13

Oscillations

TOPIC 1

Simple Harmonic Motion

01 A body is executing simple harmonic motion with frequency n, the frequency of its potential energy is **[NEET 2021]**

(a) n (b) $2n$ (c) $3n$ (d) $4n$

Ans. ***(b)***

In simple harmonic motion, both kinetic energy and potential energy attains their maximum value two times in one complete oscillation. Hence, frequency of kinetic energy and potential energy is 2 for one complete oscillation. So, the frequency of the potential energy of a body executing SHM with frequency n is $2n$.

02 Identify the function which represents a periodic motion. **[NEET (Oct.) 2020]**

(a) $e^{\omega t}$ (b) $\log_e(\omega t)$

(c) $\sin\omega t + \cos\omega t$ (d) $e^{-\omega t}$

Ans. ***(c)***

$\sin\omega t$ and $\cos\omega t$, both are periodic function of period $\frac{2\pi}{\omega}$.

We know that, sum of two periodic functions is also a periodic function, hence, $\sin\omega t + \cos\omega t$ represents periodic motion.

03 The phase difference between displacement and acceleration of a particle in a simple harmonic motion is **[NEET (Sep.) 2020]**

(a) $\frac{3\pi}{2}$ rad (b) $\frac{\pi}{2}$ rad

(c) zero (d) π rad

Ans. ***(d)***

In SHM, equation of displacement of a particle is $y = a\sin\omega t$ and equation of acceleration of a particle is

$$A = a\omega^2 \sin\omega t = a\omega^2 \sin(\omega t + \pi)$$

$\therefore$ Phase difference between displacement and acceleration of a particle is

$$= (\omega t + \pi) - \omega t = \pi \text{ rad}$$

Hence, correct option is (d).

04 The distance covered by a particle undergoing SHM in one time period is (amplitude = A) **[NEET (Odisha) 2019]**

(a) zero (b) A

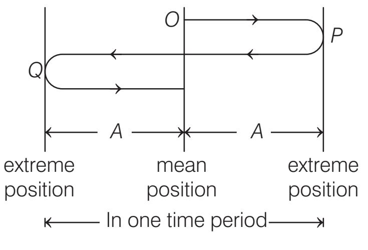
(c) 2A (d) 4A

Ans. ***(d)***

In a simple harmonic motion (SHM) the particle oscillates about its mean position on a straight line.

The particle moves from its mean position (O) to an extreme position (P) and then return to its mean position covering same distance of A.

Then by the conservative force, it is moved in opposite direction to a point Q by distance A and then back to mean position covering a distance of A. This comprises of one time period as shown below



Hence, in one time period it covers a distance of

$$x = OP + PO + OQ + QO$$
$$= A + A + A + A = 4A$$

05 Average velocity of a particle executing SHM in one complete vibration is **[NEET (National) 2019]**

(a) $A\omega$ (b) $\frac{A\omega^2}{2}$

(c) zero (d) $\frac{A\omega}{2}$

Ans. ***(c)***

The average velocity of a particle executing simple harmonic motion (SHM) is

$$v_{av} = \frac{\text{Total displacement}}{\text{Time interval}} = \frac{x_f - x_i}{T}$$

where, x_f and x_i are the initial and final position of the particle executing SHM.

As, in vibrational motion, the particle executes SHM about its mean position. So, after one complete vibration of the particle, it will reaches its initial position, i.e.

Displacement, $x_f - x_i = 0$

$$\therefore \quad v_{av} = \frac{0}{T}$$

Hence, the average velocity is zero.

06 The displacement of a particle executing simple harmonic motion is given by

$$y = A_0 + A\sin\omega t + B\cos\omega t$$

Then the amplitude of its oscillation is given by **[NEET (National) 2019]**

(a) $\sqrt{A^2 + B^2}$ (b) $\sqrt{A_0^2 + (A+B)^2}$

(c) $A + B$ (d) $A_0 + \sqrt{A^2 + B^2}$

Ans. ***(a)***

The displacement of given particle is

$$y = A_0 + A\sin\omega t + B\cos\omega t \quad \text{...(i)}$$

The general equation of SHM can be given as

$$x = a\sin\omega t + b\cos\omega t \quad \text{...(ii)}$$

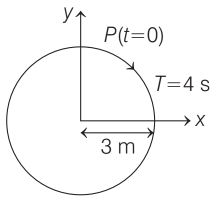
So, from Eqs. (i) and (ii), we can say that A_0 be the value of mean position, at which $y = 0$.

$\therefore$ Amplitude, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

As two function sine and cosine have phase shift to 90°.

$\therefore R = \sqrt{A^2 + B^2}$ $\qquad [\because \cos 90° = 0]$

07 The radius of circle, the period of revolution, initial position and sense of revolution are indicated in the below fig. **[NEET (National) 2019]**



y-projection of the radius vector of rotating particle P is

(a) $y(t) = 4\sin\left(\frac{\pi t}{2}\right)$, where y in m

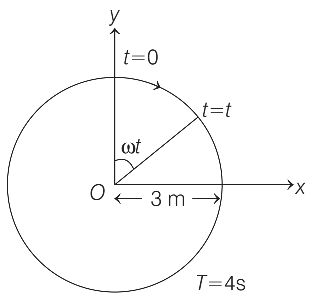
(b) $y(t) = 3\cos\left(\frac{3\pi t}{2}\right)$, where y in m

(c) $y(t) = 3\cos\left(\frac{\pi t}{2}\right)$, where y in m

(d) $y(t) = -3\cos 2\pi t$, where y in m

Ans. ***(c)***

Let O be the centre of circle, then at $t = 0$, the displacement y is maximum and have value 3 m.



As, the general equation of displacement of a particle will be in the form

$y = A\cos\omega t$

Here, $A = 3$ m

Then, $\omega = \frac{2\pi}{T} = \frac{2\pi}{4}$ $\qquad$ [given, $T = 4$ s]

$= \frac{\pi}{2}$

$\therefore \qquad y = 3\cos\left(\frac{\pi}{2}t\right)$ (in metre)

08 A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then, its time period in seconds is **[NEET 2017]**

(a) $\frac{\sqrt{5}}{\pi}$ $\qquad$ (b) $\frac{\sqrt{5}}{2\pi}$

(c) $\frac{4\pi}{\sqrt{5}}$ $\qquad$ (d) $\frac{2\pi}{\sqrt{3}}$

Ans. ***(c)***

Thinking Process Magnitude of velocity of particle when it is at displacement x from mean position

$= \omega\sqrt{A^2 - x^2}$

Also, magnitude of acceleration of particle in SHM

$= \omega^2 x$

Given, when $x = 2\,\text{cm}$

$$|\mathbf{v}| = |\mathbf{a}| \Rightarrow \omega\sqrt{A^2 - x^2} = \omega^2 x$$

$$\Rightarrow \qquad \omega = \frac{\sqrt{A^2 - x^2}}{x} = \frac{\sqrt{9-4}}{2}$$

$\Rightarrow$ Angular velocity $\omega = \frac{\sqrt{5}}{2}$

$\therefore$ Time period of motion

$T = \frac{2\pi}{\omega} = \frac{4\pi}{\sqrt{5}}$ s

09 When two displacements represented by $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are superimposed, the motion is **[CBSE AIPMT 2015]**

(a) not a simple harmonic

(b) simple harmonic with amplitude $\frac{a}{b}$

(c) simple harmonic with amplitude $\sqrt{a^2 + b^2}$

(d) simple harmonic with amplitude $\frac{(a + b)}{2}$

Ans. ***(c)***

Given, $\quad y_1 = a\sin\omega t$

$$y_2 = b\cos\omega t = b\sin\left(\omega t + \frac{\pi}{2}\right)$$

The resultant displacement is given by

$y = y_1 + y_2 = \sqrt{a^2 + b^2}\,\sin(\omega t + \phi)$

Hence, the motion of superimposed wave is simple harmonic with amplitude $\sqrt{a^2 + b^2}$.

10 A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are v_1 and v_2, respectively. Its time period is **[CBSE AIPMT 2015]**

(a) $2\pi\sqrt{\frac{x_1^2 + x_2^2}{v_1^2 + v_2^2}}$ $\qquad$ (b) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

(c) $2\pi\sqrt{\frac{v_1^2 + v_2^2}{x_1^2 + x_2^2}}$ $\qquad$ (d) $2\pi\sqrt{\frac{v_1^2 - v_2^2}{x_1^2 - x_2^2}}$

Ans. ***(b)***

Let A be the amplitude of oscillation, then

$v_1^2 = \omega^2(A^2 - x_1^2)$ $\qquad$ ...(i)

$v_2^2 = \omega^2(A^2 - x^2)$ $\qquad$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$

$$\Rightarrow \quad \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

11 A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then, its time period of vibration will be **[CBSE AIPMT 2015]**

(a) $\frac{\beta^2}{\alpha^2}$ $\qquad$ (b) $\frac{\alpha}{\beta}$ $\qquad$ (c) $\frac{\beta^2}{\alpha}$ $\qquad$ (d) $\frac{2\pi\beta}{\alpha}$

Ans. ***(d)***

For a particle executing SHM, we have maximum acceleration,

$\alpha = A\omega^2$ $\qquad$ ...(i)

where, A is maximum amplitude and ω is angular velocity of a particle.

Maximum velocity, $\beta = A\omega$ $\qquad$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\alpha}{\beta} = \frac{A\omega^2}{A\omega} \Rightarrow \frac{\alpha}{\beta} = \omega = \frac{2\pi}{T}$$

i.e. $T = \frac{2\pi\beta}{\alpha}$

Thus, its time period of vibration,

$T = \frac{2\pi\beta}{\alpha}$

12 The oscillation of a body on a smooth horizontal surface is represented by the equation,

$$X = A\cos(\omega t)$$

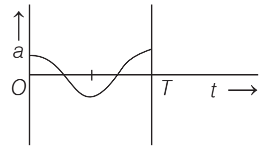
where, $X =$ displacement at time t

$\omega =$ frequency of oscillation

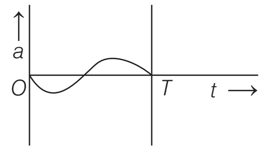
Which one of the following graphs shows
correctly the variation a with t?

[CBSE AIPMT 2014]

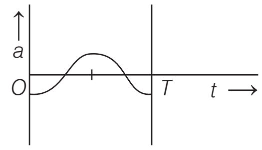
(a)



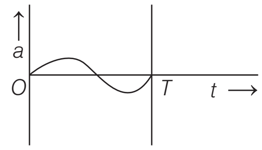
(b)



(c)



(d)



Here, $a =$ acceleration at time t
$T =$ Time period

Ans. *(c)*

As, $x = A \cos\omega t$

$\therefore \; v = \frac{dx}{dt} = -A\omega \sin\omega t$ ...(i)

and $a = \frac{d^2x}{dt^2} = -A\omega^2 \cos\omega t$ ...(ii)

We can find the correct graph by putting different values of t in Eq. (ii).

At $t = 0$, $a = -A\omega^2$

At $t = \frac{T}{4}$, $a = -A\omega^2 \cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right) = 0$

At $t = \frac{T}{2}$, $a = -A\omega^2 \cos\left(\frac{2\pi}{T} \times \frac{T}{2}\right)$

$= -A\omega^2 \cos\pi = +A\omega^2$

At $t = \frac{3T}{4}$, $a = -A\omega^2 \cos\left(\frac{2\pi}{T} \times \frac{3T}{4}\right) = 0$

At $t = T$, $a = -A\omega^2 \cos\left(\frac{2\pi}{T} \times T\right) = -A\omega^2$

This condition is represented by graph in option (c).

13 Out of the following functions representing motion of a particle which represents SHM?

[CBSE AIPMT 2011]

I. $y = \sin\omega t - \cos\omega t$

II. $y = \sin^3 \omega t$

III. $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

IV. $y = 1 + \omega t + \omega^2 t^2$

(a) Only (IV) does not represent SHM
(b) (I) and (III)
(c) (I) and (II)
(d) Only (I)

Ans. *(b)*

For a simple harmonic motion,

$$a \propto \frac{d^2y}{dt^2} \propto -y$$

Hence, equations $y = \sin\omega t - \cos\omega t$ and $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$ are satisfying this condition and equation $y = 1 + \omega t + \omega^2 t^2$ is not periodic and $y = \sin^3 \omega t$ is periodic but not simple hormonic motion.

14 The displacement of a particle along the x-axis is given by $x = a\sin^2 \omega t$. The motion of the particle corresponds to

[CBSE AIPMT 2010]

(a) simple harmonic motion of frequency ω/π
(b) simple harmonic motion of frequency $3\omega/2\pi$
(c) non-simple harmonic motion
(d) simple harmonic motion of frequency $\omega/2\pi$

Ans. *(c)*

For a particle executing SHM

Acceleration $(a) \propto -\omega^2$ displacement (x) ...(i)

Given $x = a\sin^2 \omega t$ ...(ii)

Differentiating the above equation w.r.t, we get $\frac{dx}{dt} = 2a\omega(\sin\omega t)(\cos\omega t)$

Again differentiating, we get

$$\frac{d^2x}{dt^2} = a = 2a\omega^2 [\cos^2 \omega t - \sin^2 \omega t]$$

$$= 2a\omega^2 \cos 2\omega t$$

The given equation does not satisfy the condition for SHM [Eq. (i)]. Therefore, motion is not simple harmonic.

15 Which one of the following equations of motion represents simple harmonic motion ?

[CBSE AIPMT 2009]

(a) Acceleration $= -k_0 x + k_1 x^2$
(b) Acceleration $= -k(x + a)$
(c) Acceleration $= k(x + a)$
(d) Acceleration $= kx$
(where, k, k_0, k_1 and a are all positive.)

Ans. *(b)*

As we know that, the condition for a body executing SHM is $F = -kx$

So, $a = \frac{F}{m} = -\frac{k}{m}x$

or $a = -\omega^2 x$

Acceleration $\propto -$(displacement)

$A \propto -y$

$A = -\omega^2 y$

$A = -\frac{k}{m}y$

$A = -ky$

Here, $y = x + a$

$\therefore$ Acceleration $= -k(x + a)$

16 Two simple harmonic motions of angular frequency 100 rad s^{-1} and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is

[CBSE AIPMT 2008]

(a) 1 : 10
(b) 1 : 10^2
(c) 1 : 10^3
(d) 1 : 10^4

Ans. *(b)*

Maximum acceleration of body executing SHM is given by

$$\alpha_{max} = \omega^2 a$$

So, for two different cases,

$$\frac{\alpha_{max_1}}{\alpha_{max_2}} = \frac{\omega_1^2}{\omega_2^2} \quad (\because a \text{ is same})$$

$$= \frac{(100)^2}{(1000)^2} = \frac{1}{10^2}$$

17 A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$. After the elapse of what fraction of the time period, the velocity of the point will be equal to half of its maximum velocity ?

[CBSE AIPMT 2008]

(a) $\frac{T}{8}$
(b) $\frac{T}{6}$
(c) $\frac{T}{3}$
(d) $\frac{T}{12}$

Ans. *(d)*

According to the question, equation of motion of SHM is

$$x = a\sin\left(\omega t + \frac{\pi}{6}\right)$$

velocity of body is given by

$$v = \frac{dx}{dt} = a\omega \cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\frac{a\omega}{2} = a\omega \cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\frac{1}{2} = \cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\omega t + \frac{\pi}{6} = \frac{\pi}{3} \qquad \left[\because \cos\frac{\pi}{3} = \frac{1}{2}\right]$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\frac{2\pi}{T} t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$$

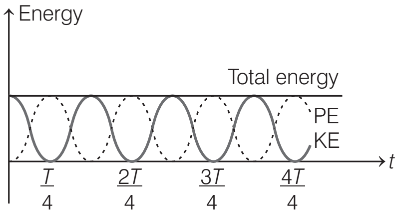
18 The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively **[CBSE AIPMT 2007]**

(a) 0 and $2K_0$ (b) $\frac{K_0}{2}$ and K_0

(c) K_0 and $2K_0$ (d) K_0 and K_0

Ans. *(d)*

In simple harmonic motion, the total energy of the particle is constant at all instants which is totally kinetic when particle is passing through the mean position and is totally potential when particle is passing through the extreme position.



The variation of PE and KE with time is shown in figure, by dotted parabolic curve and solid parabolic curve respectively.

Figure indicates that maximum values of total energy, KE and PE of SHM are equal.

Now, $\text{KE} = K_0 \cos^2 \omega t$

$\therefore$ $\text{KE}_{max} = K_0$

So, $\text{PE}_{max} = K_0$

and $(E)_{Total} = K_0$

19 A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is **[CBSE AIPMT 2007]**

(a) $\frac{T}{4}$ (b) $\frac{T}{8}$ (c) $\frac{T}{12}$ (d) $\frac{T}{2}$

Ans. *(c)*

Let displacement equation of particle executing SHM is

$$x = a \sin \omega t$$

As particle travels half of the amplitude from the equilibrium position, so

$$x = \frac{a}{2}$$

Therefore, $\frac{a}{2} = a \sin \omega t$

or $\sin \omega t = \frac{1}{2} = \sin \frac{\pi}{6}$

or $\omega t = \frac{\pi}{6}$ or $t = \frac{\pi}{6\omega}$

or $t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)}$ $\left(\text{as}, \omega = \frac{2\pi}{T}\right)$

or $t = \frac{T}{12}$

Hence, the particle travels half of the amplitude from the equilibrium in $\frac{T}{12}$ s.

20 A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is **[CBSE AIPMT 2005]**

(a) 3 Hz (b) 2 Hz
(c) 4 Hz (d) 1 Hz

Ans. *(d)*

Maximum speed of a particle executing SHM is given by,

$$v_{max} = a\,\omega = a\,(2\,\pi n) \quad \Rightarrow n = \frac{v_{max}}{2\,\pi a}$$

where, a = amplitude of oscillation

n = frequency of oscillation

Here, $v_{max} = 31.4$ cm/s, $a = 5$ cm

Substituting, the given values, we have

$$n = \frac{31.4}{2 \times 3.14 \times 5} = 1 \text{ Hz}$$

21 Which one of the following statements is true for the speed v and the acceleration α of a particle executing simple harmonic motion? **[CBSE AIPMT 2004]**

(a) When v is maximum, α is maximum

(b) Value of α is zero, whatever may be the value of v

(c) When v is zero, α is zero

(d) When v is maximum, α is zero

Ans. *(d)*

In simple harmonic motion, the displacement equation is, $x = a \sin \omega t$

where, a is the amplitude of the motion.

Velocity, $v = \frac{dx}{dt} = a\,\omega \cos \omega t$

$$v = a\omega \sqrt{1 - \sin^2 \omega t}$$

$$v = \omega \sqrt{a^2 - x^2} \quad \text{...(i)}$$

Acceleration, $\alpha = \frac{dv}{dt} = \frac{d}{dt}(a\omega \cos \omega t)$

$$\alpha = -a\omega^2 \sin \omega t$$

$$\alpha = -\omega^2 x \quad \text{...(ii)}$$

When $x = 0, v = a\omega = v_{max}$

$$\alpha = 0 = \alpha_{min}$$

When $x = a, v = 0 = v_{min}$

$$\alpha = -\omega^2 a = \alpha_{max}$$

Hence, it is clear that when v is maximum, then α is minimum (i.e. zero) or *vice-versa*.

22 The potential energy of a simple harmonic oscillator when the particle is half way to its end point is **[CBSE AIPMT 2003]**

(a) $\frac{1}{4} E$ (b) $\frac{1}{2} E$

(c) $\frac{2}{3} E$ (d) $\frac{1}{8} E$

Ans. *(a)*

Potential energy of a simple harmonic oscillator

$$U = \frac{1}{2} m\omega^2 x^2$$

Kinetic energy of a simple harmonic oscillator

$$K = \frac{1}{2} m\omega^2 (a^2 - x^2)$$

Here, x = Displacement from mean position

a = Maximum displacement (or amplitude) from mean position

Total energy is

$$E = U + K$$
$$= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 (a^2 - x^2)$$
$$= \frac{1}{2} m\omega^2 a^2$$

When the particle is half way to its end point i.e. at half of its amplitude, then

$$x = \frac{a}{2}$$

Hence, potential energy

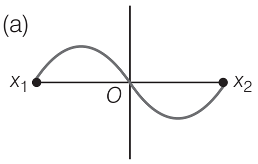
$$U = \frac{1}{2} m\omega^2 \left(\frac{a}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2} m\omega^2 a^2\right)$$

$$\Rightarrow \quad U = \frac{E}{4}$$

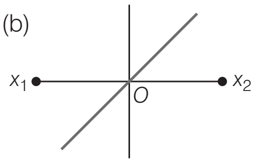
(where, E is the total energy)

23 A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph **[CBSE AIPMT 2003]**

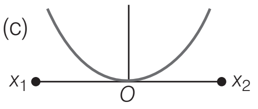
(a)



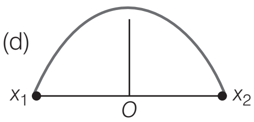
(b)



(c)



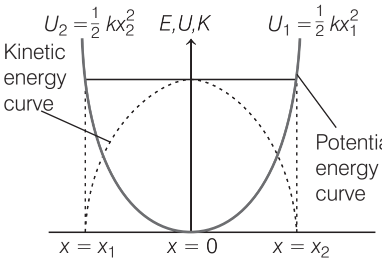
(d)



Ans. *(c)*

Potential energy is given by $U = \frac{1}{2} kx^2$

The corresponding graph is shown in figure.



At equilibrium position $(x = 0)$, potential energy is minimum. At extreme positions x_1 and x_2, its potential energies are

$$U_1 = \frac{1}{2} kx_1^2$$

and $$U_2 = \frac{1}{2} kx_2^2$$

In the above graph, the dotted line (curve) is shown for kinetic energy. This graph shows that kinetic energy is maximum at mean position and zero at extreme positions x_1 and x_2.

24 The displacement of particle between maximum potential energy position and maximum kinetic energy position in simple harmonic motion is **[CBSE AIPMT 2002]**

(a) $\pm \frac{a}{2}$ (b) $\pm a$

(c) $\pm 2a$ (d) ± 1

Ans. *(b)*

Expression of kinetic energy is

$$K = \frac{1}{2} k(a^2 - x^2) \quad \text{...(i)}$$

Expression of potential energy is

$$U = \frac{1}{2} kx^2 \quad \text{...(ii)}$$

where, $k = m\omega^2$

We observe that at mean position $(x = 0)$, kinetic energy is maximum $\left(\frac{1}{2} ka^2\right)$ and potential energy is minimum (zero). Also at extreme positions $(x = \pm a)$, kinetic energy is zero and potential energy is maximum $\left(\frac{1}{2} ka^2\right)$. Thus, displacement between positions of maximum potential energy and maximum kinetic energy is $\pm a$.

NOTE

Kinetic energy is zero at extreme positions but potential energy at mean position need not be zero. It is minimum at mean position.

25 In SHM restoring force is $F = -kx$, where k is force constant, x is displacement and a is amplitude of motion, then total energy depends upon **[CBSE AIPMT 2001]**

(a) k, a and m (b) k, x, m

(c) k, a (d) k, x

Ans. *(c)*

In SHM, the total energy

= potential energy + kinetic energy

or $E = U + K$

$$= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 (a^2 - x^2)$$

$$= \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} ka^2$$

where, $k =$ force constant $= m\omega^2$

Thus, total energy depends on k *and* a.

26 Two simple harmonic motions given by, $x = a \sin(\omega t + \delta)$ and $y = a \sin\left(\omega t + \delta + \frac{\pi}{2}\right)$ act on a particle simultaneously, then the motion of particle will be **[CBSE AIPMT 2000]**

(a) circular anti-clockwise

(b) circular clockwise

(c) elliptical anti-clockwise

(d) elliptical clockwise

Ans. *(b)*

Two simple harmonic motions can be written as

$$x = a \sin(\omega t + \delta) \quad \text{...(i)}$$

and $$y = a \sin\left(\omega t + \delta + \frac{\pi}{2}\right)$$

or $$y = a \cos(\omega t + \delta) \quad \text{...(ii)}$$

Squaring and adding Eqs. (i) and (ii), we obtain

$$x^2 + y^2 = a^2 [\sin^2(\omega t + \delta) + \cos^2(\omega t + \delta)]$$

or $x^2 + y^2 = a^2$ $(\because \sin^2\theta + \cos^2\theta = 1)$

This is the equation of a circle.

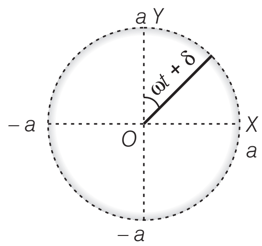
At $(\omega t + \delta) = 0$; $x = 0, y = a$

At $(\omega t + \delta) = \frac{\pi}{2}$; $x = a, y = 0$

At $(\omega t + \delta) = \pi$; $x = 0, y = -a$

At $(\omega t + \delta) = \frac{3\pi}{2}$; $x = -a, y = 0$

At $(\omega t + \delta) = 2\pi$; $x = 0, y = a$



Thus, it is obvious that motion of particle is traversed in clockwise direction.

27 Two simple harmonic motions with the same frequency act on a particle at right angles i.e. along X-axis and Y-axis. If the two amplitudes are equal and the phase difference is $\pi/2$, the resultant motion will be **[CBSE AIPMT 1997]**

(a) a circle

(b) an ellipse with the major axis along Y-axis

(c) an ellipse with the major axis along X-axis

(d) a straight line inclined at 45° to the X-axis

Ans. *(a)*

The two simple harmonic motions can be written as

$$x = a\sin\omega t \quad ...(i)$$

and $$y = a\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$y = a\cos\omega t \quad ...(ii)$$

On squaring and adding Eqs. (i) and (ii), we obtain

$$x^2 + y^2 = a^2(\sin^2\omega t + \cos^2\omega t)$$

or $x^2 + y^2 = a^2$

This is the equation of a circular motion with radius a.

NOTE

Simple harmonic motion is of two types.
1. Linear simple harmonic motion
2. Angular simple harmonic motion

28 A particle starts simple harmonic motion from the mean position. Its amplitude is a and time period is T. What is its displacement when its speed is half of its maximum speed? **[CBSE AIPMT 1996]**

(a) $\frac{\sqrt{2}}{3}a$ (b) $\frac{\sqrt{3}}{2}a$

(c) $\frac{2}{\sqrt{3}}a$ (d) $\frac{a}{\sqrt{2}}$

Ans. *(b)*

Velocity of the particle executing SHM at any instant is defined as the time rate of change of its displacement at that instant.

Let the displacement of the particle at an instant t is given by

$$x = a\sin\omega t$$

$$\therefore \text{Velocity } v = \frac{dx}{dt} = \frac{d(a\sin\omega t)}{dt}$$

$$= a\omega\cos\omega t = a\omega\sqrt{(1-\sin^2\omega t)}$$

$$= a\omega\sqrt{\left(1-\frac{x^2}{a^2}\right)} = \omega\sqrt{(a^2-x^2)}$$

At mean position, $x = 0$

$$\therefore \quad v_{max} = \omega a$$

According to problem, $v = \frac{v_{max}}{2} = \frac{a\omega}{2}$

But $v = \omega\sqrt{a^2 - x^2}$

$$\therefore \quad \frac{a\omega}{2} = \omega\sqrt{a^2 - x^2} \quad \text{or} \quad x = \frac{\sqrt{3}}{2}a$$

29 In a simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic? **[CBSE AIPMT 1996]**

(a) Zero (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Ans. *(d)*

Total energy of the particle executing SHM at instant t is given by

$$E = \frac{1}{2}m\omega^2a^2 \quad ...(i)$$

and kinetic energy of the particle at instant t is given by

$$E_K = \frac{1}{2}m\omega^2(a^2 - x^2) \quad ...(ii)$$

when $$x = \frac{a}{2},\ E_K = \frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right)$$

$$= \frac{1}{2}m\omega^2 \times \frac{3}{4}a^2$$

or $$E_K = \frac{1}{2}\times\frac{3}{4}m\omega^2a^2 \quad ...(iii)$$

From Eqs. (i) and (iii)

$$\frac{E_K}{E} = \frac{3}{4} \Rightarrow E_K = \frac{3}{4}E$$

30 A body executes SHM with an amplitude a. At what displacement from the mean position, the potential energy of the body is one-fourth of its total energy? **[CBSE AIPMT 1995]**

(a) $\frac{a}{4}$

(b) $\frac{a}{2}$

(c) $\frac{3a}{4}$

(d) Some other fraction of a

Ans. *(b)*

Potential energy of the body executing SHM is given by

$$U = \frac{1}{2}m\omega^2x^2$$

where symbols have their usual meaning.

Total energy of the body executing SHM is

$$E = \frac{1}{2}m\omega^2a^2$$

According to problem,

$$U = \frac{1}{4}E$$

$$\therefore \quad \frac{1}{2}m\omega^2x^2 = \frac{1}{4}\times\frac{1}{2}m\omega^2a^2$$

or $$x^2 = \frac{a^2}{4} \quad \text{or} \quad x = \frac{a}{2}$$

31 Which one of the following is a simple harmonic motion? **[CBSE AIPMT 1994]**

(a) Ball bouncing between two rigid vertical walls
(b) Particle moving in a circle with uniform speed
(c) Wave moving through a string fixed at both ends
(d) Earth spinning about its own axis

Ans. *(c)*

Problem Solving Strategy To calculate the time period of combined oscillation, calculate the beat produced from the given frequencies.

In transverse wave motion individual particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion. Wave moving through a string fixed at both ends executes SHM.

32 A wave has SHM (simple harmonic motion) whose period is 4s while another wave which also possesses SHM has its period 3 s. If both are combined, then the resultant wave will have the period equal to **[CBSE AIPMT 1993]**

(a) 4 s (b) 5 s
(c) 12 s (d) 3 s

Ans. *(c)*

Problem Solving Strategy To calculate the time period of combined oscillation, calculate the beat produced from the given frequencies.

When both waves are combined, then beats are produced. Frequency of beats will be $= \nu_1 - \nu_2$

$$= \frac{1}{T_1} - \frac{1}{T_2} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

Hence, time period = 12 s

33 A simple harmonic oscillator has an amplitude a and time period T. The time required by it to travel from $x = a$ to $x = \frac{a}{2}$ is **[CBSE AIPMT 1992]**

(a) $\frac{T}{6}$ (b) $\frac{T}{4}$

(c) $\frac{T}{3}$ (d) $\frac{T}{2}$

Ans. *(a)*

Equation of SHM is

$$x = a\sin\omega t \quad \text{or} \quad x = a\sin\left(\frac{2\pi}{T}\right)t$$

when $x = a$, then

$$a = a\sin\left(\frac{2\pi}{T}\right)t$$

or $\sin\left(\frac{2\pi}{T}\right)t = 1$

or $\sin\left(\frac{2\pi}{T}\right)t = \sin\frac{\pi}{2} \Rightarrow t = \frac{T}{4}$

when $x = \frac{a}{2}$, then

$$\frac{a}{2} = a\sin\left(\frac{2\pi}{T}\cdot t\right)$$

or $\sin\left(\frac{2\pi}{T}t\right) = \sin\frac{\pi}{6}$ or $t = \frac{T}{12}$

Hence, time taken to travel from

$$x = a \text{ to } x = \frac{a}{2} = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$$

34 A body is executing SHM. When the displacements from the mean position is 4 cm and 5 cm, the corresponding velocities of the body is 10 cm/s and 8 cm/s. Then, the time period of the body is **[CBSE AIPMT 1991]**

(a) 2π sec (b) $\frac{\pi}{2}$ sec

(c) π sec (d) $\frac{3\pi}{2}$ sec

Ans. *(c)*

Velocity of the particle executing SHM at any instant is defined as the time rate of change of its displacement at that instant. It is given by

$$v = \omega\sqrt{(a^2 - x^2)}$$

where, x is displacement of the particle. is acceleration and ω is angular frequency.

Case I $10 = \omega\sqrt{a^2 - 16}$ ...(i)

Case II $8 = \omega\sqrt{a^2 - 25}$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{5}{4} = \frac{\sqrt{a^2 - 16}}{\sqrt{a^2 - 25}} \quad \text{or} \quad \frac{25}{16} = \frac{a^2 - 16}{a^2 - 25}$$

or $16a^2 - 256 = 25a^2 - 625$

or $a^2 = \frac{369}{9}$

Putting value of a^2 in Eq. (i), we get

$$10 = \omega\sqrt{\left(\frac{369}{9} - 16\right)}$$

or $\omega = \frac{10 \times 3}{15} = 2$ rad/s

$\therefore$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ sec

35 The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement x from the mean position, if its kinetic energy is T and potential energy is U, then the ratio of T to U is **[CBSE AIPMT 1991]**

(a) $\left(\frac{a^2 - x^2\omega^2}{x^2\omega^2}\right)$ (b) $\frac{x^2\omega^2}{(a^2 - x^2\omega^2)}$

(c) $\frac{(a^2 - x^2)}{x^2}$ (d) $\frac{x^2}{(a^2 - x^2)}$

Ans. *(c)*

Consider a particle of mass m, executing linear SHM with amplitude a and constant angular frequency ω. Suppose t second after starting from the mean position, the displacement of the particle is x, which is given by

$$x = a\sin\omega t$$

So, potential energy of particle is

$$U = \frac{1}{2}m\omega^2 x^2 \quad \text{...(i)}$$

and kinetic energy of particle is

$$T = \frac{1}{2}m\omega^2(a^2 - x^2) \quad \text{...(ii)}$$

From Eqs. (i) and (ii) $\frac{T}{U} = \frac{a^2 - x^2}{x^2}$

36 A particle moving along the X-axis executes simple harmonic motion, then the force acting on it is given by **[CBSE AIPMT 1988]**

(a) $-AKx$ (b) $A\cos Kx$

(c) $A\exp(-Kx)$ (d) AKx

where, A and K are positive constants.

Ans. *(a)*

If a particle executing simple harmonic motion, has a displacement x from its equilibrium position, at an instant the magnitude of the restoring force F acting on the particle at that instant is given by

$$F = -kx$$

where, k is known as force constant.

Hence, in given options, option (a) is correct. Here, $k = AK$

TOPIC 2
Some Systems Exceuting SHM

37 A spring is stretched by 5 cm by a force 10 N. The time period of the oscillations when a mass of 2 kg is suspended by it is **[NEET 2021]**

(a) 0.0628 s (b) 6.28 s

(c) 3.14 s (d) 0.628 s

Ans. *(d)*

Given, the mass of suspended, $m = 2$ kg

The spring is stretched, $x = 5$ cm = 0.05 m

The constant force applied on the spring, $F = 10$ N

As we know that, spring force,

$$F = kx \Rightarrow 10\text{ N} = k(0.05\text{ m})$$

$\Rightarrow$ $k = 200$ N/m

Now, time period of the oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T = 2\pi\sqrt{\frac{2}{200}}$$

Time period, $T = 0.628$ s

38 A mass falls from a height 'h' and its time of fall 't' is recorded in terms of time period T of a simple pendulum. On the surface of earth it is found that $t = 2$ T. The entire set up is taken on the surface of another planet whose mass is half of earth and radius the same. Same experiment is repeated and corresponding times noted as t' and T'. **[NEET (Odisha) 2019]**

(a) $t' = \sqrt{2}T'$ (b) $t' > 2T'$

(c) $t' < 2T'$ (d) $t' = 2T'$

Ans. *(d)*

The distance covered by the mass falling from height 'h' during its time of fall 't' is given by

$$s = h = ut + \frac{1}{2}gt^2$$

As, $u = 0 \Rightarrow h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$ ...(i)

The time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{...(ii)}$$

where, l is the length of the pendulum. From Eq. (i) and (ii), since 'h' and 'l' are constant so, we can conclude that,

$$t \propto \frac{1}{\sqrt{g}} \text{ and } T \propto \frac{1}{\sqrt{g}} \quad \therefore \frac{t}{T} = 1$$

Thus, the ratio of time of fall and time period of pendulum is independent of value of gravity (g) or any other parameter like mass and radius of the planet. Thus, the relation between t' and T' on another planet irrespective of its mass or radius will remains same as it was on earth i.e.

$$t' = 2T'$$

39 A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is $20\ m/s^2$ at a distance of 5 m from the mean position. The time period of oscillation is **[NEET 2018]**

(a) 2 s (b) π s (c) 2π s (d) 1 s

Ans. ***(b)***

The acceleration of particle/body executing SHM at any instant (at position x) is given as

$$a = -\omega^2 x$$

where, ω is the angular frequency of the body.

$$\Rightarrow \quad |a| = \omega^2 x \quad ...(i)$$

Here, $x = 5\text{m}, |a| = 20\ \text{ms}^{-2}$

Substituting the given values in Eq. (i), we get

$$20 = \omega^2 \times 5$$

$$\Rightarrow \quad \omega^2 = \frac{20}{5} = 4$$

or $\omega = 2\ \text{rad s}^{-1}$

As, we know that

Time period, $T = \frac{2\pi}{\omega}$ ...(ii)

$\therefore$ Substituting the value of ω in Eq. (ii), we get

$$T = \frac{2\pi}{2} = \pi\ \text{s}$$

40 A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k'. If they are connected in parallel and force constant is k'', then $1k' : k''$ is **[NEET 2017]**

(a) 1 : 6 (b) 1 : 9 (c) 1 : 11 (d) 1 : 14

Ans. ***(c)***

When the spring is cut into pieces, they will have the new force constant. The spring is divided into 1 : 2 : 3 ratio.

When the pieces are connected in series, the resultant force constant

$$\frac{1}{v'} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$\frac{1}{v'} = \frac{1}{x} + \frac{1}{2x} + \frac{1}{3x}$$

$$v' = \frac{6x}{11}$$

In parallel, the net force constant

$$K'' = x + 2x + 3x = 6x = 11K$$

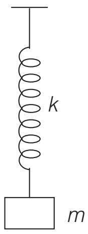
The required ratio

$$\frac{K}{K''} = \frac{6x/11}{6x} = 1 : 11$$

41 A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is **[NEET 2016]**

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{16}{9}$ (d) $\frac{9}{16}$

Ans. ***(d)***



As we know that

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

Case I

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad ...(i)$$

Case II When the mass m is increased by 1 kg, then $= m + 1$

From Eqs. (ii) and (i), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \quad \frac{5}{3} = \sqrt{\frac{m+1}{m}} \Rightarrow \frac{25}{9} = \frac{m+1}{m}$$

$$\Rightarrow \frac{25}{9} = 1 + \frac{1}{m} \Rightarrow \frac{1}{m} = \frac{16}{9}$$

$$\therefore \quad m = \frac{9}{16}\ \text{kg}$$

42 The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be **[CBSE AIPMT 2010]**

(a) T (b) $T/\sqrt{2}$ (c) $2T$ (d) $\sqrt{2}T$

Ans. ***(d)***

Time period of spring pendulum,

$$T = 2\pi\sqrt{\frac{M}{k}}.$$

If mass is doubled then time period

$$T' = 2\pi\sqrt{\frac{2M}{k}} = \sqrt{2}T$$

43 A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a and time period T. The speed of the pendulum at $x = \frac{a}{2}$ will be **[CBSE AIPMT 2009]**

(a) $\frac{\pi a\sqrt{3}}{2T}$ (b) $\frac{\pi a}{T}$ (c) $\frac{3\pi^2 a}{T}$ (d) $\frac{\pi a\sqrt{3}}{T}$

Ans. ***(d)***

Concept Use the equation of motion of a body executing SHM.

i.e. $x = a \sin\omega t$

As we know, the velocity of body executing SHM is given by

$$v = \frac{dx}{dt} = a\omega\cos\omega t = a\omega\sqrt{1 - \sin^2\omega t}$$

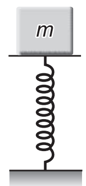
$$= \omega\sqrt{a^2 - x^2}$$

Here, $x = \frac{a}{2}$

$$\therefore \quad v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3a^2}{4}}$$

$$= \frac{2\pi}{T}\frac{a\sqrt{3}}{2} = \frac{\pi a\sqrt{3}}{T}$$

44 A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is $200\ \text{N/m}$. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan? (Take $g = 10\ \text{m/s}^2$) **[CBSE AIPMT 2007]**



(a) 8.0 cm
(b) 10.0 cm
(c) Any value less than 12.0 cm
(d) 4.0 cm

Ans. ***(b)***

Let the minimum amplitude of SHM be a. Restoring force on spring

$$F = ka$$

Restoring force is balanced by weight mg of block. For mass to execute simple harmonic motion of amplitude a.

$\therefore \quad ka = mg \quad \text{or} \quad a = \frac{mg}{k}$

Here, $m = 2 \text{ kg}, k = 200 \text{ N/m},$

$g = 10 \text{ m/s}^2$

$\therefore \quad a = \frac{2 \times 10}{200} = \frac{10}{100} \text{ m}$

$= \frac{10}{100} \times 100 \text{ cm} = 10 \text{ cm}$

Hence, minimum amplitude of the motion should be 10 cm, so that the mass gets detached from the pan.

45 A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium, it undergoes oscillation with a time period T. Then **[CBSE AIPMT 2006]**

(a) $T \propto \sqrt{\rho}$ (b) $T \propto \frac{1}{\sqrt{A}}$

(c) $T \propto \frac{1}{\rho}$ (d) $T \propto \frac{1}{\sqrt{m}}$

Ans. *(b)*

Let block be displaced through x m, then weight of displaced water or upthrust, (upwards) is given by Archimedes principle

$F_b = -Ax\rho g$

where, A is the a
rea of cross-section of the block and ρ is its density. This must be equal to force (=ma) applied, where, m is the mass of the block and a is the acceleration.

$\therefore \quad ma = -Ax\rho g \quad \text{or} \quad a = -\frac{A\rho g}{m} x = -\omega^2 x$

This is the equation of simple harmonic motion. Time period of oscillation,

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{A\rho g}} \Rightarrow T \propto \frac{1}{\sqrt{A}}$

46 Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by **[CBSE AIPMT 2004]**

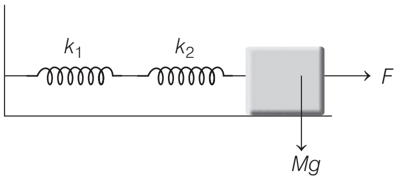
(a) $\sqrt{k_1 k_2}$ (b) $\frac{(k_1 + k_2)}{2}$

(c) $k_1 + k_2$ (d) $\frac{k_1 k_2}{(k_1 + k_2)}$

Ans. *(d)*

Let us consider two springs of spring constants k_1 and k_2 joined in series as shown in figure.

Under a force F, they will stretch by y_1 and y_2.



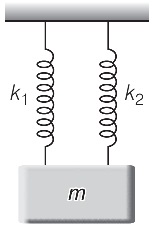
So, $\quad y = y_1 + y_2$

or $\quad \frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$

But as springs are massless, so force on them must be same i.e. $F_1 = F_2 = F$.

So, $\quad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1 k_2}{k_1 + k_2}$

47 A mass is suspended separately by two springs of spring constants k_1 and k_2 in successive order. The time periods of oscillations in the two cases are T_1 and T_2 respectively. If the same mass be suspended by connecting the two springs in parallel, (as shown in figure) then the time period of oscillations is T. The correct relation is **[CBSE AIPMT 2002]**



(a) $T^2 = T_1^2 + T_2^2$
(b) $T^{-2} = T_1^{-2} + T_2^{-2}$
(c) $T^{-1} = T_1^{-1} + T_2^{-1}$
(d) $T = T_1 + T_2$

Ans. *(b)*

Problem Solving Strategy Calculate the effective force constant of parallel spring, then by putting the values of time period $T = 2\pi\sqrt{\frac{M}{K}}$, we get the new time period of spring.

We can write time period for a vertical spring-block system as

$T = 2\pi\sqrt{\frac{l}{g}}$

Here, l is extension in the spring when the mass m is suspended from the spring.

This can be seen as under :

$kl = mg$

(in equilibrium position)

$\Rightarrow \quad \frac{m}{k} = \frac{l}{g}$

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}}$

$\therefore \quad T_1 = 2\pi\sqrt{\frac{m}{k_1}}$

$\Rightarrow \quad k_1 = 4\pi^2 \frac{m}{T_1^2}$ ...(i)

$T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\Rightarrow \quad k_2 = 4\pi^2 \frac{m}{T_2^2}$ ...(ii)

Since, springs are in parallel, effective force constant

$k = k_1 + k_2$

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

$\Rightarrow \quad k_1 + k_2 = 4\pi^2 \frac{m}{T^2}$ ...(iii)

Substituting values of k_1 and k_2 from Eqs. (i) and (ii) in Eq. (iii), we get

$4\pi^2 \frac{m}{T_1^2} + 4\pi^2 \frac{m}{T_2^2} = 4\pi^2 \frac{m}{T^2}$

$\Rightarrow \quad \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

or $\quad T^{-2} = T_1^{-2} + T_2^{-2}$

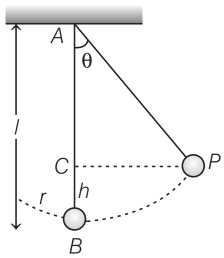
48 A pendulum is displaced to an angle θ from its equilibrium position, then it will pass through its mean position with a velocity v equal to **[CBSE AIPMT 2000]**

(a) $\sqrt{2gl}$ (b) $\sqrt{2gl \sin\theta}$

(c) $\sqrt{2gl \cos\theta}$ (d) $\sqrt{2gl(1 - \cos\theta)}$

Ans. *(d)*

If l is the length of pendulum and θ the angular amplitude, then height



$h = AB - AC$

$= l - l\cos\theta$

$= l(1 - \cos\theta)$ ...(i)

At point P (maximum displacement position i.e. extreme position), potential energy is maximum and kinetic energy is zero. At point B (mean or equilibrium position) potential energy is minimum and kinetic energy is maximum, so from principle of conservation of energy.

$$(PE + KE) \text{ at } P = (KE + PE) \text{ at } B$$

or $\quad mgh + 0 = \frac{1}{2} mv^2 + 0$

or $\quad v = \sqrt{2gh}$ ...(ii)

Substituting the value of h from Eq. (i) into Eq. (ii), we get

$$v = \sqrt{2gl\,(1 - \cos\theta)}$$

49 The time period of a simple pendulum is 2 s. If its length is increased by 4 times, then its period becomes **[CBSE AIPMT 1999]**

(a) 16 s (b) 12 s
(c) 8 s (d) 4 s

Ans. ***(d)***

Time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l = length of pendulum

g = acceleration due to gravity

i.e. $\quad T \propto \sqrt{l}$

Hence, $\quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$ ...(i)

Given, $\quad l_2 = 4l_1, T_1 = 2\,s$

Substituting the values in Eq. (i), we get

$$T_2 = \sqrt{\frac{4l_1}{l_1}} \times 2 = 2 \times 2 = 4\,s$$

50 A mass m is vertically suspended from a spring of negligible mass, the system oscillates with a frequency n. What will be the frequency of the system, if a mass $4m$ is suspended from the same spring ? **[CBSE AIPMT 1998]**

(a) $\frac{n}{4}$ (b) $4n$
(c) $\frac{n}{2}$ (d) $2n$

Ans. ***(c)***

Time period of spring-mass system, is given by

$$T = 2\pi\sqrt{\left(\frac{\text{displacement}}{\text{acceleration}}\right)}.$$

$\therefore$ Frequency, $n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \text{...(i)}$$

In case of vertical spring-mass system, in equilibrium position

$$kl = mg \Rightarrow \frac{g}{l} = \frac{k}{m}$$

where, l = extension in the spring and

m = mass of the suspended body

k = spring constant or force constant of spring.

$\therefore$ From Eq. (i), we have

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ or } n \propto \frac{1}{\sqrt{m}} \text{ or } \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$$

but $m_1 = m$, $m_2 = 4m$, $n_1 = n$ (given)

$\therefore \quad \frac{n}{n_2} = \sqrt{\frac{4m}{m}} = 2$ or $n_2 = \frac{n}{2}$

Alternative

As we know that

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(for spring mass system)}$$

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

So, for two different masses suspended with same spring.

$$n_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}}$$

$\left[\begin{array}{l} k \text{ is same for both the} \\ \text{cases as spring is same} \end{array}\right]$

$$n_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}}$$

so, $\quad \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$

here, $\quad m_2 = 4m_1$

so, $\quad \frac{n_1}{n_2} = \sqrt{\frac{4m_1}{m_1}} = \frac{2}{1}$

$\Rightarrow \quad n_1 = 2n_2$

$\Rightarrow \quad n_2 = \frac{n_1}{2} = \frac{n}{2} \quad [n_1 = n]$

51 Two simple pendulums of length 0.5 m and 2.0 m respectively are given small linear displacement in one direction at the same time. They will again be in the same phase when the pendulum of shorter length has completed oscillations **[CBSE AIPMT 1998]**

(a) 5 (b) 1
(c) 2 (d) 3

Ans. ***(c)***

For the pendulum to be again in the same phase, there should be difference of one complete oscillation.

If smaller pendulum completes n oscillations the larger pendulum will complete $(n-1)$ oscillations, so

Time period of n oscillations of first

= Time period of $(n-1)$ oscillations of second

i.e. $\quad nT_1 = (n-1)T_2$

or $\quad n 2\pi\sqrt{\frac{l_1}{g}} = (n-1)\, 2\pi\sqrt{\frac{l_2}{g}}$

or $\quad n\sqrt{l_1} = (n-1)\sqrt{l_2}$

or $\quad \frac{n}{n-1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{2.0}{0.5}}$

or $\quad \frac{n}{n-1} = 2$ or $n = 2n - 2$

$\therefore \quad n = 2$

52 A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will **[CBSE AIPMT 1997]**

(a) first increase and then decrease
(b) first decrease and then increase
(c) increase continuously
(d) decrease continuously

Ans. ***(a)***

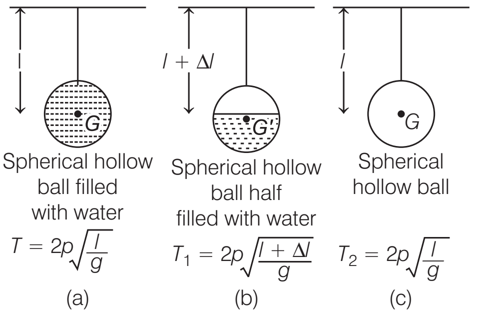
Problem Solving Strategy Compare the time period of two different oscillation.

Time period of simple pendulum

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)} \propto \sqrt{l}$$

where, l is effective length.

(i.e. distance between centre of suspension and centre of gravity of bob)



Initially, centre of gravity is at the centre of sphere [Fig. (a)]. When water leaks the centre of gravity goes down until it is half-filled [Fig. (b)], then it begins to go up and finally it again goes at the centre [Fig. (c)]. That is effective length first increases and then decreases. As $T \propto \sqrt{l}$, so time period first increases and then decreases.

53 A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its **[CBSE AIPMT 1996]**

(a) maximum potential energy is 160 J
(b) maximum potential energy is 100 J
(c) maximum potential energy is zero
(d) minimum potential energy is 100 J

Ans. *(a)*

The potential energy of a particle executing SHM is given by,

$$U = \frac{1}{2} m\omega^2 x^2$$

U is maximum, when $x = a$ = amplitude of vibration i.e. the particle is passing from the extreme position and is minimum when $x = 0$, i.e. the particle is passing from the mean position

$$U_{max} = \frac{1}{2} m\omega^2 a^2 \quad ...(i)$$

Also, total energy of the particle at instant t is given by

$$E = \frac{1}{2} m\omega^2 a^2 \quad ...(ii)$$

So, when E = 160 J, then maximum potential energy of particle will also be 160 J.

Alternative

$(KE)_{max} = (PE)_{max}$
= Total Mechanical Energy

So, Total Mechanical Energy = 160 J

54 A particle is subjected to two mutually perpendicular simple harmonic motions such that its x and y coordinates are given by $x = 2\sin\omega t, y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$

The path of the particle will be **[CBSE AIPMT 1994]**

(a) a straight line (b) a circle
(c) an ellipse (d) a parabola

Ans. *(c)*

Here, the phase difference between waves is $\frac{\pi}{4}$. So, the resultant path of particle will be ellipse.

55 If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 m/s^2 at any time, the angular frequency of the oscillator is equal to **[CBSE AIPMT 1992]**

(a) 10 rad/s (b) 0.1 rad/s
(c) 100 rad/s (d) 1 rad/s

Ans. *(a)*

Time period of body executing SHM is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x}{a}} \quad ...(i)$$

where, x is displacement of the particle and a is acceleration of the particle.

From Eq. (i)

$$\omega = \sqrt{\frac{a}{x}} \text{ or } \omega^2 = \frac{a}{x}$$

Here, $a = 2.0$ m/s^2

$x = 0.02$ m

$\therefore \quad \omega^2 = \frac{2.0}{0.02}$

or $\omega^2 = 100$

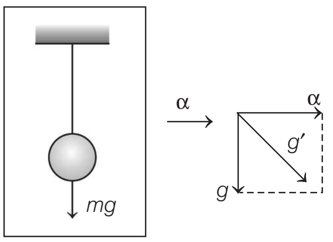
or $\omega = 10$ rad/s

56 A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration α, then the time period is given by $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$, where g is equal to **[CBSE AIPMT 1991]**

(a) g (b) $g - \alpha$
(c) $g + \alpha$ (d) $\sqrt{(g^2 + \alpha^2)}$

Ans. *(d)*

Problem Solving Strategy Apply vector formula to determine resultant acceleration of the both.



The bob is now under the combined action of two accelerations, g vertically downwards and α along the horizontal.

$\therefore$ Resultant acceleration

$$g' = \sqrt{g^2 + \alpha^2}$$

57 The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along **[CBSE AIPMT 1990]**

(a) circle (b) figure of eight
(c) straight line (d) ellipse

Ans. *(c)*

Let the SHM's be

$x = a\sin\omega t \quad ...(i)$

and $y = b\sin(\omega t + \pi)$

or $y = -b\sin\omega t \quad ...(ii)$

From Eqs. (i) and (ii)

$$\frac{x}{a} = \sin\omega t \text{ and } -\frac{y}{b} = \sin\omega t$$

$$\therefore \quad \frac{x}{a} = -\frac{y}{b} \text{ or } y = -\frac{b}{a}x$$

It is an equation of a straight line.

58 A mass m is suspended from the two coupled springs connected in series. The force constant for springs are k_1 and k_2. The time period of the suspended mass will be **[CBSE AIPMT 1990]**

(a) $T = 2\pi\sqrt{\frac{m}{k_1 - k_2}}$

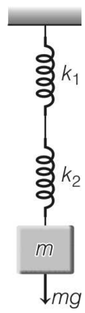
(b) $T = 2\pi\sqrt{\frac{mk_1k_2}{k_1 + k_2}}$

(c) $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

(d) $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$

Ans. *(d)*

Problem Solving Strategy Derive an expression from the given values which must be simillar to $a = -\omega^2 x$. Then calculate time period from the values in place of ω.



The situation is shown in figure. Consider two springs of spring constants k_1 and k_2. Here, the body of weight mg is suspended at the free end of the two springs in series combination. When the body is pulled downwards through a little distance y, the two springs suffer different extensions say y_1 and y_2. But the restoring force is same in each spring.

$\therefore \quad F = -k_1 y_1$

and $\quad F = -k_2 y_2$

or $\quad y_1 = -\dfrac{F}{k_1}$

and $\quad y_2 = -\dfrac{F}{k_2}$

$\therefore$ Total extension, $y = y_1 + y_2$

$$= -\frac{F}{k_1} - \frac{F}{k_2}$$

$$= -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

or $$F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right) y$$

If k is the spring constant of series combination of springs then

$$F = -ky$$

$$\therefore \quad k = \frac{k_1 k_2}{k_1 + k_2}$$

If the body is left free after pulling down, it will execute SHM of period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

TOPIC 3

Forced, Damped Oscillations and Resonance

59 The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are **[CBSE AIPMT 2012]**

(a) kg ms^{-1} (b) kg ms^{-2}
(c) kg s^{-1} (d) kg s

Ans. ***(c)***

Given,

Damping force $\propto$ velocity

$$F \propto v$$

$$F = kv \Rightarrow k = \frac{F}{v}$$

$$\text{Unit of } k = \frac{\text{unit of } F}{\text{unit of } v} = \frac{\text{kg - ms}^{-2}}{\text{ms}^{-1}} = \text{kg s}^{-1}$$

60 When a damped harmonic oscillator completes 100 oscillations, its amplitude is reduced to $\frac{1}{3}$ of its initial value. What will be its amplitude when it completes 200 oscillations ? **[CBSE AIPMT 2002]**

(a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

Ans. ***(d)***

In case of damped vibration, amplitude at any instant t is given by

$$a = a_0 \, e^{-bt}$$

where, a_0 = initial amplitude

b = damping constant

Case I $t = 100T$ and $a = \dfrac{a_0}{3}$

$$\therefore \quad \frac{a_0}{3} = a_0 \, e^{-b\,(100\,T)}$$

$$\Rightarrow \quad e^{-100\,bT} = \frac{1}{3}$$

Case II $t = 200T$

$$a = a_0 \, e^{-bt} = a_0 \, e^{-b\,(200\,T)}$$

$$= a_0 \, (e^{-100\,bT})^2 = a_0 \times \left(\frac{1}{3}\right)^2 = \frac{a_0}{9}$$

Thus, after 200 oscillations, amplitude will become $\frac{1}{9}$ times.

61 A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force $F \sin \omega t$. If the amplitude of the particle is maximum for $\omega = \omega_1$, and the energy of the particle is maximum for $\omega = \omega_2$, then **[CBSE AIPMT 1989]**

(a) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$
(b) $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$
(c) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$
(d) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$

Ans. ***(c)***

In harmonic oscillator, the energy is maximum at $\omega_2 = \omega_0$ and amplitude is maximum at frequency $\omega_1 < \omega_0$ in the presence of damping, so $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$.