} = 1$

or $\quad AB\sin\theta = 1$

$AB\sin 90° = 1 \text{ or } AB = 1$

$\Rightarrow \quad A = 1 \text{ and } B = 1$

So, $\mathbf{A}$ and $\mathbf{B}$ are perpendicular unit vectors.

61. Required area of parallelogram,

$\mathbf{A}\times\mathbf{B} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}})\times(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= -2\hat{\mathbf{k}} - \hat{\mathbf{j}} - 6(-\hat{\mathbf{k}}) - 2\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}} - 6\,(-\hat{\mathbf{i}})$

$= 4\,\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$

$|\mathbf{A}\times\mathbf{B}| = \sqrt{4^2+8^2+4^2} = \sqrt{32+64}$

$= \sqrt{96} = 4\sqrt{6}$ units

62. Given, $\quad |\mathbf{A}\times\mathbf{B}| = \sqrt{3}\mathbf{A}\cdot\mathbf{B}$

$\Rightarrow \quad AB\sin\theta = \sqrt{3}AB\cos\theta$

or $\quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60°$

63. As, $\mathbf{A}\cdot\mathbf{B} = 0$ so, $\mathbf{A}$ is perpendicular to $\mathbf{B}$. Also $\mathbf{A}\cdot\mathbf{C} = 0$ means $\mathbf{A}$ is perpendicular to $\mathbf{C}$. Since $\mathbf{B}\times\mathbf{C}$ is perpendicular to $\mathbf{B}$ and $\mathbf{C}$, then clearly $\mathbf{A}$ is parallel to $\mathbf{B}\times\mathbf{C}$.

64. As, $\mathbf{F}_1 = F_1\,\hat{\mathbf{j}}$; $\mathbf{F}_1\times\mathbf{F}_2$ is equal to zero only, if angle between $\mathbf{F}_1$ and $\mathbf{F}_2$ is either 0° or 180°. So, $\mathbf{F}_2$ will be $4\,\hat{\mathbf{j}}$. (In direction of Y-axis) $\qquad (\because \hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = 0)$

65. Area $= |\mathbf{A}\times\mathbf{B}| = |(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\times(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) = |10\,\hat{\mathbf{k}}| = 10$ units

66. Area of parallelogram $= |\mathbf{A}\times\mathbf{B}|$

$AB\sin\theta = \dfrac{1}{2}AB$

$\therefore \quad \sin\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 30°$

67. The required unit vector should be

$\hat{\mathbf{n}} = \dfrac{\mathbf{A}\times\mathbf{B}}{AB\sin\theta}$

$= \dfrac{A\hat{\mathbf{A}}\times B\hat{\mathbf{B}}}{AB\sin\theta} = \dfrac{\hat{\mathbf{A}}\times\hat{\mathbf{B}}}{\sin\theta}$

68. $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A}) = 0$ (According to rules for scalar triple product)

69. Since, $(\mathbf{A}\times\mathbf{B}) = -(\mathbf{B}\times\mathbf{A})$, so $\mathbf{C} = \mathbf{D}$ *i.e.*, $\mathbf{C}$ and $\mathbf{D}$ are antiparallel to each other, *i.e.* $\theta = 180°$.

Round II

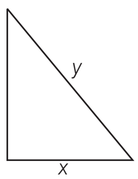
1. The resultant of 6 N along OC and 6 N along OA is

$R = \sqrt{6^2+6^2} = \sqrt{72}$ N along OB

The resultant of $\sqrt{72}$ N along OB and $\sqrt{72}$ N along OG is

$R' = \sqrt{72+72} = 12$ N along OE.

2. Given, $x + y = 16$, Also $y^2 = 8^2 + x^2$



or $\quad y^2 = 64 + (16 - y)^2 \qquad (\because x = 16 - y)$

or $\quad y^2 = 64 + 256 + y^2 - 32\,y$

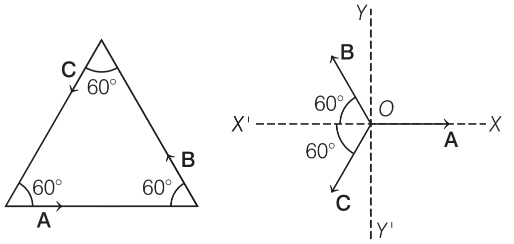
or $\quad 32\,y = 320$

or $\quad y = 10$ N

$\therefore \quad x + 10 = 16$

or $\quad x = 6$ N

3. The three vectors **A**, **B** and **C** are represented as shown in figure (a) where $A = 1, B = 2$ and $C = 3$. Here the sides of the equilateral triangle represent only the directions and not the magnitudes of the vectors.



(a) (b)

In figure (b), these vector are drawn from a common point, O and they are lying in XY-plane. Resolving these vectors into two rectangular components along XY-axis and Y-axis, we have, the X-component of resultant vector as

$$R_X = |\mathbf{A}| + |\mathbf{B}| \cos(180° - 60°) + |\mathbf{C}| \cos(180° + 60°)$$
$$= 1 - 2\cos 60° - 3\cos 60°$$
$$= 1 - 2 \times \frac{1}{2} - 3 \times \frac{1}{2} = -\frac{3}{2}$$

Y-component of resultant vector is

$$R_Y = 0 + |\mathbf{B}| \sin(180° - 60°) + |\mathbf{C}| \sin(180° + 60°)$$
$$= 0 + 2\sin 60° - 3\sin 60° = -\sin 60° = -\sqrt{3}/2$$

Magnitude of resultant vector,

$$R = \sqrt{R_X^2 + R_Y^2} = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3} \text{ units}$$

4. Given, $|\mathbf{A}| = |\mathbf{B}|$ or $A = B$ ...(i)

Let magnitude of $(\mathbf{A} + \mathbf{B})$ is R and for $(\mathbf{A} - \mathbf{B})$ is R'.

Now, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

and $R^2 = A^2 + B^2 + 2AB\cos\theta$

$R^2 = 2A^2 + 2A^2 \cos\theta$ ...(ii)

[∵ using Eq. (i)]

Again, $\mathbf{R}' = \mathbf{A} - \mathbf{B}$

$\Rightarrow$ $R'^2 = A^2 + B^2 - 2AB\cos\theta$

$R'^2 = 2A^2 - 2A^2 \cos\theta$ ...(iii)

[∵ using Eq. (i)]

Given, $R = nR'$ or $\left(\frac{R}{R'}\right)^2 = n^2$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{n^2}{1} = \frac{1 + \cos\theta}{1 - \cos\theta}$$

or $$\frac{n^2 - 1}{n^2 + 1} = \frac{(1 + \cos\theta) - (1 - \cos\theta)}{(1 + \cos\theta) + (1 - \cos\theta)}$$

$\Rightarrow$ $$\frac{n^2 - 1}{n^2 + 1} = \frac{2\cos\theta}{2} = \cos\theta$$

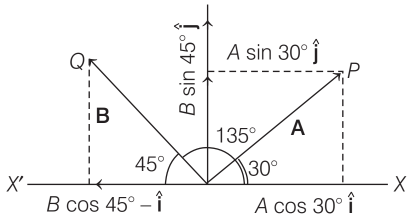
or $$\theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$$

5. Here, $\mathbf{A} = \mathbf{OP} = 10$ units along OP

$\mathbf{B} = \mathbf{OQ} = 10$ units along OQ

$\angle XOP = 30°$ and $\angle XOQ = 135°$

$\therefore$ $\angle QOX' = 180° - 135° = 45°$



Resolving **A** and **B** into two rectangular components we have $A\cos 30°$ along OX and $A\sin 30°$ along OY. While $B\cos 45°$ along OX' and $B\sin 45°$ along OY'.

Resultant of components of forces along X-axis.

$$= (A\cos 30° - B\cos 45°)\,\hat{\mathbf{i}}$$
$$= (10 \times \sqrt{3}/2 - 10 \times 1/\sqrt{2})\,\hat{\mathbf{i}} = 1.59\,\hat{\mathbf{i}}$$

Resultant of components forces along Y-axis

$$= (A\sin 30° + B\sin 45°)\,\hat{\mathbf{j}}$$
$$= \left(10 \times \frac{1}{2} + 10\frac{1}{\sqrt{2}}\right)\hat{\mathbf{j}} = 12.07\,\hat{\mathbf{j}}$$

6. Using $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

$\because$ $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$,

$\therefore$ $A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\Rightarrow$ $$\cos\theta = \frac{\mathbf{A} \cdot \hat{\mathbf{i}}}{\sqrt{\mathbf{3}}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$= \frac{1.732}{3} = 0.5773 = \cos 54°44'$$

or $\theta = 54°44' \simeq 55°$

7. Let, **C** be a vector perpendicular to **A** and **B**, then as per question, $k\mathbf{C} = \mathbf{A} \times \mathbf{B}$

or $$k = \frac{(\mathbf{A} \times \mathbf{B})}{\mathbf{C}} = \frac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{(6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})}$$

$$= \frac{(42\hat{\mathbf{i}} + 14\hat{\mathbf{j}} - 21\hat{\mathbf{k}})}{(6\hat{\mathbf{i}} + 2\hat{\mathbf{i}} - 3\hat{\mathbf{k}})} = 7$$

8. Here, $A_1 = A_2 = 1$

and $A_1^2 + A_2^2 + 2A_1A_2\cos\theta = (\sqrt{3})^2 = 3$

or $1 + 1 + 2 \times 1 \times 1 \times \cos\theta = 3$

or $\cos\theta = \frac{1}{2}$

Now, $(\mathbf{A}_1 - \mathbf{A}_2) \cdot (2\mathbf{A}_1 + \mathbf{A}_2) = 2A_1^2 - A_2^2 - A_1A_2\cos\theta$

$$= 2 \times 1^2 - 1^2 - 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$$

9. Here, $\mathbf{A} \cdot \mathbf{B} = (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}) = 0$

Hence, **A** and **B** are mutually perpendicular to each other. Vector product of two vectors is not commutative, hence $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$.

10. (a) If $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$, then $|\mathbf{A}| = \sqrt{3^2 + 4^2} = 5$

(b) $W = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot 6\hat{\mathbf{j}} = 24$ **J**

(c) $|\mathbf{A} \times \mathbf{B}|$ = Area of parallelogram whose two adjacent sides are represented by two vectors **A** and **B**.

(d) Component of force F in the direction making an angle $\theta = F\cos\theta = 20\cos 60^\circ = 20 \times \frac{1}{2} = 10$ N

11. Here, $\mathbf{A} = (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}})$

Let, $\mathbf{B} = (\hat{\mathbf{i}} - \hat{\mathbf{j}})$

Then, $$\hat{\mathbf{B}} = \frac{\mathbf{B}}{B} = \frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{(1)^2 + (-1)^2}} = \frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}$$

Component of **A** along the direction of **B** is

$$\mathbf{A} \cdot \hat{\mathbf{B}} = (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}) \cdot \frac{(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{2}} = \frac{(a_x - a_y)}{\sqrt{2}}$$

12. As, $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$ or $(\mathbf{A} \times \mathbf{B}) - (\mathbf{B} \times \mathbf{A}) = 0$

or $$(\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{B}) = 0$$

or $$2(\mathbf{A} \times \mathbf{B}) = 0$$

or $$2AB\sin\theta\,\hat{\mathbf{n}} = 0$$

As, $$A \neq 0$$

nor $$B \neq 0$$

So, $$\sin\theta = 0$$

Hence, $$\theta = 0^\circ \text{ or } \pi$$

13. If $|\mathbf{A}| = |\mathbf{B}| = x$, then $|\mathbf{C}| = \sqrt{2}\,x$.

Now, $$\mathbf{A} + \mathbf{B} = -\mathbf{C}$$

or $$(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} + \mathbf{B}) = (-\mathbf{C}) \cdot (-\mathbf{C})$$

$\Rightarrow$ $$A^2 + B^2 + 2AB\cos\theta = C^2$$

$\Rightarrow$ $$x^2 + x^2 + 2x^2\cos\theta = 2x^2$$

or $$\cos\theta = 0 \quad \text{or} \quad \theta = 90^\circ$$

Again, $\mathbf{A} + \mathbf{C} = -\mathbf{B}$

$\Rightarrow$ $$(\mathbf{A} + \mathbf{C}) \cdot (\mathbf{A} + \mathbf{C}) = -\mathbf{B} \cdot \mathbf{B}$$

or $$\mathbf{A} \cdot \mathbf{A} + \mathbf{C} \cdot \mathbf{C} + 2\mathbf{A} \cdot \mathbf{C} = B^2$$

or $$x^2 + 2x^2 + 2x^2\sqrt{2}\cos\theta = x^2$$

or $$\cos\theta = -\frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\theta = 135^\circ$$

Again, $$\mathbf{B} + \mathbf{C} = -\mathbf{A}$$

or $$(\mathbf{B} + \mathbf{C}) \cdot (\mathbf{B} + \mathbf{C}) = (-\mathbf{A}) \cdot (-\mathbf{A})$$

or $$x^2 + 2x^2 + 2x^2\sqrt{2}\cos\theta = x^2$$

or $$\cos\theta = \frac{-2x^2}{2x^2\sqrt{2}\cos\theta} = -\frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\theta = 135^\circ$$

14. As, $\mathbf{A} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 5\hat{\mathbf{i}} - p\hat{\mathbf{j}}$

$\therefore$ $$A = \sqrt{2^2 + 4^2} = \sqrt{20}$$

and $$B = \sqrt{5^2 + p^2}$$

Now, $$\mathbf{A} \cdot \mathbf{B} = 10 - 4p$$

If $\mathbf{A} \,||\, \mathbf{B}$, then

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 0^\circ = AB$$

$$10 - 4p = \sqrt{20}\sqrt{25 + p^2}$$

Squaring, $100 + 16p^2 - 80p$

$$= 20(25 + p^2) = 500 + 20p^2$$

or $$20p^2 - 16p^2 + 80p + 400 = 0$$

or $$p^2 + 20p + 100 = 0$$

or $$(p + 10)^2 = 0$$

$\therefore$ $$p = -10$$

$\therefore$ $$\mathbf{B} = 5\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$$

$$B = \sqrt{5^2 + (10)^2} = \sqrt{125} = 5\sqrt{5}$$

15. As, $\mathbf{A} = A\hat{\mathbf{A}}$ and $\mathbf{B} = B\hat{\mathbf{B}}$. Let θ be the angle between **A** and **B** as per question,

$$\cos\alpha = \frac{(A\hat{\mathbf{A}} + B\hat{\mathbf{B}}) \cdot (A\hat{\mathbf{B}} + B\hat{\mathbf{A}})}{|A\hat{\mathbf{A}} + B\hat{\mathbf{B}}||A\hat{\mathbf{B}} + B\hat{\mathbf{A}}|}$$

or $$\cos\alpha = \frac{2AB + (A^2 + B^2)\cos\theta}{\sqrt{(A^2 + B^2 + 2AB\cos\theta)^2}}$$

or $$2AB + (A^2 + B^2)\cos\theta = (A^2 + B^2)\cos\alpha + 2AB\cos\theta\cos\alpha$$

or $$2AB(1 - \cos\alpha\cos\theta) = (A^2 + B^2)(\cos\alpha - \cos\theta)$$

or $$\frac{2AB}{A^2 + B^2} = \frac{\cos\alpha - \cos\theta}{1 - \cos\alpha\cos\theta}$$

or $$\frac{2AB + (A^2 + B^2)}{(A^2 + B^2) - 2AB} = \frac{(\cos\alpha - \cos\theta) + (1 - \cos\alpha\cos\theta)}{(1 - \cos\alpha\cos\theta) - (\cos\alpha - \cos\theta)}$$

or $$\frac{(A + B)^2}{(A - B)^2} = \frac{(1 + \cos\alpha)(1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\alpha)} = \frac{\tan^2\theta/2}{\tan^2\alpha/2}$$

or $$\tan\frac{\alpha}{2} = \left(\frac{A - B}{A + B}\right)\tan\frac{\theta}{2}$$

16. For vector $\mathbf{A}_1 + \mathbf{A}_2$, we have

$$|\mathbf{A}_1 + \mathbf{A}_2|^2 = (\mathbf{A}_1 + \mathbf{A}_2) \cdot (\mathbf{A}_1 + \mathbf{A}_2) \qquad [\because \mathbf{x} \cdot \mathbf{x} = |\mathbf{x}|^2]$$

$\Rightarrow$ $$|\mathbf{A}_1 + \mathbf{A}_2|^2 = |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2\mathbf{A}_1 \cdot \mathbf{A}_2$$

Given, $|\mathbf{A}_1| = 3$, $|\mathbf{A}_2| = 5$ and $|\mathbf{A}_1 + \mathbf{A}_2| = 5$

So, we have, $(5)^2 = 9 + 25 + 2\mathbf{A}_1 \cdot \mathbf{A}_2$

$\Rightarrow$ $$\mathbf{A}_1 \cdot \mathbf{A}_2 = -\frac{9}{2}$$

Now, $(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$

$$= 6|\mathbf{A}_1|^2 - 4\mathbf{A}_1 \cdot \mathbf{A}_2 + 9\mathbf{A}_1 \cdot \mathbf{A}_2 - 6|\mathbf{A}_2|^2$$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5\mathbf{A}_1 \cdot \mathbf{A}_2$$

Substituting values, we have

$(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$

$$= 6(9) - 6(25) + 5\left(-\frac{9}{2}\right) = -118.5$$

17. If the three vectors are coplanar, then their scalar triple product is zero. So, $(\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B} = 0$

or $[(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})] \cdot [5\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}] = 0$

or $$[(13\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}] \cdot [5\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}] = 0$$

or $$65 - 4a + 7 = 0 \text{ or } a = 18$$

18. Given, $\mathbf{A}+\mathbf{B}+\mathbf{C}=0$, then $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are in one plane and are represented by the three sides of a triangle taken in one order.

(a) $\therefore \quad \mathbf{B}\times(\mathbf{A}+\mathbf{B}+\mathbf{C})=\mathbf{B}\times 0=0$

or $\quad \mathbf{B}\times\mathbf{A}+\mathbf{B}\times\mathbf{B}+\mathbf{B}\times\mathbf{C}=0$

or $\quad \mathbf{B}\times\mathbf{A}+0+\mathbf{B}\times\mathbf{C}=0$

or $\quad \mathbf{A}\times\mathbf{B}=\mathbf{B}\times\mathbf{C} \qquad \ldots(\text{i})$

$\therefore \quad (\mathbf{A}\times\mathbf{B})\times\mathbf{C}=(\mathbf{B}\times\mathbf{C})\times\mathbf{C};$

It cannot be zero.

If $\mathbf{B}\,||\,\mathbf{C}$, then $\mathbf{B}\times\mathbf{C}=0$, then $(\mathbf{B}\times\mathbf{C})\times\mathbf{C}=0$

Thus, option (a) is correct.

(b) $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}=(\mathbf{B}\times\mathbf{C})\cdot\mathbf{C}=0$

If $\mathbf{B}\,||\,\mathbf{C}$, then $\mathbf{B}\times\mathbf{C}=0$, then $(\mathbf{B}\times\mathbf{C})\times\mathbf{C}=0$

Thus, option (b) is correct.

(c) $(\mathbf{A}\times\mathbf{B})=\mathbf{D}=AB\sin\theta\,\mathbf{D}$. The direction of $\mathbf{D}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$.

$(\mathbf{A}\times\mathbf{B})\times\mathbf{C}=\mathbf{D}\times\mathbf{C}$. Its direction is in the plane of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. Thus, option (c) is correct.

(d) If $C^2=A^2+B^2$, then the angle between $\mathbf{A}$ and $\mathbf{B}$ is $90°$.

$\therefore \quad (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}=(AB\sin 90°\,\mathbf{D})\cdot\mathbf{C}=AB\,(\mathbf{D}\cdot\mathbf{C})$

$= ABC\cos 90° = 0.$

Thus, option (d) is false.

19. If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other.

$$R=\sqrt{a^2+b^2}$$

If a and b are opposite to each other,

$$\frac{R}{\sqrt{2}}=a-b$$

$$\Rightarrow \quad \frac{R^2}{2}=a^2+b^2-2ab$$

$$\Rightarrow \quad a^2+b^2=2a^2+2b^2-4ab$$

$$\Rightarrow \quad a^2+b^2-4ab=0$$

$$\Rightarrow \quad \frac{a^2+b^2}{ab}=4$$

$$\text{or} \quad \frac{a}{b}+\frac{b}{a}=4$$

20. Given, sum of $\mathbf{P}$ and $\mathbf{Q}$ is $\mathbf{R}$. Let angle between $\mathbf{P}$ and $\mathbf{Q}$ is β, then resultant of $\mathbf{P}$ and $\mathbf{Q}$,

$$|\mathbf{R}|=\sqrt{|\mathbf{P}|^2+|\mathbf{Q}|^2+2|\mathbf{P}||\mathbf{Q}|\cos\beta}$$

As, $\quad |\mathbf{R}|=|\mathbf{P}| \qquad$ (given)

So, $\quad |\mathbf{P}|^2=|\mathbf{P}|^2+|\mathbf{Q}|^2+2|\mathbf{P}||\mathbf{Q}|\cos\beta$

or $\quad |\mathbf{P}|\cos\beta=-\dfrac{Q}{2} \qquad \ldots.(\text{i})$

If resultant of $2\mathbf{P}$ and $\mathbf{Q}$ makes angle θ with $\mathbf{Q}$, then angle θ is given by

$$\tan\theta=\frac{|2\mathbf{P}|\sin\beta}{|\mathbf{Q}|+|2\mathbf{P}|\cos\beta}$$

Substituting the value of $|\mathbf{P}|\cos\beta$ from Eq. (i) in above equation, we get

$$\tan\theta=\infty\Rightarrow\theta=\frac{\pi}{2}=90°$$

21. If three vectors are coplanar, then $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=0$.

$$\Rightarrow \quad (2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 2 & -3\\ 3 & -y & 5\end{vmatrix}=0$$

$$\Rightarrow \quad (2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot\{\hat{\mathbf{i}}\,(10-3y)-\hat{\mathbf{j}}(5+9)+\hat{\mathbf{k}}\,(-y-6)\}=0$$

$$\Rightarrow \quad (2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot\{(10-3y)\hat{\mathbf{i}}-14\hat{\mathbf{j}}-(6+y)\hat{\mathbf{k}}\}=0$$

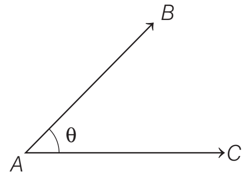
$$\Rightarrow \quad 2\,(10-3y)+14-(6+y)=0$$

$$\Rightarrow \quad 20-6y+14-6-y=0$$

$$\Rightarrow \quad 28-7y=0$$

$$\therefore \quad y=4$$

22. Here, $\mathbf{AB}$ = position vector of B – position vector of A



$$=(\hat{\mathbf{i}}-3\hat{\mathbf{j}}-5\,\hat{\mathbf{k}})-(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

$$=-\hat{\mathbf{i}}-4\hat{\mathbf{j}}-6\,\hat{\mathbf{k}}$$

and $\quad \mathbf{AC}$ = position vector of C – position vector of A

$$=(4\hat{\mathbf{i}}-4\hat{\mathbf{j}}-4\,\hat{\mathbf{k}})-(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=2\hat{\mathbf{i}}-5\hat{\mathbf{j}}-5\,\hat{\mathbf{k}}$$

$\because \quad \mathbf{AC}\cdot\mathbf{AB}=|\mathbf{AC}|\,|\mathbf{AB}|\cos\theta$

$$\therefore \quad \cos\theta=\frac{\mathbf{AC}\cdot\mathbf{AB}}{|\mathbf{AC}||\mathbf{AB}|}$$

$$=\frac{(-2+20+30)}{\sqrt{4+25+25}\,\sqrt{1+16+36}}$$

$$=\frac{48}{\sqrt{29}\,\sqrt{53}}$$

$$=\frac{48}{\sqrt{1537}}=\frac{8n}{\sqrt{1537}}$$

$$\therefore \quad n=6$$

23. The volume of parallelopiped is $|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$

$$=|(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\,\hat{\mathbf{k}})\cdot\{(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})\times(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})\}|$$

$$=\left|(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\,\hat{\mathbf{k}})\cdot\left\{\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}\right\}\right|$$

$$=(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\,\hat{\mathbf{k}})\cdot\{\hat{\mathbf{i}}\,(4-1)-\hat{\mathbf{j}}\,(2+3)+\hat{\mathbf{k}}\,(-1-6)\}|$$

$$=|(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\,\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}}-5\hat{\mathbf{j}}-7\,\hat{\mathbf{k}})|$$

$$=|6+15-28|$$

$$=7\text{ m}^3$$