06

Laws of Motion and Friction

IN THIS CHAPTER ....

Force

Force is a push or pull which

(i) generates or tends to generate motion in a body at rest.
(ii) stops or tends to stop a body in motion.
(iii) increases or decreases the magnitude of velocity of the moving body.
(iv) changes or tries to change the direction of a moving body.
(v) tends to change the shape of the body.

Based on the nature of interaction between two bodies, forces may be broadly classified into two types

(a) **Non-contact Forces** These are the forces that act between two bodies separated by a distance without any actual contact, e.g. gravitational force, electrostatic force.
(b) **Contact Forces** These are the forces that act between two bodies in contact, e.g. tension, normal reaction, friction, etc.

Inertia

The inability of a body to change by itself its state of rest or state of uniform motion along a straight line is called *inertia* of the body.

As inertia of a body is measured by the mass of the body. Heavier the body, greater the force required to change its state and hence, greater is its inertia. It is of three types (i) inertia of rest (ii) inertia of motion (iii) inertia of direction.

Newton's Laws of Motion

According to Newton, there are three laws' of motion

First Law of Motion (Law of Inertia)

It states that a body continues to be in a state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change of state. This is also called law of inertia.

If $F = 0 \Rightarrow v = \text{constant} \Rightarrow a = 0$

- This law defines force.
- The body opposes any external change in its state of rest or of uniform motion.

e.g. When a stationary vehicle suddenly moves, then the passengers inside the vehicle fall backward.

Linear Momentum

It is measured as the product of the mass of the body and its velocity.

The momentum of a body of mass m moving with a velocity $\mathbf{v}$ is given by $\mathbf{p} = m\mathbf{v}$.

Its unit is kg-ms^{-1} and dimensional formula is $[\text{ML T}^{-1}]$

Second Law of Motion

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts. According to second law,

$$F \propto \frac{dp}{dt} \quad \text{or} \quad F = k\frac{dp}{dt}$$

where, k is constant.

Second law can be written as $F = \dfrac{dp}{dt} = ma$

The SI unit of force is newton (N) and in CGS system is dyne.

$$1\text{ N} = 10^5 \text{ dyne}$$

Impulse

It is defined as the product of the average force and the time for which the force acts.

Impulse, $\mathbf{I} = \mathbf{F}_{av}\, t$

Impulse is also equal to the total change in momentum of the body during the impact.

Impulse, $\mathbf{I} = \mathbf{p}_2 - \mathbf{p}_1$

Impulse = Change in momentum

Third Law of Motion

To every action, there is an equal and opposite reaction.

i.e. $\mathbf{F}_{12} = -\mathbf{F}_{21}$

Action and reaction are mutually opposite and act on two different bodies.

e.g. Jumping of a man from a boat onto the bank of a river, Jerk is produced in a gun when bullet is fired from it.

Example 1. *A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at the rate of 1 kgs^{-1} and with a speed of 5 ms^{-1}. The initial acceleration of the block is*

(a) 2 ms^{-2} (b) 2.5 ms^{-2} (c) 3 ms^{-2} (d) 3.5 ms^{-2}

Sol. (*b*) The water releasing from jet striking the block at the rate of 1 kgs^{-1} with a speed of 5 ms^{-1} will exert a force on the block,

$$F = v\frac{dm}{dt} = 5 \times 1 = 5\text{ N}$$

Under the action of this force of 5 N, the block of mass 2 kg will move with an acceleration given by

$$a = \frac{F}{m} = \frac{5}{2} = 2.5 \text{ ms}^{-2}$$

Example 2. *A bullet of mass 0.04 kg moving with a speed of 90 ms^{-1} enters a having wooden block and is stopped after a distance of 60 cm. The average resistive force exerted by the block on the bullet is*

(a) 100 *N* (b) 270 *N* (c) 50 *N* (d) 298 *N*

Sol. (*b*) The retardation *a* of the bullet is given by

$$a = -\frac{u^2}{2s} = \frac{-90 \times 90}{2 \times 0.6} \text{ms}^{-2} = -6750 \text{ ms}^{-2}$$

The retarding force by the second law of motion is

$$F = ma = 0.04 \times 6750 = 270 \text{ N}$$

Note The actual resistive force and therefore, retardation of the bullet may not be uniform. The answer therefore only indicates the average resistive force.

Example 3. *A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0.15 kg, the impulse imparted to the ball is* **[NCERT Exemplar]**

(a) 1.5 *Ns* (b) 3.6 *Ns* (c) 7.2 *Ns* (d) 10.2 *Ns*

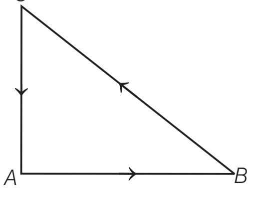
Sol. (*b*) Change in momentum $= m\,\Delta v$

Given, $m = 0.15$ kg

$\therefore \quad \Delta p = 0.15 \times 12 - (-0.15 \times 12) = 3.6 \text{ N s}$

In the direction from the batsman to the bowler.

Example 4. *Three forces start acting simultaneously on a particle moving with velocity v. These forces are represented in magnitude and direction by the three sides of a ΔABC (as shown). The particle will now move with velocity* **[AIEEE 2003]**



(a) less than *v*

(b) greater than *v*

(c) $|v|$ in the direction of largest force *BC*

(d) *v*, remaining unchanged

Sol. (*d*) Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order. Hence, by Newton's second law,

$$\mathbf{F} = m\frac{d\mathrm{v}}{dt} \Rightarrow m\frac{dv}{dt} = 0 \Rightarrow \frac{dv}{dt} = 0$$

$\Rightarrow$ v = constant, hence v remains unchanged.

Principle of Conservation of Linear Momentum

It states that if no external force is acting on a system, the momentum of the system remains constant.

If no force is acting, then $\mathbf{F} = 0$

$$\therefore \qquad \frac{d\mathbf{p}}{dt} = 0 \Rightarrow \mathbf{p} = \text{constant}$$

$$\text{or} \qquad m_1 v_1 = m_2 v_2 = \text{constant}$$

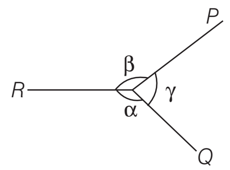
Equilibrium of Concurrent Forces

- If all the forces working on a body are acting on the same point, then they are said to be concurrent.
- A body under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.
- When a particle is at rest or moving with constant velocity in an inertial frame of reference, the net force on it, i.e. the vector sum of all the forces acting on it must be zero.

  i.e. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$ or $\Sigma\mathbf{F} = 0$

  In component form, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

  Lami's Theorem Three forces P, Q and R are acting on a body are in equilibrium, if

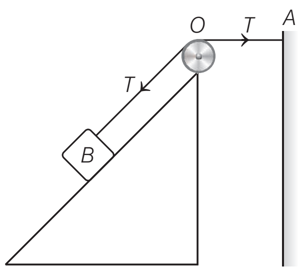


$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Common Forces in Mechanics

(i) Tension (T)

When a body is connected through a string or rope, a force may act on the body by the string or rope due to the tendency of extension. This force is called tension.



If the string is massless, then

(i) the tension T has the same magnitude at all points throughout the string.

(ii) the magnitude of acceleration of any number of masses connected through string is always same.

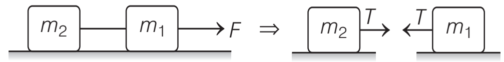
(iii) if there is friction between string and pulley, tension is different on two sides of the pulley but if there is no friction between pulley and string, tension will be same on both sides of the pulley.

Note (i) If string slacks, tension in string becomes zero.

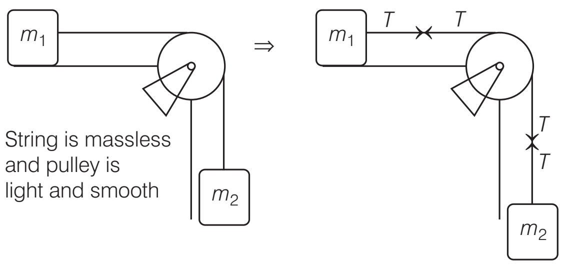
(ii) The direction of tension on a body or pulley is always away from the point of contact.

The direction of tension (T) in some cases are shown below

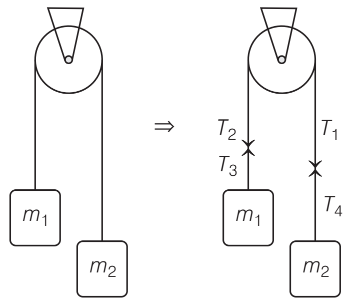
(a)



(b)

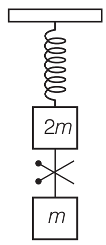


(c)



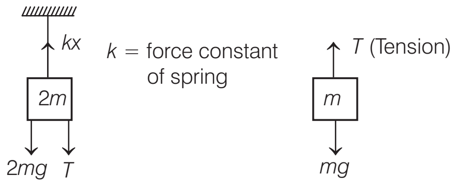
String is not massless and there is friction between pulley and string

Example 5. *System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of masses 2m and m, just after the string is cut will be*



(a) *g*/2 *upwards*, *g downwards*
(b) *g upwards*, *g*/2 *downwards*
(c) *g upwards*, 2*g downwards*
(d) 2*g upwards*, *g downwards*

Sol. (*b*) Before cutting the string, free body diagrams of masses m and $2m$ are given as

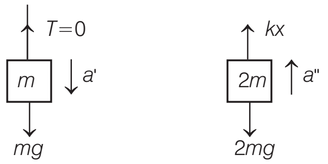


$$\Rightarrow \quad T + 2mg = kx \quad \ldots(i)$$
$$T = mg \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$mg + 2mg = kx$$
$$\Rightarrow \quad kx = 3mg \quad \ldots(iii)$$

After cutting the string, free body diagrams of masses m and $2m$ are given as



After cutting the string, mass m starts freely falling with gravitational acceleration (g).

i.e. $\quad a' = g$

Equation of motion for mass $2m$,

$$kx - 2mg = 2ma''$$
$$\Rightarrow \quad 3mg - 2mg = 2ma'' \quad \text{[from Eq. (i)]}$$
$$\Rightarrow \quad mg = 2ma''$$
$$\Rightarrow \quad a'' = \frac{g}{2}$$

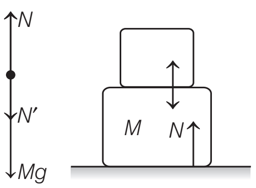
(ii) The Normal Reaction Force

Normal reaction is a contact force between two surfaces, which is always perpendicular to the surfaces in contact.

Example 6. *In the above problem, if another block of mass 2 kg is placed over the first block, the normal reaction between the first block and the ground surface in new case is*

(a) 40 N (b) 60 N (c) 20 N (d) 10 N

Sol. (*b*)



$$N - N' = Mg$$
$$\Rightarrow \quad N = (M + m)\,g$$
$$\Rightarrow \quad N = (4 + 2) \times 10 = 60 \text{ N}$$

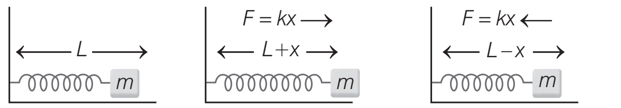
(iii) Spring Force

The resistive force developed in spring, when its length is changed, is called spring force.

$$F \propto -x$$

Spring force, $\quad F = -kx$

where, k = spring constant and x = change in length of the spring.



Spring in natural length (a) | Spring in the condition of elongation (b) | Spring in the condition of compression (c)

(iv) Weight of a Body in a Lift

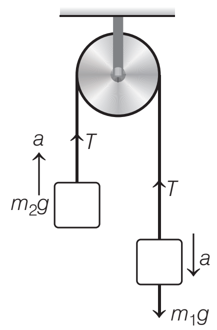
Earth attracts every body towards its centre. The force of attraction exerted by the earth on the body is called *gravity force*. If m be the mass of the body, then the gravity force on it is mg. Normally, the weight of a body is equal to the gravity force, $w = mg$.

- If lift is accelerating upward with acceleration a, then apparent weight of the body is $R = m(g + a)$.
- If lift is accelerating downward at the rate of acceleration a, then apparent weight of the body is $R = m(g - a)$.
- If lift is moving upward or downward with constant velocity, then apparent weight of the body is equal to actual weight, *i.e.* $R = mg$.
- If the lift is falling freely under the effect of gravity ($g = a$), then it is called weightlessness condition ($R = 0$).

Connected Motion

When two objects of masses m_1 and m_2 tied at the ends of an inextensible string which passes over the light and frictionless pulley. Suppose $m_1 > m_2$, the heavier object m_1 moves downward and lighter object m_2 moves upward.

If a be the common acceleration of two objects, since pulley is frictionless and light, then the tension in the string will be same on both sides of pulley.



For mass m_1, equation of motion is, $m_1 g - T = m_1 a$

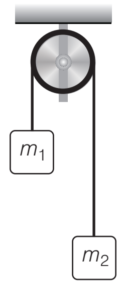
Similarly for the mass m_2, equation of motion is

$$T - m_2 g = m_2 a$$

$$\therefore \quad a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

Hence, tension (T) in the string, $T = \left(\dfrac{2\, m_1 m_2}{m_1 + m_2}\right) g$

Example 7. *Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move ?* ($g = 9.8\ ms^{-2}$) ***[AIEEE 2004]***



(a) $0.2\ ms^{-2}$ *(b)* $9.8\ ms^{-2}$ *(c)* $5\ ms^{-2}$ *(d)* $4.8\ ms^{-2}$

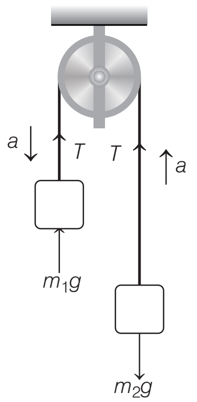
Sol. (*a*) On releasing, the motion of the system will be according to figure,

$$m_1 g - T = m_1 a \quad \ldots\text{(i)}$$

and $$T - m_2 g = m_2 a \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \quad \ldots\text{(iii)}$$

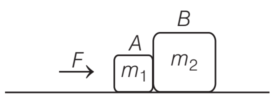


Here, $m_1 = 5$ kg, $m_2 = 4.8$ kg, $g = 9.8\ \text{ms}^{-2}$

$\therefore$ $$a = \left(\frac{5 - 4.8}{5 + 4.8}\right) \times 9.8 = \frac{0.2}{9.8} \times 9.8 = 0.2\ \text{ms}^{-2}$$

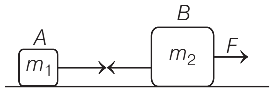
Some Important Cases of Connected Motion

(i) **Motion of blocks in contact**



Acceleration, $$a = \frac{F}{m_1 + m_2}$$

(ii) **Motion of blocks connected by massless string**

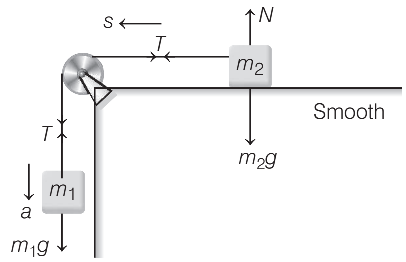


Acceleration, $$a = \frac{F}{m_1 + m_2}$$

and tension, $$T = \frac{m_1 F}{m_1 + m_2}$$

(iii) **Pulley-mass system**

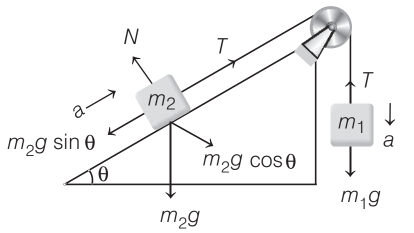
(a) Body accelerated on a horizontal surface by a falling body,



Acceleration, $$a = \left(\frac{m_1}{m_1 + m_2}\right) g$$

Tension, $$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g$$

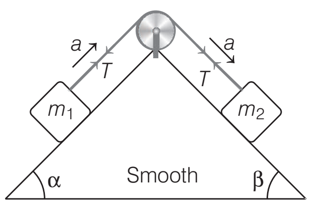
(b) Motion on a smooth inclined plane



$\therefore$ $$a = \left(\frac{m_1 - m_2 \sin\theta}{m_1 + m_2}\right) g$$

and $$T = \frac{m_1 m_2 (1 + \sin\theta) g}{(m_1 + m_2)}$$

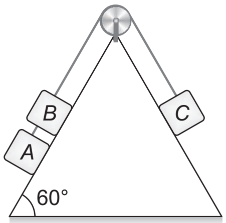
(c) Body accelerated on a wedge due to a another falling body ($m_2 > m_1$)



$$T = \frac{m_1 m_2}{m_1 + m_2} (\sin\alpha + \sin\beta)\, g$$

and $$a = \left(\frac{m_2 \sin\beta - m_1 \sin\alpha}{m_2 + m_1}\right) g$$

Example 8. *As shown in figure A, B and C are 1 kg, 3 kg and 2 kg, respectively. The acceleration of the system is*



(a) 5 ms^{-2} (b) 4.11 ms^{-2} (c) 4 ms^{-2} (d) 5.11 ms^{-2}

Sol. (*b*) Net pulling force

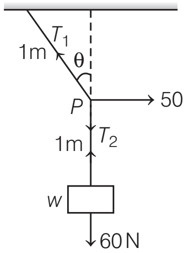
$$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$$

$$= 1\times 10\frac{\sqrt{3}}{2} + 3\times 10\frac{\sqrt{3}}{2} - 2\times 10\times\frac{1}{2} = 24.66\,\text{N}$$

Total mass being pulled $= 1 + 3 + 2 = 6$ kg

$\therefore$ Acceleration of the system, $a = \dfrac{24.66}{6} = 4.11\,\text{ms}^{-2}$

Example 9. *A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction is applied at the mid-point P of the rope as shown. The angle that rope makes with the vertical in equilibrium is*

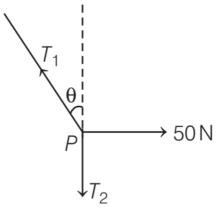


(a) 10° (b) 20° (c) 30° (d) 40°

Sol. (*d*) Making the free body diagram of P and w, we consider the point of equilibrium of the weight w.

$\therefore$ $$T_2 = 6\times 10 = 60\,\text{N}$$

Consider the equilibrium of the point P under the action of three forces, the tensions T_1 and T_2 and the horizontal force 50 N. The horizontal and vertical components of the resultant force must vanish separately.



$$T_1 \cos\theta = T_2 = 60\ \text{N}$$
$$T_1 \sin\theta = 50\ \text{N}$$

which gives that

$$\tan\theta = \frac{5}{6}$$

or $$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 40°$$

Friction

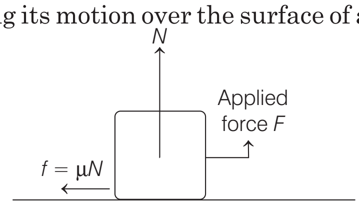
Whenever an object actually slides or rolls over the surface of another body or tends to do, a force opposing the relative motion acts between these two surfaces in contact. It is known as friction or force due to friction. Force of friction acts tangential to the surfaces in contact.

Types of friction
- External friction
  - Static friction
  - Limiting friction
  - Kinetic friction
    - Sliding friction
    - Rolling friction
- Internal friction

(i) **Static friction** is a self-adjusting force and is always equal and opposite to the applied force.

The static friction between two contact surfaces is given by $f_s \le \mu_s N$, where N is the normal force between the contact surfaces and μ_s is the coefficient of static friction, which depends on the nature of the surfaces.

(ii) **Limiting friction** is the limiting (maximum) value of static friction when a body is just on the verge of starting its motion over the surface of another body.



The force of limiting friction f_l between the surfaces of two bodies is directly proportional to the normal reaction at the point of contact. Mathematically,

$$f_l \propto N \text{ or } f_l = \mu_l N \Rightarrow \mu_l = \frac{f_l}{N}$$

where, μ_l is the *coefficient of limiting friction* for the given surfaces in contact.

(iii) **Kinetic friction** is the opposing force that comes into play, when one body actually slides over the surface of another body.

Force of friction f_k is directly proportional to the normal reaction N and the ratio $\dfrac{f_k}{N}$ is called *coefficient of kinetic* μ_k.

Whenever limiting friction is converted into kinetic friction, body starts its motion with an abrupt uncontrolled movement.

- **Sliding friction** is opposing force that comes into play when one body actually slides over the surface of the other body is called *sliding friction*.

  e.g. A book is moving over a horizontal table.

- **Rolling friction** is the opposing force that comes into play, when one body of symmetric shape (wheel or cylinder or disc, etc.) rolls over the surface of another body.

  Force of rolling friction f_r is directly proportional to the normal reaction N and inversely proportional to the radius (r) of wheel. Thus,

$$f_r \propto \frac{N}{r} \text{ or } f_r = \mu_r \frac{N}{r}$$

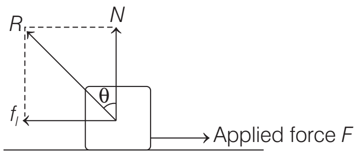
  The constant μ_r is known as the *coefficient of rolling friction*, μ_r has the unit and dimensions of length.

  Magnitude wise $\mu_r << \mu_k$ or μ_l

Angle of Friction

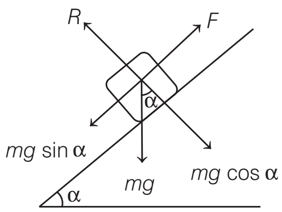
It is defined as the angle θ at which the resultant R of the force of limiting friction f_l and normal reaction N, subtends with the normal reaction.

The tangent of the angle of friction is equal to the coefficient of friction, i.e. $\mu = \tan\theta$.



Angle of Repose (α)

If a body is placed on an inclined plane and it is just on the point of sliding down, then the angle of inclination of the plane with the horizontal is called the *angle of repose* (α).



In limiting condition, $F = mg\sin\alpha$ and $R = mg\cos\alpha$

So, $$\frac{F}{R} = \tan\alpha$$

$\therefore$ $$\frac{F}{R} = \mu_s = \tan\alpha$$

Thus, the coefficient of limiting friction is equal to tangent of the angle of repose.

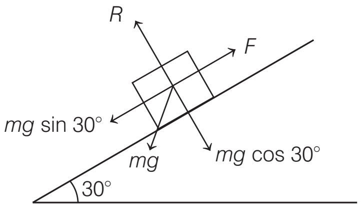
As well as, $\alpha = \theta$

i.e. Angle of repose = Angle of friction

Example 10. *A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is ($g = 10$ m/s^2)* **[AIEEE 2004]**

(a) 2.0 (b) 4.0 (c) 1.6 (d) 2.5

Sol. (a) Let mass of the block be m.



Frictional force in rest position,

$$F = mg \sin 30°$$

(this is static frictional force and may be less than the limiting frictional force)

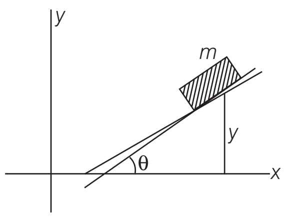
$\therefore$ $$10 = m \times 10 \times \frac{1}{2}$$

or $$m = \frac{2\times 10}{10} = 2 \text{ kg}$$

Example 11. *A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is* **[JEE Main 2014]**

(a) $\frac{1}{6}$ m (b) $\frac{2}{3}$ m (c) $\frac{1}{3}$ m (d) $\frac{1}{2}$ m

Sol. (a) A block of mass m is placed on a surface with a vertical cross-section, then



$$\tan\theta = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan\theta,\ 0.5 = x^2/2$$

$\Rightarrow$ $$x^2 = 1 \Rightarrow x = \pm 1$$

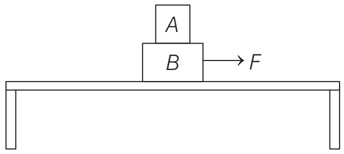
Now, putting the value of x in $y = x^3/6$, we get

When $x = 1$	When $x = -1$
$\therefore\ y = \frac{(1)^3}{6} = \frac{1}{6}$	$y = \frac{(-1)^3}{6} = \frac{-1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is $\frac{1}{6}$ m.

Example 12. *Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2.*

The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is (Take, $g = 10\ m/s^2$) ***[JEE Main 2019]***



(a) 12 N (b) 16 N (c) 8 N (d) 40 N

Sol. (b) Acceleration a of system of blocks A and B is

$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{F - f_1}{m_A + m_B}$$

where, f_1 = friction between B and the surface

$$= \mu(m_A + m_B)g$$

So, $$a = \frac{F - \mu(m_A + m_B)g}{(m_A + m_B)} \quad \ldots\text{(i)}$$

Here, $\mu = 0.2$, $m_A = 1\,\text{kg}$, $m_B = 3\ \text{kg}$, $g = 10\ \text{ms}^{-2}$

Substituting the above values in Eq. (i), we have

$$a = \frac{F - 0.2(1+3)\times 10}{1+3}$$

$$a = \frac{F-8}{4} \quad \ldots\text{(ii)}$$

Due to acceleration of block B, a pseudo force F' acts on A.

This force F' is given by $F' = m_A a$.

where, a is acceleration of A and B caused by net force acting on B.

For A to slide over B; pseudo force on A, *i.e.* F' must be greater than friction between A and B.

$$\Rightarrow \quad m_A a \ge f_2$$

We consider limiting case,

$$m_A a = f_2 \Rightarrow m_A a = \mu(m_A)g$$

$$\Rightarrow \quad a = \mu g = 0.2 \times 10 = 2\ \text{ms}^{-2} \quad \ldots\text{(iii)}$$

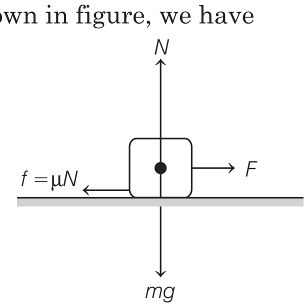
Putting the value of a from Eq. (iii) into Eq. (ii), we get

$$\frac{F-8}{4} = 2$$

$$\therefore \quad F = 16\ \text{N}$$

Some Special Cases of Friction

***Case* I Acceleration of a block on a horizontal surface** As shown in figure, we have



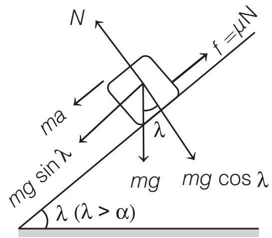
$$N = mg \text{ and } f = \mu N = \mu\, mg$$

$\therefore$ Net acceleration produced,

$$a = \frac{F - f}{m} = \frac{F - \mu\, mg}{m} = \frac{F}{m} - \mu g$$

where, μ = coefficient of (kinetic) friction between the two surfaces in contact.

***Case* II Acceleration of a block sliding down a rough inclined plane** Let there be an inclined plane having angle of inclination λ, which is more than the angle of repose α. Then, as shown in figure



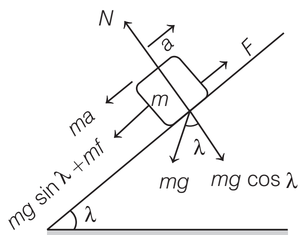
$$N = mg\cos\lambda \quad \text{and} \quad f = \mu N = \mu\, mg\cos\lambda$$

$\therefore$ Net accelerating force down the inclined plane,

$$mg\sin\lambda - f = mg\sin\lambda - \mu\, mg\cos\lambda = ma$$

Acceleration, $a = g(\sin\lambda - \mu\cos\lambda)$

***Case* III Retardation of a block sliding up a rough inclined plane** In this case, various forces have been shown in figure



$$N = mg\cos\lambda$$

Force of friction, $f = \mu N = \mu\, mg\cos\lambda$

and net retarding force $= mg\sin\lambda + f$

$$= mg\sin\lambda + \mu\, mg\cos\lambda$$

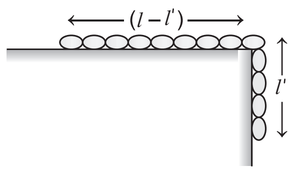
$\therefore$ External force needed (up the inclined plane) to maintain sliding motion $F = mg(\sin\lambda + \mu\cos\lambda)$

In the absence of external force, the motion of given block will be retarded and the value of retardation will be

$$a = g(\sin\lambda + \mu\cos\lambda)$$

***Case* IV Maximum length of hung chain** A uniform chain of length l is placed on the table in such a manner that its l' part is hanging over the edge of table without sliding. Since, the chain has uniform linear density, therefore coefficient of friction,

$$\mu = \frac{m_2}{m_1} = \frac{\text{mass hanging from the table}}{\text{mass lying on the table}}$$



$\therefore$ For this case, we can rewrite the above expression in the following manner,

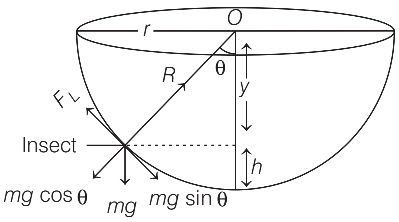
$$\mu = \frac{\text{length hanging from the table}}{\text{length lying on the table}}$$

(as chain has uniform linear density)

$$\therefore \qquad \mu = \frac{l'}{l - l'}$$

By solving, $\quad l' = \dfrac{\mu l}{(\mu + l)}$

***Case* V Motion of an insect in the rough bowl** The insect crawls up the bowl upto a certain height h only till the component of its weight along the bowl is balanced by limiting frictional force.



Let m = mass of the insect,
r = radius of the bowl
and μ = coefficient of friction.
For limiting condition at point A,

$$R = mg\cos\theta \qquad \text{...(i)}$$

and $$F_L = mg\sin\theta \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = \frac{F_L}{R} = \mu$$

$$\therefore \qquad \frac{\sqrt{r^2 - y^2}}{y} = \mu$$

or $$y = \frac{r}{\sqrt{1 + \mu^2}}$$

Example 13. *A man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 ms^{-2}. If the coefficient of static friction between the man's shoes and the belt is 0.2, upto what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man = 65 kg)*

(a) 1.25 ms^{-2} (b) 1.96 ms^{-2} (c) 2.5 ms^{-2} (d) 3.6 ms^{-2}

Sol. (*b*) As the man is standing stationary w.r.t. the horizontal conveyor belt, he is also accelerating at 1 ms^{-2}, the acceleration of the belt. Thus,

Acceleration of the man, $a = 1 ms^{-2}$

Mass of the man, $M = 65$ kg

Therefore, net force on the man, $ma = 65 \times 1 = 65$ N

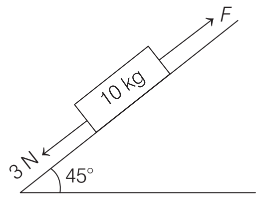
The limiting friction between the shoes of the man and the belt is given by $F = \mu Mg = 0.2 \times 65 \times 9.8$ N

If the man can remain stationary upto an acceleration say a', then

$$Ma' = F$$

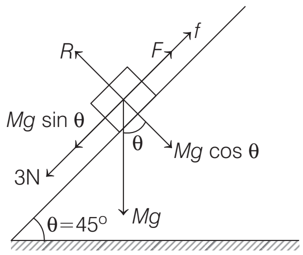
or $$a' = \frac{F}{M} = \frac{0.2 \times 65 \times 9.8}{65} = 1.96 \text{ ms}^{-2}$$

Example 14. *A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force F, such that the block does not move downward) ? (Take, $g = 10 ms^{-2}$)* ***[JEE Main 2019]***



(a) 32 N (b) 25 N (c) 23 N (d) 18 N

Sol. (*a*) Free body diagram, for the given figure is as follows:



For the block to be in equilibrium, i.e. so that it does not move downward, then $\Sigma f_x = 0$

$\therefore \quad 3 + Mg \sin\theta - F - f = 0$ or $3 + Mg \sin\theta = F + f$

As, frictional force, $\quad f = \mu R$

$$\therefore \qquad 3 + Mg \sin\theta = F + \mu R \qquad \text{...(i)}$$

Similarly, $\quad \Sigma f_y = 0$

$$-Mg \cos\theta + R = 0 \text{ or } Mg\cos\theta = R \qquad \text{...(ii)}$$

Substituting the value of R from Eq. (ii) to Eq. (i), we get

$$3 + Mg \sin\theta = F + \mu(Mg \cos\theta) \qquad \text{...(iii)}$$

Here, $\quad M = 10$ kg, $\theta = 45°$, $g = 10$ m/s^2

and $\quad \mu = 0.6$

Substituting these values in Eq. (iii), we get

$3 + (10 \times 10 \sin 45°) - (0.6 \times 10 \times 10 \cos 45°) = F$

$$\Rightarrow \qquad F = 3 + \frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}} = 3 + \frac{40}{\sqrt{2}}$$

$$= 3 + 20\sqrt{2} = 31.8 \text{ N or } F \simeq 32 \text{ N}$$

Practice Exercise

ROUND I Topically Divided Problems

Newton's Laws of Motion

1. A cricket ball of mass 150 g collides straight with a bat with a velocity of 10 ms^{-1}. Batsman hits it straight back with a velocity of 20 ms^{-1}. If ball remains in contact with bat for 0.1s, then average force exerted by the bat on the ball is

(a) 15 N (b) 45 N
(c) 150 N (d) 4.5N

2. A disc of mass 10 g is kept floating horizontally in air by firing bullets, each of mass 5 g, with the same velocity at the same rate of 10 bullets per second. The bullets rebound with the same speed in positive direction. The velocity of each bullet at the time of impact is

(a) 196 cms^{-1} (b) 98 cms^{-1}
(c) 49 cms^{-1} (d) 392 cms^{-1}

3. A satellite in force free space sweeps stationary interplanetary dust at a rate $dM/dt = \alpha v$, where M is the mass, v is the velocity of the satellite and α is a constant. What is the deacceleration of the satellite?

(a) $-2\,\alpha v^2/M$ (b) $-\alpha v^2/M$
(c) $+\,\alpha v^2/M$ (d) $-\,\alpha v^2$

4. The engine of a car produces an acceleration of $6\,ms^{-2}$ in the car. If this car pulls another car of the same mass, then the acceleration would be

(a) 6 ms^{-2} (b) 12 ms^{-2}
(c) 3 ms^{-2} (d) 1.5 ms^{-2}

5. A body of mass 2 kg travels according to law $x(t) = pt + qt^2 + rt^3$, where $p = 3\,ms^{-1}$, $q = 4\,ms^{-2}$ and $r = 5\,ms^{-3}$. The force acting on the body at $t = 2$ s is **[NCERT Exemplar]**

(a) 136 N (b) 134 N
(c) 158 N (d) 68 N

6. In a rocket of mass 1000 kg fuel is consumed at a rate of 40 kg/s. The velocity of the gases ejected from the rocket is 5×10^4 m/s. The thrust on the rocket is

(a) 2×10^3 N (b) 5×10^4 N
(c) 2×10^6 N (d) 2×10^9 N

7. An open carriage in a goods train is moving with a uniform velocity of 10 ms^{-1}. If the rain adds water with zero velocity at the rate of 5 kgs^{-1}, then the additional force applied by the engine to maintain the same velocity of the train is

(a) 0.5 N (b) 2.0 N (c) 50 N (d) 25 N

8. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying a force and the ball goes upto 2 m height further find, the magnitude of the force. (Take, $g = 10\,m/s^2$)

(a) 16 N (b) 20 N
(c) 22 N (d) 44 N

9. A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 m/s to 3.5 m/s. The direction of motion of the body remains unchanged. What is the magnitude and direction of the force?

(a) 0.18 N, along the direction of motion
(b) 0.18 N, opposite to the direction of motion
(c) 0.28 N, along the direction of motion
(d) 0.28 N, opposite to the direction of motion

10. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body

(a) 2 m/s^2 at an angle 37° to force
(b) 2 m/s^2 at an angle 57° to force
(c) 4 m/s^2 at angle 37° to force
(d) 4 m/s^2 at an angle 57° to force

11. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T, so that its momentum changes from p to $3p$. Here, k is a constant.
The value of T is **[JEE Main 2019]**

(a) $\sqrt{\frac{2p}{k}}$ (b) $2\sqrt{\frac{p}{k}}$
(c) $\sqrt{\frac{2k}{p}}$ (d) $2\sqrt{\frac{k}{p}}$

Conservation of Linear Momentum and Impulse

12. If a force of 250 N act on body the momentum acquired is 125 kg-m/s. What is the period for which force acts on the body?

(a) 0.5 s (b) 0.2 s (c) 0.4 s (d) 0.25 s

13. 100 g of an iron ball having velocity 10 ms^{-1} collides with wall at an angle 30° and rebounds with the same angle. If the period of contact between the ball and wall is 0.1s, then the average force experienced by the wall is

(a) 10 N (b) 100 N
(c) 1.0 N (d) 0.1 N

14. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[AIEEE 2006]**

(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

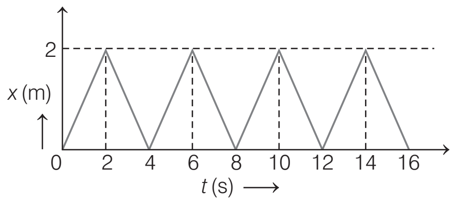
15. Conservation of momentum in a collision between particles can be understood from **[NCERT Exemplar]**

(a) conservation of energy
(b) Newton's first law only
(c) Newton's second law only
(d) Both Newton's second and third laws

16. A bag of sand of mass m is suspended by a rope. A bullet of mass $\frac{m}{20}$ is fired at it with a velocity v and gets embedded into it. The velocity of the bag finally is

(a) $\frac{v}{20}\times 21$ (b) $\frac{20\,v}{21}$
(c) $\frac{v}{20}$ (d) $\frac{v}{21}$

17. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[AIEEE 2010]**



(a) 0.4 N- s (b) 0.8 N-s
(c) 1.6 N-s (d) 0.2 N-s

18. An object at rest in space suddenly explodes into three parts of same mass. The momentum of the two parts are $2p\hat{\mathbf{i}}$ and $p\hat{\mathbf{j}}$. The momentum of the third part

(a) will have a magnitude $p\sqrt{3}$
(b) will have a magnitude $p\sqrt{5}$
(c) will have a magnitude p
(d) will have a magnitude $2p$

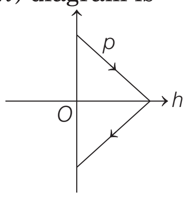
19. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? **[AIEEE 2004]**

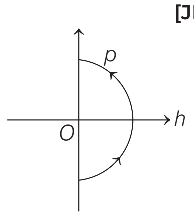
(a) 1 (b) 4
(c) 2 (d) 3

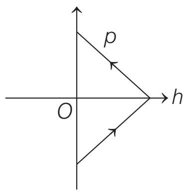
20. A man weighing 60 kg is standing on a trolley weighing 240 kg. The trolley is resting on frictionless horizontal rails. If the man starts walking on the trolley with a velocity of 1 ms^{-1}, then after 4 s, his displacement relative to the ground is

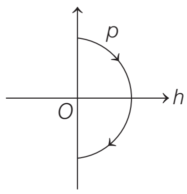
(a) 6 m (b) 4.8 m
(c) 3.2 m (d) 2.4 m

21. A ball is thrown vertically up (taken as + Z-axis) from the ground. The correct momentum-height (p-h) diagram is **[JEE Main 2019]**

(a) 

(b) 

(c) 

(d) 

22. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball? (Mass of the ball is 0.15 kg)

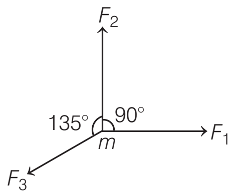
(a) 4 kg-m/s
(b) 6 kg-m/s
(c) 2 kg-m/s
(d) 5 kg-m/s

23. A shell is fired from a cannon with velocity v ms^{-1} at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed in m/s of the piece immediately after the explosion is

(a) $3\,v\cos\theta$ (b) $2\,v\cos\theta$
(c) $\frac{3\,v}{2}\cos\theta$ (d) $\frac{\sqrt{3}\,v\cos\theta}{2}$

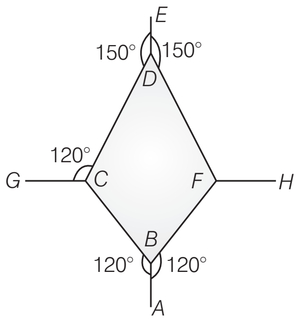
Equilibrium of Forces

24. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in figure, the acceleration produced is



(a) $(\sqrt{2}-1)\,a$ (b) $(\sqrt{2}+1)\,a$
(c) $\sqrt{2}\,a$ (d) a

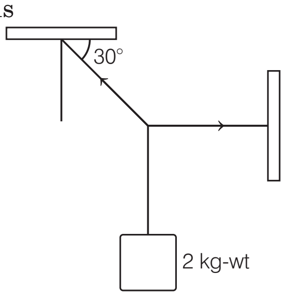
25. The following figure is the part of a horizontally stretched net section AB which is stretched with a force of 10 N. The tension in the section BC and BF are



(a) 10 N, 11 N
(b) 10 N, 6 N
(c) 10 N, 10 N
(d) Cannot calculate due to insufficient data

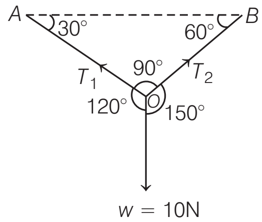
26. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then, F equals (Take, $g = 10\ \text{ms}^{-2}$ and the rope to be massless) **[JEE Main 2020,19]**
(a) 75 N (b) 70 N (c) 100 N (d) 90 N

27. A body of weight 2 kg is suspended as shown in figure. The tension T_1 in the horizontal string (in kg-wt) is



(a) $2\sqrt{3}$ (b) $\sqrt{3}/2$ (c) $\sqrt{3}$ (d) 2

28. A ball of mass 1 kg hangs in equilibrium from two strings OA and OB as shown in figure. What are the tensions in strings OA and OB? (Take, $g = 10\ \text{ms}^{-2}$)



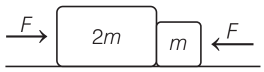
(a) 5 N, zero (b) Zero, N
(c) 5 N, $5\sqrt{3}$ N (d) $5\sqrt{3}$ N, 5 N

29. A piece of wire is bent in the shape of a parabola $y = kx^2$ (Y -axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the X-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the Y-axis is

(a) $\dfrac{a}{gk}$ (b) $\dfrac{a}{2gk}$ (c) $\dfrac{2a}{gk}$ (d) $\dfrac{a}{4gk}$

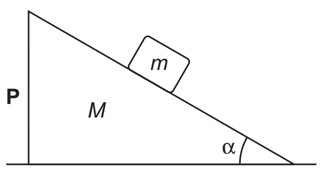
Motion of Connected Bodies

30. Two blocks are in contact on a frictionless table. One has mass m and other $2m$. A force f is applied on $2m$ as shown in figure. Next the same force F is applied from the right on m. In the two cases respectively, the force of contact between the two blocks will be



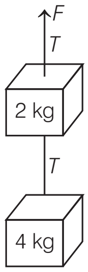
(a) 2 : 1 (b) 1 : 3
(c) 1 : 2 (d) 3 : 1

31. A wooden wedge of mass M and inclination angle α rests on a smooth floor. A block of mass m is kept on wedge. A force **P** is applied on the wedge as shown in figure, such that a block remains stationary with respect to wedge. The magnitude of force **P** is



(a) $(M+m)\,g\tan\alpha$ (b) $g\tan\alpha$
(c) $mg\cos\alpha$ (d) $(M+m)\,g\,\text{cosec}\,\alpha$

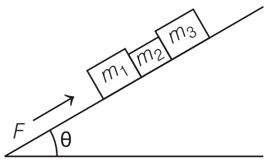
32. Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force applied on the upper string produces an acceleration of 2 m/s^2 in the upward direction in both the blocks. If T and T' be the tensions in the two parts of the string, then (Take, $g = 9.8\ m/s^2$)



(a) $T = 70.8$ N and $T' = 47.2$ N
(b) $T = 58.8$ N and $T' = 47.2$ N
(c) $T = 70.8$ N and $T' = 58.8$ N
(d) $T = 70.8$ N and $T' = 0$

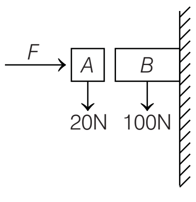
33. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then, the true statement about the scale reading is **[AIEEE 2003]**
(a) both the scales read M kg each
(b) the scale of the lower one reads M kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the readings will be M kg
(d) both the scales read $M/2$ kg

34. Three blocks are placed at rest on a smooth inclined plane with force acting on m_1 parallel to the inclined plane. Find the contact force between m_2 and m_3.



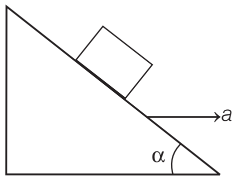
(a) $\frac{(m_1 + m_2 + m_3)F}{m_3}$ (b) $\frac{m_3 F}{m_1 + m_2 + m_3}$
(c) $F - (m_1 + m_2)g$ (d) None of these

35. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is **[JEE Main 2015]**



(a) 100 N (b) 80 N
(c) 120 N (d) 150 N

36. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then, a is equal to **[AIEEE 2005]**



(a) $\frac{g}{\tan\alpha}$ (b) g cosec
(c) g (d) $g\tan\alpha$

37. An elevator and its load have a total mass of 800 kg. The elevator is originally moving downwards at 10 ms^{-1}, it slows down to stop with constant acceleration in a distance of 25 m. Find the tension T in the supporting cable while the elevator is being brought to rest. (Take, g = 10 ms^{-2})
(a) 8000 N (b) 1600 N
(c) 9600 N (d) 6400 N

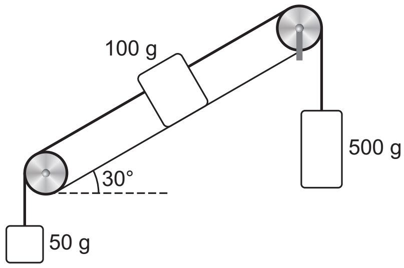
38. Two blocks of masses $m_1 = 4$ kg and $m_2 = 2$ kg are connected to the ends of a string which passes over a massless, frictionless pulley. The total downward thrust on the pulley is nearly
(a) 27 N (b) 54 N
(c) 0.8 N (d) zero

39. A man wants to slide down a rope. The breaking load for the rope $\frac{2}{3}$rd of the weight of the man. With what minimum acceleration should fireman slide down?
(a) $\frac{g}{4}$ (b) $\frac{g}{3}$
(c) $\frac{2g}{3}$ (d) $\frac{g}{6}$

40. A sphere is accelerated upward by a cord whose breaking strength is four times its weight. The maximum acceleration with which the sphere can move up without breaking the cord is
(a) g (b) $3g$
(c) $2g$ (d) $4g$

41. A body of mass 0.05 kg is observed to fall with an acceleration of $9.5\ ms^{-2}$. The opposite force of air on the body is (Take, $g = 9.8\ ms^{-2}$)
(a) 0.015 N
(b) 0.15 N
(c) 0.030 N
(d) zero

42. The acceleration of the 500 g block in figure is

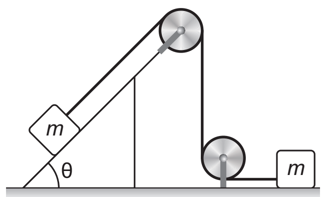


(a) $\frac{6g}{13}$ downwards (b) $\frac{7g}{13}$ downwards
(c) $\frac{8g}{13}$ downwards (d) $\frac{9g}{13}$ upwards

43. A frictionless inclined plane of length l having inclination θ is placed inside a lift which is accelerating downward with an acceleration $a\,(< g)$. If a block is allowed to move down the inclined plane from rest, then the time taken by the block to slide from top of the inclined plane to the bottom of the inclined plane is

(a) $\sqrt{\frac{2l}{g}}$ (b) $\sqrt{\frac{2l}{g-a}}$
(c) $\sqrt{\frac{2l}{g+a}}$ (d) $\sqrt{\frac{2l}{(g-a)\sin\theta}}$

44. For the system shown in figure, the pulleys are light and frictionless. The tension in the string will be



(a) $\frac{2}{3}mg\sin\theta$ (b) $\frac{3}{2}mg\sin\theta$
(c) $\frac{1}{2}mg\sin\theta$ (d) $2\,mg\sin\theta$

45. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 ms^{-2}, the reading of the spring balance will be **[AIEEE 2003]**

(a) 24 N (b) 74 N
(c) 15 N (d) 49 N

46. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is **[AIEEE 2003]**

(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$ (c) P (d) $\frac{PM}{M+m}$

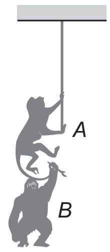
47. Two weights w_1 and w_2 are suspended from the ends of a light string over a smooth fixed pulley. If the pulley is pulled up with acceleration g, the tension in the string will be

(a) $\frac{4\,w_1w_2}{w_1+w_2}$ (b) $\frac{2\,w_1w_2}{w_1+w_2}$
(c) $\frac{w_1-w_2}{w_1+w_2}$ (d) $\frac{w_1w_2}{2\,(w_1+w_2)}$

48. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively **[AIEEE 2002]**

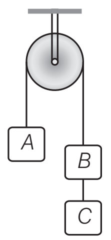
(a) g, g (b) $g-a, g-a$
(c) $g-a, g$ (d) a, g

49. The monkey B shown in figure is holding on to the tail of the monkey A which is climbing up a rope. The masses of the monkeys A and B are 5 kg and 2 kg respectively. If A can tolerate a tension of 30 N in its tail, what force should it apply on the rope in order to carry the monkey B with it? (Take, $g = 10\ \text{ms}^{-2}$)



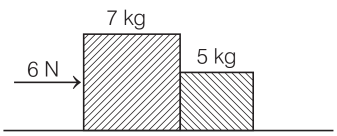
(a) 105 N (b) 108 N
(c) 10.5 N (d) 100 N

50. Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure. The tension in the string connecting weight B and C is



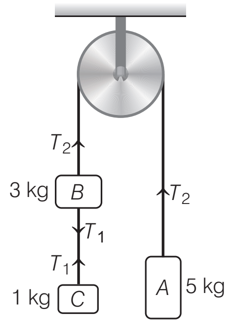
(a) zero (b) 13 N
(c) 3.3 N (d) 19.6 N

51. Two block of masses 7 kg and 5 kg are placed in contact with each other on a smooth surface. If a force of 6 N is applied on a heavier mass the force on the lighter mass is



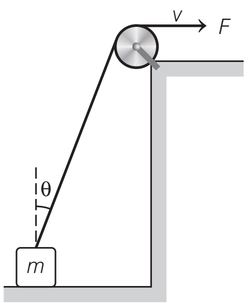
(a) 3.5 N (b) 2.5 N
(c) 7 N (d) 5 N

52. Refer to the system shown in figure. The acceleration of the masses is



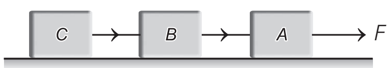
(a) $\frac{g}{3}$ (b) $\frac{g}{6}$ (c) $\frac{g}{9}$ (d) $\frac{g}{12}$

53. A block is dragged on a smooth horizontal plane with the help of a light rope which moves with a velocity v as shown in figure. The horizontal velocity of the block is



(a) v
(b) $v \sin \theta$
(c) $\frac{v}{\sin \theta}$
(d) $\frac{v}{\cos \theta}$

54. Three identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2$ N with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in newton) in the string between the blocks B and C? **[AIEEE 2002]**

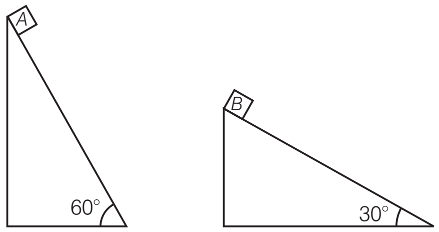


(a) 9.2
(b) 7.8
(c) 4
(d) 9.8

55. A blumb bob is hung from the ceiling of a train compartment. The train moves on an inclined track of inclination 30° with horizontal. Acceleration of train up the plane is $a = 9/2$. The angle which the string supporting the bob makes with normal to the ceiling in equilibrium is

(a) 30°
(b) $\tan^{-1} \frac{2}{\sqrt{3}}$
(c) $\tan^{-1} \frac{\sqrt{3}}{2}$
(d) $\tan^{-1} 2$

56. Two fixed frictionless inclined plane making the angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[AIEEE 2010]**



(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

Friction and Motion on Inclined Surface

57. A chain lies on a rough horizontal table. It starts sliding when one-fourth of its length hangs over the edge of the table. The coefficient of static friction between the chain and the surface of the table is

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{4}$
(d) $\frac{1}{5}$

58. The coefficient of friction between a body and the surface of an inclined plane at 45° is 0.5 if $g = 9.8 \text{ m/s}^2$. The acceleration of the body downwards (in m/s^2) is

(a) $\frac{4.9}{\sqrt{2}}$
(b) $4.9\sqrt{2}$
(c) $19.2\sqrt{2}$
(d) 4.9

59. A fireman of mass 60 kg slides down a pole. He is pressing the pole with a force of 600 N. The coefficient of friction between the hands and the pole is 0.5 with what acceleration with the fireman slide down? (Take, $g = 10 \text{ m/s}^2$)

(a) 1 m/s^2
(b) 2.5 m/s^2
(c) 10 m/s^2
(d) 5 m/s^2

60. A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 10 s. Then, the coefficient of friction is **[AIEEE 2003]**

(a) 0.02
(b) 0.03
(c) 0.06
(d) 0.01

61. A box of mass m kg is placed on the rear side of an open truck accelerating at 4 ms^{-2}. The coefficient of friction between the box and the surface below it is 0.4.

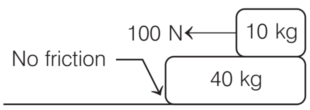
The net acceleration of the box with respect to the truck is zero. The value of m is
(Take, $g = 10\ \text{ms}^{-2}$)

(a) 4 kg (b) 8 kg
(c) 9.78 kg (d) It could be any value

62. A body of mass 40 kg resting on a rough horizontal surface is subjected to a force P which is just enough to start the motion of the body. If $\mu_s = 0.5$, $\mu_k = 0.4$, $g = 10\ \text{ms}^{-2}$ and the force P is continuously applied on the body, then the acceleration of the body is

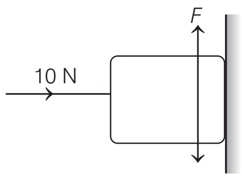
(a) zero (b) $1\ \text{ms}^{-2}$
(c) $2\ \text{ms}^{-2}$ (d) $2.4\ \text{ms}^{-2}$

63. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab. The static coefficient of friction between the block and the slab is 0.60 while the kinetic coefficient of friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8\ \text{ms}^{-2}$, the resulting acceleration of the slab will be



(a) $1.47\ \text{ms}^{-2}$ (b) $1.69\ \text{ms}^{-2}$
(c) $9.8\ \text{ms}^{-2}$ (d) $0.98\ \text{ms}^{-2}$

64. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 20 N (b) 50 N
(c) 100 N (d) 2 N

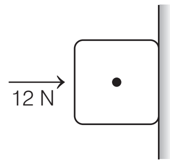
65. Consider a car moving on a straight road with a speed of $100\ \text{ms}^{-1}$. The distance at which car can be stopped, is (Take, $\mu_k = 0.5$)

(a) 800 m (b) 1000 m
(c) 100 m (d) 400 m

66. A block of mass 1 kg is at rest on a horizontal table. The coefficient of static friction between the block and the table is 0.5. If $g = 10\ \text{ms}^{-2}$, then the magnitude of the force acting upwards at an angle of 60° from the horizontal that will just start the block moving is

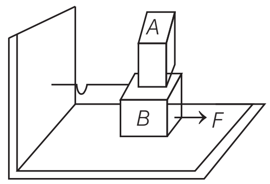
(a) 5 N (b) 5.36 N
(c) 74.6 N (d) 10 N

67. A block of weight 5 N is pushed against a vertical wall by a force 12 N. The coefficient of friction between the wall and block is 0.6. The magnitude of the force exerted by the wall on the block is



(a) 12 N (b) 5 N
(c) 7.2 N (d) 13 N

68. A block A with mass 100 kg is resting on another block B of mass 200 kg. As shown in figure, a horizontal rope tied to a wall holds it. The coefficient of friction between A and B is 0.2 while coefficient of friction between B and the ground is 0.3. The minimum required force F to start moving B will be

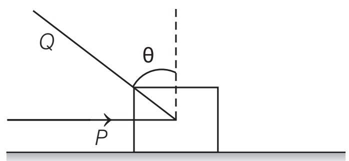


(a) 900 N (b) 100 N
(c) 1100 N (d) 1200 N

69. A wooden box of mass 8 kg slides down an inclined plane of inclination 30° to the horizontal with a constant acceleration of $0.4\ \text{ms}^{-2}$. What is the force of friction between the box and inclined plane? (Take, $g = 10\ \text{ms}^{-2}$)

(a) 36.8 N
(b) 76.8 N
(c) 65.6 N
(d) 97.8 N

70. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined at an angle θ to the vertical. The block will remain in equilibrium, if the coefficient of friction between it and the surface is



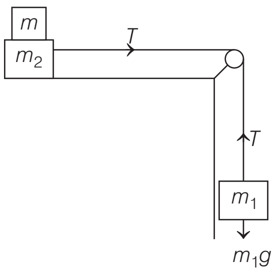
(a) $\dfrac{P + Q\sin\theta}{mg + Q\cos\theta}$ (b) $\dfrac{P\cos\theta + Q}{mg - Q\sin\theta}$

(c) $\dfrac{P + Q\cos\theta}{mg + Q\sin\theta}$ (d) $\dfrac{P\sin\theta - Q}{mg - Q\cos\theta}$

71. A partly hanging uniform chain of length L is resting on a rough horizontal table. l is the maximum possible length that can hang in equilibrium. The coefficient of friction between the chain and table is

(a) $\frac{l}{L-l}$ (b) $\frac{L}{l}$ (c) $\frac{l}{L}$ (d) $\frac{lL}{L+l}$

72. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is **[JEE Main 2018]**

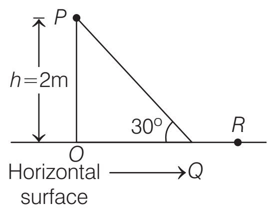


(a) 18.3 kg (b) 27.3 kg (c) 43.3 kg (d) 10.3 kg

73. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$, then the ratio $\frac{F_1}{F_2}$ is

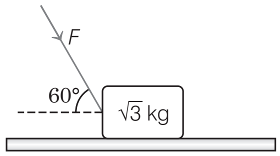
(a) 4 (b) 1 (c) 2 (d) 3

74. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ. The particle is released, from rest , from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(= QR)$, are respectively close to **[JEE Main 2016]**



(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

75. A block of mass $\sqrt{3}$ kg resting on a horizontal surface. A force F is applied on the block as shown in figure. If coefficient of friction between the block be $\frac{1}{2\sqrt{3}}$ what can be the maximum value of force F, so that block does not start moving?
(Take, $g = 10\ \text{ms}^{-2}$)



(a) 20 N (b) 10 N
(c) 12 N (d) 15 N

76. A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is

(a) 20% (b) 25%
(c) 35% (d) 15%

77. A car starts from rest to cover a distance s. The coefficient of friction between the road and the tyres is m. The minimum time in which the car can cover the distance is proportional to

(a) μ (b) $\sqrt{\mu}$
(c) $1/\mu$ (d) $1/\sqrt{\mu}$

78. The coefficient of kinetic friction between a 20 kg box and the floor is 0.40. How much work does a pulling force do on the box in pulling it 8.0 m across the floor at constant speed? The pulling force is directed $37°$ above the horizontal

(a) 343 J (b) 482 J
(c) 14.4 J (d) None of these

79. A block moves down a smooth inclined plane of inclination θ. Its velocity on reaching the bottom is v. If it slides down a rough inclined plane of same inclination, its velocity on reaching the bottom is v/n, where n is a number greater than 1. The coefficient of friction is given by

(a) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)$

(b) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)$

(c) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)^{1/2}$

(d) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)^{1/2}$

ROUND II Mixed Bag

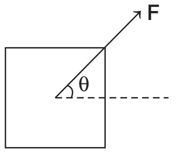
Only One Correct Option

1. 80 railway wagons all of same mass 5×10^3 kg are pulled by an engine with a force of 4×10^5 N. The tension in the coupling between 30th and 31st wagon from the engine is
(a) 25×10^4 N (b) 40×10^4 N
(c) 20×10^4 N (d) 32×10^4 N

2. A gardner waters the plants by a pipe of diameter 1 mm. The water comes out at the rate of 10 $cm^3 s^{-1}$. The reactionary force exerted on the hand of the gardner is
(a) zero
(b) 1.27×10^{-2} N
(c) 1.27×10^{-4} N
(d) 0.127 N

3. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is
(a) F_1/m (b) F_2F_3/mF_1
(c) $(F_2 - F_3)/m$ (d) F_2/m

4. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_k, then the block's acceleration a is given by (g is acceleration due to gravity) **[JEE Main 2021]**



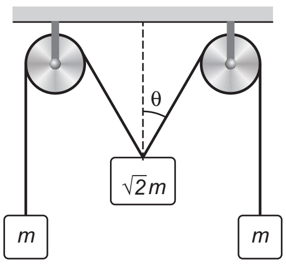
(a) $-\dfrac{F}{m}\cos\theta - \mu_K\left(g - \dfrac{F}{m}\sin\theta\right)$
(b) $\dfrac{F}{m}\cos\theta - \mu_K\left(g - \dfrac{F}{m}\sin\theta\right)$
(c) $\dfrac{F}{m}\cos\theta - \mu_K\left(g + \dfrac{F}{m}\sin\theta\right)$
(d) $\dfrac{F}{m}\cos\theta + \mu_K\left(g - \dfrac{F}{m}\sin\theta\right)$

5. A 5 kg stationary bomb is exploded in three parts having mass 1 : 1 : 3 respectively. Parts having same mass move in perpendicular directions with velocity 39 ms^{-1}, then the velocity of bigger part will be
(a) $10\sqrt{2}$ ms^{-1} (b) $\dfrac{10}{\sqrt{2}}$ ms^{-1}
(c) $13\sqrt{2}$ ms^{-1} (d) $\dfrac{15}{\sqrt{2}}$ ms^{-1}

6. Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to rope at the mid-point which now no more remains horizontal. The minimum tension required to completely straighten the rope is
(a) 15 kg (b) (15/2) kg
(c) 5 kg (d) infinitely large

7. A mass of 6 kg is suspended by a rope of length 2 m from a ceiling. A force of 50 N is applied in the horizontal direction at the mid-point of the rope. The angle made by the rope, with the vertical, in equilibrium position will be (Take, $g = 10\,ms^{-2}$, neglect the mass of the rope)
(a) 90° (b) 60°
(c) 50° (d) 40°

8. The pulley and strings shown in figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

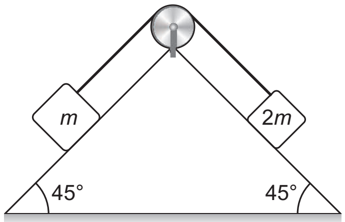


(a) 0° (b) 30°
(c) 45° (d) 60°

9. A 24 kg block resting on a floor has a rope tied to its top. The maximum tension, the rope can withstand without breaking is 310 N. The minimum time in which the block can be lifted a vertical distance of 4.6 m by pulling on the rope is
(a) 1.2 s (b) 1.3 s
(c) 1.7 s (d) 2.3 s

10. A rope of mass 0.1 kg is connected at the same height of two opposite walls. It is allowed to hang under its own weight. At the constant point between the rope and the wall, the rope makes an angle $\theta = 10°$ with respect to horizontal. The tension in the rope at its mid-point between the wall is
(a) 2.78 N (b) 2.56 N
(c) 2.82 N (d) 2.71 N

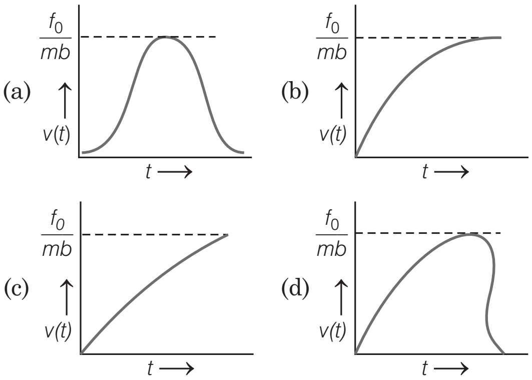
11. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless at 45° to the horizontal on both the side. If the coefficient of friction between the block A and the wedge is 2/3 and that between the block B and the wedge is 1/3 and both blocks A and B are released from rest, the acceleration of A will be **[UP SEE 2008]**



(a) $-1\ m/s^2$ (b) $1.2\ m/s^2$
(c) $0.2\ m/s^2$ (d) $0\ m/s^2$

12. A block of mass 5 kg is moving horizontally at a speed of $1.5\ ms^{-1}$. A vertically upward force 5 N acts on it for 4 s. What will be the distance of the block from the point where the force starts acting?
(a) 2 m (b) 6 m
(c) 8 m (d) 10 m

13. A particle of mass m is at rest at the origin at time $g = 0$. It is subjected to a force $F(t) = f_0 e^{-bt}$ in the x-direction. Its speed $v(t)$ is depicted by which of the following curves?

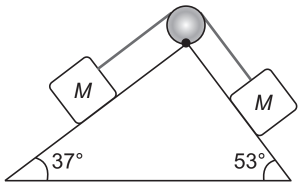


14. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be the perfect uniform circular disc, the acceleration of the mass m. If the string does not slip on the pulley, is
(a) g (b) $\frac{2}{3}g$
(c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

15. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. (Take, $g = 10\ m/s^2$)
(a) 4 N (b) 16 N
(c) 20 N (d) 22 N

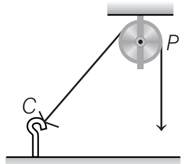
16. A block at rest slides down a smooth inclined plane which makes an angle 60° with the vertical and it reaches the ground in t_1 seconds. Another block is dropped vertically from some point and reaches the ground in t_2 seconds, then the ratio of $t_1 : t_2$ is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) $1 : \sqrt{2}$

17. The acceleration of system of two bodies over the wedge as shown in figure.



(a) $1\ ms^{-2}$ (b) $2\ ms^{-2}$ (c) $0.5\ ms^{-2}$ (d) $10\ ms^{-2}$

18. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope?

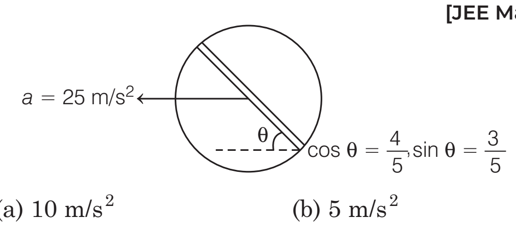


(a) 16 (b) 6 (c) 4 (d) 80

19. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2, where v is its speed. The maximum height attained by the ball is **[JEE Main 2020]**

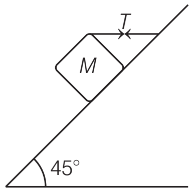
(a) $\frac{1}{k}\tan^{-1}\left(\frac{ku^2}{2g}\right)$ (b) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$
(c) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$ (d) $\frac{1}{2k}\tan^{-1}\left(\frac{ku^2}{g}\right)$

20. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The coefficient of friction between the block and all surfaces of groove in contact is $\mu = \frac{2}{5}$, the disc has an acceleration of $25\ m/s^2$. Find the acceleration of block with respect to disc. **[JEE Main 2021]**



(a) $10\ m/s^2$ (b) $5\ m/s^2$
(c) $20\ m/s^2$ (d) $1\ m/s^2$

21. A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact is (Take, g = 10 ms^{-2})



(a) 1/2 (b) 3/4
(c) 2/3 (d) 1/4

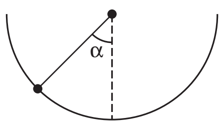
22. A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

(a) $\dfrac{1}{1-n^2}$ (b) $1-\dfrac{1}{n^2}$
(c) $\sqrt{1-\dfrac{1}{n^2}}$ (d) $\sqrt{\dfrac{1}{1-n^2}}$

23. A body of mass M is kept on a rough horizontal surface (friction coefficient μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on the body is F, where
(a) $F = Mg$
(b) $F = \mu MgF$
(c) $Mg \le f \le Mg\sqrt{1+\mu^2}$
(d) $Mg \ge f \ge Mg\sqrt{1+\mu^2}$

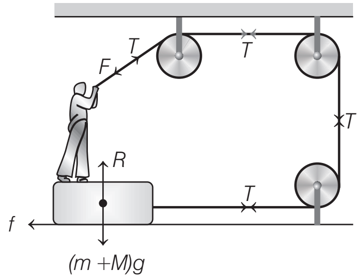
24. The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if coefficient of friction for the lower half is given by
(a) $2\sin\phi$ (b) $2\cos\phi$
(c) $2\tan\phi$ (d) $\tan\phi$

25. An insect crawls up a hemispherical surface very slowly, figure. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(a) $\cot\alpha = 3$ (b) $\sec\alpha = 3$
(c) $\operatorname{cosec}\alpha = 3$ (d) None of these

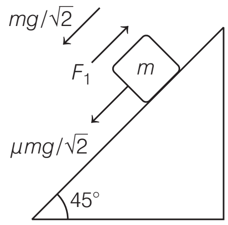
26. A man of mass M is standing on a board of mass m. The friction coefficient between the board and the floor is μ, shown in figure. The maximum force that the man can exert on the rope so that the board does not move is



(a) $\mu\,(m+M)\,g$ (b) $\dfrac{\mu\,(m+M)\,g}{\mu+1}$
(c) $\dfrac{\mu\,(m+M)\,g}{\mu-1}$ (d) None of these

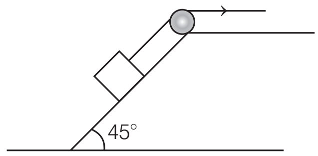
27. If coefficient of friction between an insect and bowl is μ and radius of the bowl is r, the maximum height to which the insect can crawl in the bowl is
(a) $r\left(1-\dfrac{1}{\sqrt{1+\mu^2}}\right)$ (b) $\dfrac{r}{\sqrt{1+\mu^2}}$
(c) $r\sqrt{1+\mu^2}$ (d) $r\,(\sqrt{1+\mu^2}-1)$

28. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is



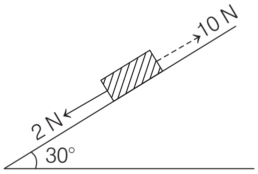
(a) 2 (b) 4 (c) 5 (d) 6

29. A block of mass 200 kg is being pulled up by men on an inclined plane at angle of 45° as shown in figure. The coefficient of static friction is 0.5. Each man can only apply a maximum force of 500 N. Calculate the number of men required for the block to just start moving up the plane.



(a) 10 (b) 15 (c) 5 (d) 3

30. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is (Take, $g = 10 \text{ m/s}^2$) **[JEE Main 2019]**



(a) $\frac{2}{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{1}{2}$

31. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

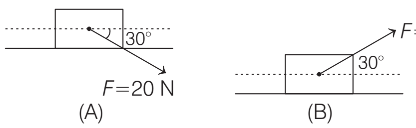
(a) $\mu_k = 1 - \frac{1}{n^2}$

(b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

(c) $\mu_s = 1 - \frac{1}{n^2}$

(d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

32. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20$ N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block, the floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be (Take, $g = 10 \text{ ms}^{-2}$) **[JEE Main 2019]**

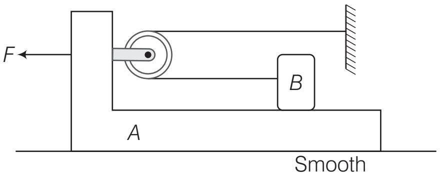


(a) 0.4 ms^{-2} (b) 3.2 ms^{-2}
(c) 0.8 ms^{-2} (d) 0 ms^{-2}

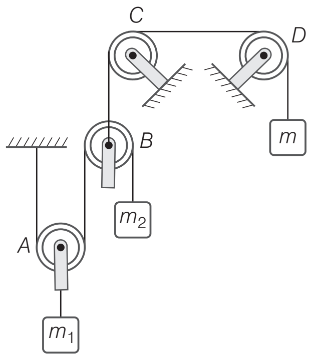
Numerical Value Questions

33. It is found that when 0.5 m was cut-off from the muzzle of a gun firing block of 50 kg, the velocity of the block was changed from 700 m/s to 600 m/s. The force exerted on the block by the powder of gas at the muzzle when expanded in the bore was 6.5×10^n N. The value of n is …… .

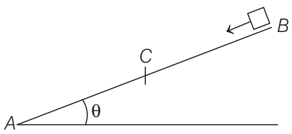
34. A block B of mass 1 kg is placed on a light plank shown in figure. A force F of 10 N is applied on the plank horizontally. The acceleration (in m/s^2) of block B is …… .



35. For the situation shown in the figure, pulleys are light and smooth. Pulleys C and D are fixed, but A and B are movable. The value of m (in kg), if m is in equilibrium is ……… .
(Take, $m_1 = 2$ kg and $m_2 = 1$ kg)

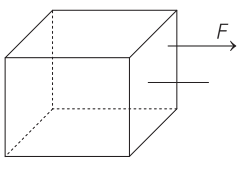


36. A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC which is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan\theta$. Then, the value of k is……… .



[JEE Main 2020]

37. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is ……… .



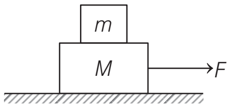
[JEE Main 2020]

38. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, the nearest integer to I is .... **[JEE Main 2020]**

39. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be .... (Round off to the nearest integer)

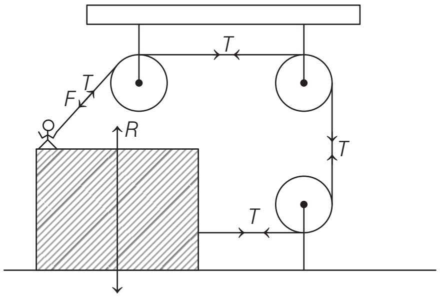
(Take, $g = 10\ \text{ms}^{-2}$) **[JEE Main 2021]**

40. Two blocks ($m = 0.5$ kg and $M = 4.5$ kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$, then the maximum horizontal force that can be applied on the larger block so that the blocks move together is .... N. (Round off to the nearest integer) (Take, $g = 9.8\ \text{ms}^{-2}$)



[JEE Main 2021]

41. A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is .... N. (Round off to the nearest integer). [Take, $g = 10\ \text{ms}^{-2}$] **[JEE Main 2021]**



Answers

Round I

1. (b)	2. (b)	3. (b)	4. (c)	5. (a)	6. (c)	7. (c)	8. (c)	9. (a)	10. (a)
11. (b)	12. (a)	13. (a)	14. (c)	15. (d)	16. (d)	17. (b)	18. (b)	19. (d)	20. (c)
21. (d)	22. (a)	23. (a)	24. (a)	25. (c)	26. (c)	27. (a)	28. (c)	29. (b)	30. (c)
31. (a)	32. (a)	33. (a)	34. (b)	35. (c)	36. (d)	37. (c)	38. (b)	39. (b)	40. (b)
41. (a)	42. (c)	43. (d)	44. (c)	45. (a)	46. (d)	47. (a)	48. (c)	49. (a)	50. (b)
51. (b)	52. (c)	53. (c)	54. (b)	55. (b)	56. (d)	57. (b)	58. (a)	59. (d)	60. (c)
61. (d)	62. (b)	63. (d)	64. (d)	65. (b)	66. (b)	67. (d)	68. (c)	69. (a)	70. (a)
71. (a)	72. (b)	73. (d)	74. (c)	75. (a)	76. (a)	77. (d)	78. (b)	79. (a)	

Round II

1. (a)	2. (d)	3. (a)	4. (b)	5. (c)	6. (d)	7. (d)	8. (c)	9. (c)	10. (c)
11. (d)	12. (d)	13. (a)	14. (b)	15. (d)	16. (b)	17. (a)	18. (c)	19. (b)	20. (a)
21. (a)	22. (b)	23. (c)	24. (c)	25. (a)	26. (b)	27. (a)	28. (c)	29. (c)	30. (b)
31. (a)	32. (c)	33. 6	34. 5	35. 2	36. 3	37. 75	38. 346	39. 5	40. 21
41. 30									

Solutions

Round I

1. $F = \dfrac{m(v-u)}{t} = \dfrac{0.15\,[20-(-10)]}{0.1} = \dfrac{0.15\times 30}{0.1} = 45\text{ N}$

2. $$2mnv = Mg$$

$\therefore$ $$v = \frac{Mg}{2mn}$$

or $$u = \frac{10\times 980}{2\times 5\times 10} = \frac{9800}{100}\text{ cms}^{-1} = 98\text{ cms}^{-1}$$

3. The force acting on the satellite is given by

$$F = \frac{d}{dt}(Mv) = M\frac{dv}{dt} + v\frac{dM}{dt}$$

$$= M\frac{dv}{dt} + v(\alpha v) \qquad \left(\because \frac{dM}{dt} = \alpha v\right)$$

We know that, net force on satellite is zero,

i.e. $M\dfrac{dv}{dt} + v(\alpha v) = 0$

$\Rightarrow$ Acceleration, $a = \dfrac{dv}{dt} = \dfrac{-v^2\alpha}{M}$

4. Force applied by engine $= 6\,m$

When two cars are pulled,

$(m+m)\,a = 6m$

or $2ma = 6m$

or $a = 3\text{ ms}^{-2}$

5. Given, $x(t) = pt + qt^2 + rt^3$ and $p = 3\text{ ms}^{-1}$, $q = 4\text{ ms}^{-2}$, $r = 5\text{ ms}^{-3}$, then $x(t) = 3t + 4t^2 + 5t^3$

$$a = \frac{d^2x(t)}{dt^2} = 8 + 30\,t$$

$\therefore$ $t = 2\text{ s}$

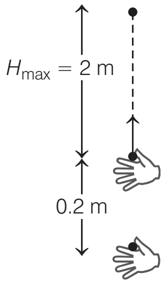
$\therefore$ $a = 8 + 30\times 2 = 68$

Now, $F = m\times a = 2\times 68 = 136\text{ N}$

6. Thrust, $F = u\left(\dfrac{dm}{dt}\right) = 5\times 10^4\times 40 = 2\times 10^6\text{ N}$

7. $F = v\dfrac{dm}{dt} = 10\times 5\text{ N} = 50\text{ N}$

8. Let the ball starts moving with velocity u and it reaches upto maximum height H_{max}, then from



$$H_{\text{max}} = \frac{u^2}{2g}$$

$\Rightarrow$ $$u = \sqrt{2g\,(H_{\text{max}})} = \sqrt{2\times 10\times 2} = 2\sqrt{10}\text{ m/s}$$

This velocity is supplied to the ball by the hand and initially the hand was at rest, so it requires this velocity during travelled distance of 0.2 m,

$\therefore$ $$a = \frac{u^2}{2s} = \frac{40}{2\times 0.2} = 100\text{ m/s}^2$$

So, upward force on the ball,

$$F = m(g+a) = 0.2\,(10+100) = 0.2\times 110 = 22\text{ N}$$

9. Mass of the body, $m = 3.0$ kg

Initial speed, $u = 2.0$ m/s

Final speed, $v = 3.5$ m/s

Time, $t = 25$ s

Force, $F = ?$

Using the first equation of motion, $v = u + at$

$\therefore$ $3.5 = 2.0 + a\times 25$

or $a = \dfrac{3.5 - 2.0}{25}\text{ m/s}^2$

Acceleration, $a = \dfrac{1.5}{25}\text{ m/s}^2$

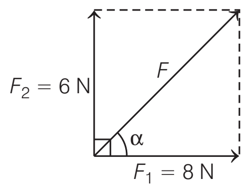
$\therefore$ Force acting on the body,

$$F = ma = 3.0\times\frac{1.5}{25} = \frac{4.5}{25}\text{ N} = 0.18\text{ N}$$

As direction of motion of the body remains unchanged, therefore the direction of force acting on the body is along the direction of motion.

10. Mass of the body, $m = 5$ kg

Force acting on body, $F_1 = 8$ N



Force perpendicular to force F_1 on the body,

$$F_2 = 6\text{ N}$$

Angle between two forces, $\theta = 90°$

Resultant force acting on the body,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(8)^2 + (6)2 + 2\times 8\times 6\times\cos 90°}$$

$$= \sqrt{64 + 36} \qquad (\because \cos 90° = 0)$$

$$= 10\text{ N}$$

If resultant force F makes an angle α with force F_1, then

$$\tan\alpha = \frac{F_2}{F_1} = \frac{6}{8} = 0.75 = \tan 36°\,33'$$

$$\alpha = 36°53'$$

Using relation, $F = ma$

$\Rightarrow$ Acceleration, $a = \dfrac{F}{m} = \dfrac{10}{5} = 2 \text{ m/s}^2$

$\therefore$ An acceleration of 2 m/s^2 is acting on body at an angle of $36°33'$ from the direction of force $F_1 = 8$ N.

11. Here, $F = kt$

When $t = 0$, linear momentum $= p$

When $t = T$, linear momentum $= 3p$

According to Newton's second law of motion,

applied force, $F = \dfrac{dp}{dt}$

or $\quad dp = F \cdot dt$

or $\quad dp = kt \cdot dt$

Now, integrate both side with proper limit,

$$\int_p^{3p} dp = k\int_0^T t\,dt \text{ or } [p]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T$$

or $\quad (3p - p) = \dfrac{1}{2} k\,[T^2 - 0]$

or $\quad T^2 = \dfrac{4p}{k}$

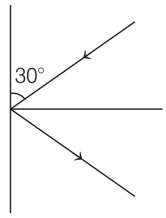
or $\quad T = 2\sqrt{\dfrac{p}{k}}$

12. Change in momentum = Impulse

$\Rightarrow \quad \Delta p = F \times \Delta t$

$\Rightarrow \quad \Delta t = \dfrac{\Delta p}{F} = \dfrac{125}{250} = 0.5$ s

13. Change in momentum,



$$\Delta p = 2\,mu \sin 30°$$
$$= 2 \times 0.1 \times 10 \times \frac{1}{2}$$
$$= 1 \text{ kg-ms}^{-1}$$

$\therefore \quad F_{\text{av}} = \dfrac{\Delta p}{\Delta t} = \dfrac{1}{0.1} = 10$ N

14. This is the question based on impulse-momentum theorem,

$$|F \cdot \Delta t| = |\text{Change in momentum}|$$

$\Rightarrow \quad F \times 0.1 = |p_f - p_i|$

As the ball will stop after catching

$$p_i = mv_i = 0.15 \times 20 = 3, \; p_f = 0$$

$\Rightarrow \quad F \times 0.1 = 3$

$\Rightarrow \quad F = 30$ N

15. Conservation of momentum in a collision between particles can be understood from both, Newton's second and third laws.

16. Applying law of conservation of momentum,

$$\frac{m}{20} v = \left(m + \frac{m}{20}\right) V$$

or $\quad V = \dfrac{v}{20} \times \dfrac{20}{21} = \dfrac{v}{21}$

17. From the graph, it is a straight line, so it is showing uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity, $v_1 = \dfrac{2}{2} = 1 \text{ ms}^{-1}$

Final velocity, $v_2 = -\dfrac{2}{2} = -1 \text{ ms}^{-1}$

$$\mathbf{p}_i = mv_1 = 0.4 \text{ N-s}$$
$$\mathbf{p}_f = mv_2 = -0.4 \text{ N-s}$$
$$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i = -0.4 - 0.4 = -0.8 \text{ N-s}$$

(where, $\mathbf{J}$ = impulse)

$\therefore \quad |\text{J}| = 0.8$ N-s

18. Combined momentum $= 2p\hat{\mathbf{i}} + p\hat{\mathbf{j}}$

Magnitude of combined momentum

$$= \sqrt{(2p)^2 + p^2} = \sqrt{5\,p^2} = \sqrt{5}p$$

This must be equal to the momentum of the third part.

19. Force exerted by machine gun on man's hand in firing a bullet = Change in momentum per second on a bullet or rate of change of momentum

$$= \left(\frac{40}{1000}\right) \times 1200 = 48 \text{ N}$$

Force exerted by man on machine gun

= Force exerted on man by machine gun

= 144 N

Hence, number of bullets fired $= \dfrac{144}{48} = 3.$

20. The trolley shall move backwards to conserve momentum. The backward momentum would be shared by both the trolley and man.

Applying conservation of momentum,

$$60 \times 1 = (240 + 60)\,v$$

or $\quad 60 = 300\,v$

or $\quad v = \dfrac{60}{300}$

$$= \frac{1}{5} \text{ ms}^{-1} = 0.2 \text{ ms}^{-1}$$

Speed of man w.r.t. ground $= (1 - 0.2) \text{ ms}^{-1} = 0.8 \text{ ms}^{-1}$

Displacement of man $= 0.8 \times 4$ m $= 3.2$ m

21. When a ball is thrown vertically upward, then the acceleration of the ball,

a = acceleration due to gravity (g) (acting in the downward direction).

Now, using the equation of motion, $v^2 = u^2 - 2\,gh$

or $\quad h = \dfrac{-v^2 + u^2}{2g} \qquad \ldots$(i)

As we know, momentum, $p = mv$ or $v = p/m$

So, substituting the value of v in Eq. (i), we get

$$h = \frac{u^2 - (p/m)^2}{2g}$$

As we know that, at the maximum height, velocity of the ball thrown would be zero.

So, for the flight when the ball is thrown till it reaches the maximum height (h).

$$v \rightarrow \text{changes from } u \text{ to } 0$$

$$\Rightarrow \qquad p \rightarrow \text{changes from } mu \text{ to } 0$$

Similarly, when it reacher it's initial point, then

$$h \rightarrow \text{changes from } h_{\max} \text{ to } 0$$

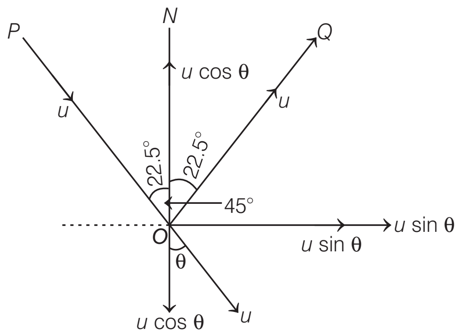
Also, $p \rightarrow$ changes from 0 to some values.

Thus, these conditions are only satisfied in the plot given in option (d).

22. Mass of the ball, $m = 0.15$ kg

Velocity of the ball, $v = 54 \text{ km/h} = 54 \times \frac{5}{18} \text{ m/s} = 15 \text{ m/s}$

Let the ball be incident along path PO. Batsman deflects the ball by an angle of 45° along both OQ.



$$\angle PON = \angle NOQ = \frac{45^\circ}{2} = 22.5^\circ$$

The horizontal component of velocity $u \sin\theta$ remains unchanged while vertical component of velocity is just reversed.

$\therefore$ Impulse imparted to the ball

$$\begin{aligned} &= \text{Change in linear momentum of the ball} \\ &= mu \cos\theta - (-mu\cos\theta) \\ &= 2\,mu\cos\theta = 2 \times 0.15 \times 15 \times \cos 22.5^\circ \\ &= 4.5 \times 0.9239 \text{ kg-m/s} = 4.16 \text{ kg-m/s} \\ &\simeq 4 \text{ kg-m/s} \end{aligned}$$

23. In case of projectile motion at the highest point,

$$(v)_{\text{vertical}} = 0$$

and $\qquad (v)_{\text{horizontal}} = v\cos\theta$

The initial linear momentum of the system will be $mv\cos\theta$. Now, as force of blasting is internal and force of gravity is vertical.

So, linear momentum of the system along horizontal is conserved.

$$p_1 + p_2 = mv\cos\theta$$

$$m_1v_1 + m_2v_2 = mv\cos\theta$$

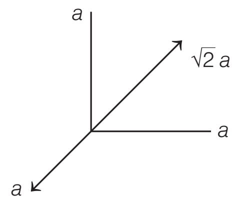
But it is given that $m_1 = m_2 = \frac{m}{2}$ and as one part retraces its path, $v_1 = -v\cos\theta$

$$\therefore \qquad \frac{1}{2}m(-v\cos\theta) + \frac{1}{2}mv_2 = mv\cos\theta$$

or $\qquad v_2 = 3\,v\cos\theta$

24. Since, acceleration acts in the direction of force, hence according to diagram,

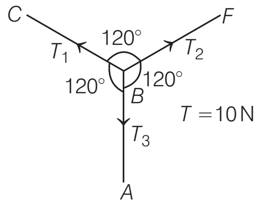
resultant acceleration $(a)_R = \sqrt{2}a - a$



$$(a)_R = (\sqrt{2} - 1)\,a$$

25. By drawing the free body diagram of point B. Let the tension in the section BC and BF be T_1 and T_2, respectively.

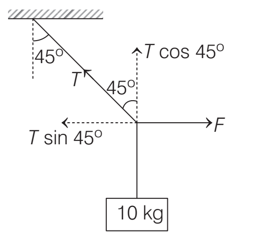
From Lami's theorem,



$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{T}{\sin 120^\circ}$$

$$\Rightarrow \qquad T = T_1 = T_2 = 10 \text{ N}$$

26. Given situation is as shown below.



We resolve tension T in string into vertical and horizontal components.

For equilibrium, $\qquad F = T\sin 45^\circ \qquad$ ...... (i)

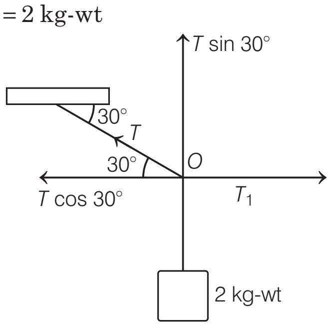
and $\qquad Mg = T\cos 45^\circ \qquad$ .... (ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{F}{Mg} = \tan 45^\circ \text{ or } F = Mg = 10 \times 10 = 100 \text{ N}$$

27. According to equilibrium of forces at point O,

$T \sin 30^\circ = 2$ kg-wt

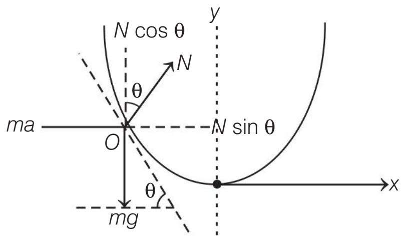


$$\Rightarrow \qquad T = 4 \text{ kg-wt}$$
$$T_1 = T\cos 30^\circ$$
$$= 4\cos 30^\circ = 2\sqrt{3}$$

28. Various forces acting on the ball are as shown in figure. The three concurrent forces are in equilibrium. Using Lami's theorem.

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{10}{\sin 90^\circ}$$
$$\frac{T_1}{\sin 30^\circ} = \frac{T_2}{\sin 60^\circ} = \frac{10}{1}$$
$$\therefore \qquad T_1 = 10\sin 30^\circ = 10 \times 0.5 = 5 \text{ N}$$
$$T_2 = 10\sin 60^\circ = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ N}$$

29. The given situation is shown in the figure



At equilibrium condition at point O.

$$N\sin\theta = ma \qquad \text{...(i)}$$
$$N\cos\theta = mg \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \qquad \tan\theta = \frac{a}{g} \qquad \text{...(iii)}$$

Given, $\qquad y = kx^2$

$$\frac{dy}{dx} = 2kx$$

But $\qquad \dfrac{dy}{dx} = \tan\theta$

Hence, $\tan\theta = 2kx \Rightarrow \dfrac{a}{g} = 2kx \Rightarrow x = \dfrac{a}{2gk}$

30. When force F is applied on $2m$ from left, contact force,

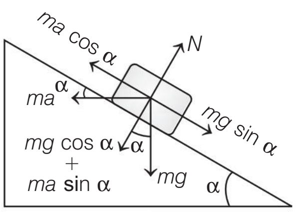
$$F_1 = \frac{m}{m+2m}F = \frac{F}{3}$$

When force F is applied on m from right, contact force,

$$F_2 = \frac{2m}{m+2m}F = \frac{2F}{3}$$
$$\therefore \qquad F_1 : F_2 = 1 : 2$$

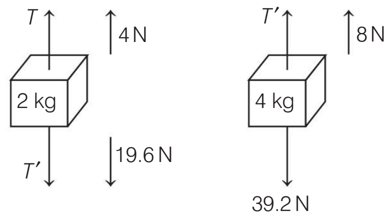
31. Since, $P = (M+m)a$

Now, as in free body diagram of block,



$$ma\cos\alpha = mg\sin\alpha$$
$$\therefore \qquad a = g\frac{\sin\alpha}{\cos\alpha} = g\tan\alpha$$
$$\text{or} \qquad P = (M+m)\,g\tan\alpha$$

32. FBD of mass 2 kg $\qquad$ FBD of mass 4 kg



$$T - T' = 19.6 = 4 \qquad \text{...(i)}$$
$$T' - 39.2 = 8 \qquad \text{...(ii)}$$

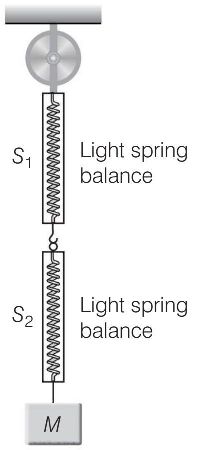
From Eq. (ii), we get

$$T' = 47.2 \text{ N}$$

and substituting T' in Eq. (i), we get

$$T = 4 + 19.6 + 47.2 = 70.8 \text{ N}$$

33. The arrangement is shown in figure.



Now, draw the free body diagram of the spring balances and block.

For equilibrium of block,

$$T_1 = Mg$$

where, $\quad T_1$ = reading of S_2.

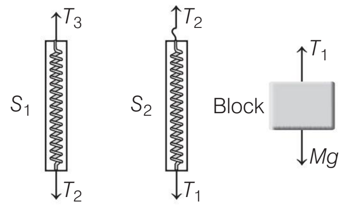
For equilibrium of S_2,

$$T_2 = T_1$$

where, $\; T_2$ = reading of S_1.

For equilibrium of S_1,

$$T_2 = T_3$$



Hence, $\qquad T_1 = T_2 = Mg$

So, both scales read M kg.

34. Acceleration of system, $a = \dfrac{\text{Net pushing force}}{\text{Total mass}}$

$$\text{or} \qquad a = \frac{F - (m_1 + m_2 + m_3)\,g\sin\theta}{(m_1 + m_2 + m_3)}$$

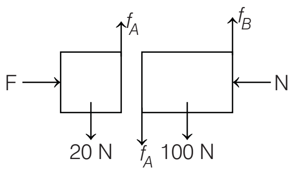
Equation of motion for m_3,

$$N - m_3 g\sin\theta = m_3 a$$

$$\text{or } N = m_3 g\sin\theta + m_3\left[\frac{F - (m_1+m_2+m_3)\,g\sin\theta}{(m_1+m_2+m_3)}\right]$$
$$= \frac{m_3 F}{m_1 + m_2 + m_3}$$

35. In vertical direction, weights are balanced by frictional forces.

Consider FBD of block A and B as shown in figure.



As the blocks are in equilibrium, balance forces are in horizontal and vertical direction.

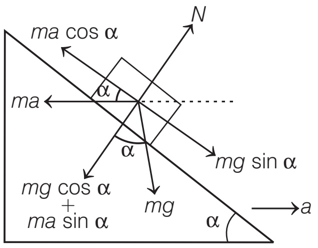
For the system of blocks (A + B),

$$F = N$$

For block A, $f_A = 20$ N and for block B,

$$f_B = f_A + 100 = 120 \text{ N}$$

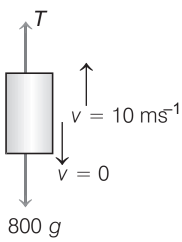
36. In the frame of wedge, the force diagram of block is shown in figure. From free body diagram of wedge,



For block to remain stationary,

$$ma \cos \alpha = mg \sin \alpha \text{ or } a = g \tan \alpha$$

37. As the elevator is going down with decreasing speed, so acceleration is in upward direction. Let it be a,



$$T - 800\,g = 800\,a$$

$$T = 800\,(g + a)$$

From $\quad v^2 = u^2 - 2as$

$\Rightarrow \quad 10^2 = 0 - 2 \times a \times 25$

$\therefore \quad a = 2 \text{ ms}^{-2}$

$\therefore \quad T = 800\,(10 + 2),$

$\therefore \quad T = 9600$ N

38. $T = \dfrac{2\,m_1 m_2}{m_1 + m_2}\,g = \dfrac{2 \times 4 \times 2 \times 10}{4 + 2}$

$$= \frac{160}{6} = 26.6 \approx 27 \text{ N}$$

Total downward thrust on the pulley

$$= 2\,T = 2 \times 27 = 54 \text{ N}$$

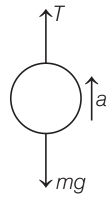
39. Tension in rope, $T <$ Breaking load, $\dfrac{2}{3}\,mg$

$\therefore \quad m\,(g - a) < \dfrac{2}{3}\,mg$

or $\quad a > \dfrac{g}{3}$

40. Here, the tension in the cord is given by

$$T = mg + ma$$



Here, upward acceleration = a

Mass of sphere = M

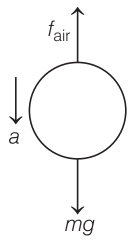
$$T = 4\,mg$$

$\Rightarrow \quad 4\,mg = mg + ma$

$\quad 3\,mg = ma$

$\Rightarrow \quad a = 3\,g$

41. Here, mass of the body,



$$M = 0.05 \text{ kg}$$

Acceleration, $g = 9.8 \text{ ms}^{-2}$, $a = 9.5 \text{ ms}^{-2}$

$\therefore \quad mg - f_{\text{air}} = ma$

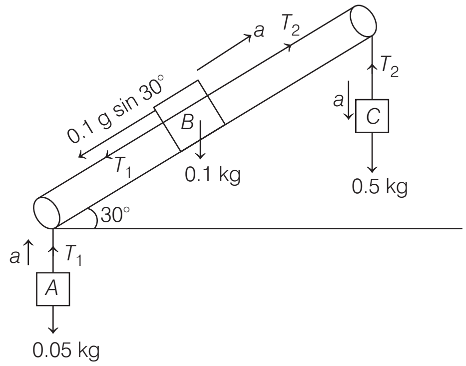
$\Rightarrow \quad f_{\text{air}} = m\,(g - a)$

$\quad = 0.05\,(9.8 - 9.5)$

$\quad = 0.015$ N

42. The given situation is shown below.

Let a be the common acceleration.



$$m_B = 100 \text{ g} = 0.1 \text{ kg}$$

$$m_C = 500 \text{ g} = 0.5 \text{ kg}$$

$$m_A = 50 \text{ g} = 0.05 \text{ kg}$$

Equation of motion for the mass A,

$$T_1 - 0.05\text{ g} = 0.05\,a \quad \text{...(i)}$$

Equation of motion for the mass B,

$$T_2 = 0.1\,g \sin 30° - T_1 = 0.1\,a$$

$$T_2 - T_1 - 0.05\,g = 0.1\,a \quad \text{...(ii)}$$

Equation of motion for the mass C,

$$0.5\,g - T_2 = 0.5\,a \quad \text{...(iii)}$$

Adding Eqs. (i) and (ii), we have

$$T_2 - 0.1\,g = 0.15\,a$$

$\Rightarrow \quad T_2 = 0.1\,g + 0.15\,a \quad$ ...(iv)

Putting the value of T_2 from Eq. (iv) in Eq. (iii), we get

$$0.5\,g - 0.1\,g - 0.15\,a = 0.5\,a$$

$\Rightarrow \quad 0.4\,g = 0.65\,a$

$\Rightarrow \quad a = \dfrac{0.4g}{0.65} = \dfrac{8g}{13}$ (downwards)

43. Effective value of acceleration due to gravity in the lift $= g - a$

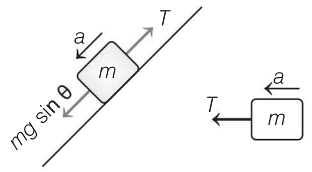
Using $\quad s = ut + \frac{1}{2}at^2$, we get

$$l = \frac{1}{2}(g - a)\sin\theta t^2, \text{ we get}$$

$$t = \sqrt{\frac{2l}{(g-a)\sin\theta}}$$

44. Let force in downward to the incline $mg\sin\theta - T = ma$

$$T = ma$$



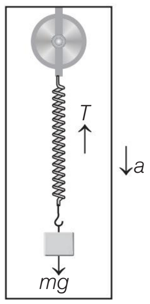
$\therefore \quad mg\sin\theta - T = T$

or $\quad 2T = mg\sin\theta$

or $\quad T = \frac{1}{2}mg\sin\theta$

45. In stationary position, spring balance reading $= mg = 49$

or $\quad m = \frac{49}{9.8} = 5$ kg

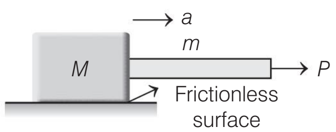


When lift moves downward, $mg - T = ma$

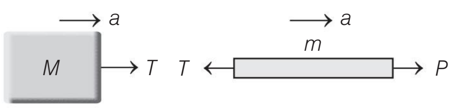
Reading of balance,

$$T = mg - ma = 5\,(9.8 - 5)$$
$$= 5 \times 4.8 = 24.0 \text{ N}$$

46. Let acceleration of system (rope + block) be a along the direction of applied force, then $a = \frac{P}{M+m}$.



Draw the FBD of block and rope as shown in the following figure



where, T is the required parameter.

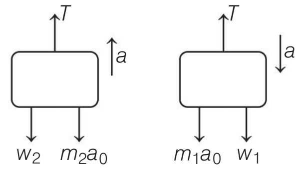
For block, $\quad T = Ma$

$\Rightarrow \quad T = \frac{MP}{M+m}$

47. For solving the problem, we assume that observer is situated in the frame of pulley (non-inertial reference frame),

$$m_1 g = w_1,\ m_2 g = w_2$$

From force diagram,



$$T - m_2a_0 - w_2 = m_2a$$

or $\quad T - m_2g - w_2 = m_2a \quad (\because a_0 = g)$

or $\quad T - 2w_2 = m_2a \quad \ldots\text{(i)}$

From force diagram, $m_1a_0 + w_1 - T = m_1a$

or $\quad m_1a_0 + w_1 - T = m_1a$

or $\quad 2w_1 - T = m_1a \quad \ldots\text{(ii)} \ (\because a_0 = g)$

From Eqs. (i) and (ii), $T = \frac{4w_1w_2}{w_1 + w_2}$

48. Apparent weight of ball,

$w' = w - R \quad$ (where, R = normal reaction)

$R = ma \quad$ (acting upward)

$w' = mg - ma = m(g - a)$

Hence, apparent acceleration in the lift is $g - a$. Now, if the man is standing stationary on the ground, then the apparent acceleration of the falling ball is g.

49. If A is climbing with constant velocity, then

$$T' = 5g + T \text{ and } T = 2g$$
$$T' = 5g + 2g = 7g$$

Suppose A is climbing with acceleration a such that

$$T = 30 \text{ N}$$
$$T - 2a = 2a$$
$$30 - 2 \times 10 = 2a$$

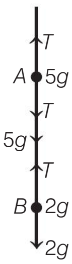
or $\quad a = 5 \text{ ms}^{-2}$

Again, $\quad T' - T - 5g = 5a$

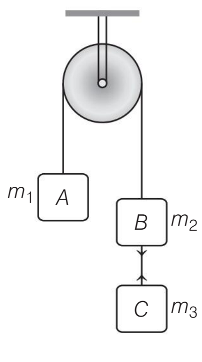
or $\quad T' = T + 5g + 5a$

or $\quad T' = (30 + 50 + 25)$ N

$\quad = 105$ N



50. Tension between m_2 and m_3 is given by



$$T = \frac{2m_1 m_2}{m_1 + m_2 + m_3} \times g$$

$$= \frac{2 \times 2 \times 2}{2+2+2} \times 9.8 = 13 \text{ N}$$

51. From Newton's second law,

$$F = ma$$

$$\Rightarrow \quad 6 = (7+5)\, a$$

$$a = \frac{1}{2} \text{m/s}^2$$

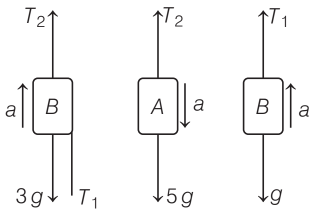
$$f' = 5 \text{ kg}$$

Now $\quad f' = 5 \times \frac{1}{2} = 2.5 \text{ N}$

52. From diagram, $5\,g - T_2 = 5\,a$ ...(i)

$$T_2 - T_1 = 3\,a \quad \text{...(ii)}$$

$$T_1 - g = a \quad \text{...(iii)}$$



Solving Eqs. (i), (ii) and (iii), we get

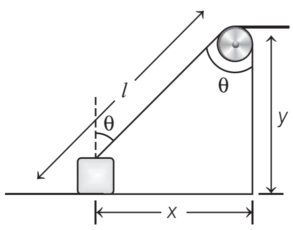
$$g = 9\,a$$

or $\quad a = \frac{g}{9}$

53. From geometry, $l^2 = x^2 + y^2$ but y is constant, hence on differentiating, we have $2\,l\frac{dl}{dt} = 2x\frac{dx}{dt}$

But $\frac{dl}{dt} = v$. Hence, horizontal velocity of block,

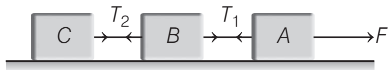
$$v_x = \frac{dx}{dt}$$



$$\Rightarrow \quad lv = x \cdot v_x$$

or $\quad v_x = \frac{l \cdot v}{x} = \frac{v}{\sin\theta}$

54. The system of masses is shown below



From the figure,

$$F - T_1 = ma \quad \text{...(i)}$$

and $\quad T_1 - T_2 = ma \quad$ ...(ii)

Eq. (i) gives,

$$10.2 - T_1 = 2 \times 0.6$$

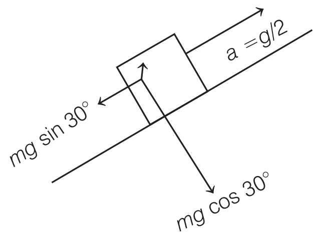
$$\Rightarrow \quad T_1 = 10.2 - 1.2 = 9 \text{N}$$

Again, from Eq. (ii), we get

$$9 - T_2 = 2 \times 0.6$$

$$\Rightarrow \quad T_2 = 9 - 1.2 = 7.8 \text{ N}$$

55.



From diagram, $T\sin\theta - mg\sin\theta = ma$

$$T\sin\theta = mg\sin\theta + \frac{mg}{2} \quad \text{...(i)}$$

$$T\cos\theta = mg\cos\theta \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\tan\theta = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{2}{\sqrt{3}}$$

56. Force applying on the block,

$$F = mg\sin\theta$$

or $\quad mg\sin\theta = ma$

$\therefore \quad a = g\sin\theta$

where, a is acceleration along the inclined plane.

$\therefore$ Vertical component of acceleration

$$= a\sin\theta = g\sin\theta \cdot \sin\theta = g\sin^2\theta$$

$\therefore$ Relative vertical acceleration of A with respect to B is

$$g\,(\sin^2 60° - \sin^2 30°\,] = \frac{g}{2} = 4.9 \text{ms}^{-2}$$

(in vertical direction)

57. Weight of chain on table $= mg - \frac{1}{4}mg = \frac{3}{4}mg$

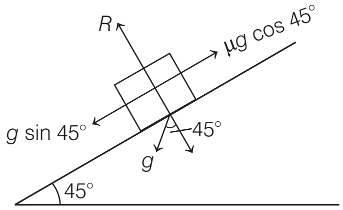
For maximum possible friction, $\frac{3}{4}\mu mg = \frac{1}{4}mg$

$\therefore \quad \mu = \frac{1}{3}$

58. Net acceleration,

$$a = g\,(\sin\theta - \mu\cos\theta) = 9.8\,(\sin 45° - 0.5\cos 45°)$$

$$= \frac{4.9}{\sqrt{2}} \text{ m/s}^2$$



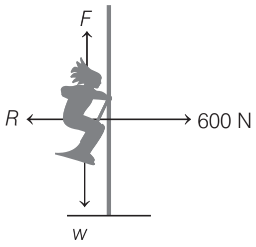
59. Net downward acceleration

$$= \frac{\text{Weight} - \text{Frictional force}}{\text{Mass}}$$

$$= \frac{mg - \mu R}{m}$$

$$= \frac{60 \times 10 - 0.5 \times 600}{60}$$

$$= \frac{300}{60} = 5 \text{ m/s}^2$$



60. Let coefficient of friction be μ, then retardation will be $\mu\, g$. From equation of motion, $v = u + at$

$\Rightarrow \quad 0 = 6 - \mu\, g \times 10$

or $\quad \mu = \frac{6}{100} = 0.06$

61. Pseudo force on the block $= m \times 4$ N (backward)

Force of friction $= 0.4 \times m \times 10$ N (forward)

Equating, $m \times 4 = 0.4 \times m \times 10 = 4\,m$

Clearly, the equation holds good for all values of m.

62. Force, $P = f_{ms} = \mu_s mg$ (when body is at rest)

When the body starts moving with acceleration a, then

$$P - f_k = ma$$

$$\mu_s mg - \mu_k mg = ma$$

or $\quad a = (\mu_0 - \mu_k)\, g$

or $\quad a = (0.5 - 0.4)\ 10$

$\quad = 0.1 \times 10 \text{ ms}^{-2} = 1 \text{ ms}^{-2}$

63. $f_{ms} = 0.6 \times 10 \times 9.8 \text{ N} = 58.8 \text{ N}$

Since, the applied force is greater than f_{ms}, therefore the block will be in motion. So, we should consider f_k.

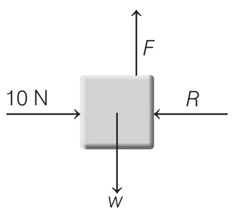
$$f_k = 0.4 \times 10 \times 9.8 \text{ N}$$

or $\quad f_k = 4 \times 9.8$ N

This would cause acceleration of 40 kg block.

Acceleration $= \frac{4 \times 9.8 \text{ N}}{40 \text{ kg}} = 0.98 \text{ ms}^{-2}$

64. Let R be the normal contact force by wall on the block.



$$R = 10 \text{ N}$$

$$F = w \quad \text{and} \quad F = \mu R$$

$\therefore \quad \mu R = w$

or $\quad w = 0.2 \times 10 = 2$ N

65. From third equation of motion,

$$v^2 = u^2 - 2as$$

Given, $\quad v = 0 \quad$ (car is stopped)

As friction provide the retardation,

$$a = \mu g,\ v = 100 \text{ ms}^{-1}$$

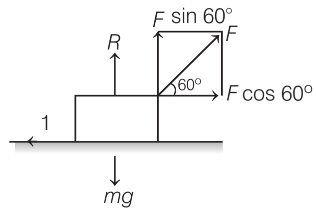
$\therefore \quad (100)^2 = 2\,\mu\, gs$

$\Rightarrow \quad s = \frac{100 \times 100}{2 \times 0.5 \times 10}$

$$= \frac{100 \times 100}{5 \times 2} = 1000 \text{ m}$$

66. $R + F \sin 60^\circ = mg$

or $\quad R = mg - \frac{\sqrt{3}\, F}{2}$



$F \cos 60^\circ = f = \mu R$

or $\quad \frac{F}{2} = 0.5\left(1 \times 10 - \frac{\sqrt{3}\, F}{2}\right)$

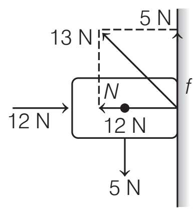
or $\quad F + \frac{\sqrt{3}\, F}{2} = 10$

or $\quad F = \frac{20}{2 + \sqrt{3}}$

or $\quad F = \frac{20}{3.732} = 5.36$ N

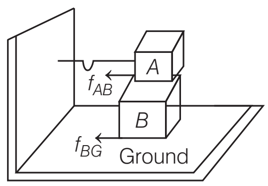
67. Wall applies 2 forces of the block (i) normal reaction, $R = 12$ N, and (ii) frictional force, $f_2 = mg = 5$ N tangentially upward.

$\therefore$ Total force exerted by wall on block



$$F = \sqrt{N^2 + f_s^2} = \sqrt{(12)^2 + (5)^2} = 13 \text{ N}$$

68. As, $F = F_{AB} + F_{BG}$

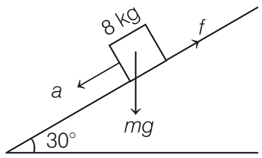


$$= \mu_{AB}\, m_a g + \mu_{BG}\, (m_A + m_B)\, g$$

$$= 0.2 \times 100 \times 10 + 0.3 \times (300) \times 10$$

$$= 200 + 900 = 1100 \text{ N}$$

69. $ma = mg\sin\theta - f$



or $\quad f = mg\sin\theta - ma = m\,(g\sin\theta - a)$

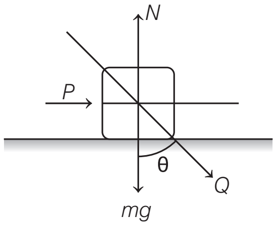
$$= 8\left(10\times\frac{1}{2} - 0.4\right)\text{ N} = 8\times 4.6\text{ N} = 36.8\text{ N}$$

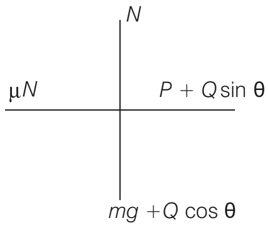
70. For equilibrium of the block,

$$N = mg + Q\cos\theta$$

$$Q\sin\theta + P = \mu N$$

$$Q\sin\theta + P = \mu\,(mg + Q\cos\theta)$$





$\therefore \quad \mu = \left(\dfrac{Q\sin\theta + P}{mg + Q\cos\theta}\right)$

71. If μ is the mass/length, then

weight of hanging length $= \mu\, lg$

Weight of chain on table $= \mu\,(L - l)\,g$

$\because \quad R = \mu\,(L - l)\,g$

$\therefore \quad f = \mu_s R = \mu_s \mu\,(L - l)\,g$

Equating $\quad \mu_s\mu\,(L - l)\,g = \mu lg$

or $\quad \mu_s = \dfrac{l}{L - l}$

72. Motion stops when pull due to $m_1 \le$ force of friction between m and m_2 and surface.

$\Rightarrow \quad m_1 g \le \mu(m_2 + m)g$

$\Rightarrow \quad 5\times 10 \le 0.15(10 + m)\times 10$

$\Rightarrow \quad m \ge 23.33$ kg

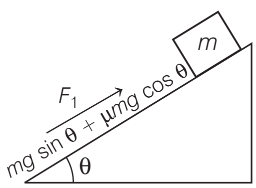
Here, nearest value is 27.3 kg.

So, $\quad m_{\text{min}} = 27.3$ kg

73. $F_1 = mg\,(\sin\theta + \mu\cos\theta)$ (as body just in position to move up, friction force downward)

$F_2 = mg(\sin\theta - \mu\cos\theta)$

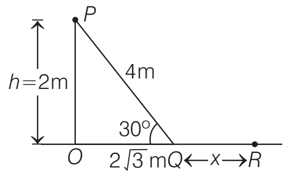
(as body just in position to slide down, friction upward)



$$\frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$= \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = 3$$

74. Energy lost over path $PQ = \mu\, mg\cos\theta \times 4$



Energy lost over path $QR = \mu mgx$

i.e. $\mu\, mg\cos 30° \times 4 = \mu mgx \qquad (\because \theta = 30°)$

$$x = 2\sqrt{3} = 3.45\text{ m}$$

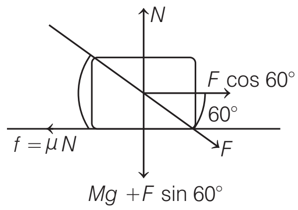
From Q to R energy loss is half of the total energy loss.

i.e. $\quad \mu mgx = \dfrac{1}{2}\times mgh$

$\Rightarrow \quad \mu = 0.29$

The values of the coefficient of friction μ and the distance $x(= QR)$ are 0.29 and 3.5.

75. From acting on block are shown in adjoining figure



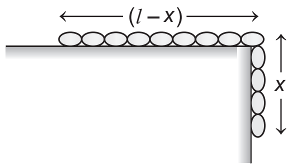
As the block does not move, hence

$F\cos 60° = f = \mu N = \mu\,(Mg + F\sin 60°)$

$\therefore \quad F\dfrac{1}{2} = \dfrac{1}{2\sqrt{3}}\left(\sqrt{3}0\times 10 + F\dfrac{\sqrt{3}}{2}\right)$

On simplification, we get $F = 20$ N

76. Let the length of chain be l and mass m. Let a part x of chain can hang over one edge of table having coefficient of friction.



$\therefore$ Pulling force, $F = \dfrac{mx}{l}g$

and friction force, $f = \mu N = \mu\dfrac{m}{l}(l - x)\,g$

For equilibrium, $\quad F = f$, hence

$$\frac{mx}{l}\cdot g = \mu\frac{m}{l}(l - x)\,g = 0.25\frac{m}{l}(l - x)\,\text{g}$$

$\Rightarrow \quad x = \dfrac{l}{5}$

$\therefore$ % change in $\dfrac{x}{l} = \dfrac{1}{5}\times 100 = 20\%$

77. Force on the car,

$$F = \mu R$$

or $\quad ma = \mu\, mg \qquad (\because R = mg)$

or $\quad a = \mu g$

Now, from second equation of motion,

$$s = ut + \frac{1}{2}at^2$$

or $\quad s = 0 + \frac{1}{2}at^2 \qquad (\because u = 0)$

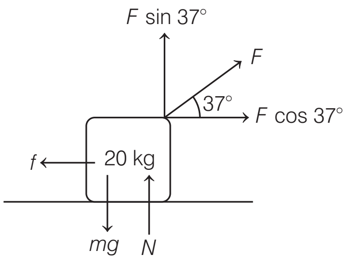
or $\quad t = \sqrt{\frac{2s}{\mu g}}$

$\therefore \quad t = \sqrt{\frac{2s}{\mu g}}$

or $\quad t \propto \frac{1}{\sqrt{\mu}}$

78. The work done by the force is $F\cos 37°$,

where $\quad F\cos 37° = f = \mu N$



In this case, $\quad N = mg - F\sin 37°$,

So that, $\quad F = \frac{\mu\, mg}{(\cos 37° + \mu \sin 37°)}$

Here, $\mu = 0.40$ and $m = 20$ kg

$\therefore \quad F = 75.4\text{ N}$

Hence, $\quad W = (75.4\cos 37°)\,(8.0) = 482\text{ J}$

79. For a smooth plane, $v = \sqrt{2g\sin\theta \cdot s}$ and for a rough plane,

$$\frac{v}{n} = \sqrt{2g\,(\sin\theta - \mu\cos\theta)\,s}$$

$\therefore \quad n = \sqrt{\frac{\sin\theta}{\sin\theta - \mu\cos\theta}}$

or $\quad n^2 = \frac{\sin\theta}{\sin\theta - \mu\cos\theta}$

$\Rightarrow \quad (n^2 - 1)\sin\theta = n^2\mu\cos\theta$

or $\quad \mu = \left(\frac{n^2-1}{n^2}\right)\tan\theta = \tan\theta\left(1 - \frac{1}{n^2}\right)$

Round II

1. Total mass of 80 wagons $= 80 \times 5 \times 10^3 = 4 \times 10^5$ kg

Acceleration, $\quad a = \frac{F}{M} = \frac{4\times 10^5}{4\times 10^5} = 1\text{ ms}^{-2}$

Tension in the coupling between 30th and 31st wagon will be due to mass of remaining 50 wagons. Now, mass of remaining 50 wagons.

$$m = 50 \times 5 \times 10^3\text{ kg} = 25 \times 10^4\text{ kg}$$

$\therefore$ Required tension, $T = mg = 25 \times 10^4 \times 1$

$$= 25 \times 10^4\text{ N}$$

2. Rate of flow water, $\frac{v}{t} = 10\text{ cm}^3\text{s}^{-1}$

Density of water, $\rho = 10^3\text{ kg/m}^3$

Cross-sectional area of pipe $OA = \pi\,(0.5 \times 10^{-3})^2$

$$\text{Force} = m\frac{dv}{dt} = \frac{mv}{t} = \frac{V\rho v}{t} = \frac{\rho v}{t} \times \frac{v}{At}$$

$$= \left(\frac{v}{t}\right)^2 \frac{l}{A} \qquad \left(\because V = \frac{v}{At}\right)$$

$$F = \frac{(10\times 10^{-6})^2 \times 10^3}{\pi \times (0.5\times 10^{-3})^2} = 0.127\text{ N}$$

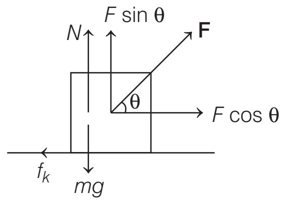
3. As F_2 and F_3 are mutually perpendicular, their

$$\text{resultant} = \sqrt{F_2^2 + F_3^2}$$

As particle is stationary under F_1, F_2, F_3, therefore $\sqrt{F_2^2 + F_3^2}$ must be equal and opposite to F_1.

The force F_1 is now removed. So, the resultant of F_2 and F_3 will now make the particle move with force equal to F_1. Hence, the acceleration is F_1/m.

4.



$$N = mg - F\sin\theta$$

$$F\cos\theta - \mu_K N = ma$$

$$F\cos\theta - \mu_K\,(mg - F\sin\theta) = ma$$

$$a = \frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$$

5. As $m_1 : m_2 : m_3 = 1 : 1 : 3$

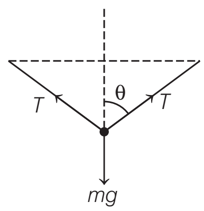
and momentum is conserved, $\sqrt{p_1^2 + p_2^2 + p_3^2} = 3v_3$

$$\sqrt{1\times 39^2 + 1 \times 39^2} = 3v_3$$

$\Rightarrow \quad 39\sqrt{2} = 3v_3$

$\Rightarrow \quad v_3 = \frac{39\sqrt{2}}{3} = 13\sqrt{2}\text{ ms}^{-1}$

6. Let T be the tension in the string. Since, the system is in equilibrium, therefore from figure,



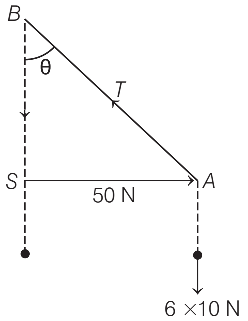
$$2T\cos\theta = mg$$

or $\quad T = mg/2\cos\theta$

The string will be straight, if $\theta = 90°$

$\therefore \quad T = mg/2\cos 90° = mg/2(0) = \infty$

7. The three forces acting on the mass at location A have been shown in figure. Since, the mass is in equilibrium, therefore the three forces acting on the mass must be represented by the three sides of a triangle in one order. Hence,



In ΔSBA, $\quad \dfrac{50}{SA} = \dfrac{6\times 10}{SB}$

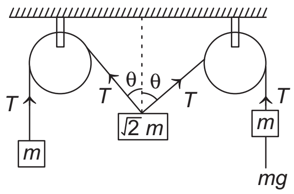
or $\quad \dfrac{SA}{SB} = \dfrac{50}{60} = \dfrac{5}{6}$

or $\quad \tan\theta = \dfrac{SA}{SB} = \dfrac{5}{6} = 0.8333 = \tan 40°$

$\therefore \quad \theta = 40°$

8. If T is tension in each part of the string holding mass $\sqrt{2}\,m$, then in equilibrium,

$$T\cos\theta + T\cos\theta = \sqrt{2}\,mg$$



$$2T\cos\theta = \sqrt{2}\,mg$$

But $\quad T = mg$

$\therefore \quad 2mg\cos\theta = \sqrt{2}\,mg$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

9. Effective upward force $= 310 - mg$

$$= 310 - 24\times 9.8 = 74.8\ \text{N}$$

Upward acceleration,

$$a = 74.8\ /\ 24 = 3.12\,\text{ms}^{-2}$$

As, $\quad s = ut + \dfrac{1}{2}at^2$

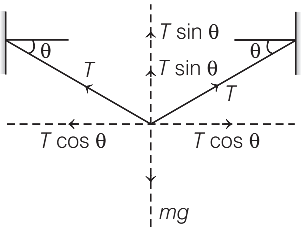
$\Rightarrow \quad 4.6 = 0 + \dfrac{1}{2}\times 3.12\times t^2$

$\Rightarrow \quad t^2 = \dfrac{4.6}{1.56} = 2.95$

$\Rightarrow \quad t = \sqrt{2.95} \approx 1.7\text{s}$

10. Mass of rope, $M = 0.1$ kg, $\theta = 10°$

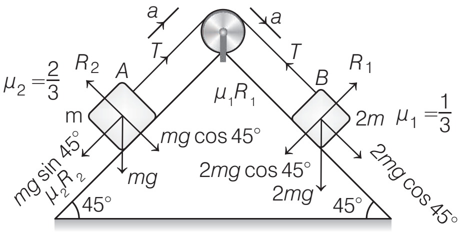
From figure,



$$2T\sin\theta = mg$$

$$T = \frac{mg}{2\sin\theta} = \frac{0.1\times 9.8}{2\sin 10°} = 2.82\ \text{N}$$

11. The situation is as shown in the figure



The equation of motion for body B,

$$2mg\sin 45° - \mu_1 R_1 - T_2 = 2ma$$

$$2mg\sin 45° - \frac{1}{3}2mg\cos 45° - T = 2ma$$

$$\Rightarrow\ 2mg\times\frac{1}{\sqrt{2}} - \frac{1}{3}2mg\times\frac{1}{\sqrt{2}} - T = 2ma$$

In this problem as $(m_B = m_A)g\sin\theta = (mg/\sqrt{2})$ is lesser than $(\mu_B m_B + \mu_A m_A)g\cos\theta = (4mg/3\sqrt{2})$, the masses will not move and hence,

acceleration of B = acceleration of $A = 0$

12. $\quad a = \dfrac{5}{5} = 1\ \text{ms}^{-2}$

Upward distance covered in 4 s,

$$y = \frac{1}{2}at^2 = \frac{1}{2}\times 1\times(4)^2 = 8\ \text{m}$$

Horizontal distance covered in 4s,

$$x = vt = 1.5\times 4 = 6\ \text{m}$$

$$s = \sqrt{x^2 + y^2} = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64} = 10\ \text{m}$$

13. $m\dfrac{dv}{dt} = f_0 e^{-bt}$

$\Rightarrow \quad \dfrac{dv}{dt} = \dfrac{f_0}{m}e^{-bt}$

$\Rightarrow \quad \displaystyle\int_0^v dv = \frac{f_0}{m}\int_0^t e^{-bt}\,dt$

$\Rightarrow \quad v = \dfrac{f_0}{m}\left(\dfrac{e^{-bt}}{-b}\right)_0^t$

$\Rightarrow \quad v = \dfrac{f_0}{mb}(1 - e^{bt})$

Hence, option (a) is correct.

14. For the motion of the block,

$$mg - T = ma$$

For the rotation of the pulley,

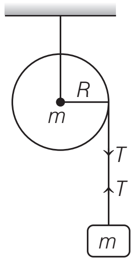
$$\tau = TR = I\alpha$$

$$T = \frac{1}{2} mR\alpha$$

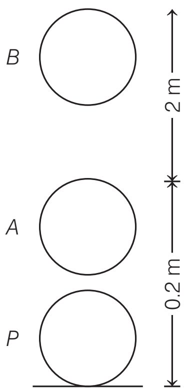
As string does not slip on the pulley,

$$a = R\alpha$$

On solving Eqs. (i),(ii) and (iii), $a = \frac{2g}{3}$



15. The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B. Let acceleration of ball during PA be a ms^{-2} (assumed to be constant) in upward direction and velocity of ball be v m/s.



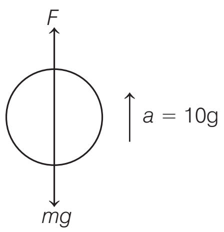
Then, for PA, $v^2 = 0^2 + 2\,a \times 0.2$

For AB, $0 = v^2 - 2 \times g \times 2$

$\Rightarrow$ $v^2 = 2\,g \times 2$

From above equations,

$$a = 10g = 100 \text{ ms}^{-2}$$

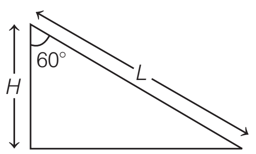


Then, for PA, FBD of ball is

$F - mg = ma$ (F is the force exerted by hand on ball)

$\Rightarrow$ $F = m(g + a) = 0.2\,(10 + 100) = 22$ N

16. Let L be the length and H be the height of the inclined plane, respectively. Acceleration of the block slide down the smooth inclined plane is



$$a = g\cos 60°$$

$\therefore$ $L = \frac{1}{2} g\cos 60°\, t_1^2$ ...(i) ($\because u = 0$)

Acceleration to another block dropped vertically down from the same inclined plane is

$$a = g$$

$\therefore$ $H = \frac{1}{2} at_2^2 = \frac{1}{2} gt_2^2$ ($\because u = 0$)

From figure, $\cos 60° = \frac{H}{L}$

$\Rightarrow$ $H = L\cos 60°$

$\therefore$ $L\cos 60° = \frac{1}{2} gt_2^2$ ...(ii)

Divide Eq. (i) by Eq. (ii), we get

$$\frac{t_1^2 \cos 60°}{t_2^2} = \frac{1}{\cos 60°}$$

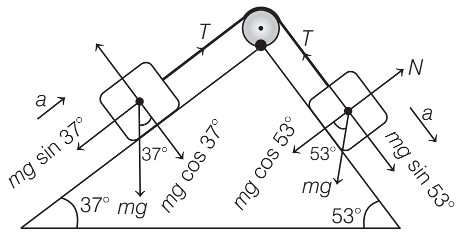
$\Rightarrow$ $\frac{t_1^2}{t_2^2} = \frac{1}{\cos^2 60°} = \frac{4}{1}$

$\Rightarrow$ $\frac{t_1}{t_2} = \frac{2}{1}$

17. Let T be the tension in the string. Let a be the acceleration of the system. The equation of motion are

$Ma = Mg\sin 53° - T$ ...(i)

and $Ma = T - mg\sin 37°$ ...(ii)



Adding Eqs. (i) and (ii), we get

$$a = \frac{mg\,(\sin 53° - \sin 37°)}{2m}$$

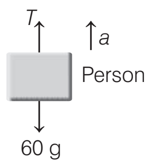
$$= g\cos 45° \sin 8°$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= 10 \times \frac{1}{\sqrt{2}} \times 0.139$$

$$= 0.98 \text{ ms}^{-1} \cong 1 \text{ ms}^{-2}$$

18. The free body diagram of the person can be drawn as



Let the person move up with an acceleration a, then

$$T - 60g = 60a$$

$$\Rightarrow \quad a_{\max} = \frac{T_{\max} - 60g}{60}$$

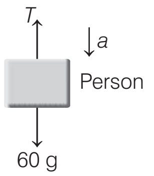
$$\text{or} \quad a_{\max} = \frac{360 - 60g}{60} \rightarrow -\text{ve value}$$

That means, it is not possible to climb up on the rope.

Even in this problem, it is not possible to remain at rest on rope.

Hence, no option is correct.

But, if they will ask for the acceleration of climbing down, then

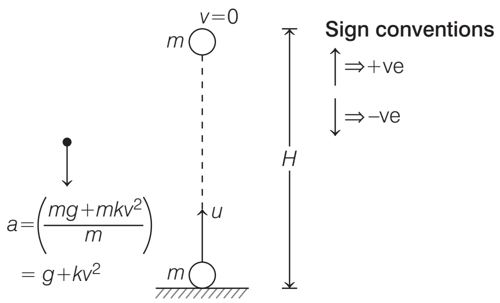


$$60g - T = 60a$$

$$\Rightarrow \quad 60g - T_{\max} = 60\, a_{\min}$$

$$\text{or} \quad a_{\min} = \frac{60g - 360}{60} = 4 \text{ ms}^{-2}$$

19.



Net force on ball = Weight of ball + Resistive force

i.e. $F_{\text{net}} = w + F_{\text{resistive}} = (-mg) + (-mkv^2)$

$= -m(g + kv^2)$

So, net acceleration of ball,

$$a = \frac{F_{\text{net}}}{m} = \frac{-m(g+kv^2)}{m} = -(g + kv^2)$$

$$\Rightarrow \quad v\frac{dv}{dy} = -(g + kv^2) \quad \Rightarrow \quad \frac{vdv}{(g+kv^2)} = -dy$$

Integrating both sides, we get

$$\int_u^0 \frac{v}{(g+kv^2)} dv = -\int_0^H dy \qquad \ldots\text{(i)}$$

Let $g + kv^2 = t \quad \Rightarrow \quad 0 + 2kvdv = dt$

$$\Rightarrow \quad vdv = \frac{1}{2k} dt$$

Lower limit

If $v = u$, then $g + ku^2 = t \quad \Rightarrow \quad t = g + ku^2$

Upper limit

If $v = 0$, then $g + k(0)^2 = t \Rightarrow t = g$

Putting these values in Eq. (i), we get

$$\int_{g+ku^2}^{g} \frac{\frac{1}{2k} dt}{t} = -\int_0^H dy$$

$$\Rightarrow \quad \frac{1}{2k}\int_{g+ku^2}^{g} \frac{1}{t}\, dt = -[y]_0^H$$

$$\Rightarrow \quad \frac{1}{2k}[\ln(t)]_{g+ku^2}^{g} = -[H - 0]$$

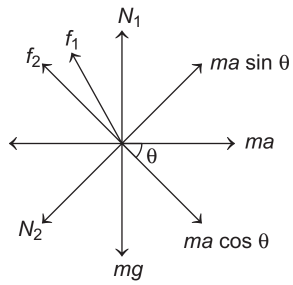
$$\Rightarrow \quad \frac{1}{2k}[\ln(g) - \ln(g + ku^2)] = -[H]$$

$$\Rightarrow \quad -\frac{1}{2k}[\ln(g) - \ln(g + ku^2)] = H$$

$$\Rightarrow \quad H = \frac{1}{2k}[\ln(g + ku^2) - \ln(g)]$$

$$= \frac{1}{2k}\left[\ln\left(\frac{g+ku^2}{g}\right)\right] = \frac{1}{2k}\ln\left(1 + \frac{ku^2}{g}\right)$$

20. The free body diagram of the given system is given as



Given, $m = 1$ kg,

$$\mu = \frac{2}{5}, a = 25 \text{ ms}^{-2}, \ \cos\theta = \frac{4}{5}, \ \sin\theta = \frac{3}{5}$$

Here, f_1 and f_2 are the two frictional forces corresponding to the two points of contact.

If a' be the acceleration of the block with respect to disc, then from above figure,

$$ma\cos\theta - f_1 - f_2 = ma'$$

$$\Rightarrow \quad ma\cos\theta - \mu N_1 - \mu N_2 = ma'$$

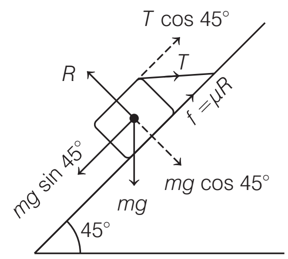
$$\Rightarrow \quad ma\cos\theta - \mu mg - \mu ma\sin\theta = ma'$$

$$\Rightarrow \quad a' = a\cos\theta - \mu g - \mu a\sin\theta$$

$$= 25 \times \frac{4}{5} - \frac{2}{5} \times 10 - \frac{2}{5} \times 25 \times \frac{3}{5}$$

$$= 20 - 4 - 6 = 10 \text{ ms}^{-2}$$

21. Figure shows free body diagram of the block,



For equilibrium, along the plane,

$$\mu R + T\cos 45° = mg\sin 45°$$

$$\mu R + \frac{T}{\sqrt{2}} = \frac{mg}{\sqrt{2}} \qquad \ldots\text{(i)}$$

For equilibrium, in direction perpendicular to inclined plane,

$$R = T\sin 45° = mg\cos 45°$$

$$= \frac{T}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$$

Put in Eq. (i), $\frac{\mu}{\sqrt{2}}(T + mg) = -\frac{1}{\sqrt{2}}(mg - T)$

$$\mu(50 + 15\times 10) = (15\times 10 - 50)$$

$$\mu = \frac{100}{200} = \frac{1}{2}$$

22. From $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$

For smooth plane, $a = g\sin\theta$

For rough plane, $a' = g(\sin\theta - \mu\cos\theta)$

$\therefore \quad t' = \sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}}$

$$= nt = n\sqrt{\frac{2s}{g\sin\theta}}$$

$\therefore \quad n^2 g(\sin\theta - \cos\theta) = g\sin\theta$

When $\quad \theta = 45°$,

$$\sin\theta = \cos\theta = 1/\sqrt{2}$$

Solving, we get $\mu = 1 - \frac{1}{n^2}$

23. Maximum force by surface when friction works,

$$F = \sqrt{f^2 + R^2} = \sqrt{\mu R^2 + R^2} = R\sqrt{\mu^2 + 1}$$

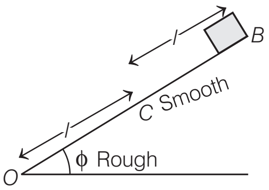
Maximum force $= R$ when there is no friction

Hence, ranging from R to $R\sqrt{\mu^2 + 1}$, we get

$$Mg \le f \le Mg\sqrt{\mu^2 + 1}$$

24. For the smooth portion BC,

$u = 0,\ s = l,\ g\sin\phi,\ u = ?$



From $\quad v^2 - u^2 = 2as$,

$$v^2 - 0 = 2g\sin\phi \times l$$

For the rough portion CO,

$$u = v = \sqrt{2g\sin\phi \cdot l}$$

$$v = 0,\ a = g(\sin\phi = \mu\cos\phi)$$

$$s = l$$

From $v^2 - u^2 = 2as$,

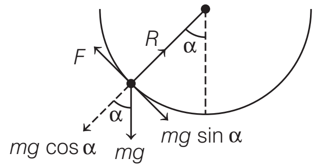
$$0 - 2gl\sin\phi = 2g(\sin\phi - \mu\cos\phi)l$$

$$-\sin\phi = \sin\phi - \mu\cos\phi$$

$$\mu\cos\phi = 2\sin\phi$$

$$\mu = 2\tan\phi$$

25. As is from figure.



$$F = mg\sin\alpha$$

$$R = mg\cos\alpha$$

$$\frac{F}{R} = \tan\alpha$$

i.e. $\quad \mu = \tan\alpha = \frac{1}{3}$

$\therefore \quad \cot\alpha = 3$

26. As is clear from figure,

$$R + T = (m + M)g$$

$$R = (m + M)g - T$$

The system will not move till,

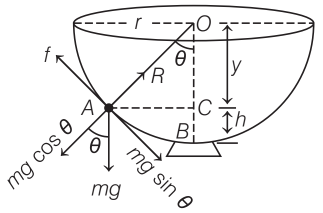
$$T \le F \text{ or } (T \le \mu R)$$

$$T \le \mu[(m + M)g = T]$$

$$T \le \frac{\mu(m + M)g}{\mu + 1}$$

$\therefore \quad F_{\max} = \frac{\mu(m + M)g}{\mu + 1}$

27. In figure O is the centre of the bowl of radius r. The insect will crawl (from B to A) till component of its weight (mg) along the bowl is balanced by the force of limiting friction (f)



i.e $\quad mg\sin\theta = f = \mu R = \mu mg\cos\theta$

or $\quad \mu = \tan\theta = \frac{AC}{OC}$

$$= \sqrt{\frac{OA^2 - OC^2}{OC}} = \frac{\sqrt{r^2 - y^2}}{y}$$

or $\quad \mu^2 = \frac{r^2 - y^2}{y^2}$

$$\mu^2 y^2 + y^2 = r^2$$

$$y = \frac{r}{\sqrt{\mu^2 + 1}}$$

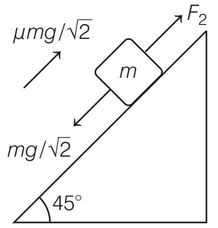
$$h = BC = OB - OC = r - y$$

$$= r - \frac{r}{\sqrt{\mu^2 + 1}} = r\left(1 - \frac{1}{\sqrt{\mu^2 + 1}}\right)$$

28.

$$f_1 = \frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}}$$

$$f_2 = \frac{mg}{\sqrt{2}} - \frac{\mu mg}{\sqrt{2}}$$



$$F_1 = 3\sqrt{2}$$

$$1 + \mu = 3 - 3\mu$$

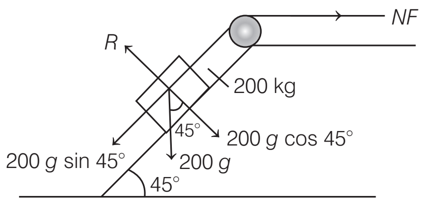
$$\Rightarrow \quad 4\mu = 2$$

$$\Rightarrow \quad \mu = \frac{1}{2}$$

$$N = 10\mu$$

$$\Rightarrow \quad N = 5$$

29. Here, mass of the block, $m = 200$ kg, coefficient of static friction, $\mu_s = 0.5 = \frac{1}{2}$



Angle to inclined plane, $\theta = 45°$

Maximum force that each man can apply, $F = 500$ N

Let N number of men are required for the block to just start moving up the plane,

$$NF = mg\sin\theta + f = mg\sin\theta + \mu_s R$$

$$= mg\sin\theta + \mu_s mg\cos\theta$$

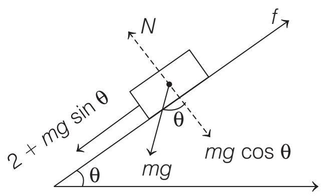
$$= mg[\sin\theta + \mu_s\cos\theta]$$

$$NF = 200 \times 10\left(\sin 45° + \frac{1}{2}\cos 45°\right)$$

$$= \frac{200 \times 10 \times 3}{2\sqrt{2}}$$

$$N = \frac{200 \times 10 \times 3}{2\sqrt{2} \times 500} = 4.2 \simeq 5$$

30. Block does not move upto a maximum applied force of 2 N down the inclined plane.

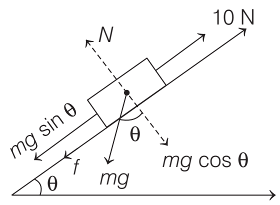


So, equating forces, we have

$$2 + mg\sin\theta = f$$

or $$2 + mg\sin\theta = \mu\, mg\cos\theta \qquad \text{...(i)}$$

Similarly, block also does not move upto a maximum applied force of 10 N up the plane.



Now, equating forces, we have

$$mg\sin\theta + f = 10 \text{ N}$$

or $$mg\sin\theta + \mu mg\cos\theta = 10 \qquad \text{...(ii)}$$

Now, solving Eqs. (i) and (ii), we get

$$mg\sin\theta = 4 \qquad \text{...(iii)}$$

and $$\mu\, mg\cos\theta = 6 \qquad \text{...(iv)}$$

Dividing Eq. (iii) by Eq. (iv), we get

$$\mu\cot\theta = \frac{3}{2}$$

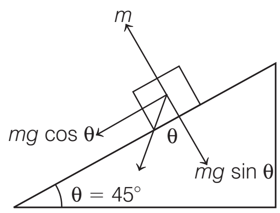
$$\Rightarrow \quad \mu = \frac{3\tan\theta}{2} = \frac{3\tan 30°}{2}$$

$$\Rightarrow \quad \mu = \frac{\sqrt{3}}{2}$$

31. When friction is absent, $ma_1 = mg\sin\theta$

$$a = g\sin\theta$$

$$\therefore \quad s_1 = \frac{1}{2}a_1 t_1^2 \qquad \text{...(i)}$$



When friction is present, friction is in opposite to the direction of motion,

$$a_2 = g\sin\theta - \mu_k\, g\cos\theta$$

$$\therefore \quad s_2 = \frac{1}{2}a_2 t_2^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$$

$$\Rightarrow \quad a_1 t_1^2 = a_2(n\,t_1)^2 \qquad (\because\ t_2 = n\,t_1)$$

or $$a_1 = n^2 a_2$$

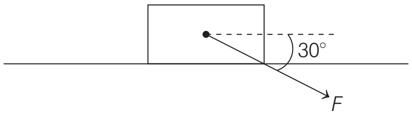
$$\Rightarrow \quad \frac{a_2}{a_1} = \frac{g\sin\theta - \mu_k g\cos\theta}{g\sin\theta} = \frac{1}{n^2}$$

or $$\frac{g\sin 45° - \mu_k g\cos 45°}{g\sin 45°} = \frac{1}{n^2}$$

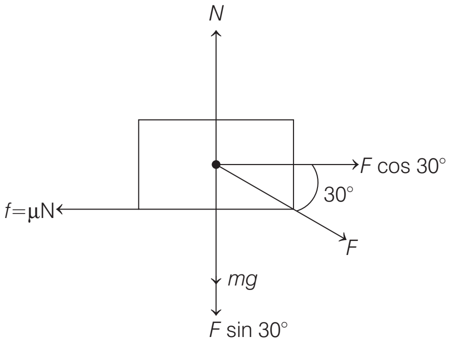
or $\quad 1-\mu_k = \dfrac{1}{n^2}$

or $\quad \mu_k = 1 - \dfrac{1}{n^2}$

32. ***Case*** **I** Block is pushed over surface



Free body diagram of block is



In this case, normal reaction,

$N = mg + F \sin 30° = 5 \times 10 + 20 \times \dfrac{1}{2} = 60$ N

(Given, $m = 5$ kg, $F = 20$ N)

Force of friction, $f = \mu N = 0.2 \times 60 \quad (\because \mu = 0.2)$

$= 12$ N

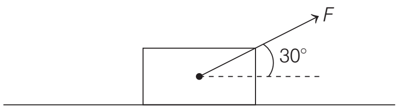
So, net force causing acceleration (a_1) is

$$F_{net} = ma_1 = F \cos 30° - f$$

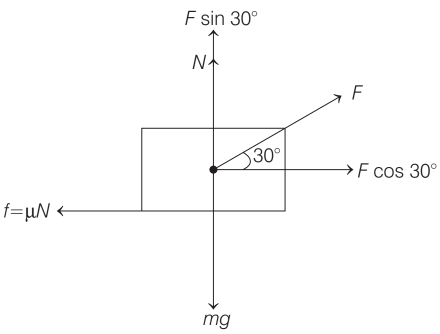
$$\Rightarrow \quad ma_1 = 20 \times \frac{\sqrt{3}}{2} - 12$$

$$\therefore \quad a_1 = \frac{10\sqrt{3} - 12}{5} \approx 1 \text{ ms}^{-2}$$

Case **II** Block is pulled over the surface



Free body diagram of block is,



Net force causing acceleration is

$$F_{net} = F \cos 30° - f$$
$$= F \cos 30° - \mu N$$
$$\Rightarrow \quad F_{net} = F \cos 30° - \mu(mg - F \sin 30°)$$

If acceleration is now a_2, then

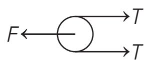
$$a_2 = \frac{F_{net}}{m}$$
$$= \frac{F \cos 30° - \mu(mg - F \sin 30°)}{m}$$
$$= \frac{20 \times \frac{\sqrt{3}}{2} - 0.2\left(5 \times 10 - 20 \times \frac{1}{2}\right)}{5}$$
$$= \frac{10\sqrt{3} - 8}{5}$$
$$\Rightarrow \quad a_2 \approx 1.8 \text{ ms}^{-2}$$

So, difference $= a_2 - a_1 = 1.8 - 1 = 0.8 \text{ ms}^{-2}$

33. With full barrel length, the muzzle velocity of block is 700 m/s and when 0.5 m is cut-off from the barrel , the muzzle velocity falls to 600 m/s. Hence, due to pressure of powder, the velocity increases from 600 m/s to 700 m/s over a distance of 0.5 m.

$$\therefore \quad 700^2 = 600^2 + 2 \times a \times 0.5$$
$$\therefore \quad a = 13 \times 10^4 \text{ m/s}^2$$
$$\therefore \quad F = ma = 50 \times 13 \times 10^4$$
$$= 6.5 \times 10^6 \text{ N}$$
$$\therefore \quad n = 6$$

34. Here, $F = 2T$ (from FBD of pulley)



$$\therefore \quad T = 5 \text{ N}$$

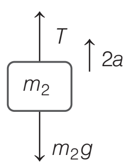
But, $\quad T = ma$

$$\therefore \quad a = \frac{T}{m} = \frac{5}{1} = 5 \text{ m/s}^2$$

35. For m_2,

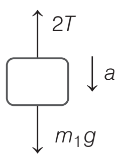
$$T - m_2 g = m_2(2a) \qquad \text{...(i)}$$

Free body diagram (FBD) of m_2,



For m_1, $\quad m_1 g - 2m_2 = m_1(a) \qquad$ ...(ii)

Free body diagram (FBD) of m_1,



From Eqs. (i) and (ii), we get

$$m_1 g - 2m_2 g = 4m_2 a + m_1 a$$

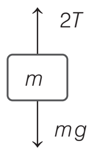
$\Rightarrow \quad a = 0 \text{ m/s}^2$

or $\quad T = m_2 g = 10 \text{ N}$

For m, $2T = mg$

$\Rightarrow \quad m = \dfrac{20}{10} = 2 \text{ kg}$

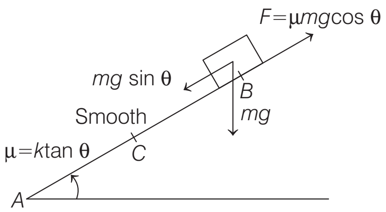
Free body diagram of m,



$$mg = 2T$$

$\Rightarrow \quad m = \dfrac{2 \times 10}{10} = 2 \text{ kg}$

36. Different forces acting on the inclined plane are shown below.



As block stops at point A, this means work done by component of weight down the plane is dissipated in doing work against friction.

$\Rightarrow \quad mg \sin\theta (AB) = \mu mg \cos\theta (AC)$

$\Rightarrow \quad mg \sin\theta (3AC) = \mu mg \cos\theta (AC)$

$\Rightarrow \quad 3 \tan\theta = \mu$

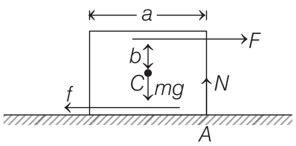
Given, $\mu = k \tan\theta$

Comparing both, we get

$\Rightarrow \quad k = 3$

37. According to the given situation,

When the minimum force F capable to topple the block is applied, the block will be on the verge of toppling.



As the block is not moving, we have friction f such that

$$f = F \qquad \text{... (i)}$$

Also, note that reaction N acts from point A as block is at the verge of toppling.

To maintain the equilibrium, net torque about centre of mass C is zero.

$\Rightarrow \quad F \cdot b + f\left(\dfrac{a}{2}\right) = N\left(\dfrac{a}{2}\right)$

Using result of Eq. (i), we get

$$f\left(\frac{a}{2} + b\right) = N\left(\frac{a}{2}\right)$$

Now, $f = \mu mg$ and $N = mg$

$\therefore \quad \mu mg\left(\dfrac{a}{2} + b\right) = mg\left(\dfrac{a}{2}\right)$

$\Rightarrow \quad \mu\left(\dfrac{a}{2} + b\right) = \dfrac{a}{2}$

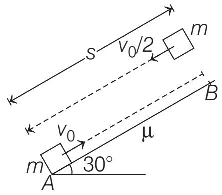
As, $\quad \mu = 0.4$

$\therefore \quad \dfrac{b}{a} = \dfrac{3}{4}$

Hence, $\quad 100 \times \dfrac{b}{a} = 100 \times \dfrac{3}{4} = 75$

38. Let μ be the coefficient of kinetic friction between the block and the inclined plane.

The free body diagram of the given situation is shown below.



While going from A to B, the acceleration of the block is

$$a_1 = g \sin 30° + \mu g \cos 30°$$

$\Rightarrow \quad a_1 = \dfrac{g}{2} + \dfrac{\mu g\sqrt{3}}{2}$

$\Rightarrow \quad a_1 = 5 + 5\sqrt{3}\,\mu \qquad \text{...(i)} \; (g = 10 \text{ ms}^{-2})$

Let s be the distance between A and B.

From third equation of motion,

$$v_0^2 - 0 = 2a_1 s$$

$\Rightarrow \quad s = \dfrac{v_0^2}{2a_1} = \dfrac{v_0^2}{2(5 + 5\sqrt{3}\,\mu)} \qquad \text{...(ii)}$

[using Eq. (i)]

When the block comes back to its initial position (i.e. from B to A), its velocity is $\dfrac{v_0}{2}$.

So, the acceleration of the block while coming from B to A is

$$a_2 = g \sin 30° - \mu g \cos 30°$$

$\Rightarrow \quad a_2 = \dfrac{g}{2} - \dfrac{\mu g\sqrt{3}}{2}$

$\Rightarrow \quad a_2 = 5 - 5\sqrt{3}\,\mu \qquad \text{...(iii)}$

Again, using third equation of motion, we get

$$\left(\frac{v_0}{2}\right)^2 - 0 = 2a_2 s$$

$\Rightarrow \quad \dfrac{v_0^2}{4} = 2a_2 s$

$$\Rightarrow \quad s = \frac{v_0^2}{8a_2}$$

$$\Rightarrow \quad s = \frac{v_0^2}{8(5 - 5\sqrt{3}\,\mu)} \quad \text{...(iv)}$$

[using Eq. (iii)]

Equating Eqs. (ii) and (iv), we get

$$\frac{v_0^2}{2(5 + 5\sqrt{3}\,\mu)} = \frac{v_0^2}{8(5 - 5\sqrt{3}\,\mu)}$$

$$\Rightarrow \quad 4(5 - 5\sqrt{3}\,\mu) = 5 + 5\sqrt{3}\,\mu$$

$$\Rightarrow \quad 20 - 20\sqrt{3}\,\mu = 5 + 5\sqrt{3}\,\mu$$

$$\Rightarrow \quad 25\sqrt{3}\,\mu = 15$$

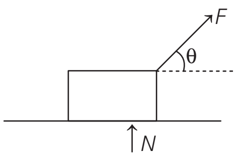
$$\Rightarrow \quad \mu = \frac{15}{25\sqrt{3}} = \frac{3}{5\sqrt{3}}$$

$$\Rightarrow \quad \mu = \frac{\sqrt{3}}{5}$$

$$\Rightarrow \quad \mu = 0.346 = \frac{346}{1000}$$

$$\therefore \quad I = 346$$

39.



$$F\cos\theta = \mu N$$

$$F\sin\theta + N = mg$$

$$\Rightarrow \quad F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

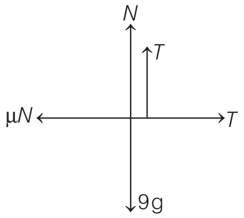
$$F_{\min} = \frac{\mu mg}{\sqrt{1 + \mu^2}} = \frac{\frac{1}{\sqrt{3}} \times 10}{\frac{2}{\sqrt{3}}} = 5\ \text{N}$$

40. $a_{\max} = \mu g = \frac{3}{7} \times 9.8$

$$F = (M + m)\,a_{\max} = 5\,a_{\max}$$

$$= 21\ \text{N}$$

41.



$$N + T = 90$$

$$T = \mu N = 0.5\,(90 - T)$$

$$1.5T = 45$$

$$T = 30\ \text{N}$$