Sample Question Paper - 44
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

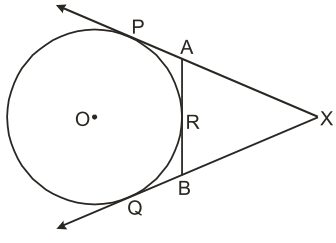
Time : 2 Hr. **Max. Marks : 40**

General Instructions :

1. The question paper consists of 14 questions divided into three Sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section - A

1. In the given figure, XP and XQ are two tangents to the circle with centre O drawn from an external point X. ARB is another tangent touching the circle at R. Prove that XA + AR = XB + BR.



OR

Prove that the tangents drawn at the end of any diameter are parallel.

2. For what value of p are $2p + 1$, 13, $5p - 3$ are the three consecutive terms of an A.P. ?

3. Solve : $abx^2 + (b^2 - ac)x - bc = 0$.

4. Two types of water tankers are available in a shop. One is in a cubic form of dimensions 1 m × 1 m × 1 m and another is in the cylindrical form of height 1 m and diameter 1 m. Calculate the volume of both the containers. (Use $\pi = 3.14$)

5. Find the mode of the following frequency distribution :

Class Interval	0–10	10-20	20-30	30-40	40-50	50-60
Frequency	2	8	10	5	4	3

6. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Section - B

7. Find the mean and median for the following data :

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	5	9	5	3

8. Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

9. The following table shows the weights (in gm) of a sample of 100 potatoes taken from a large consignment.

Weight	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110	110 – 120	120 – 130
Frequency	8	10	12	16	18	14	12	10

Calculate the cumulative frequency and determine the median class.

OR

The contents of 100 matchboxes were checked to determine the number of matchsticks they contained.

Matchboxes	35	36	37	38	39	40	41
Matchsticks	6	10	18	25	21	12	8

Calculate the mean of the number of matchsticks per box and determine how many extra matchsticks would have to be added to the total contents of the 100 boxes to bring up the mean to 39.

10. The angle of depression of a car standing on the ground from the top of a 75 m high tower is 30°. Find the distance of the car from the base of the tower (in m).

Section - C

11. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq. m, find the amount shared by each school to set-up the tents.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

From a solid cylinder of height 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take $\pi = 3.14$]

12. The horizontal distance between two poles is 15 m. The angle of depression of the top of the first pole as seen from the top of the second pole is 30°. If the height of the second pole is 24 m, find the height of the first pole. [Use $\sqrt{3} = 1.732$]

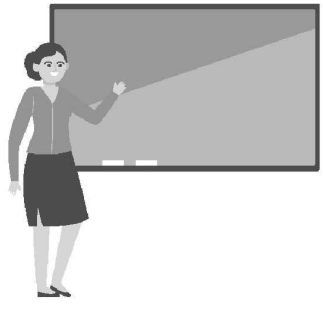
13. Amit was playing a number card game. In the game, some number cards (having both +ve or –ve numbers) are arranged in a row such that they are following an arithmetic progression. On his first turn, Amit picks up 6th and 14th card and finds their sum to be –76. On the second turn he picks up 8th and 16th card and finds their sum to be – 96.

Based on the above information, answer the following questions.

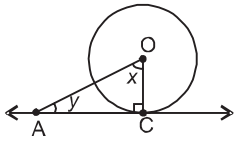
(i) What is the difference between the numbers on any two consecutive cards?

(ii) What is the sum of 9^{th} and 15^{th} card ?

14. For class of 10 students, a teacher planned a game for the revision of chapter circles with some questions written on the board, which are to be answered by the students. For each correct answer, a student will get a reward. Some of the questions are given below.



(i) In the given figure, $x + y =$



(ii) If PA and PB are two tangents drawn to a circle with centre O from P such that $\angle PBA = 50°$, then $\angle OAB =$

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section - A

1. As XP and XQ are two tangents from X, so

$$XP = XQ \quad ...(i)$$

Also, $AP = AR$ ...(ii)

[Tangents from the external point A]

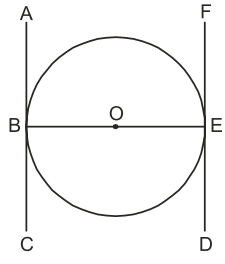
and $BR = BQ$ [Tangents from the external point B] ...(iii)

Now, $XP = XQ$ [from (i)]

$\Rightarrow$ $XA + AP = XB + BQ$

$\Rightarrow$ $XA + AR = XB + BR$ [from (ii) and (iii)]

OR



As AC and DF are tangents to B and E, the opposite points of the diameter BE.

$\therefore$ $\angle ABO = \angle OBC = \angle DEO = \angle OEF = 90°$

[the tangent at a point is always perpendicular to the radius at that point]

Hence, $\angle ABO = \angle OED$,

which are alternate interior angles.

$\therefore$ AC || DF.

2. For $2p + 1$, 13, $5p - 3$ to be consecutive, the common difference should be the same.

Thus $13 - (2p + 1) = 5p - 3 - 13$

$\Rightarrow$ $12 - 2p = 5p - 16$

$\Rightarrow$ $7p = 28$

$\Rightarrow$ $p = 4$.

3. Given,

$$abx^2 + (b^2 - ac)x - bc = 0$$

$\Rightarrow$ $abx^2 + b^2x - acx - bc = 0$

$\Rightarrow$ $bx(ax + b) - c(ax + b) = 0$

$\Rightarrow$ $(ax + b)(bx - c) = 0$

$\Rightarrow$ $x = -\frac{b}{a}$ and $\frac{c}{b}$.

4. Dimensions of cubic tank, $l = 1$ m, $b = 1$ m, $h = 1$ m

$\therefore$ Volume of cubic tank $= lbh = 1 \times 1 \times 1 = 1\ m^3$

Height of cylindrical container $= 1$ m

$$\text{Radius} = \frac{1}{2}\text{m}$$

$\therefore$ Volume of cylindrical tank $= \pi r^2 h$

$$= 3.14 \times \frac{1}{2} \times \frac{1}{2} \times 1$$

$$= 0.785\ m^3.$$

5.

Class Interval	0–10	10-20	20-30	30-40	40-50	50-60	
Frequency (f_i)	2	8	10	5	4	3	$N = \Sigma f_i = 32$

Since, 20 – 30 has the highest frequency, it is the modal class.

So, $l = 20$, $f = 10$, $f_1 = 8$, $f_2 = 5$, $h = 10$

Thus, $$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 20 + \frac{10-8}{20-8-5} \times 10$$

$$= 20 + \frac{2}{7} \times 10$$

$$= 20 + 2.86 = 22.86$$

6. As the numbers that are divisible by both 2 and 5, have their last digits as always 0. Hence, the series is 110, 120, 130, …, 990.

Here, $a = 110$, $d = 10$ and $t_n = 990$

We know that, $t_n = a + (n-1)d$

$\Rightarrow$ $990 = 110 + (n-1)10$

$\Rightarrow$ $(n-1)10 = 880$

$\Rightarrow$ $n - 1 = 88$

$\Rightarrow$ $n = 89$

Thus, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 is 89.

Section - B

7.

Class	Frequency (f_i)	X_i	f_iX_i	$c.f_i$
0 – 4	3	2	6	3
4 – 8	5	6	30	$cf = 8$
8 – 12	$f = 9$	10	90	17
12 – 16	5	14	70	22
16 – 20	3	18	54	25
	$\Sigma f_i = 25$		$\Sigma f_iX_i = 250$	

$$\text{Mean } (\overline{X}) = \frac{\Sigma f_i X_i}{\Sigma f_i}$$

$$= \frac{250}{25} = 10$$

Now, $N = \Sigma f_i = 25$

$$\frac{N}{2} = \frac{25}{2} = 12.5$$

Cumulative frequency just above 12.5 is 17 which lies in class interval 8 – 12.

$\therefore$ Median class is 8 – 12.

Here, $l = 8$, $h = 4$, $cf = 8$, $f = 9$

$$\text{Median} = l + \frac{\frac{N}{2} - c.f}{f} \times h$$

$$= 8 + \frac{12.5-8}{9} \times 4$$

$$= 8 + \frac{4.5}{9} \times 4$$

$$= 8 + 2 = 10$$

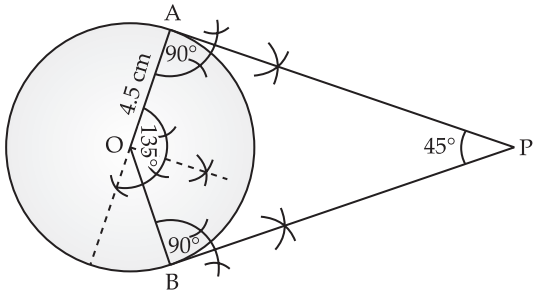
8. Steps of construction :

Step I : Draw a circle of radius 4.5 cm, which centre O.

Step II : Draw a radius OA.

Step III : Draw an angle $\angle$AOB of 135° *i.e.*, (180° – 45°)

Step IV : At point A and B, draw an angle of 90°
Step V : The two lines at point A and B intersect at P.
Step VI : Then, AD and BP are the required tangents.



Draw $\angle AOB = 135°$

$\angle OAP = 90°$

$\angle OBP = 90°$

$\therefore$ PA and PB are the required tangents.

9. The cumulative frequency is as shown below :

Weight	Frequency	Cumulative Frequency
50 – 60	8	8
60 – 70	10	18
70 – 80	12	30
80 – 90	16	46
90 – 100	18	64
100 – 110	14	78
110 – 120	12	90
120 – 130	10	100
	$N = \Sigma f_i = 100$	

Now $N = \Sigma f_i = 100$

$\Rightarrow$ $\frac{N}{2} = \frac{100}{2} = 50$

The cumulative frequency just above 50 is 64.

Hence, the median class is 90 – 100.

OR

Class Interval (x_i)	Frequency (f_i)	($f_i \times x_i$)
35	6	210
36	10	360
37	18	666
38	25	950
39	21	819
40	12	480
41	8	328
	$\Sigma f_i = 100$	$\Sigma(f_i \times x_i) = 3813$

Thus, $\text{Mean} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i} = \frac{3813}{100} = 38.13$

Now, to make the Mean = 39, $\Sigma(f_i \times x_i)$ should be 3900.

So the number of matchsticks to be added = 3900 – 3813 = 87.

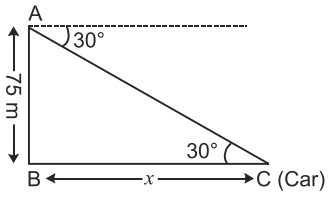
10. Let the distance of the car from the base of the tower be x m.

Height of tower = 75 m

$$\tan 30° = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{x}$$

$$\Rightarrow \quad x = 75\sqrt{3}$$



Thus, distance of the car from the base of the tower is $75\sqrt{3}$ m.

Section - C

11. Radius of the base of cylinder (r) = 2.8 m

Radius of the base of the cone (r) = 2.8 m

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m

Slant height of conical part (l) $= \sqrt{r^2 + H^2}$

$$= \sqrt{(2.8)^2 + (2.1)^2}$$

$$= \sqrt{7.84 + 4.41}$$

$$= \sqrt{12.25} = 3.5 \text{ m}$$

Area of canvas used to make tent = CSA of cylinder + CSA of cone

$$= 2\pi rh + \pi rl$$

$$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5$$

$$= 61.6 + 30.8$$

$$= 92.4 \text{ m}^2$$

Cost of 1500 tents at ₹ 120 per sq. m = 1500 × 120 × 92.4

= ₹ 16,632,000

Share of each school to set-up the tents $= \frac{16632000}{50}$

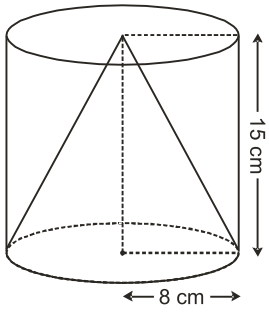
= ₹ 332,640.

OR

Given, height of solid cylinder and cone (h) = 15 cm

Diameter of cylinder and cone = 16 cm

∴ Radius of cylinder and cone (r) = 8 cm



Thus, curved surface area of cylinder

$$= 2\pi rh$$

$$= 2 \times \pi \times 8 \times 15 \text{ cm}^2$$

$$= 240\ \pi \text{ cm}^2$$

and curved surface area of cone

$$= \pi(8)(\sqrt{(8)^2 + (15)^2})\ \text{cm}^2$$

$$= \pi(8)(\sqrt{64 + 225})\ \text{cm}^2$$

$$= \pi(8)(\sqrt{289})\ \text{cm}^2$$

$$= \pi(8)\ (17)\ \text{cm}^2$$

$$= 136\pi\ \text{cm}^2$$

and the area of the top of cylinder

$$= \pi(8)^2\ \text{cm}^2$$

$$= 64\pi\ \text{cm}^2$$

Hence, the total surface area of the remaining solid

$$= 240\pi + 136\pi + 64\pi$$

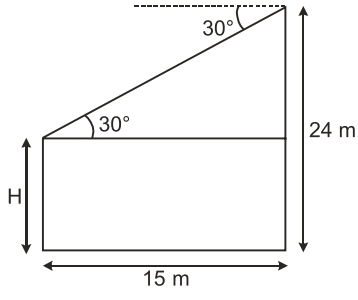
$$= 440\pi\ \text{cm}^2$$

$$= 440(3.14)\ \text{cm}^2$$

$$= 1381.6\ \text{cm}^2.$$

12. Let the height of the first pole be H m.

Thus, $\tan 30° = \dfrac{24 - H}{15} = \dfrac{1}{\sqrt{3}}$



$\Rightarrow \quad \sqrt{3}(24 - H) = 15$

$\Rightarrow \quad 24 - H = 5\sqrt{3}$

$\Rightarrow \quad H = 24 - 5\sqrt{3}$

$= [24 - 5(1.732)]$ m

$= (24 - 8.66)$ m

$= 15.34$ m

So, the height of the first pole is 15.34 m.

13. (i) Let the numbers on the cards be a, $a + d$, $a + 2d$, ...

According to questions, we have

$$(a + 5d) + (a + 13d) = -76$$

$\Rightarrow \quad 2a + 18d = -76$

$\Rightarrow \quad a + 9d = -38$ ...(i)

And $(a + 7d) + (a + 15d) = -96$

$\Rightarrow \quad 2a + 22d = -96$

$\Rightarrow \quad a + 11d = -48$ ...(ii)

From equations (i) and (ii), we get

$$2d = -10 \Rightarrow d = -5$$

(ii) From (i), $a + 9(-5) = -38 \Rightarrow a = 7$

Number on 9^{th} card $= a + 8d$

$= 7 + 8 \times -5$

$= 7 - 40 = -33$

Number on 15^{th} card $= a + 14d$

$= 7 + 14 \times -5$

$= 7 - 70 = -63$

Sum of 9^{th} and 15^{th} card $= -33 + (-63)$

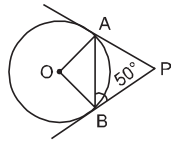
$= -96$

14. (i) In ΔOAC, $\angle OCA = 90°$ [Since, radius at the point of contact is perpendicular to tangent]

$\Rightarrow \quad \angle OAC + \angle AOC = 90°$

$\Rightarrow \quad x + y = 90°$

(ii) Since, OB $\perp$ PB [Since, radius at the point of contact is perpendicular to tangent]



and $\angle PBA = 50°$ (Given)

$\therefore$ $\angle OBA = 90° - 50° = 40°$

Also, $OA = OB$ [Radius of circle]

$\therefore$ $\angle OAB = \angle OBA = 40°$ [Angle opposite to equal sides are equal]