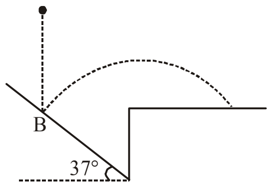
GUIDED REVISION

SECTION-I

Single Correct Answer Type **9 Q. [3 M (–1)]**

1. A small ball is dropped from a height h onto a rigid frictionless plate at B and rebounds. Find the maximum height reached after rebound if the coefficient of restitution between the ball and the plate is e = 0.75:



(A) 0 (B) 3h /25 (C) 3h/125 (D) 9h/625

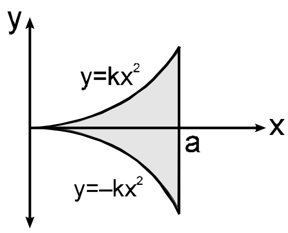
2. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time ?

(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

3. Two particles bearing mass ratio n : 1 are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is :

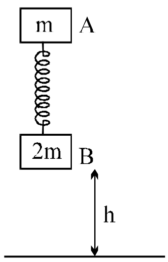
(A) $(n-1)^2 g$ (B) $\left(\frac{n+1}{n-1}\right)^2 g$ (C) $\left(\frac{n-1}{n+1}\right)^2 g$ (D) $\left(\frac{n+1}{n-1}\right) g$

4. A thin uniform sheet of metal of uniform thickness is cut into the shape bounded by the line x = a and $y = \pm kx^2$, as shown. Find the coordinates of the centre of mass.



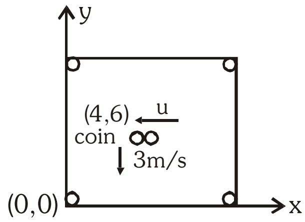
(A) $\left(\frac{1}{4}a, 0\right)$ (B) $\left(\frac{3}{4}a, 0\right)$ (C) $\left(\frac{5}{4}a, 0\right)$ (D) None of these

5. From what minimum height h must the system be released when spring is unstretched so that after perfectly inelastic collision (e = 0) with ground, B may be lifted off the ground (Spring constant = k).



(A) mg/(4k) (B) 4mg/k (C) mg/(2k) (D) none

6. On a smooth carom board, a coin moving in negative y–direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x–axis. The line joining centres of the coin and the striker just before the collision is parallel to x–axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be



(A) (1.2, 0) (B) (2, 0) (C) (3, 0) (D) None of these

7. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude 19.6 m/s^2. [Exhaust speed of fuel = 980 m/s.]

(A) 240 kg s^{-1} (B) 60 kg s^{-1} (C) 120 kg s^{-1} (D) None

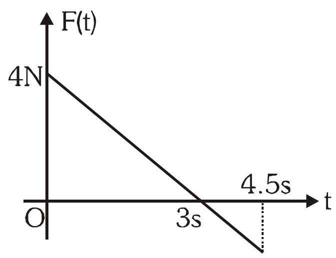
8. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. **[JEE Main-2013]**

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(A) Statement–I is true, Statement–II is true, Statement–II is a correct explanation of Statement–I.
(B) Statement–I is true, Statement–II is true, Statement–II is a not correct explanation of Statement–I.
(C) Statement–I is true, Statement–II is false.
(D) Statement–I is false, Statement–II is true

9. A block of mass 2 kg is free to move along the x–axis. It is at rest and from t=0 onwards it is subjected to a time–dependent force F(t) in the x–direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is **[IIT-JEE-2010]**



(A) 4.50 J (B) 7.50 J (C) 5.06 J (D) 14.06 J

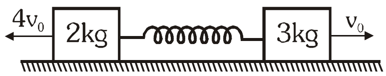
Multiple Correct Answer Type 2 Q. [4 M (–1)]

10. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.
(A) The minimum kinetic energy of the system is 1 joule.
(B) The maximum elastic potential energy of the system is 2 joule.
(C) Momentum and total kinetic energy of the system are conserved at every instant.
(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

Linked Comprehension Type **(3 Para × 3Q.) [3 M (-1)]**
(Single Correct Answer Type)

Paragraph for Questions no. 11 to 13

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



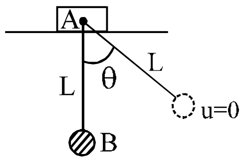
11. The maximum energy stored in the spring in the subsequent motion will be
(A) $5v_0^2$ (B) $15v_0^2$ (C) zero (D) $10v_0^2$

12. Maximum speed of 3 kg block in the subsequent motion will be
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

13. Maximum speed of 2 kg block in the subsequent motion will be
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

Paragraph for Question No. 14 to 16

A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from lower most position & then released



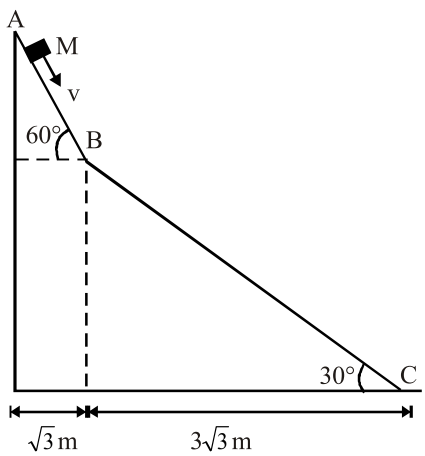
14. The displacement of block when ball reaches the lower most position is :-
(A) $\frac{L\sin\theta}{2}$ (B) $L\sin\theta$ (C) L (D) none of these

15. Maximum velocity of block during subsequent motion of the system after release of ball is
(A) $[gl(1-\cos\theta)]^{1/2}$ (B) $[2gl(1-\cos\theta)]^{1/2}$
(C) $[gl\cos\theta]^{1/2}$ (D) informations are insufficient to decide

16. The displacement of centre of mass of A + B system till the string becomes vertical is
(A) zero (B) $\frac{L}{2}(1-\cos\theta)$ (C) $\frac{L}{2}(1-\sin\theta)$ (D) none of these

Paragraph for Question no. 17 to 19

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$) **[IIT-JEE 2008]**



17. The speed of the block at point B immediately after it strikes the second incline is :
(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

18. The speed of the block at point C, immediately before it leaves the second incline is :

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

19. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is :

(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) $-\sqrt{15}$ m/s

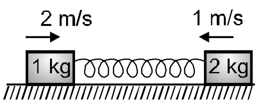
SECTION-IV

Matrix Match Type (4 × 5) **2 Q. [8 M (for each entry +2(0)]**

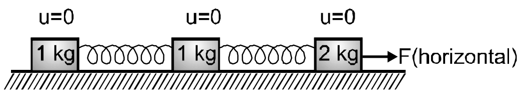
1. Match the column:

In all cases in **column–I**, the blocks/plank/trolley are placed on the smooth horizontal surface.

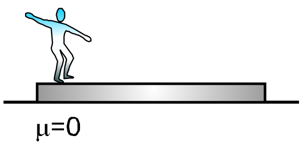
Column–I	**Column–II**
(A) The initial velocities given to the blocks when spring is relaxed are as shown	(P) Centre of mass of the complete system shown will not move horizontally



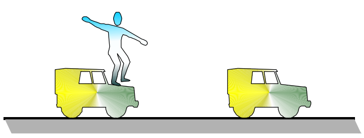
(B) A constant force is applied on 2 kg block. Springs are initially relaxed	(Q) Centre of mass of the complete system shown will move horizontally



(C) Initially system is at rest. Man starts moving on a large plank with constant velocity.	(R) Mechanical energy of the system will be conserved

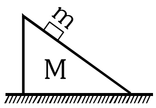
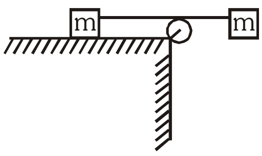
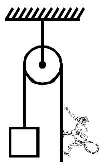
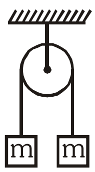


(D) A man standing on one of the trolleys (initially at rest) jumps to the other with relative velocity of 4 m/s horizontally	(S) Mechanical energy of the system will increase



(T) Linear momentum of the complete system will always remain constant

2. In each situation of column–I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column – I with the corresponding results in column–II

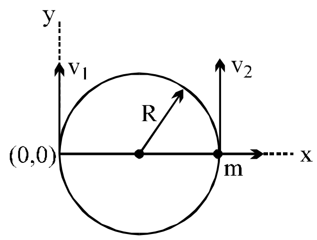
Column I	Column II
(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system 	(P) Shifts towards right
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system 	(Q) Shifts downwards
(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey+block system 	(R) Shifts upwards
(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards 	(S) Does not shift

Subjective Type 11 Q. [4 M (0)]

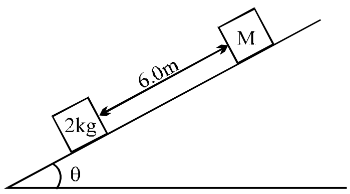
1. Two bodies of same mass tied with an inelastic string of length l lie together. One of them is projected vertically upwards with velocity $\sqrt{6g\ell}$. Find the maximum height up to which the centre of mass of system of the two masses rises.

2. A 24 kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.
(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)
(ii) How much energy was released during the explosion?

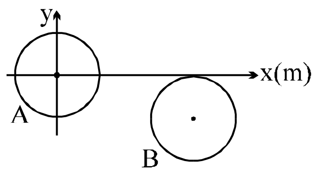
3. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time. **[IIT-JEE' 2003]**



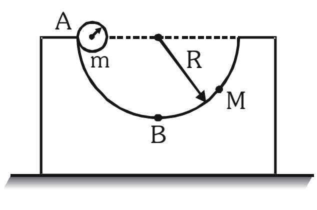
4. Two blocks of mass 2kg and M are at rest on an inclined plane and are separated by a distance of 6m as shown. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2kg block is given a velocity of 10 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1 m/s when it reaches its initial position. The other block M after the collision moves 0.5m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M. [Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\ m/s^2$]



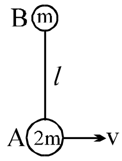
5. Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,–R,R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.



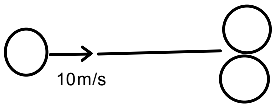
6. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track?



7. Two masses A and B connected with an inextensible string of length ℓ lie on a smooth horizontal plane. A is given a velocity of v m/s along the ground perpendicular to line AB as shown in figure. Find the tension in string during their subsequent motion.



8. A ball with initial speed of 10 m/s collides elastically with two other identical ball whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other. All the three ball are lying on a smooth horizontal table. The first ball is aimed directly at the contact point of the other two balls All the balls are smooth. Find the velocities of the three balls after the collision.



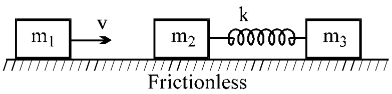
9. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity v along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find the

(i) velocities of m_1, m_2 and m_3 immediately after impact.

(ii) maximum kinetic energy of m_3.

(iii) minimum kinetic energy of m_2.

(iv) maximum compression of the spring.



10. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a smooth wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.

11. Two particles, each of mass m, are connected by a light inextensible string of length 2ℓ. Initially they lie on a smooth horizontal table at points A and B distant ℓ apart. The particle at A is projected across the table with velocity u. Find the speed with which the second particle begins to move if the direction of u is :-

(i) along BA.

(ii) at an angle of 120° with AB.

(iii) perpendicular to AB.

In each case calculate (in terms of m & u) the impulsive tension in the string.

SECTION-I

Single Correct Answer Type 9 Q. [3 M (–1)]

1. Ans. (D) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (B)**
5. Ans. (B) **6. Ans. (B)** **7. Ans. (C)** **8. Ans. (D)**
9. Ans. (C)

Multiple Correct Answer Type 2 Q. [4 M (–1)]

10. Ans. (A,B,D)

Linked Comprehension Type (3 Para × 3Q.) [3 M (-1)]
(Single Correct Answer Type)

11. Ans. (B) **12. Ans. (C)** **13. Ans. (D)** **14. Ans. (A)**
15. Ans. (A) **16. Ans. (B)** **17. Ans. (B)** **18. Ans. (B)**
19. Ans. (C)

SECTION-IV

Matrix Match Type (4 × 5) 2 Q. [8 M (for each entry +2(0)]\

1. Ans. (A)→(P,R,T); (B)→(Q,S); (C)→(P,S,T); (D)→(P, S, T)
2. Ans. (A)-Q; (B)-P, Q; (C)-R; (D) S

Subjective Type 11 Q. [4 M (0)]

1. Ans. ℓ **2. Ans.** (i) 360 m, (ii) 10800 J

3. Ans. $\vec{P}_{PM} = m\,\vec{v}_{PM}$
$= -mv_2 \sin\omega t\,\hat{i} + m(v_2\cos\omega t - v_1)\,\hat{j}$

4. Ans. e = $(5 + \sqrt{3})/8$, M = $26/\sqrt{3}$ kg

5. Ans. $(\hat{i} + \sqrt{3}\hat{j})$ m/s, $(3m\hat{i} - \sqrt{3}m\hat{j})$ kg-m/s

6. Ans. $\dfrac{m(R-r)}{M+m}$, $m\sqrt{\dfrac{2g(R-r)}{M(M+m)}}$

7. Ans. $\dfrac{2mv^2}{3\ell}$

8. Ans. –2m/s, 6.93 m/s $\angle 30°$

9. Ans. (i) v/2, v/2, 0; (ii) $2mv^2/9$; (iii) $mv^2/72$; (d) $x = \sqrt{m/6k}\;v$

10. Ans. $m(-3\hat{i} + 4\hat{j})$, e = $\dfrac{9}{16}$

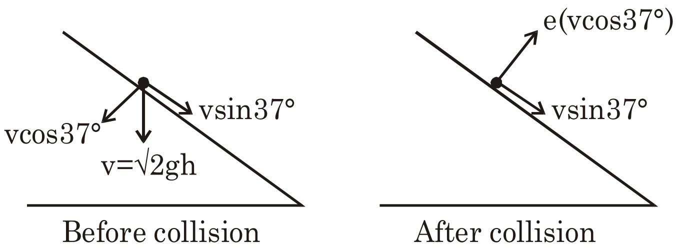
11. Ans. (i) $\dfrac{u}{2}$, $\dfrac{mu}{2}$ (ii) $\dfrac{u\sqrt{13}}{8}$, $\dfrac{mu\sqrt{13}}{8}$ (iii) $\dfrac{u\sqrt{3}}{4}$, $\dfrac{mu\sqrt{3}}{4}$

SOLUTIONS
SECTION-I

Single Correct Answer Type **9 Q. [3 M (–1)]**

1. Ans. (D)

Sol.



$\Rightarrow$ After collision, net velocity in vertical direction will be

$v_y = (e\ v\cos 37°)\cos 37° - (v\sin 37°)\sin 37°$

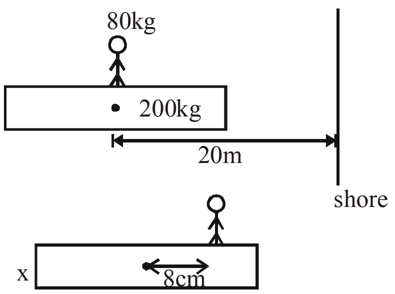
$$= \frac{3}{4}v \times \left(\frac{4}{5}\right)^2 - v\left(\frac{3}{5}\right)^2$$

$$\Rightarrow v_y = \frac{3v}{25}$$

$$\Rightarrow h_{max} = \frac{v_y^2}{2g} = \frac{9v^2}{1250g} = \frac{9}{1250g} \times 2gh = \frac{9h}{625}$$

2. Ans. (C)

Sol. Let the displacement of boat be x towards the shore



$80(x + 8) + 200x = 0$

$$\Rightarrow x = \frac{-640}{280} = -\frac{16}{7}$$

$$\therefore \ (\Delta x)_{mon} = 8 - \frac{16}{7} = \frac{40}{7}$$

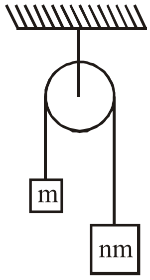
$\therefore$ New distance from shone

$$= 20 - \frac{40}{7} = \frac{100}{7} = 14.28\text{m} \approx 14.3\text{ m}$$

3. Ans. (C)

Sol. $nmg - T = nma$ ....(i)

$T - mg = ma$ ....(ii)



$$T = \frac{2nmg}{(n+1)}$$

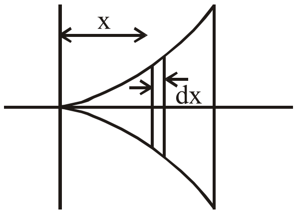
For a_{CM}

$(n + 1)\ mg - 2T = (n + 1)m\ a_{CM}$

$$a_{CM} = \frac{(n-1)^2}{(n+1)^2} g$$

4. Ans. (B)

Sol. $$x_{cm} = \frac{\int (dm)(r)}{\int dm}$$



$$\int rdm = \int_0^a 2kx^2 \times \rho x dx$$

$$= 2k\rho \int_0^a x^3 dx = \frac{2k\rho a^4}{4}$$

$$\int dm = \int 2kx^2 \rho dx$$

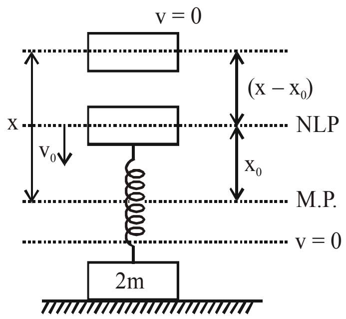
$$= \frac{2k\rho a^3}{3}$$

$$x_{cm} = \frac{3}{4} a$$

$y_{cm} = 0$ as object is symmetric about $x = 0$

5. Ans. (B)

Sol. (A)

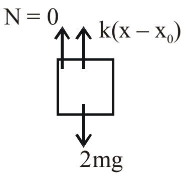


$v_0 = \sqrt{2gh}$ just after

$kx_0 = mg$

$$x_0 = \frac{mg}{k}$$

compression from MP = Elongation from MP = x
To lift 2m block



$$kx - kx_0 = 2mg \Rightarrow kx - k.\frac{mg}{k} = 2mg$$

$$\Rightarrow x = \frac{3mg}{k}$$

$$\therefore\ x + x_0 = \frac{3mg}{k} + \frac{mg}{k} = \frac{4mg}{k}$$

Now from W.E.T. $\Rightarrow$ from NLP to max compression

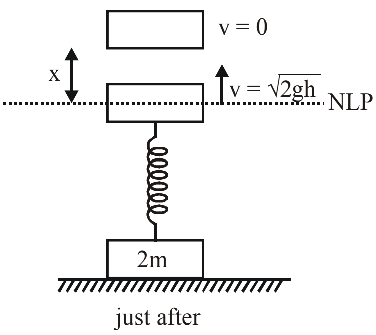
$$mg\ (x + x_0) + \frac{1}{2}k\left[0^2 - \left(x + x_0\right)^2\right] = 0 - \frac{1}{2}mv_0^2$$

$$\Rightarrow mg\ .\ \frac{4mg}{k} - \frac{1}{2}k.\frac{\left(4mg\right)^2}{k^2} = -\frac{1}{2}mv_0^2$$

$$\Rightarrow -\frac{4\left(mg\right)^2}{k} = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \frac{8m^2g^2}{k} = m.2gh\ \Rightarrow h = \frac{4mg}{k}$$

Alternate Solution



After collision with the ground and during upward journey of upper block, shown will be the condition.

$$\frac{1}{2}m\left(2gh\right) = mgx + \frac{1}{2}kx^2 \quad ...(i)$$

$$kx = 2mg \qquad ...(ii)$$

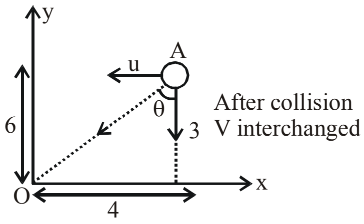
From (i) & (ii)

$$h = \frac{4mg}{k}$$

6. Ans. (B)

Sol. When striker strikes the coin, due to same mass and elastic collision, velocity of striker will be transferred to coin i.e. Coin starts to move with u along –ve x-axis.

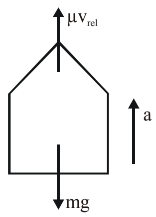
Resultant of u & 3 is along AO.



$$\tan\theta = \frac{4}{6} = \frac{u}{3} \Rightarrow u = 2 \text{ m/s}$$

$$\text{striker } v = \begin{cases} u = 2 \Rightarrow \text{Before} \\ 0 \Rightarrow \text{After} \end{cases}$$

7. Ans. (C)



Sol. Initial $\Rightarrow$ mass = 4000

$\mu v_{rel} - mg = ma$

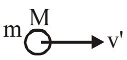
$\mu \times 980 - 4000 \times 9.8 = 4000 \times 19.6$

$\Rightarrow \mu = 40 + 80 = 120$

8. Ans. (D)

Sol. 

Energy loss will be maximum when collision will be perfectly elastic



(By momentum) $mv = (m + M)v'$

$$\Rightarrow v' = \frac{mv}{m+M}$$

Maximum energy loss = $K_i - K_f$

$$= \frac{1}{2} mv^2 - \frac{1}{2}(m + M)v'^2$$

$$= \frac{1}{2} mv^2 - \frac{1}{2}(m + M)\frac{m^2v^2}{(m+M)^2}$$

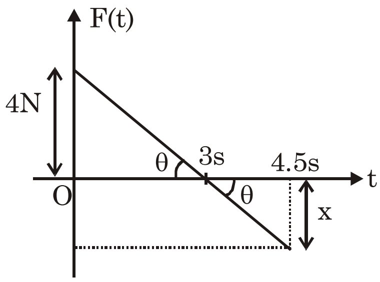
$$= \frac{1}{2}mv^2\left[1 - \frac{m}{m+M}\right]$$

$$= \left(\frac{M}{m+M}\right)\frac{1}{2}mv^2$$

statement 1 is false.

9. Ans. (C)

Sol. Area of F-t graph gives change in momentum and above area is taken positive and down area is negative.



$$\tan\theta = \frac{4}{3} = \frac{x}{1.5} \Rightarrow x = 2$$

$$\Rightarrow \text{Area} = \frac{1}{2} \times 3 \times 4 + \frac{1}{2}(-2)(1.5) = mv$$

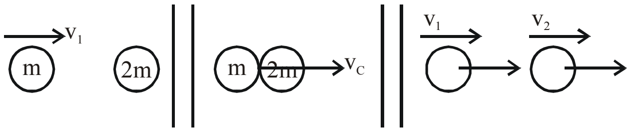
$$\Rightarrow 6 - 1.5 = 2v$$

$$\Rightarrow v = \frac{4.5}{2} = 2.25$$

$$\Rightarrow \text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}2 \times (2.25)^2 = 5.06\,\text{J}$$

Multiple Correct Answer Type 2 Q. [4 M (–1)]

10. Ans. (A,B,D)

Sol. 

At maximum compression (PE $\Rightarrow$ Max $\Rightarrow$ KE $\Rightarrow$ Min)

$3mv_C = mv_1$

$\Rightarrow v_C = \frac{v_1}{3}$

$\frac{1}{2}mv_1^2 = 3$, $(KE)_{min} = \frac{1}{2}.3mv_C^2 = \frac{3}{2}m\left(\frac{v_1}{3}\right)^2 = \frac{1}{6}mv_1^2 = \frac{1}{3}\times\left(\frac{1}{2}mv_1^2\right) = \frac{1}{3}\times 3$

$\Rightarrow (KE)_{min} = 1J$

$(PE)_{max} = 3 - 1 = 2J$

(C) P $\Rightarrow$ Always conserved

KE $\Rightarrow$ During collision changes

(D) During collision PE increases, becomes max. & then decreases.

During deformation

$PE\uparrow$ & $KE\downarrow \Rightarrow \left(\frac{PE}{KE}\right)\uparrow \Rightarrow \frac{KE}{PE}\downarrow$

During restitution, PE $\downarrow$, KE$\uparrow$

$\Rightarrow\left(\frac{KE}{PE}\right)\uparrow$

Linked Comprehension Type (3 Para × 3Q.) [3 M (-1)]
(Single Correct Answer Type)

11. Ans. (B)

12. Ans. (C)

13. Ans. (D)

Sol. (11 to 13)

Maximum energy will be stored in the spring when blocks will be moving with same velocity.

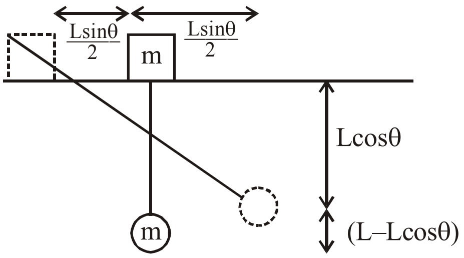
$v_c = \frac{8v_0 - 3v_0}{5} = v_0$ towards left

So, $U_{max} = \frac{1}{2}\times 2\times 16v_0^2 + \frac{1}{2}\times 3\times v_0^2 - \frac{1}{2}\times 5\times v_0^2$

Velocity of centre of mass is v_0 towards left and both the blocks 2kg & 3 kg will perform simple harmonic motion with respect to centre of mass with speed at mean position $3v_0$ and $2v_0$ respectively with respect to centre of mass. So, maximum speed of 2 kg block in ground frame will be $3v_0 + v_0 = 4v_0$ and maximum speed of 3 kg block will be $2v_0 + v_0 = 3v_0$.

14. Ans. (A)

Sol. x_{com} will not change since F_{ext} is zero in horizontal direction. At lower most point ball will be right beneath the block & COM will be on the vertical line as shown in figure.



15. Ans. (A)

Sol. Since momentum is conserved in x direction

$m_{block}v_x = m_{ball}\ v_x'$

$v_x = v_x'$

Applying work energy theorem.

$$mg\ (L - L\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$v = \sqrt{g\ell(1-\cos\theta)}$$

16. Ans. (B)

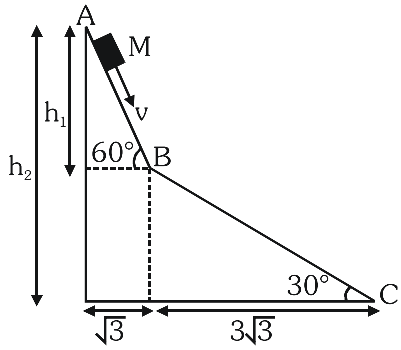
Sol. $(y_{cm})_{initial} = L/2\ \cos\theta$

$(y_{cm})_{final} = L/2$

$\Delta y_{cm} = L/2\ (1-\cos\theta)$

17. Ans. (B)

Sol. $\frac{h_1}{\sqrt{3}} = \tan 60° \Rightarrow h_1 = 3m$



$\frac{h_2 - h_1}{3\sqrt{3}} = \tan 30°$

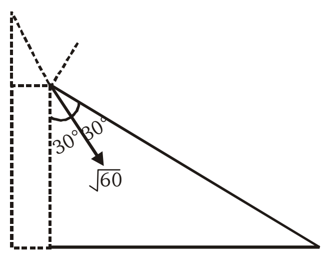
$\Rightarrow h_2 - h_1 = 3$

$\Rightarrow h_2 = 6m$

Velocity of block just before collision at B

$= \sqrt{2gh} = \sqrt{2\times 10\times 3} = \sqrt{60}\ ms^{-1}$

For totally inelastic collision velocity of block along normal to BC becomes zero and since there is no impulse along BC so momentum (velocity) along BC remains unchanged. speed of block just after collision



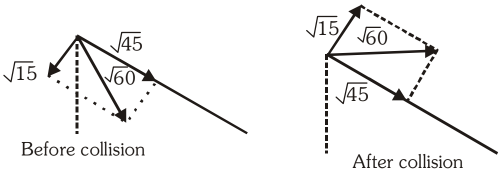
$$v_B = \sqrt{60}\cos 30° = \sqrt{60}\times\frac{\sqrt{3}}{2} = \sqrt{45}ms^{-1}$$

18. Ans. (B)

Sol. $v_c^2 = v_B^2 + 2g(h_2 - h_1) \Rightarrow v_c = \sqrt{45 + 2 \times 10 \times 3} = \sqrt{105}\ ms^{-1}$

19. Ans. (C)

Sol.



vertical component of velocity = $\sqrt{15}\sin 60° - \sqrt{45}\sin 30° = 0$

SECTION-IV

Matrix Match Type (4 × 5) **2 Q. [8 M (for each entry +2(0)]**

1. Ans. (A)→(P,R,T); (B)→(Q,S); (C)→(P,S,T); (D)→(P, S, T)

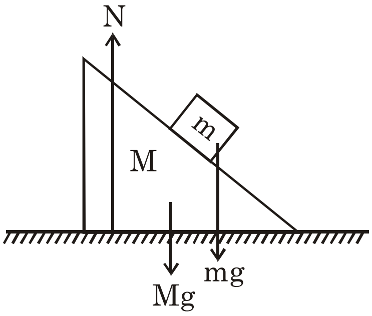
Sol. No external force along horizontal in A, C, D so center of mass will remain at rest and linear momentum will always remain constant.

In B work is done by F so mechanical energy increases.

In C, D internal forces of man does work so mechanical energy increases

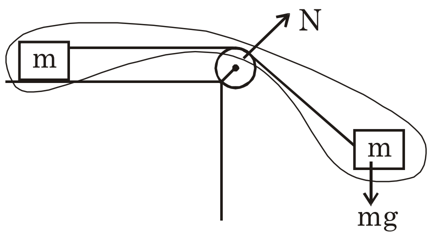
2. Ans. (A)-Q; (B)-P, Q; (C)-R; (D) S

Sol. (A)



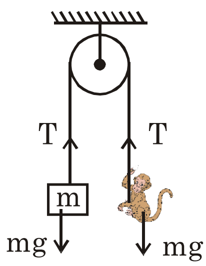
No external external force in horizontal direction. So COM does not move in horizontal direction. Since 'm' comes downward but M does not move in vertical direction, So COM moves vertically downward.

(B)



External force (N) has a component towards right. So COM will be displaced towards right- Due to mg, COM moves downward.

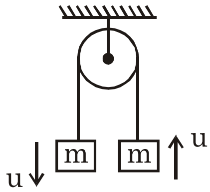
(C)



Since monkey is moving upward, so T > Mg.

So block will also move upward.

Since both are moving upward, so COM will move upward only.

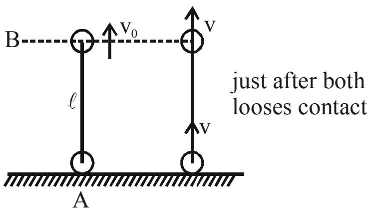
(D) 

$$v_{CM} = \frac{mu + m(-u)}{2m} = 0$$

$\Rightarrow$ COM does not move.

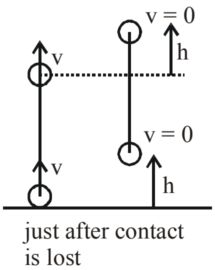
Subjective Type   11 Q. [4 M (0)]

1. **Ans.** ℓ

Sol. 

$v_0^2 = 6\,g\ell - 2g\ell \Rightarrow v_0 = \sqrt{4g\ell}$

$2mv = mv_0 \Rightarrow v = \frac{v_0}{2} = \sqrt{g\ell}$



just after both looses contact $\Rightarrow v_{cm} = \frac{mv + mv}{2m} = v = \sqrt{g\ell}$ & $a_{cm} = g$

Now just after both looses contact

$\Rightarrow 0^2 = v_{cm}^2 - 2gh$

$\Rightarrow h = \frac{\ell}{2}$ = displacement of each block after both looses contact.

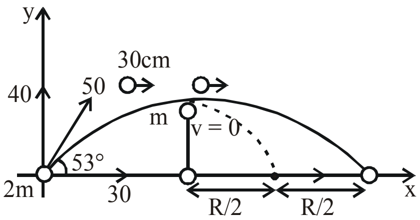
Displacement of COM till only one block moves upward is

$\Rightarrow (\Delta x_{cm})_1 = \frac{m\ell}{2m} = \frac{\ell}{2}$

$\therefore$ Total upward displacement of COM is $= \frac{\ell}{2} + \frac{\ell}{2} = \ell$

2. **Ans.** (i) 360 m, (ii) 10800 J

Sol.



(i) Range R = $\dfrac{50^2 \times 2\sin 53°.\cos 53°}{10}$ = 250 × 2 × .6 × .8

R = 240 m

$\therefore$ d = R + $\dfrac{R}{2}$ = $\dfrac{3R}{2}$ = 360 m

(ii) From $P_f = P_i$ in horizontal

2m × 30 = mv

$\Rightarrow$ v = 60 m/s

Just before explosion

$K_i = \dfrac{1}{2}.2m.v_x^2$ 2m = 24 $\Rightarrow$ m = 12

$= \dfrac{1}{2} \times 24 \times (30)^2 = \dfrac{1}{2} \times 21600J = 10800J$

Just after explosion

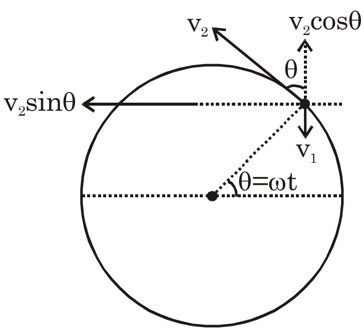
$k_f = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 12 \times 60^2 = 21600J$

$\therefore$ Energy released = $k_f - k_i$ = 21600 – 10800 = 10800 J

3. **Ans.** $\vec{P}_{PM} = m\bar{v}_{PM}$

$= -mv_2 \sin \omega t\,\hat{i} + m(v_2 \cos \omega t - v_1)\,\hat{j}$

Sol.



$\omega = \dfrac{v_2}{R}$

$\vec{v}_{rel} = (-v_2 \cdot \sin\theta)\hat{i} + (v_2 \cos\theta - v_1)\hat{j}$

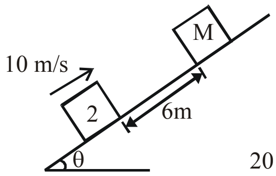
$= (-v_2 \cdot \sin\omega t)\hat{i} + (v_2 \cos\omega t - v_1)\hat{j}$

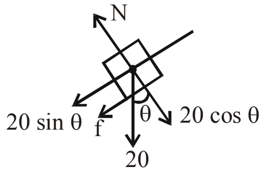
$\vec{P}_{rel} = m \cdot \vec{v}_{rel}$

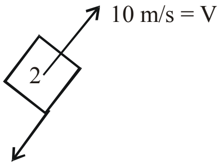
$= -mv_2 \sin\omega t\,\hat{i} + m(v_2 \cos\omega t - v_1)\hat{j}$

4. **Ans.** e = (5 + √3)/8, M = 26/√3 kg

Sol.







During upward motion of 2 kg

$f = \mu \cdot 20\cos\theta = .25 \times 20\cos\theta$

$f = 5$

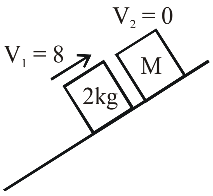
$$\therefore\ a_{2kg} = \frac{20\sin\theta + 5}{2} = \frac{20 \times 0.05 + 5}{2} = 3\ \text{m/s}^2$$

velocity of 2kg just before collision is

$V_1 = \sqrt{10^2 - 2\times3\times6} = \sqrt{64}$

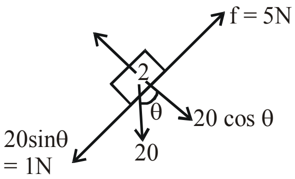
$V_1 = 8$ m/s

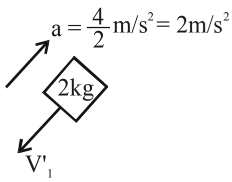
∴ Condition of just before collision



Just after collision

During downward motion of 2kg



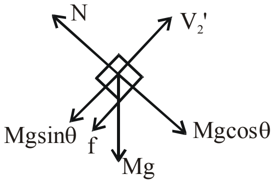


During motion of 2kg from P to initial position

$\Rightarrow 1^2 = V_1'^{\,2} - 2 \times 2 \times 6$

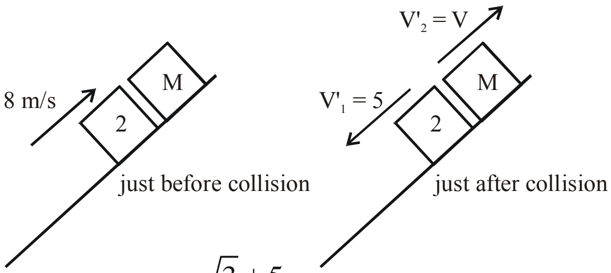
$\Rightarrow V_1'^{\,2} = 25 \Rightarrow V_1' = 5$m/s

Upward motion of M



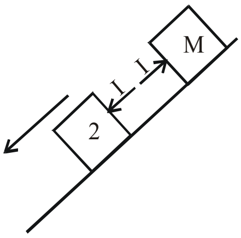
$$a = \frac{Mg\sin\theta + \mu Mg\cos\theta}{M} = g(\sin\theta + \mu\cos\theta) = 10(.05 + .25 \times 1) = 3\text{m/s}^2$$

Now $0^2 = V_2'^{\,2} - 2 \times 3 \times 0.5 \Rightarrow V_2' = \sqrt{3}\text{m/s}$



$$e = \frac{\sqrt{3}+5}{8}$$

To calculate (M =)

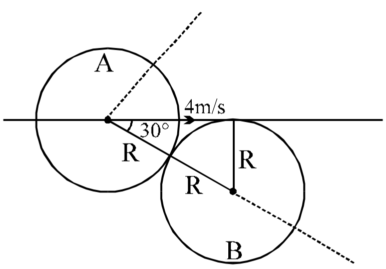


During collision
Impulse on 2 kg is
$= (\Delta P)_{2kg}$
$= 2[5 - (-8)] = 26$ N.S.
impulse on Mkg is = 26
$M \,.\, \sqrt{3} = 26$

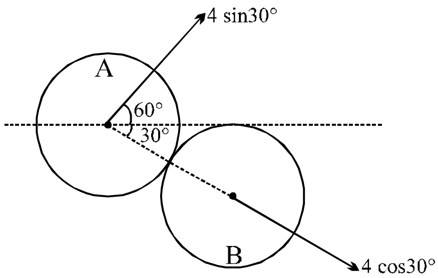
$$\Rightarrow M = \frac{26}{\sqrt{3}}\,\text{kg}$$

5. **Ans.** $(\hat{i} + \sqrt{3}\hat{j})$ m/s, $(3m\hat{i} - \sqrt{3}m\hat{j})$ kg-m/s

Sol. **Before collision**



After collision

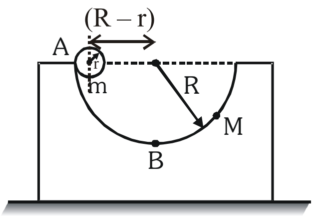


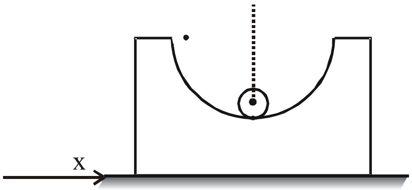
$v_A = 4 \sin 30° \,[\cos 60\,\hat{i} + \sin 60\,\hat{j}]$; $v_A = \hat{i} + \sqrt{3}\,\hat{j}$

$\overrightarrow{J_{A\,on\,B}} = m\,\overrightarrow{V_{B_f}} - \overrightarrow{V_{B_i}} = m\,[4\cos 30°(\cos 30°\hat{i} - \sin 30°\,\hat{j}) - 0] = (3mi - \sqrt{3}mj)$ kg-m/s

6. **Ans.** $\dfrac{m(R-r)}{M+m}$, $m\sqrt{\dfrac{2g(R-r)}{M(M+m)}}$

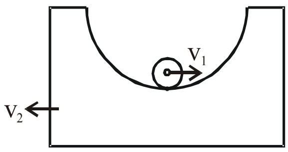
Sol.





$\Rightarrow$ M . x + m(x + R – r) = 0

$$\Rightarrow x = \frac{m(R-r)}{(M+m)}$$



v_1 & v_2 $\Rightarrow$ w.r.t. ground $\Rightarrow$

$(P)_H$ $\Rightarrow$ Conserved $\Rightarrow$ $P_f = P_i$

$mv_1 - Mv_2 = 0$

$mv_1 = Mv_2$ ....(i)

Energy conservation [for system]

$$mg(R-r) = \frac{1}{2}Mv_2^2 + \frac{1}{2}mv_1^2 \quad \text{....(ii)}$$

$$\text{from (1)} \Rightarrow v_2 = \frac{m}{M}.v_1$$

$$\text{from (2)}\ \ mg(R-r) = \frac{1}{2}M\left(\frac{m}{M}v_1\right)^2 + mv_1^2 \quad \text{....(ii)}$$

$$mg(R-r) = \frac{1}{2}mv_1^2\left(1+\frac{m}{M}\right)$$

$$\Rightarrow v_1 = \sqrt{\frac{2mg(R-r)M}{m(M+m)}}$$

$$\Rightarrow v_2 = \frac{m}{M}v_1 = \frac{m}{M}\sqrt{\frac{2Mg(R-r)}{M+m}}$$

$$\Rightarrow v_2 = m\sqrt{\frac{2g(R-r)}{M(m+M)}}$$

7. **Ans.** $\dfrac{2mv^2}{3\ell}$

Sol. In frame of COM $\Rightarrow$ (B moves in a circle of radius $\dfrac{2\ell}{3}$ and A moves in a circle of radius $\dfrac{\ell}{3}$

$F_{ext} = 0$

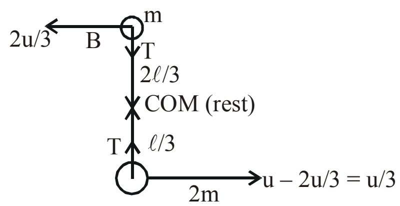
$\Rightarrow a_{cm} = 0$

$v_{cm} = \dfrac{2mu}{3m} = \dfrac{2u}{3}$

In free of COM

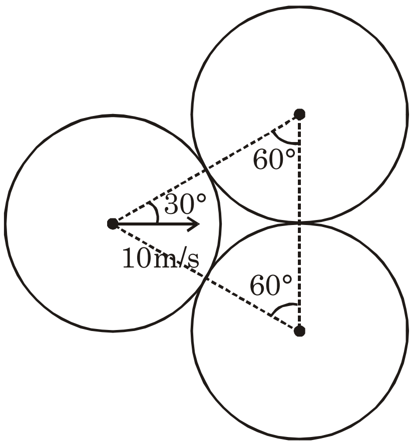
for A $\Rightarrow$ T = $\dfrac{2m(u/3)^2}{\ell/3} = \dfrac{2mu^2}{3\ell}$

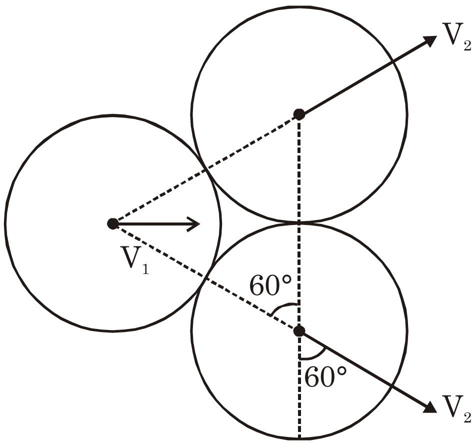
or for B $\Rightarrow$ T = $\dfrac{m(2u/3)^2}{2\ell/3} = \dfrac{2mu^2}{3\ell}$



8. **Ans.** –2m/s, 6.93 m/s $\angle 30°$

Sol.





Before collision

From momentum conservation

$m(10) = mv_1 + (mv_2 \sin 60°) \times 2$

$\Rightarrow 10 = v_1 + \sqrt{3}v_2$ ... (i)

Elastic collision $e = 1 = \dfrac{v_2 - v_1 \cos 30°}{10 \cos 30°}$

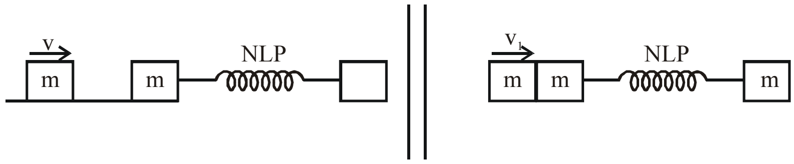
$\Rightarrow v_2 - \dfrac{\sqrt{3}v_1}{2} = 5\sqrt{3}$ ... (ii)

Solving (i) & (ii)

$v_1 = -2$, $v_2 = 4\sqrt{3} \Rightarrow v_2 = 6.93$ m/s

angle $\theta = 30°$ from horizontal

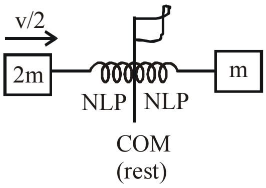
9. **Ans.** (i) v/2, v/2, 0; (ii) $2mv^2/9$; (iii) $mv^2/72$; (d) $x = \sqrt{m/6k}\, v$

Sol. (i) 

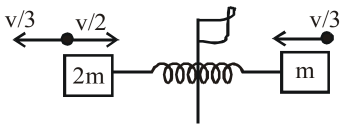
$mv = 2mv_1 \Rightarrow v_1 = v/2$

just after collision $\Rightarrow v_{m_1} = v_{m_2} = v/2$

$v_{m_3} = 0$

(ii) 

$$v_{cm} = \frac{2m.\frac{v}{2}}{3m} = \frac{v}{3} \rightarrow (\text{right})$$

$\therefore$ In COM frame 

Range in (COM frame) $\left[-\frac{v}{6}, \frac{v}{6}\right]$ $\left[-\frac{v}{3}, \frac{v}{3}\right]$

Range in (ground) $\left[-\frac{v}{6}+\frac{v}{3}, \frac{v}{6}+\frac{v}{3}\right]$ $\left[-\frac{v}{3}+\frac{v}{3}, \frac{v}{3}+\frac{v}{3}\right]$

$\left[\frac{v}{6}, \frac{v}{2}\right]$ $\left[0, \frac{2v}{3}\right]$

$\therefore$ Max. velocity of m_3 during its motion

$$\therefore (k_3)_{max} = \frac{1}{2}.m.\left(\frac{2v}{3}\right)^2 = \frac{2}{9}mv^2$$

$$\therefore (k_3)_{max} = \frac{2}{9}mv^2$$

(iii) According to option (ii)

$\left[\frac{v}{6}, \frac{v}{2}\right]$

minimum velocity of combined mass after collision = $\frac{v}{6}$

$\Rightarrow (v_2)_{min} = \frac{v}{6}$

(min of m_2)

$$\therefore\ (k_2)_{min} = \frac{1}{2}\times m\left(\frac{v}{6}\right)^2 = \frac{mv^2}{72}$$

max. compression $\Rightarrow$ at max. compression $\Rightarrow v_1 = v_2 = v_3 = v_c$

$$\frac{1}{2}uv_{rel}^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}\times\frac{2m.m}{3m}\left(\frac{v}{2}\right)^2 = \frac{1}{2}kx^2$$

$$\Rightarrow x = v\sqrt{\frac{m}{6k}}$$

OR

solve from basic

$$2m\frac{v}{2} = 3mv_c \Rightarrow v_c = \frac{v}{3}$$

$$\text{wet } \frac{1}{2}k\left(O^2 - x^2\right) = \frac{1}{2}m.v_c^2$$

$$\Rightarrow -\frac{1}{2}kx^2 = \frac{3}{2}m\left(\frac{v}{3}\right)^2 - \frac{mv^2}{4}$$

$$\Rightarrow -kx^2 = -\frac{mv^2}{6} \Rightarrow x = v\sqrt{\frac{m}{6k}}$$

(iv) Initial condition is just after collision of m_1 & m_2 mox less of KE in cllision of m_1 & m_2

10. **Ans.** $m\left(-3\hat{i}+4\hat{j}\right)$, $e = \frac{9}{16}$

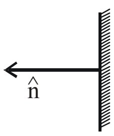
Sol. (i) $\vec{P}_i = m\vec{v}_i = m\left(4\hat{i}-\hat{j}\right)$

$\vec{P}_f = m\vec{v}_f = m\left(\hat{i}+3\hat{j}\right)$

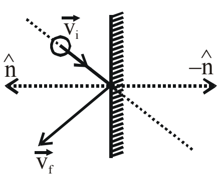
$\therefore$ Impulse $1 = \vec{P}_f - \vec{P}_i = m\left(-3\hat{i}+4\hat{j}\right)$

(ii) Impulse is imported on ball in the direction of common normal between wall & ball

$\therefore$ unit vector along normal to wall



$$\hat{n} = \hat{i} = \frac{\vec{j}}{|\vec{j}|} = \frac{m\left(-3\hat{i}+4\hat{j}\right)}{m.5} = \left(\frac{-3\hat{i}+4\hat{j}}{5}\right)$$



$\therefore$ component in $\vec{v}_i$ in the direction of $(-\hat{n})$ is = $\dfrac{v_i.(-\hat{n})}{|\hat{n}|=1}$

$$= \frac{1}{5}(12+4) = \frac{16}{5} \Rightarrow \text{R.V.A. just before collision}$$

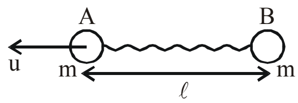
Component of $\vec{v}_f$ in the direction of $\hat{n}$ is = $\dfrac{\vec{v}_f.\hat{n}}{|\hat{n}|=1} = \dfrac{1}{5}(-3+12) = \dfrac{9}{5}$

R.V.S just after collision

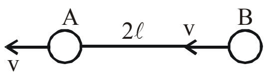
$\therefore$ e = $\dfrac{\text{R.V.S.}}{\text{R.V.A}} = \dfrac{9/5}{16/5}$

e = $\dfrac{9}{16}$

11. **Ans.** (i) $\dfrac{u}{2}$, $\dfrac{mu}{2}$ (ii) $\dfrac{u\sqrt{13}}{8}$, $\dfrac{mu\sqrt{13}}{8}$ (iii) $\dfrac{u\sqrt{3}}{4}$, $\dfrac{mu\sqrt{3}}{4}$



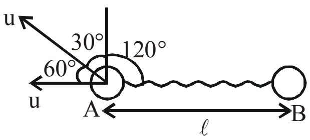
Sol. (i)

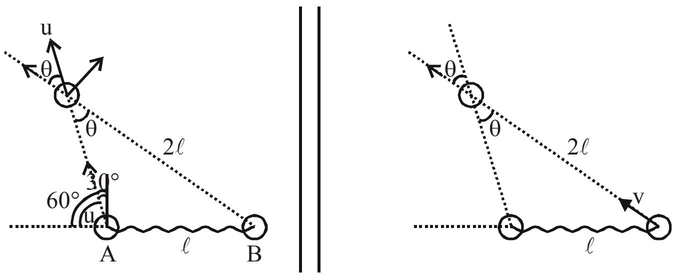


2mv = mu $\Rightarrow$ v = $\dfrac{u}{2}$

$\therefore$ impulse of tension = $(\Delta P)_B$ = mv = mu/2

(ii)





Just before string is tight

Just after string is tight

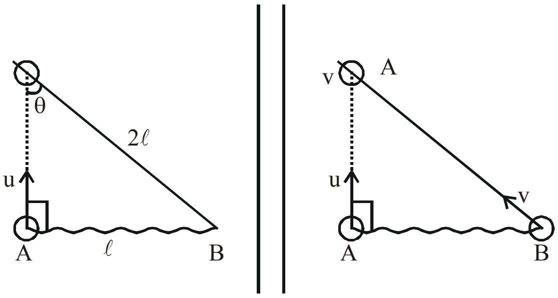
Sine rule $\Rightarrow$ $\dfrac{\sin\theta}{\ell} = \dfrac{\sin 120°}{2\ell}$ $\Rightarrow \sin\theta = \dfrac{\sqrt{3}}{4}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{4}$

2m.v = mu cos θ

$$v = \frac{u\cos\theta}{2} = \frac{u}{2} \times \frac{\sqrt{13}}{4} = \frac{u\sqrt{13}}{8}$$

$\therefore$ Impulse of tension = $(\Delta P)_B = \dfrac{m.u\sqrt{13}}{8}$

(iii) 

Just before

$\sin\theta = \dfrac{\ell}{2\ell} \Rightarrow \theta = 30°$

from $\vec{P}$ conservation along string just after tight

$2mv = mu\cos\theta = mu\cos 30°$

$$v = \frac{u\sqrt{3}}{4}$$

$\therefore$ Impulse of tension = $(\Delta P)_B = mv = \dfrac{mu\sqrt{3}}{4}$